Universidad Veracruzana

Generalized Michel parameters in the presence of massive Dirac and Majorana neutrinos for radiative leptonic decay

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Outline

- **Fermi theory for leptonic decay** $\ell \to \ell' \, \overline{\nu_{\ell'}} \, \nu_\ell$
- **Radiative process** $\ell \to \ell' \, \overline{\nu_{\ell'}} \, \nu_\ell \, \gamma$
- **Adding mass to neutrinos**
- **Derived radiative decay rate distribution**
- **Comments and further work**

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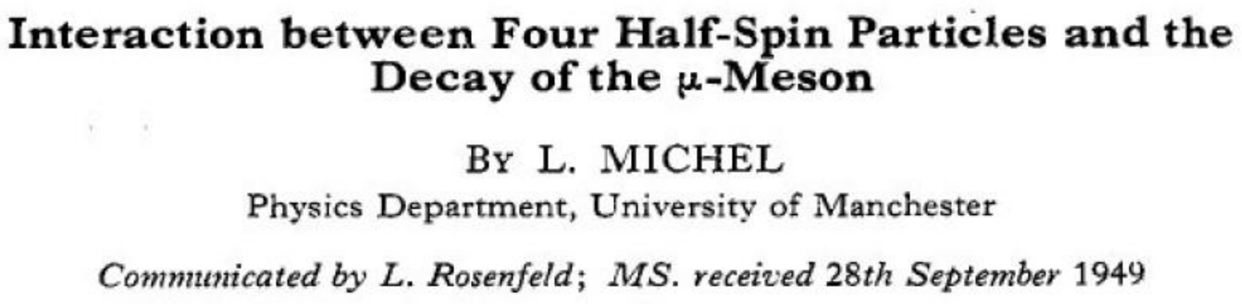
Origins of Michel Parameters

At the earlier 50s, **[Louis Michel]** focused on exploring the μ-decay in general to achieve a full description of the decay that aligns to the experimental results.

His proposal was to introduce a set of parameters that classify the different types of interactions by relating the coupling constants associated with each possible contribution (Wilson coefficients).


Interaction between Four Half-Spin Particles and the Decay of the μ-Meson

By L. MICHEL
Physics Department, University of Manchester

Communicated by L. Rosenfeld; MS. received 28th September 1949

[Louis Michel] L Michel. “Interaction between Four Half-Spin Particles and the Decay of the μ - Meson”. In: Proceedings of the Physical Society. Section A 63.5 (May 1950), p. 514. doi: 10.1088/0370-1298/63/5/311.

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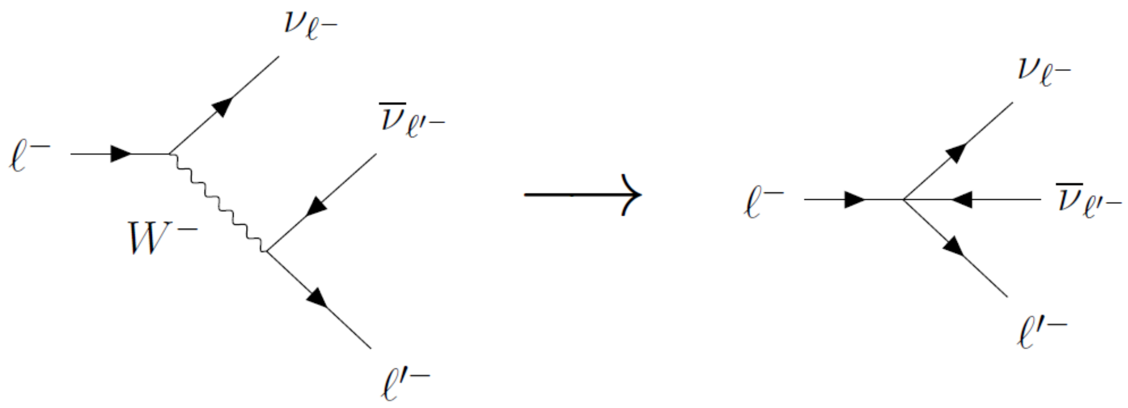

Approaching the problem from the *Effective Field Theory* (EFT) framework, he built the most general, local, derivative-free, lepton-number conserving, four-lepton interaction Hamiltonian, consistent with locality and Lorentz invariance

$$\mathcal{H} = 4\frac{G_{\ell\ell'}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\vec{\ell}_\epsilon \Gamma^n (\nu_{\ell'})_\sigma\right] [(\bar{\nu}_\ell)_\lambda \Gamma_n \ell_\omega]$$

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$d\Gamma$ distribution

Where there are 3 relevant elements:

- $\Gamma^n = \{\mathbb{I}, \gamma^\mu, \sigma^{\mu\nu}/\sqrt{2}\}$
- $G_{\ell\ell'}$: Effective leptonic coupling
- $g^n_{\epsilon\omega}$: Wilson coefficients
- $(\epsilon,\omega)/(\sigma,\lambda)$: (Lepton)/(Neutrino) chirality labels

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From this Hamiltonian, considering the polarization degree $\mathcal{P}_\ell$ of the parent lepton ℓ, the decay rate distribution is given in terms of the Michel parameters ρ, ξ, δ, η inside functions $F(x)$ and $A(x)$ as in **[A. Pich]**:

$$\frac{d^2\Gamma_{\ell\to\ell'}}{dx\, d\cos\theta} = \frac{m_\ell\,\omega^4}{2\pi^3}\, G^2_{\ell'\ell}\, \sqrt{x^2 - x_0^2}\, \left\{F(x) - \frac{\xi}{3}\mathcal{P}_\ell \sqrt{x^2 - x_0^2}\, \cos\theta\, A(x)\right\}$$

$$F(x) = x(1-x) + \frac{2}{9}\rho\left(4x^2 - 3x - x_0^2\right) + \eta\, x_0(1-x)\,,$$

$$A(x) = 1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right).$$

[A.Pich] A.Pich, Precision Tau Physics (2013), Prog.Part.Nucl.Phys. 75 (2014) 41-85.

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Michel parameters

While diagonal contributions on these parameters denote only 1 type of interaction, the presence of interference contributions imply that two different interactions are having place in the process, that's the relevance of η parameter.

$$\alpha^+ \equiv |g^V_{RL}|^2 + \frac{1}{16}|g^S_{RL} + 6\,g^T_{RL}|^2\,,$$

$$\beta^+ \equiv |g^V_{RR}|^2 + \frac{1}{4}|g^S_{RR}|^2\,,$$

$$\gamma^+ \equiv \frac{3}{16}|g^S_{RL} - 2\,g^T_{RL}|^2\,,$$

$$\rho = \frac{3}{4}(\beta^+ + \beta^-) + (\gamma^+ + \gamma^-)\,,$$

$$\xi = 3(\alpha^- - \alpha^+) + (\beta^- - \beta^+) + \frac{7}{3}(\gamma^+ - \gamma^-)\,,$$

$$\xi\delta = \frac{3}{4}(\beta^- - \beta^+) + (\gamma^+ - \gamma^-)\,,$$

$$\eta = \frac{1}{2}\mathrm{Re}\left[g^V_{LL}g^{S*}_{RR} + g^V_{RR}g^{S*}_{LL} + g^V_{LR}\left(g^{S*}_{RL} + 6\,g^{T*}_{RL}\right) + g^V_{RL}\left(g^{S*}_{LR} + 6\,g^{T*}_{LR}\right)\right]\,,$$

Bounds on the Michel parameters for pure leptonic decay with massless neutrinos are given on PDG.

The SM ($g^V_{LL} = 1$) predict the following values:

- $\rho = \delta = 3/4$
- $\eta = 0$
- $\xi = 1$

	$\mu^- \to e^-\bar{\nu}_e\nu_\mu$	$\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$	$\tau^- \to e^-\bar{\nu}_e\nu_\tau$	$\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau$
ρ	0.74979 ± 0.00026	0.763 ± 0.020	0.747 ± 0.010	0.745 ± 0.008
η	0.057 ± 0.034	0.094 ± 0.073	—	0.013 ± 0.020
ξ	$1.0009\,^{+\,0.0016}_{-\,0.0007}$	1.030 ± 0.059	0.994 ± 0.040	0.985 ± 0.030
$\xi\delta$	$0.7511\,^{+\,0.0012}_{-\,0.0006}$	0.778 ± 0.037	0.734 ± 0.028	0.746 ± 0.021
ξ'	1.00 ± 0.04	—	—	—
ξ''	0.65 ± 0.36	—	—	—

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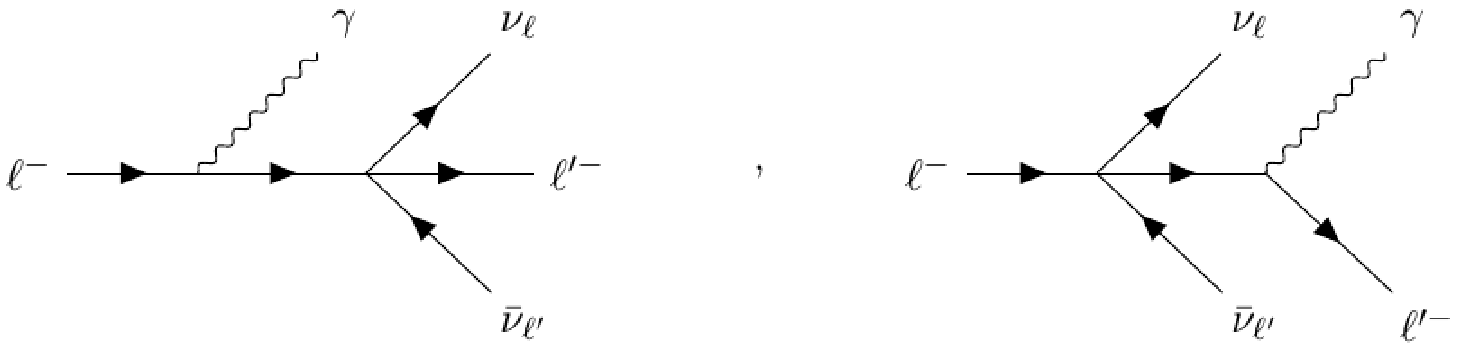
Radiative Process $\ell \to \ell' \overline{\nu_{\ell'}} \nu_\ell \gamma$

Inner Bremsstrahlung of 4-leptons interaction

The lowest dimension effective operator describing the leptonic decay with the emission of one photon is giving by an inner Bremsstrahlung-like process, where the photon couples to the parent/daughter lepton.

These couplings come straight from the photon-leptons interaction in the SM Lagrangian, i.e. gauge invariance is fulfilled.



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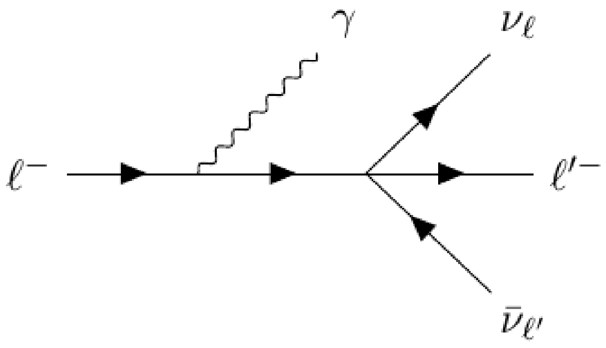

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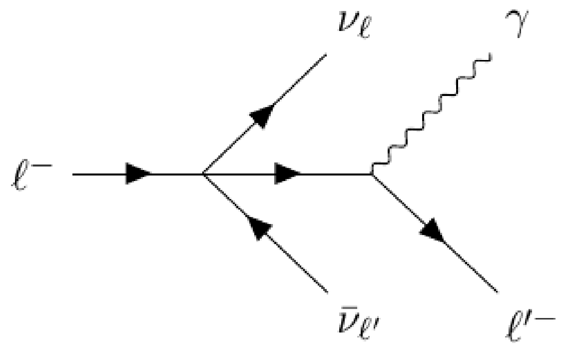

[Arbuzov-Kopilova] have already done an EFT description of the radiative leptonic decay, giving a solid result for the decay rate distribution and the corresponding Michel parameters, considering massless neutrinos.

$$\frac{d\Gamma(\mu^\pm \to e^\pm \bar{\nu}\nu\gamma)}{dx\, dy\, d\Omega_e\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha_{\text{QED}}}{64\pi^3} \frac{\beta_e}{y} \left[F(x,y,d) \mp \beta_e P_\mu \cos\theta_e G(x,y,d) \mp P_\mu \cos\theta_\gamma H(x,y,d) \right],$$

$$\Gamma_0 = \frac{G_{\text{Fermi}}^2 m_\mu^5}{192\pi^3}, \qquad d = 1 - \beta_e \cos\theta_{e\gamma}, \qquad \beta_e = \sqrt{1 - \frac{m_e^2}{E_e^2}},$$

(**Note:** P_μ is the μ polarization, not a four-vector)

[Arbuzov-Kopilova] A. B. Arbuzov and T. V. Kopylova. “Michel parameters in radiative muon decay”. In: *JHEP* 09 (2016), p. 109. doi: 10.1007/JHEP09(2016)109. arXiv: 1605.06612 [hep-ph].

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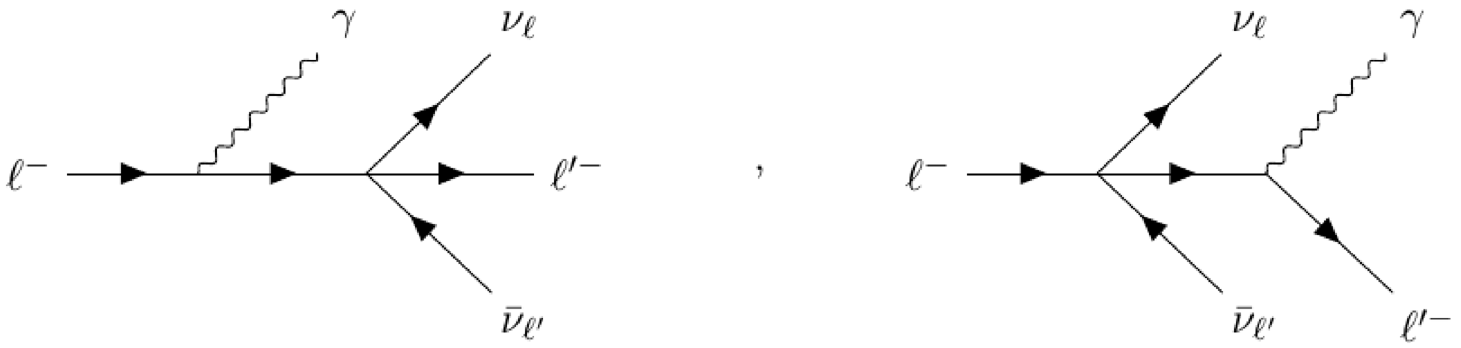

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Due to the complexity of the functions parametrizing the decay rate, they organized the isotropic and anisotropic functions according to the degree of suppression by the lepton-mass ratio.

$$\mathcal{F}(x,y,d) = \sum_{k=1}^{5} \left(\frac{m_e}{m_\mu}\right)^k \mathcal{F}^{(k)}, \qquad \mathcal{F} \equiv F,\ G,\ H.$$

$$F^{(1)} = \beta\left(\frac{24x(x+y-1)}{d} - \frac{4y^2}{d} + 4xy^2 - 12x(xy+x+y-1) - x^2y^2d + 6x^2yd\right) + \alpha\left(\frac{12x(1-x-y)}{d} + 6x(xy+x+y-1) - 3x^2yd\right),$$

They recovered the Michel parameters of the non-radiative case, together with additional ones

$$\rho = \frac{3b+6c}{16} = \frac{3}{4} + \frac{3}{4}\cdot\frac{(2c-a)}{16}, \qquad \eta = \frac{\alpha - 2\beta}{16}, \qquad \bar{\eta} = \frac{a+2c}{16},$$

$$\kappa\xi = \frac{a_1 + 2c_1}{16}, \qquad \delta\xi = \frac{6c_1 - 3b_1}{16}, \qquad \xi = \frac{14c_1 - 3a_1 - 4b_1}{16},$$

$$\alpha = 8\mathrm{Re}\left(g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR}) + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL})\right), \qquad \beta = -4\mathrm{Re}(g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}),$$

$$a = 16(|g^V_{RL}|^2 + |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 + |g^S_{LR} + 6g^T_{LR}|^2, \quad a_1 = 16(|g^V_{RL}|^2 - |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 - |g^S_{LR} + 6g^T_{LR}|^2,$$

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Radiative result

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Adding mass to neutrinos

Dirac and Majorana considerations

To incorporate neutrino masses within this EFT, including the effects due to the Dirac or Majorana nature of neutrinos, one can express the lepton fields appearing in the effective Hamiltonian in the mass basis.

$$\nu_{\ell L} = \sum_j U_{\ell j} N_{jL}, \quad \nu_{\ell R} = \sum_j V_{\ell j} N_{jR}, \quad \begin{cases} \mathcal{H} = 4\frac{G_{\ell\ell'}}{\sqrt{2}} \sum_{j,k} \left\{ g^S_{LL} \left[\overline{\ell'}_L V_{\ell' j} N_{jR}\right] \left[\overline{N}_{kR} V^*_{\ell k} \ell_L\right] + g^V_{LL} \left[\overline{\ell'}_L \gamma^\mu U_{\ell' j} N_{jL}\right] \left[\overline{N}_{kL} U^*_{\ell k} \gamma_\mu \ell_L\right] \right. & \text{(Dirac neutrinos)} \\ \mathcal{H} = 4\frac{G_{\ell\ell'}}{\sqrt{2}} \sum_{j,k} \left\{ g^S_{LL} V_{\ell' j} V^*_{\ell k} \left[\overline{\ell'}_L N_{jR}\right] \left[N_{kR} \ell_L\right] + g^V_{LL} U_{\ell' j} U^*_{\ell k} \left[\overline{\ell'}_L \gamma^\mu N_{jL}\right] \left[N_{kL} \gamma_\mu \ell_L\right] \right. & \text{(Majorana neutrinos)} \end{cases}$$

Generalized Michel parameters are obtained, providing information about the relative strengths associated with all possible interaction types involving massive neutrinos.

For this to be possible, one may assume that the new sterile neutrinos have non-negligible mixings with the active neutrinos, and some of them are sufficiently light to be produced on-shell.

Adding mass to neutrinos

Non-radiative massive-neutrinos result

Results on the non-radiative case are carefully explained in **[Márquez-López-Roig]:**

$$\frac{d\Gamma}{dx\,d\cos\theta} = \sum_{j,k} \frac{m_\ell}{4\pi^3}\omega^4 G_{\ell\ell'}^2 \sqrt{x^2 - x_0^2}$$
$$\times \Big(\big(F_{IS}(x) + F'_{IS}(x) + F''_{IS}(x)\big) - \mathcal{P}\cos\theta\big(F_{AS}(x) + F'_{AS}(x) + F''_{AS}(x)\big)\Big)$$
$$\times \left[1 + \hat{\zeta} \cdot \vec{\mathcal{P}}_{\ell'}(x,\theta)\right],$$

[Márquez-López-Roig] Juan Manuel Márquez, Gabriel López Castro, and Pablo Roig. “Michel parameters in the presence of massive Dirac and Majorana neutrinos”. In: *Journal of High Energy Physics* 2022.11 (Nov. 2022). issn: 1029-8479. doi: 10 . 1007 / jhep11(2022) 117. url: http://dx.doi.org/10.1007/JHEP11(2022)117.

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They propose ∼14 new Michel parameters in terms of products of Wilson coefficients times mixing-matrix elements.

$$
\begin{aligned}
(f^S_{RR})_{jk} &\equiv g^S_{RR} U_{\ell' j} U^*_{\ell k} \quad , \\
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$$
\begin{aligned}
(\rho)_{jk} = {} & \frac{3}{4}(|(f^V_{LL})_{jk}|^2 + \frac{1}{4}|(f^S_{LL})_{jk}|^2) + \frac{3}{16}|(f^S_{LR})_{jk} - 2(f^T_{LR})_{jk}|^2 + (R \leftrightarrow L) + \epsilon\frac{3}{16}\,\mathrm{Re}\Big[- (f^S_{LR})_{jk} \\
& (f^S_{LR})^*_{kj} + 4(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + 4(f^S_{LL})_{jk}(f^V_{LL})^*_{kj} - 4(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + (L \leftrightarrow R)\Big], \\
(\xi)_{jk} = {} & 3(|(f^V_{LR})_{jk}|^2 + \frac{1}{16}|(f^S_{LR})_{jk} + 6(f^T_{LR})_{jk}|^2) + |(f^V_{LL})_{jk}|^2 + \frac{1}{4}|(f^S_{LL})_{jk}|^2 - \frac{7}{16}|(f^S_{LR})_{jk} - 2(f^T_{LR})_{jk}|^2 \\
& - (R \leftrightarrow L) + \epsilon\,\mathrm{Re}\Big[- (f^S_{RR})_{jk}(f^V_{RR})^*_{kj} + \frac{17}{2}(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + \frac{1}{2}(f^S_{LR})_{jk}(f^T_{LR})^*_{kj} + 3(f^V_{LR})_{jk} \\
& (f^V_{LR})^*_{kj} + \frac{5}{8}(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} - (L \leftrightarrow R)\Big], \\
(\eta)_{jk} = {} & \frac{1}{2}\,\mathrm{Re}[(f^V_{LL})_{jk}(f^S_{RR})^*_{jk} + (f^V_{RR})_{jk}(f^S_{LL})^*_{jk} + (f^V_{LR})_{jk}((f^S_{RL})^*_{jk} + 6(f^T_{RL})^*_{jk}) + (f^V_{RL})_{jk}((f^S_{LR})^*_{jk} \\
& + 6(f^T_{LR})^*_{jk})] + \frac{\epsilon}{8}\,\mathrm{Re}\Big[4(f^S_{LR})_{jk}(f^V_{RL})^*_{kj} + 24(f^T_{LR})_{jk}(f^V_{RL})^*_{kj} + (f^S_{LL})_{jk}(f^S_{RR})^*_{kj} + 4(f^V_{LL})_{jk} \\
& (f^V_{RR})^*_{kj} + (L \leftrightarrow R)\Big],
\end{aligned}
$$

[Márquez-López-Roig] Juan Manuel Márquez, Gabriel López Castro, and Pablo Roig. “Michel parameters in the presence of massive Dirac and Majorana neutrinos”. In: *Journal of High Energy Physics* 2022.11 (Nov. 2022). issn: 1029-8479. doi: 10 . 1007 / jhep11(2022) 117. url: http://dx.doi.org/10.1007/JHEP11(2022)117.

Outline

- Fermi theory for leptonic decay $\ell \to \ell' \overline{\nu_{\ell'}} \nu_\ell$
- Radiative process $\ell \to \ell' \overline{\nu_{\ell'}} \nu_\ell \gamma$
- Adding mass to neutrinos
- **Derived radiative decay rate distribution**
- Comments and further work

Radiative decay rate distribution obtained

Radiative case with massive-neutrinos

Despite the strong theoretical and experimental motivation for extending the previous result, to the best of our knowledge, no differential decay rate distribution for radiative leptonic decay including massive neutrinos has been propose to date, within the effective framework parameterized by Michel parameters.

(i.e. we have work to do)

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Despite the strong theoretical and experimental motivation for extending the previous result, to the best of our knowledge, no differential decay rate distribution for radiative leptonic decay including massive neutrinos has been propose to date, within the effective framework parameterized by Michel parameters.

(i.e. we have work to do)

The effective framework is the same as in the non-radiative case

$$\nu_{\ell L} = \sum_j U_{\ell j} N_{jL}, \quad \nu_{\ell R} = \sum_j V_{\ell j} N_{jR},$$

(Dirac neutrinos)

$$\mathcal{H} = 4\frac{G_{\ell\ell'}}{\sqrt{2}} \sum_{j,k} \left\{ g^S_{LL} \left[\vec{\ell}_L V_{\ell' j} N_{jR}\right] \left[\overline{N}_{kR} V^*_{\ell k} \ell_L\right] + g^V_{LL} \left[\vec{\ell}_L \gamma^\mu U_{\ell' j} N_{jL}\right] \left[\overline{N}_{kL} U^*_{\ell k} \gamma_\mu \ell_L\right]\right.$$

(Majorana neutrinos)

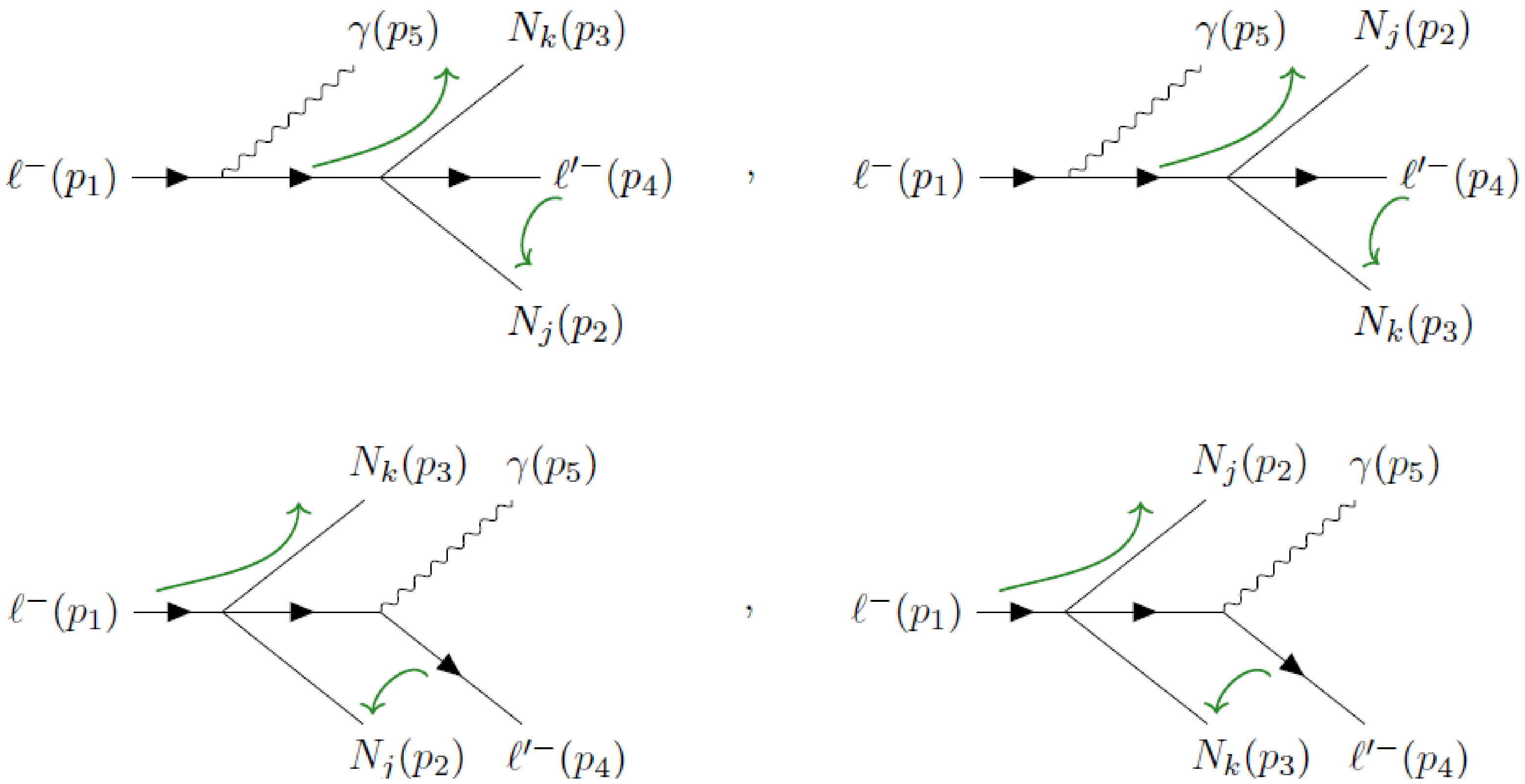
$$\mathcal{H} = 4\frac{G_{\ell\ell'}}{\sqrt{2}} \sum_{j,k} \left\{ g^S_{LL} V_{\ell' j} V^*_{\ell k} \left[\vec{\ell}_L N_{jR}\right] \left[N_{kR} \ell_L\right] + g^V_{LL} U_{\ell' j} U^*_{\ell k} \left[\vec{\ell}_L \gamma^\mu N_{jL}\right] \left[N_{kL} \gamma_\mu \ell_L\right]\right.$$

Due to the coupling of the photon, and the indistinguishability of neutrinos in the Majorana case, we have *4 relevant diagrams*

Radiative decay rate distribution obtained

Dirac and Majorana diagrams

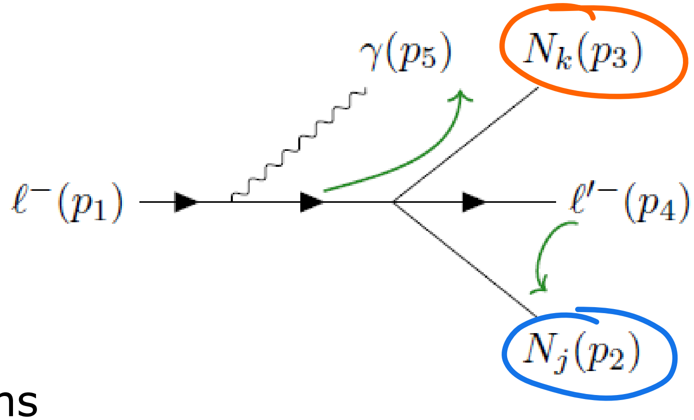
The green arrows indicate the convention for fermionic chains, since Feynman rules for LNV processes proposed by **[A. Denner]** are implemented

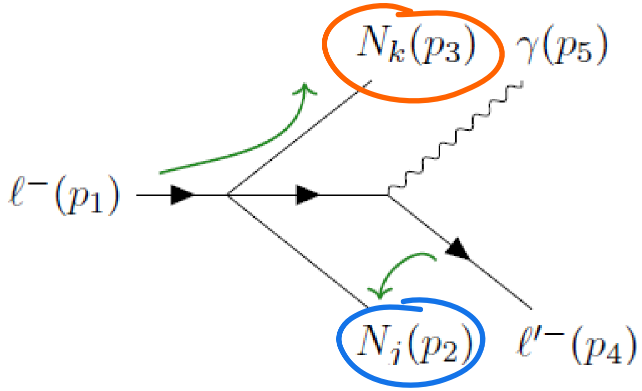


[A. Denner] Ansgar Denner et al. "Feynman rules for fermion number violating interactions". In: *Nucl. Phys. B* 387 (1992), pp. 467–481. doi: 10.1016/0550-3213(92)90169-C.

The green arrows indicate the convention for fermionic chains, since Feynman rules for LNV processes proposed by **[A. Denner]** are implemented

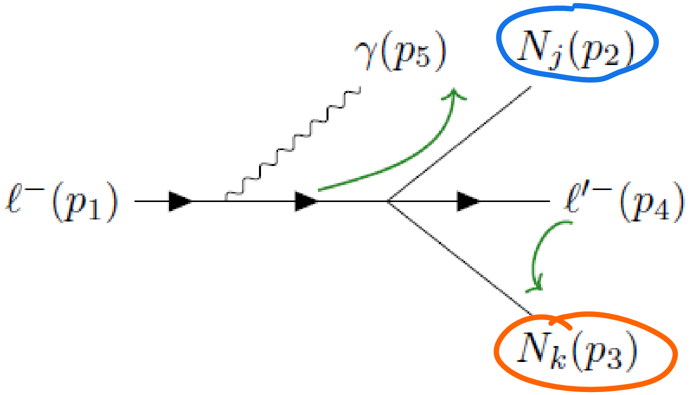
Dirac diagrams

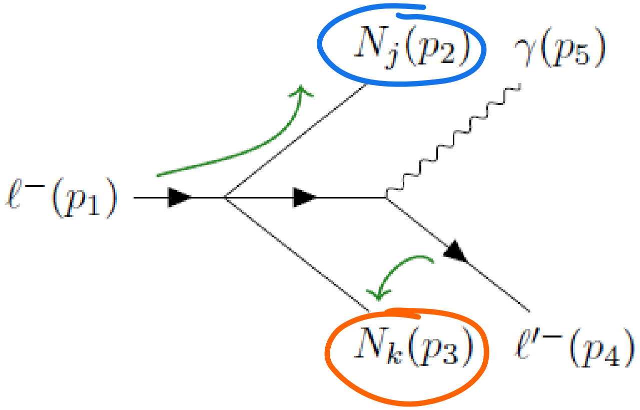




Majorana diagrams

$(-\epsilon)$





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Radiative decay rate distribution obtained

Main result

The goal was to split the distribution in polarized (due to daughter lepton and photon) and isotropic parts, each organized according to the order of neutrino mass suppression $M_\nu = m_j/m_\ell$, and each of these parts classified by the order of daughter-lepton mass suppression $M_\ell = m_{\ell'}/m_\ell$.

These last functions are parameterized in terms of the new generalized Michel parameters found

$$\frac{d\Gamma(\ell^- \to \ell'^- \bar{\nu}_{\ell'} \nu_\ell \gamma)}{dx\, dy\, d\Omega_{\ell'}\, d\Omega_\gamma} = \Gamma_{0\,(\ell\ell')} \frac{\alpha_{\rm QED}}{64\,\pi^3} \frac{\beta_{\ell'}}{4y} \sum_{j,k} \left[F(x,y,d) + P_\ell \cos\theta_{\ell'} G(x,y,d) + P_\ell \cos\theta_\gamma H(x,y,d)\right]$$

$$\Gamma_{0\,(\ell\ell')} \equiv \frac{G_{\ell\ell'} m_\ell^5}{192\pi^3} \quad , \quad d \equiv 1 - \beta_{\ell'} \cos\theta_{\ell'\gamma} \quad , \quad \beta_{\ell'} = \sqrt{1 - \frac{m_{\ell'}^2}{m_\ell^2}}$$

$$\mathcal{F} = \sum_{p=0}^{2} (M_\nu)^p\, \mathcal{F}_{(p)} \qquad \mathcal{F}_{(p)} = \sum_{r=0}^{6} (M_\ell)^r\, \mathfrak{f}_{(p)}^{(r)}(x,y,d,\beta_{\ell'})$$

The key result is the successful isolation of linear combinations of weighted Wilson coefficients, the generalized Michel parameters.

Radiative decay rate distribution obtained

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Radiative decay rate distribution obtained

Main result: Generalized Michel Parameters

A total of 28 parameters were found for the isotropic part.

What was found <u>for no neutrino mass suppression</u>:

- Characteristic parameters of Dirac/Majorana-like behavior.
- Same grouping patterns for Wilson coefficients as in the non-radiative result with massive neutrinos.
- Many linear combinations of small groups of Wilson coefficients.
- Splitting of the massless-case parameters.

$$
\begin{aligned}
(N_\alpha)_{jk} =& |(f^S_{LL})_{jk}|^2 + |(f^S_{RR})_{jk}|^2 + |(f^S_{LR})_{jk}|^2 + |(f^S_{RL})_{jk}|^2 + 20|(f^T_{LR})_{jk}|^2 + 20|(f^T_{RL})_{jk}|^2 \\
&+ 4|(f^V_{LL})_{jk}|^2 + 4|(f^V_{RR})_{jk}|^2 + 8|(f^V_{LR})_{jk}|^2 + 8|(f^V_{RL})_{jk}|^2 \\
(N_\beta)_{jk} =& |(f^S_{LL})_{jk}|^2 + |(f^S_{RR})_{jk}|^2 + 4(|(f^V_{LL})_{jk}|^2 + |(f^V_{RR})_{jk}|^2) \\
(N_\gamma)_{jk} =& \mathrm{Re}[(f^V_{RL})_{jk}(f^S_{LR})^*_{jk} + (f^V_{LR})_{jk}(f^S_{RL})^*_{jk} + 6((f^T_{LR})_{jk}(f^V_{RL})^*_{jk} + (f^T_{RL})_{jk}(f^V_{LR})^*_{jk}) \\
&+ \epsilon\,((f^V_{RL})_{jk}(f^S_{LR})^*_{kj} + (f^V_{LR})_{jk}(f^S_{RL})^*_{kj} + 6((f^T_{LR})_{jk}(f^V_{RL})^*_{kj} + (f^T_{RL})_{jk}(f^V_{LR})^*_{kj}))] \\
(N_\delta)_{jk} =& 5(|(f^T_{RL})_{jk}|^2 + |(f^T_{LR})_{jk}|^2) + |(f^V_{RR})_{jk}|^2 + |(f^V_{LL})_{jk}|^2 + 2(|(f^V_{RL})_{jk}|^2 + |(f^V_{LR})_{jk}|^2) \\
(N_\varepsilon)_{jk} =& |(f^S_{RR})_{jk}|^2 + |(f^S_{LL})_{jk}|^2 + |(f^S_{RL})_{jk}|^2 + |(f^S_{LR})_{jk}|^2 + 4(|(f^V_{RR})_{jk}|^2 + |(f^V_{LL})_{jk}|^2) \\
(\upsilon)_{jk} =& \mathrm{Re}[2((f^V_{RR})_{jk}(f^S_{LL})^*_{jk} + (f^V_{LL})_{jk}(f^S_{RR})^*_{jk}) + \epsilon\,((f^S_{LL})_{jk}(f^S_{RR})^*_{kj} + 4(f^V_{LL})_{jk}(f^V_{RR})^*_{kj})] \\
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(\Sigma^+)_{jk} =& \mathrm{Re}[(f^S_{LR})_{jk}(f^S_{LL})^*_{jk} + (f^S_{RL})_{jk}(f^S_{RR})^*_{jk} + 2\epsilon\,((f^V_{LL})_{jk}(f^S_{LR})^*_{kj} + (f^V_{RR})_{jk}(f^S_{RL})^*_{kj})] \\
(\Sigma^-)_{jk} =& (\Sigma^+)_{jk}|_{RR\leftrightarrow LL} \\
(\Sigma'^+)_{jk} =& \mathrm{Re}[2(f^V_{LL})_{jk}(f^S_{LR})^*_{jk} + (f^V_{RR})_{jk}(f^S_{RL})^*_{jk}) + \epsilon((f^S_{LR})_{jk}(f^S_{LL})^*_{kj} + (f^S_{RL})_{jk}(f^S_{RR})^*_{kj})] \\
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\end{aligned}
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Radiative decay rate distribution obtained

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What was found for no neutrino mass suppression:

- Characteristic parameters of Dirac/Majorana-like behavior.
- Same grouping patterns for Wilson coefficients as in the non-radiative result with massive neutrinos.
- Many linear combinations of small groups of Wilson coefficients.
- Splitting of the massless-case parameters.

$$
\begin{aligned}
(N_\alpha)_{jk} =& |(f^S_{LL})_{jk}|^2 + |(f^S_{RR})_{jk}|^2 + |(f^S_{LR})_{jk}|^2 + |(f^S_{RL})_{jk}|^2 + 20|(f^T_{LR})_{jk}|^2 + 20|(f^T_{RL})_{jk}|^2 \\
&+ 4|(f^V_{LL})_{jk}|^2 + 4|(f^V_{RR})_{jk}|^2 + 8|(f^V_{LR})_{jk}|^2 + 8|(f^V_{RL})_{jk}|^2 \\
(N_\beta)_{jk} =& |(f^S_{LL})_{jk}|^2 + |(f^S_{RR})_{jk}|^2 + 4(|(f^V_{LL})_{jk}|^2 + |(f^V_{RR})_{jk}|^2) \\
(N_\gamma)_{jk} =& \mathrm{Re}[(f^V_{RL})_{jk}(f^S_{LR})^*_{jk} + (f^V_{LR})_{jk}(f^S_{RL})^*_{jk} + 6((f^T_{LR})_{jk}(f^V_{RL})^*_{jk} + (f^T_{RL})_{jk}(f^V_{LR})^*_{jk}) \\
&+ \epsilon\,((f^V_{RL})_{jk}(f^S_{LR})^*_{kj} + (f^V_{LR})_{jk}(f^S_{RL})^*_{kj} + 6((f^T_{LR})_{jk}(f^V_{RL})^*_{kj} + (f^T_{RL})_{jk}(f^V_{LR})^*_{kj}))] \\
(N_\delta)_{jk} =& 5(|(f^T_{RL})_{jk}|^2 + |(f^T_{LR})_{jk}|^2) + |(f^V_{RR})_{jk}|^2 + |(f^V_{LL})_{jk}|^2 + 2(|(f^V_{RL})_{jk}|^2 + |(f^V_{LR})_{jk}|^2) \\
(N_\varepsilon)_{jk} =& |(f^S_{RR})_{jk}|^2 + |(f^S_{LL})_{jk}|^2 + |(f^S_{RL})_{jk}|^2 + |(f^S_{LR})_{jk}|^2 + 4(|(f^V_{RR})_{jk}|^2 + |(f^V_{LL})_{jk}|^2) \\
(\upsilon)_{jk} =& \mathrm{Re}[2((f^V_{RR})_{jk}(f^S_{LL})^*_{jk} + (f^V_{LL})_{jk}(f^S_{RR})^*_{jk}) + \epsilon\,((f^S_{LL})_{jk}(f^S_{RR})^*_{kj} + 4(f^V_{LL})_{jk}(f^V_{RR})^*_{kj})] \\
(\tau)_{jk} =& \epsilon\,\mathrm{Re}[(f^T_{LR})_{jk}(f^T_{LR})^*_{kj} + (f^T_{RL})_{jk}(f^T_{RL})^*_{kj}] \\
(\sigma)_{jk} =& \epsilon\,\mathrm{Re}[(f^S_{LR})_{jk}(f^S_{LR})^*_{kj} + (f^S_{RL})_{jk}(f^S_{RL})^*_{kj}] \\
(\Sigma V_{\mathrm{M}})_{jk} =& \epsilon\,\mathrm{Re}[(f^V_{LL})_{jk}(f^S_{LL})^*_{kj} + (f^V_{RR})_{jk}(f^S_{RR})^*_{kj}] \\
(\Sigma T_{\mathrm{D}})_{jk} =& \mathrm{Re}[(f^T_{LR})_{jk}(f^S_{LR})^*_{jk} + (f^T_{RL})_{jk}(f^S_{RL})^*_{jk}] \\
(\Sigma^+)_{jk} =& \mathrm{Re}[(f^S_{LR})_{jk}(f^S_{LL})^*_{jk} + (f^S_{RL})_{jk}(f^S_{RR})^*_{jk} + 2\epsilon\,((f^V_{LL})_{jk}(f^S_{LR})^*_{kj} + (f^V_{RR})_{jk}(f^S_{RL})^*_{kj})] \\
(\Sigma^-)_{jk} =& (\Sigma^+)_{jk}|_{RR\leftrightarrow LL} \\
(\Sigma'^+)_{jk} =& \mathrm{Re}[2(f^V_{LL})_{jk}(f^S_{LR})^*_{jk} + (f^V_{RR})_{jk}(f^S_{RL})^*_{jk}) + \epsilon((f^S_{LR})_{jk}(f^S_{LL})^*_{kj} + (f^S_{RL})_{jk}(f^S_{RR})^*_{kj})] \\
(\Sigma'^-)_{jk} =& (\Sigma'^+)_{jk}|_{RR\leftrightarrow LL}
\end{aligned}
$$

Radiative decay rate distribution obtained

Phenomenological analysis

To study the implications of the existence of one heavy neutrino, the suppression arising from mixing parameters and mass ratios was investigated under current experimental and theoretical constraints **[a,b,c]** *(plugged on the references)*

Neutrino flavor mix	Mass m_j (MeV)	Mixing $\|U_{\ell 4}\|^2$
$\ell = e$	$0.001 - 0.45$	10^{-3}
	$10 - 55$	10^{-8}
	$135 - 350$	10^{-6}
$\ell = \mu$	$10 - 30$	10^{-4}
	$70 - 300$	10^{-5}
	$175 - 300$	10^{-8}
$\ell = \tau$	$100 - 1.3 \times 10^3$	$10^{-5} - 10^{-2}$
	$1 \times 10^3 - 60 \times 10^3$	$10^{-5} - 10^{-3}$

(Recall that $\boldsymbol{M_\nu} \equiv \frac{\boldsymbol{m_j}}{\boldsymbol{m_\ell}}$, with m_j the neutrino mass, m_ℓ the parent-lepton mass)

Neutrino production	Decay mode	$M_\nu \|U_{\ell 4}\|^2$ suppression order	
		Linear (M_ν)	Quadratic (M_ν^2)
Both light	$\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$	10^{-8}	10^{-16}
	$\tau^- \to X^- \bar{\nu}_X \nu_\tau \gamma$	10^{-9}	10^{-18}
1 Heavy ($\ell = e$)	$\mu^- \to e^- \bar{N}_4 \nu_\mu \gamma$	$10^{-8} - 10^{-6}$ 10^{-10} 10^{-6}	$10^{-16} - 10^{-14}$ 10^{-17} 10^{-14}
	$\tau^- \to e^- \bar{N}_4 \nu_\tau \gamma$	$10^{-9} - 10^{-7}$ 10^{-10} 10^{-7}	$10^{-18} - 10^{-16}$ 10^{-19} 10^{-16}
1 Heavy ($\ell = \mu$)	$\mu^- \to e^- \bar{\nu}_e N_4 \gamma$	10^{-5} $10^{-6} - 10^{-5}$ 10^{-8}	10^{-13} $10^{-14} - 10^{-13}$ 10^{-16}
	$\tau^- \to \mu^- \bar{N}_4 \nu_\tau \gamma$	10^{-6} $10^{-7} - 10^{-6}$ 10^{-9}	10^{-15} $10^{-16} - 10^{-15}$ 10^{-18}
1 Heavy ($\ell = \tau$)	$\tau^- \to X^- \bar{\nu}_X N_4 \gamma$	$10^{-4} - 10^{-1}$ $10^{-5} - 10^{-3}$	$10^{-13} - 10^{-9}$ $10^{-14} - 10^{-12}$

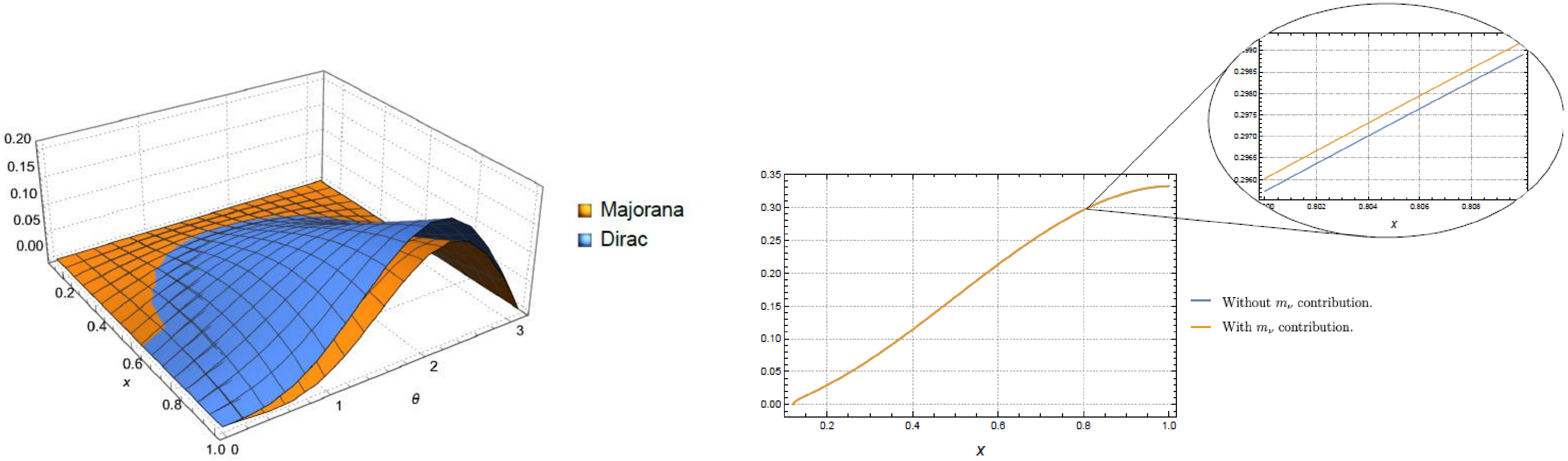
Outline

- **Fermi theory for leptonic decay** $\ell \to \ell' \overline{\nu_{\ell'}} \nu_\ell$
- **Radiative process** $\ell \to \ell' \overline{\nu_{\ell'}} \nu_\ell \gamma$
- **Adding mass to neutrinos**
- **Derived radiative decay rate distribution**
- **Comments and further work**

Comments and further work

Further work

- The polarized differential decay rate was also derived, but it was not considered for the phenomenological analysis.
- Further work will include a phenomenological study of SM-like interaction ($|g_{LL}^V| = 1$, others 0).
- The aim will be to conduct a phenomenological study as robust as that of **[Márquez-López-Roig]**.



Doubly differential decay rate (left) and energy spectrum (right) for the $V \pm A$ theory with P_{T_2} polarization for the non-radiative decay, extracted from **[Márquez-López-Roig]**

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Thanks!

Questions?

References

- **[Louis Michel]** L Michel. “Interaction between Four Half-Spin Particles and the Decay of the μ - Meson”. In: Proceedings of the Physical Society. Section A 63.5 (May 1950), p. 514. doi: 10.1088/0370-1298/63/5/311.
- **[Márquez-López-Roig]** Juan Manuel Márquez, Gabriel López Castro, and Pablo Roig. “Michel parameters in the presence of massive Dirac and Majorana neutrinos”. In: Journal of High Energy Physics 2022.11 (Nov. 2022). issn: 1029-8479. doi: 10 . 1007 / jhep11(2022) 117.
- **[Arbuzov-Kopilova]** A. B. Arbuzov and T. V. Kopylova. “Michel parameters in radiative muon decay”. In: JHEP 09 (2016), p. 109. doi: 10.1007/JHEP09(2016)109. arXiv: 1605.06612 [hep-ph].
- **[Kuno-Okada]** Yoshitaka Kuno and Yasuhiro Okada. “Muon decay and physics beyond the standard model”. In: Rev. Mod. Phys. 73 (2001), pp. 151–202. doi: 10.1103/RevModPhys.73. 151. arXiv: hep-ph/9909265.
- **[a]** Andrew Kobach and Sean Dobbs. “Heavy Neutrinos and the Kinematics of Tau Decays”. In: Phys. Rev. D 91.5 (2015), p. 053006. doi: 10.1103/PhysRevD.91. 053006. arXiv: 1412.4785 [hep-ph].
- **[b]** S. P. Das et al. “Heavy Neutrinos and Lepton Flavour Violation in Left-Right Symmetric Models at the LHC”. In: Phys. Rev. D 86 (2012), p. 055006. doi: 10.1103/ PhysRevD.86.055006. arXiv: 1206.0256 [hep-ph].
- **[c]** J. P. Lees et al. “Search for heavy neutral leptons using tau lepton decays at BaBaR”. In: Phys. Rev. D 107.5 (2023), p. 052009. doi: 10.1103/PhysRevD.107.052009. arXiv: 2207.09575 [hep-ex].