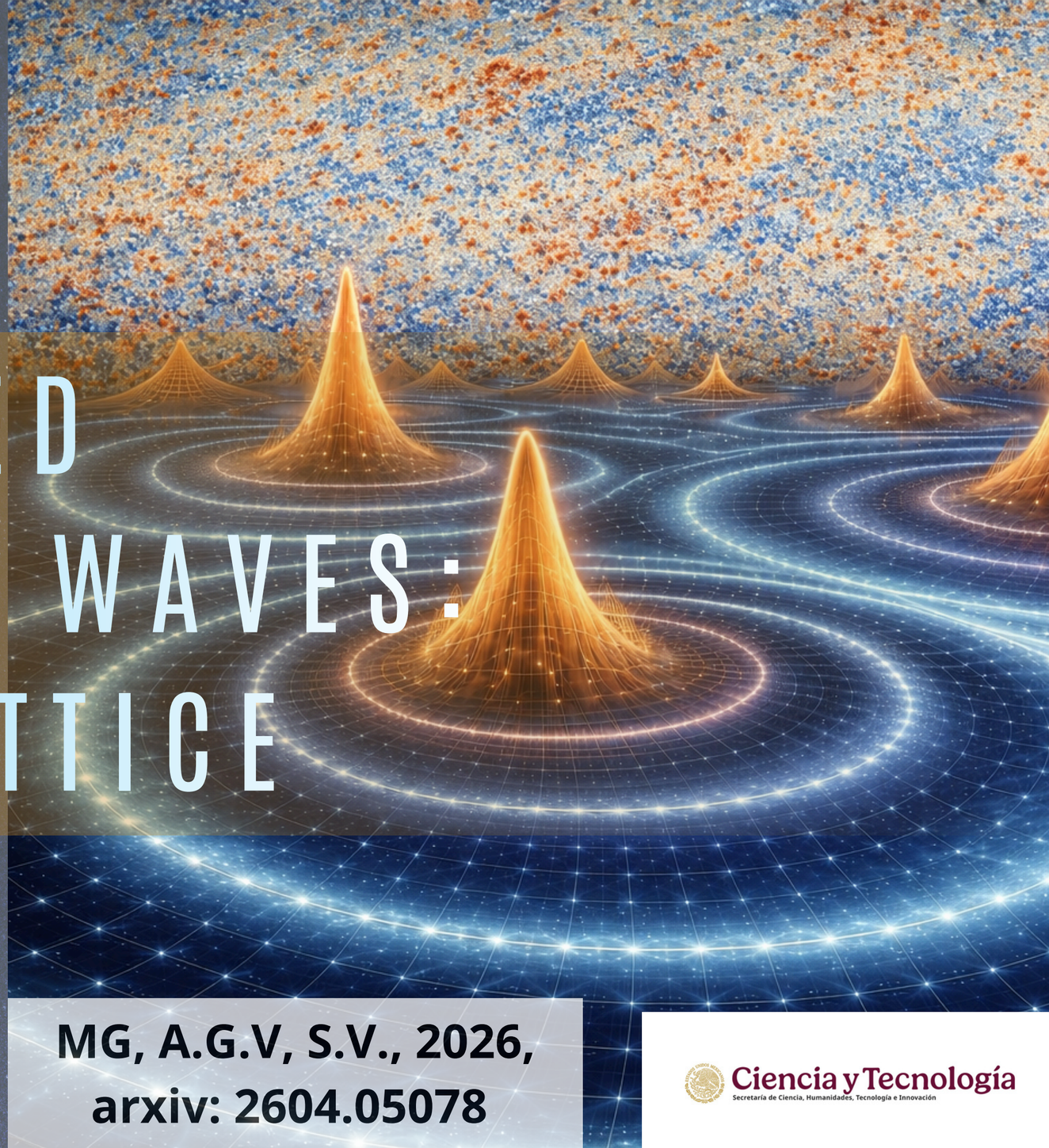
SCALAR-INDUCED GRAVITATIONAL WAVES: BEYOND THE LATTICE



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MG, A.G.V, S.V., 2026, arxiv: 2604.05078

OBJECTIVE

Compute the stochastic gravitational wave background producee by the perturbations of a scalar spectator field...

MOTIVATION

MOTIVATION

Many BSM models introduce new scalar fields, such as:

- axions or ALPs,
- moduli,
- dark sectors,
- scalar singlets,
- curvatons,
- or spectator fields.

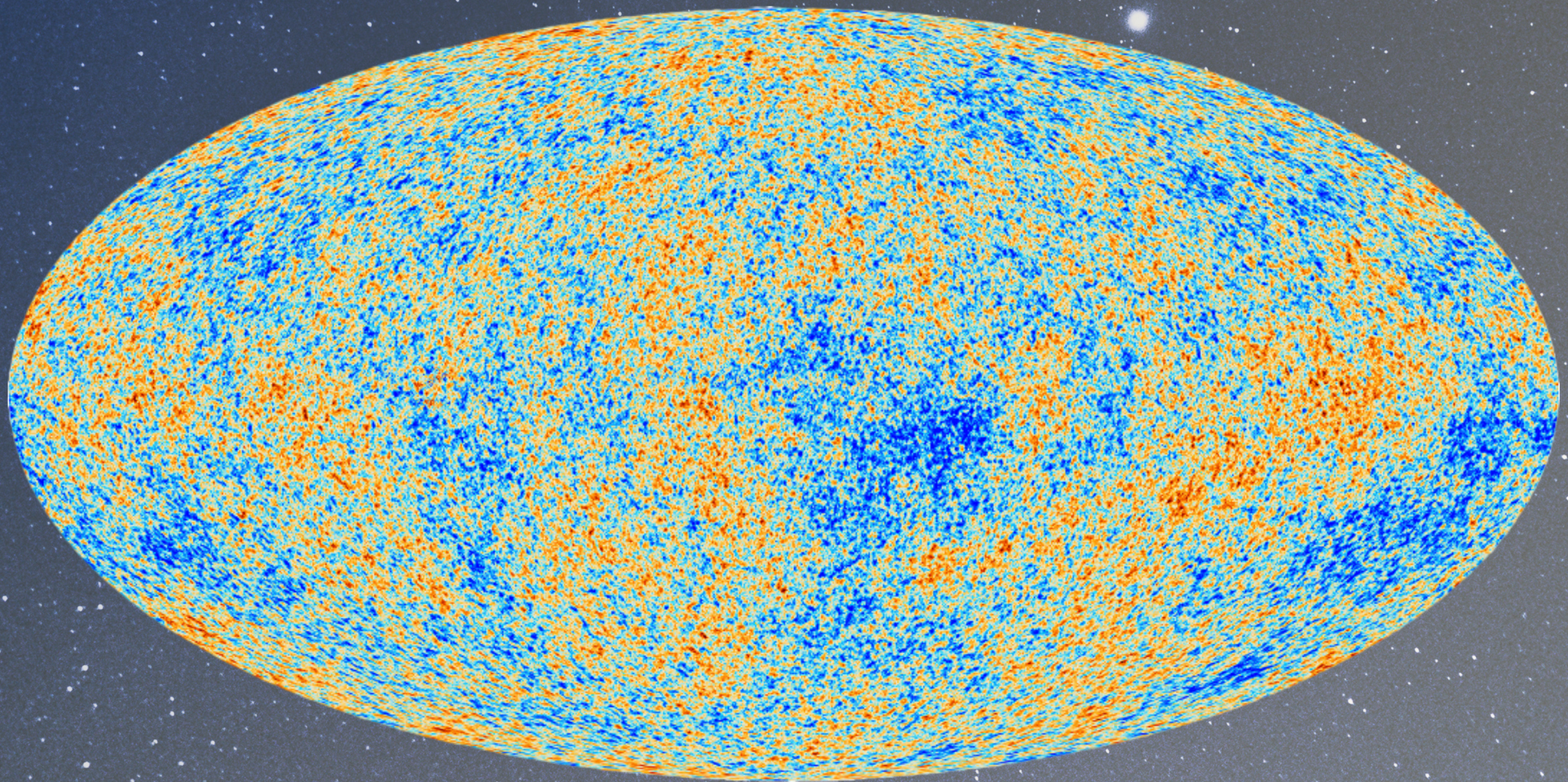
MOTIVATION

All of them can leave a gravitational signature!

COSMOLOGICAL PRINCIPLE

COSMOLOGICAL PRINCIPLE

The universe is homogeneous and isotropic...



COSMOLOGICAL PRINCIPLE

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \text{sen}^2\theta d\varphi^2\right]$$

- K = 1, close universe
- K = 0, spatially flat universe
- K = -1, open universe

Baumann (2022)

$a(t)$ is the scale factor; measures the expansion of the universe

INFLATION

INFLATION

An accelerated expansion era in the early universe...

INFLATION

An accelerated expansion era in the early universe...

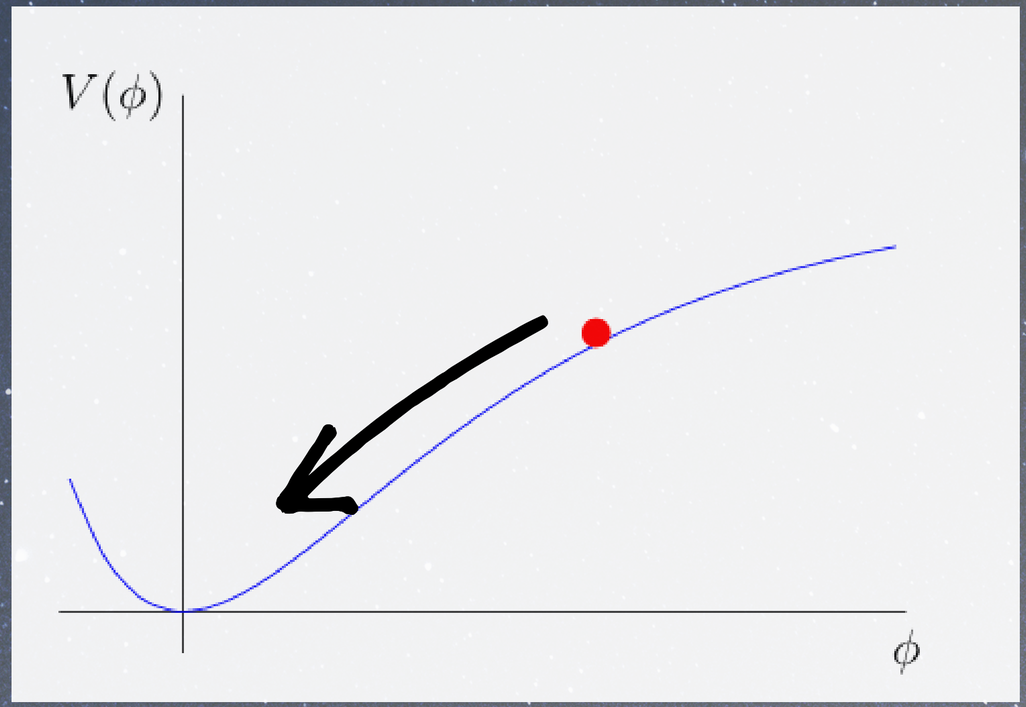
$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[-\frac{M_p^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$M_p^{-2} = 8\pi G, \quad g = \det(g_{\mu\nu})$$

Mukhanov (2005)

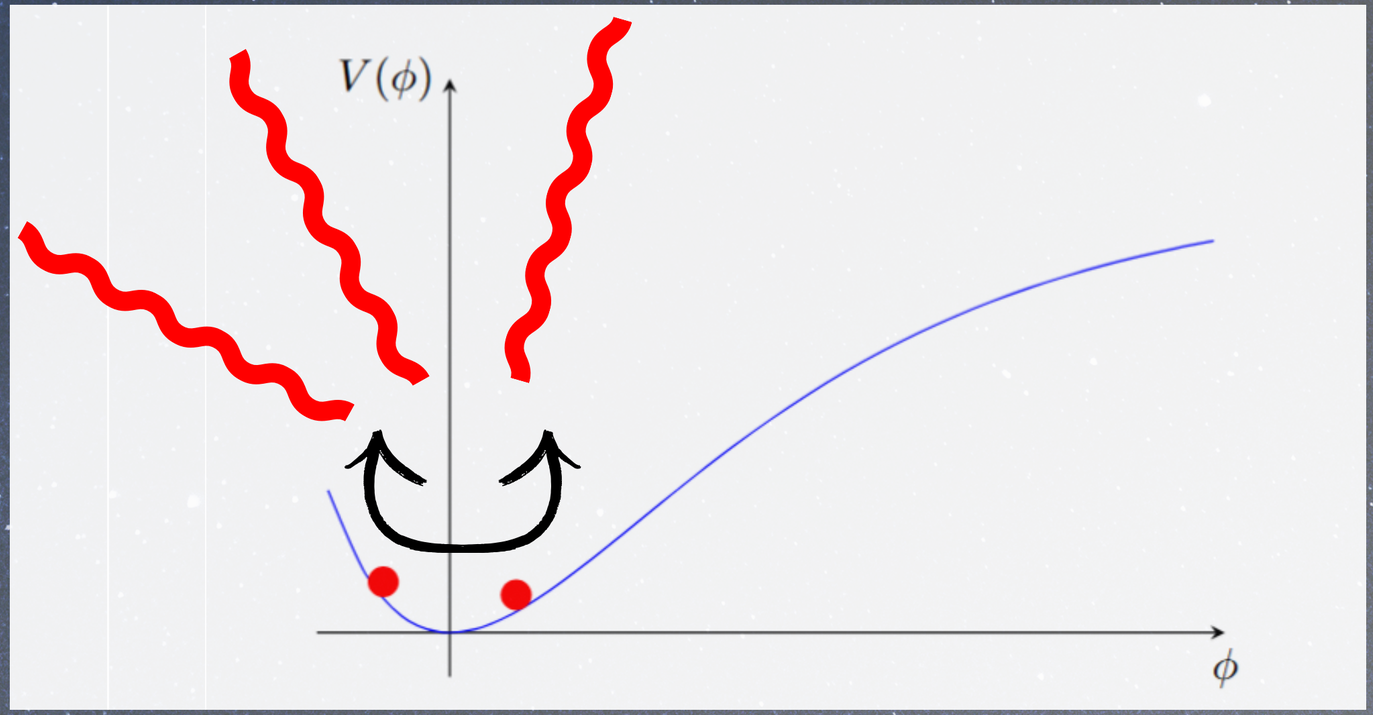
INFLATION

An accelerated expansion era in the early universe...



INFLATION

After the end of inflation, the universe is left empty, requiring a reheating era...



INFLATION

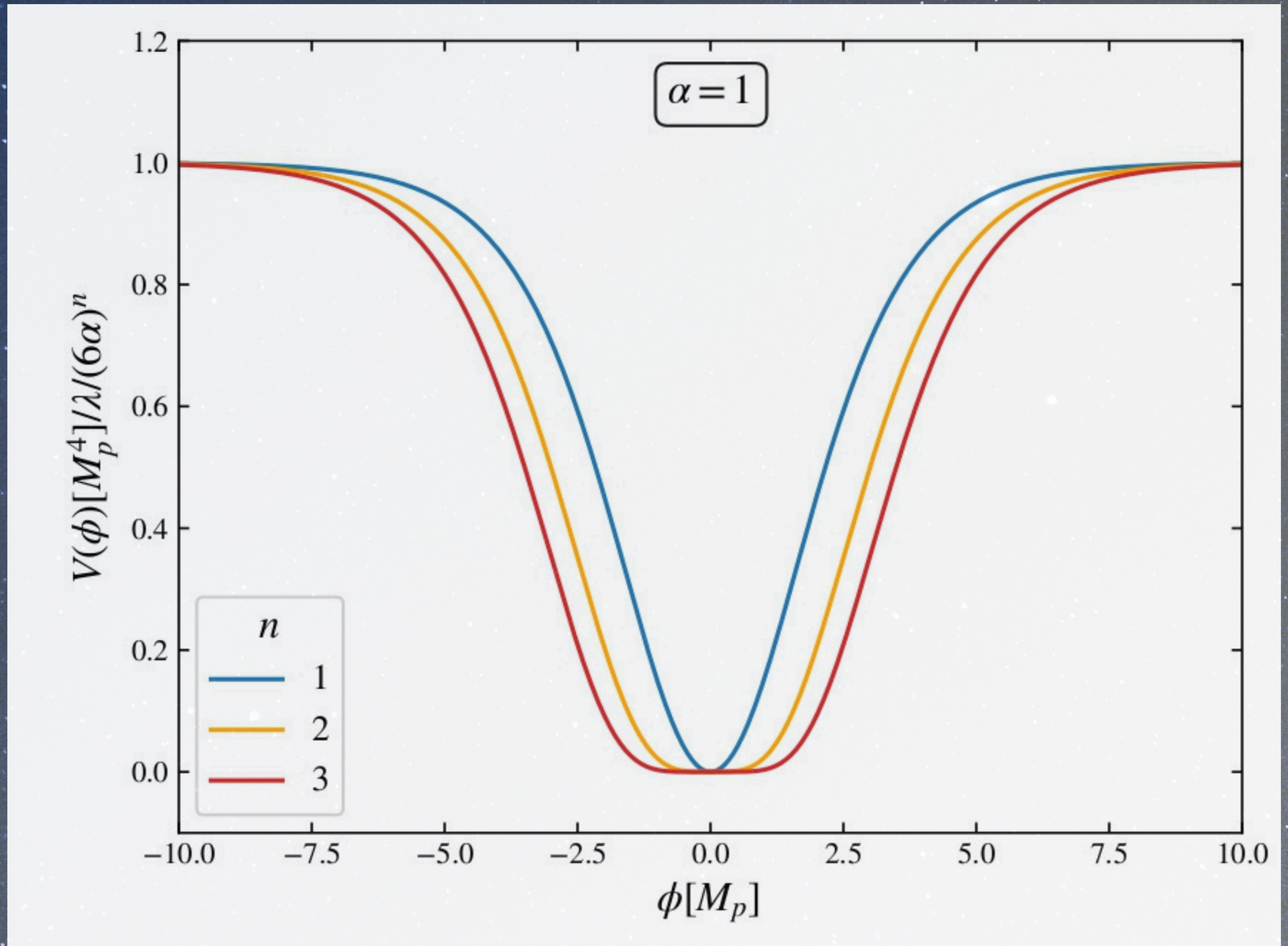
T model:

$$V(\phi) = \lambda M_p^4 \left[\sqrt{6} \tanh \left(\frac{\phi}{\sqrt{6} M_p} \right) \right]^{2n}$$

Compatible with CMB measurements under certain choice of parameters

Kallosh & Linde (2013), Ellis et al. (2022)

INFLATION


$\alpha = 1$
$V(\phi)[M_p^4]/\lambda/(6\alpha)^n$
$\phi[M_p]$
n
1
2
3
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

INFLATION

$$n = 1$$

Around the minimum,

$$V(\phi) \simeq \lambda \phi^2 M_p^2 \implies m_\phi = \sqrt{2\lambda} M_p$$

INFLATION

$$\ddot{\phi} - \frac{\nabla^2}{a^2}\phi + 3H\dot{\phi} + V_{,\phi} = 0$$

$H = \frac{\dot{a}}{a}$ is the Hubble parameter; it measures the velocity of expansion

$$3M_p^2 H^2 = \rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)$$

EARLY PARTICLE PRODUCTION

EARLY PARTICLE PRODUCTION

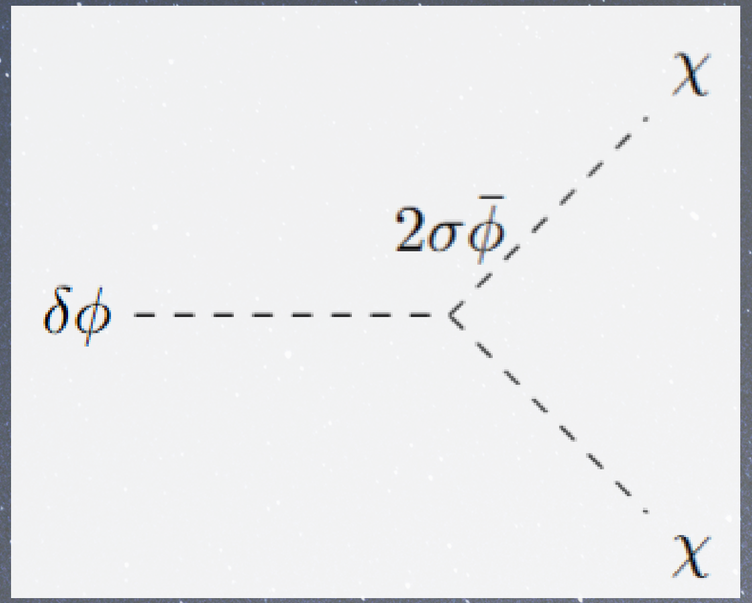
$$S_\chi[\chi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{2} \sigma \phi^2 \chi^2 \right]$$

Kolb & Long (2024)

EARLY PARTICLE PRODUCTION

$$\phi = \bar{\phi} + \delta\phi \qquad \langle\phi\rangle = \bar{\phi}$$

$$\phi^2\chi^2 \longrightarrow \bar{\phi}^2\chi^2 + 2\bar{\phi}\delta\phi\chi^2 + \mathcal{O}(\delta\phi^2)$$



EARLY PARTICLE PRODUCTION

$$d\vartheta = \frac{dt}{a^3(t)}$$

Parker time

$$\frac{d^2}{d\vartheta^2}\chi_k + \omega_k^2 \chi_k = 0$$

$$\omega_k^2(\vartheta) = a^4(\vartheta)k^2 + a^6(\vartheta)\left[m_\chi^2 + \sigma\phi(\vartheta)^2\right]$$

Parker & Toms (2009)

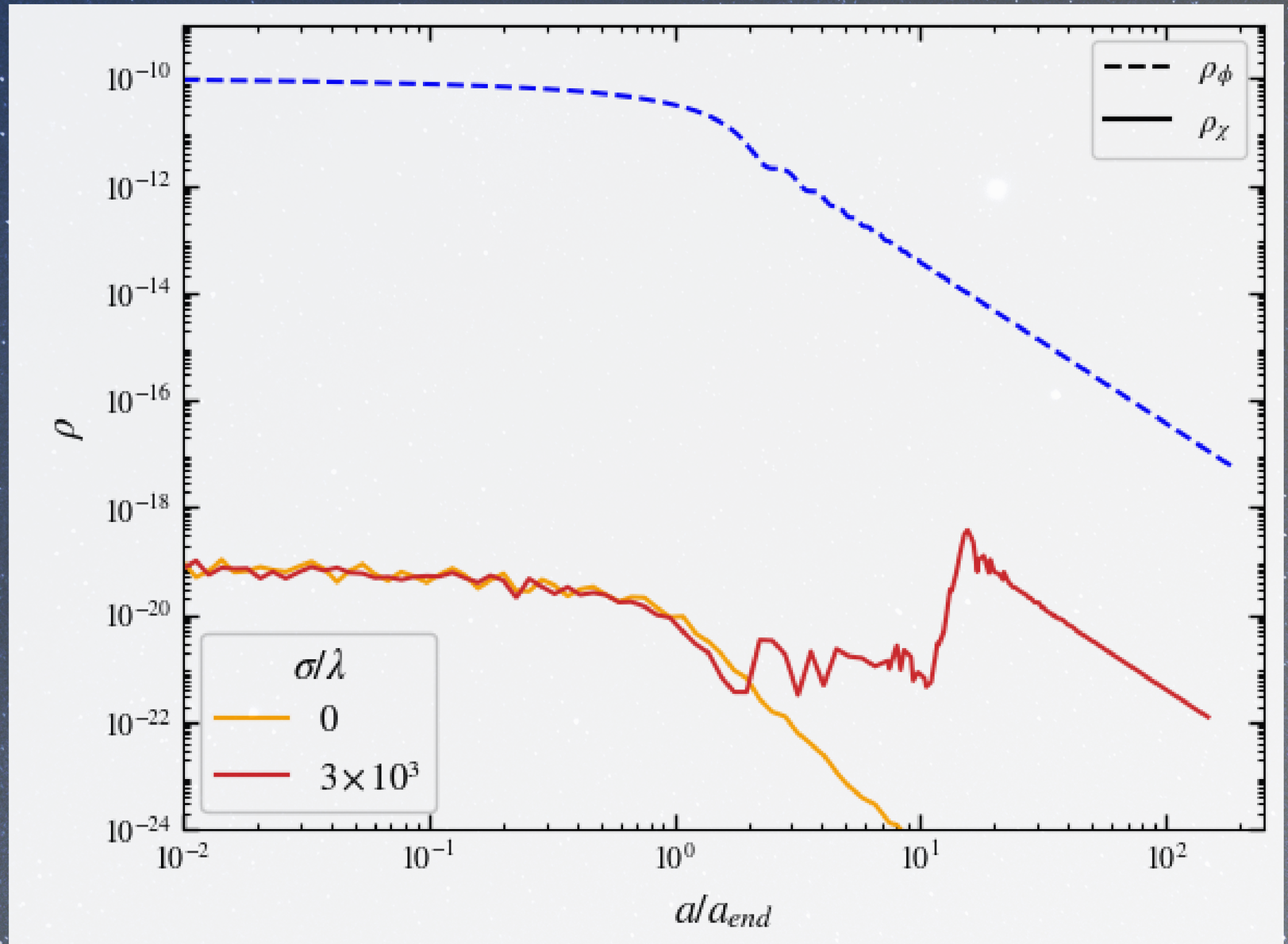
EARLY PARTICLE PRODUCTION

Particle production occurs when the quickly change in the frequency does not allow the modes to adjust to this variation.

When the latter occur, the vacuum defined in one moment does not coincide with later times, this maladjustment manifests as particle creation.

Birrell & Davies (1982)

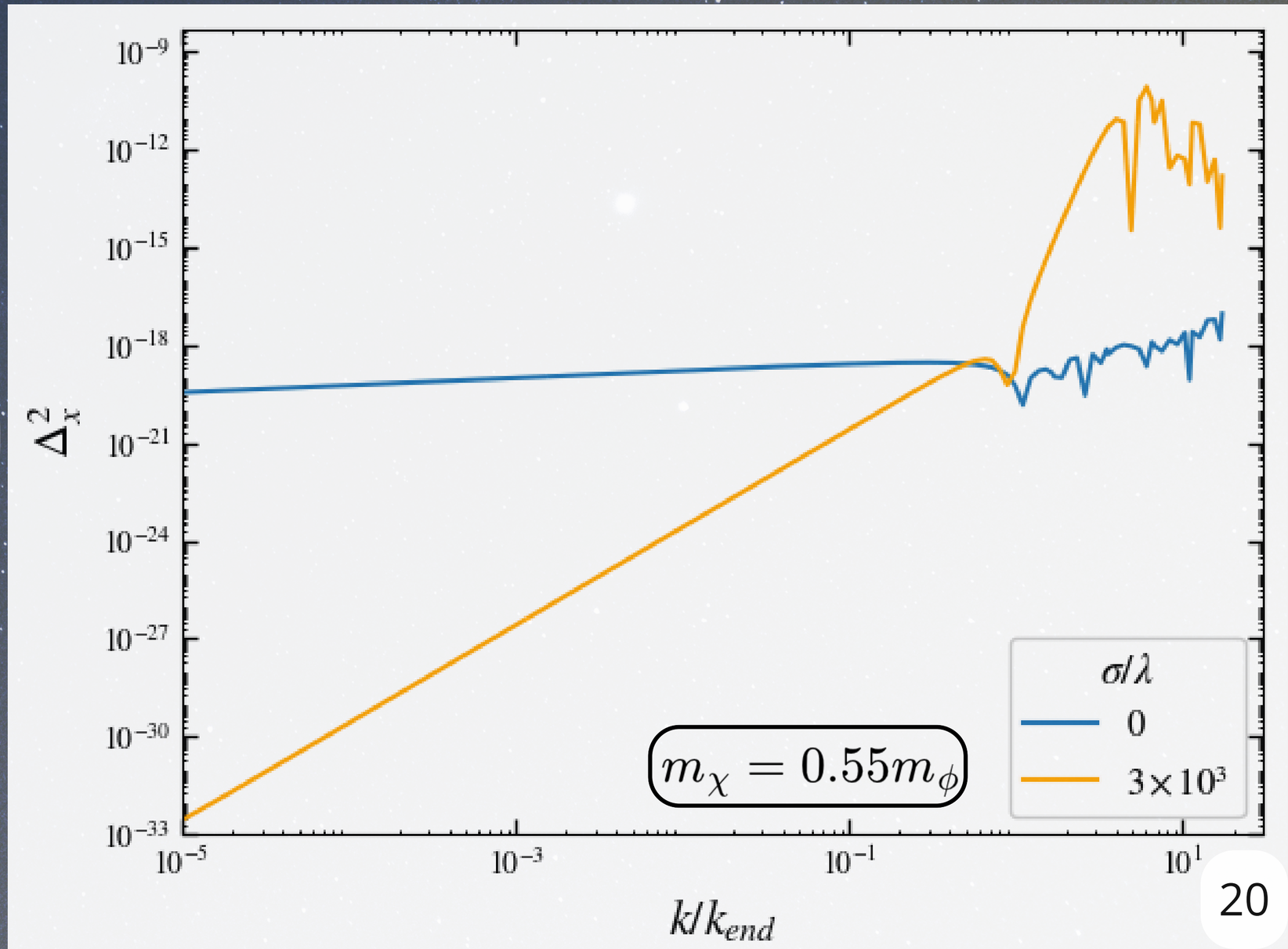
EARLY PARTICLE PRODUCTION



EARLY PARTICLE PRODUCTION

$$\langle \chi^2 \rangle = \int \frac{dk}{k} \Delta^2_\chi$$

$$= \int \frac{dk}{k} \frac{k^3}{2\pi^2} \left(|\chi_k|^2 - \frac{1}{2\omega_k} \right)$$



GRAVITATIONAL WAVES

GRAVITATIONAL WAVES

Propagating perturbations of the spacetime metric

$$g_{\mu\nu} = g^{FLRW}_{\mu\nu} + h_{\mu\nu}$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2} T_{ij}^{\rm TT}$$

GRAVITATIONAL WAVES

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Domènech (2021)

$$T_{ij}^{\rm TT} = \frac{1}{a^2}\mathcal{P}_{ij}^{ab}\{\partial_a\chi\partial_b\chi\}$$

GRAVITATIONAL WAVES

Energy density of produced gravitational waves

$$\rho_{\mathrm{GW}} = \frac{M_p^2}{4} \left\langle \dot{h}_{ij} \dot{h}_{ij} \right\rangle$$

Maggiore (2007)

GRAVITATIONAL WAVES

$$\rho_{\rm GW} \sim \langle hh \rangle, \quad h \sim \chi\chi$$

GRAVITATIONAL WAVES

$$\rho_{\mathrm{GW}} \sim \langle hh \rangle, \quad h \sim \chi\chi$$

$$\Longrightarrow$$

$$\rho_{\mathrm{GW}} \sim \langle \chi\chi\chi\chi \rangle$$

Quadrupole moment

GRAVITATIONAL WAVES

$$\frac{\rho_{GW}}{\rho_c} = \int d(\log k)\Omega_{GW}(k)$$

$$\Omega_{GW}(k) = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \Delta_h^2(k)$$

Maggiore (2007)

GRAVITATIONAL WAVES

Master Formula

$$\Omega_{GW}(k, N) = \mathcal{I}(k, N)^2 g(k)$$

GRAVITATIONAL WAVES

Master Formula

$$\Omega_{GW}(k, N) = \mathcal{I}(k, N)^2 g(k)$$

Contains all the temporal dependence

Contains all the spectral information of the field

GRAVITATIONAL WAVES

$$g(k) = \frac{1}{k^6}\int_0^\infty dp \int_{|k-p|}^{k+p} dq \left[k^4 - 2k^2(p^2+q^2) + (p^2-q^2)^2\right]^2 \Delta_\chi^2(p)\Delta_\chi^2(q)$$

$$\mathcal{J}(k,N) = \frac{1}{2\sqrt{12}}\left(\frac{k}{a(N)H(N))}\right)^2 \int_0^N dN' \frac{G_k(N,N')\mathcal{T}_\chi(N')}{(a(N')/a_{end})^3}\left(\frac{k}{a(N')H(N')}\right)^2$$

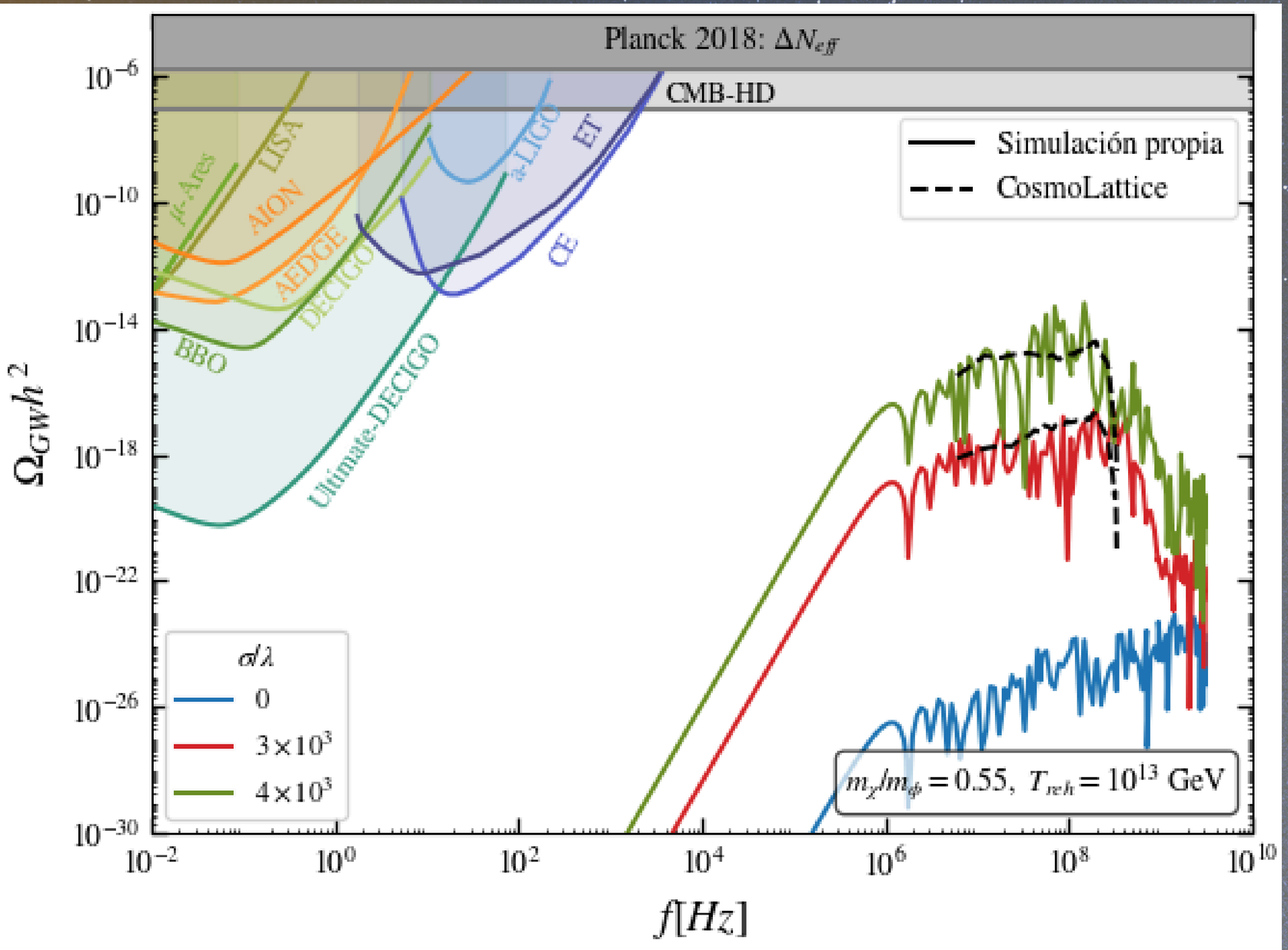
$$\frac{d^2}{dN^2}G_k(N,N') + \left(3 + \frac{\dot{H}}{H^2}\right)G_k(N,N') + \left(\frac{k}{a(N)H(N)}\right)^2 G_k(N,N') = \delta(N-N')$$

GRAVITATIONAL WAVES

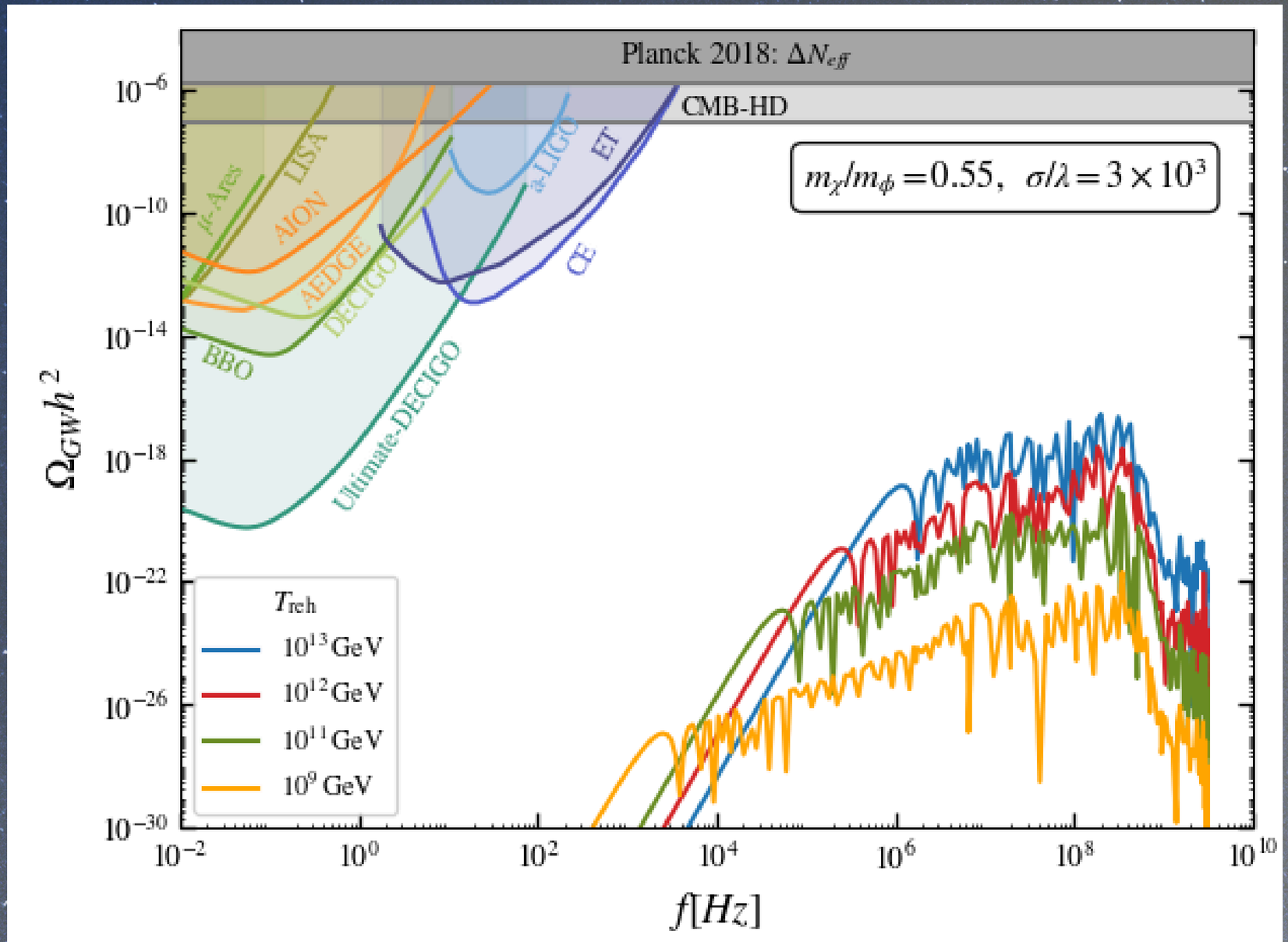
$$\Delta^2_\chi(k,N) = \left(\frac{a_{end}}{a(N)}\right)^3 \mathcal{T}_\chi(N)\Delta^2_\chi(k)$$

RESULTS

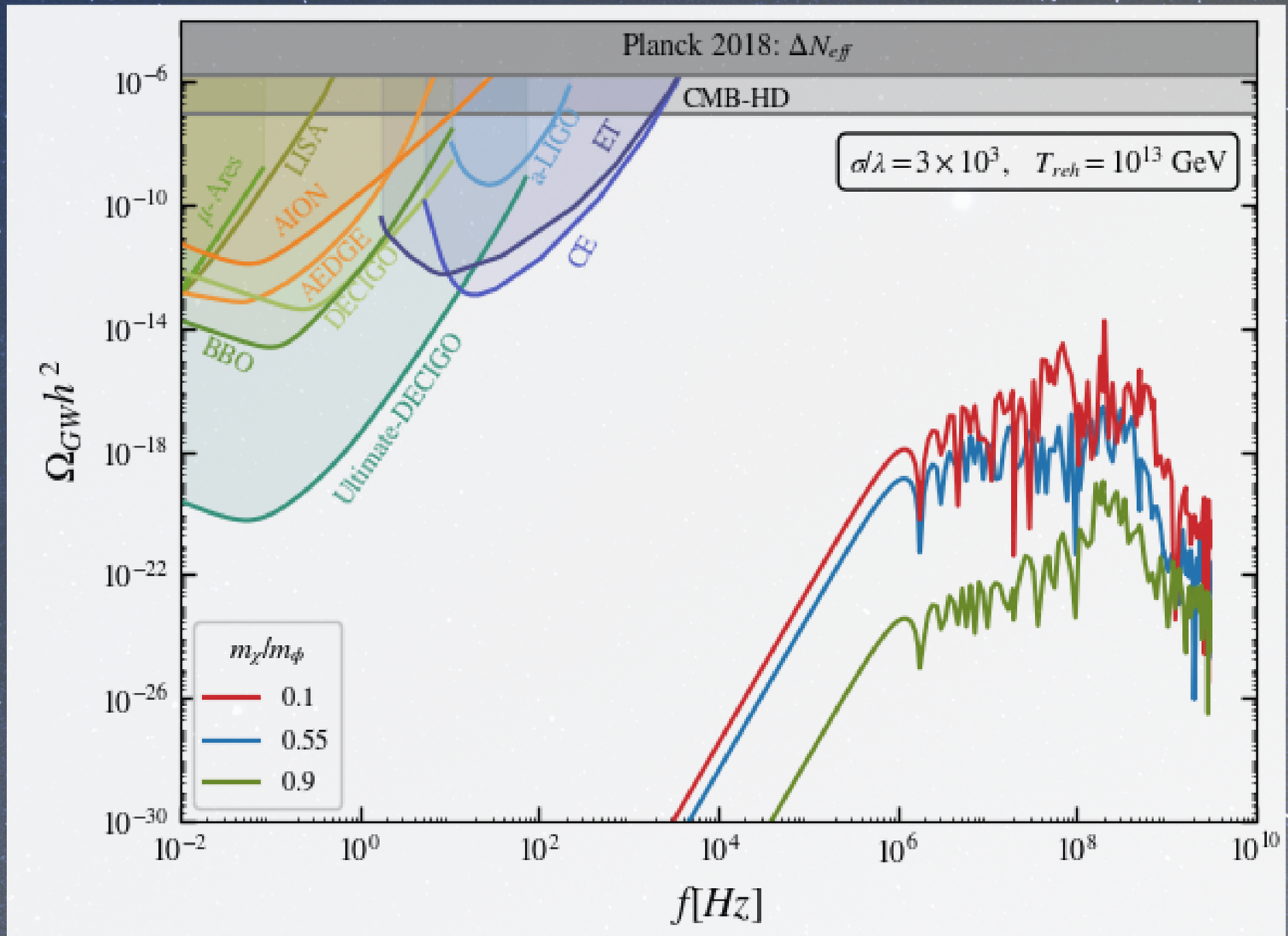
RESULTS


Planck 2018: ΔN_{eff}
CMB-HD
Simulación propia
CosmoLattice
α/λ
0
3×10^3
4×10^3
$m_\chi/m_\phi = 0.55,\ T_{reh} = 10^{13}$ GeV
$\Omega_{GW}h^2$
$f[Hz]$
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-26}
10^{-30}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
LISA
AION
AEDGE
DECIGO
BBO
Ultimate-DECIGO
aLIGO
ET
CE

RESULTS


Planck 2018: ΔN_{eff}
CMB-HD
$m_\chi/m_\phi = 0.55,\ \sigma/\lambda = 3\times10^3$
μ-Ares
LISA
AION
AEDGE
DECIGO
BBO
Ultimate-DECIGO
a-LIGO
ET
CE
$T_{\rm reh}$
10^{13} GeV
10^{12} GeV
10^{11} GeV
10^{9} GeV
$\Omega_{GW}h^2$
$f[Hz]$
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-26}
10^{-30}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}

RESULTS


Planck 2018: ΔN_{eff}
CMB-HD
$\alpha/\lambda = 3\times10^3$, $T_{reh} = 10^{13}$ GeV
μ-Ares
LISA
AION
AEDGE
DECIGO
BBO
Ultimate-DECIGO
aLIGO
ET
CE
m_χ/m_ϕ
0.1
0.55
0.9
$\Omega_{GW}h^2$
$f[Hz]$
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-26}
10^{-30}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}

CONCLUSIONS

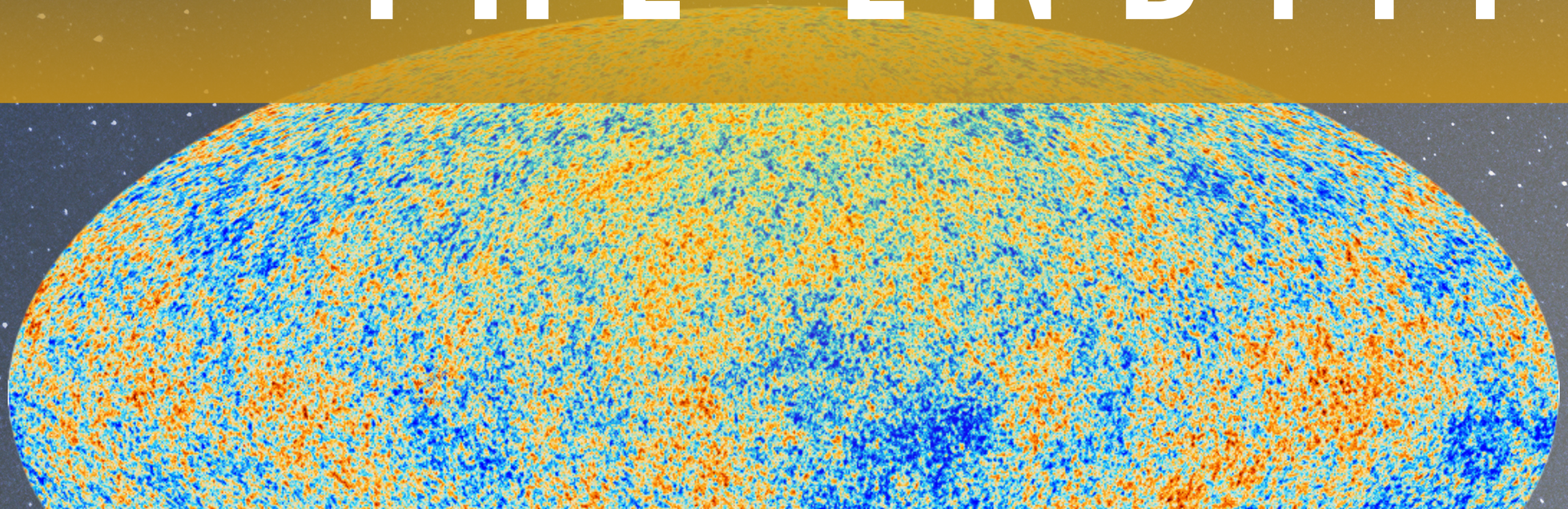
CONCLUSIONS

- The GWs are fundamental to study the early universe
- We study two scenarios of early particle production
- We developed a method to compute GWs spectral with the Master Formula
- The Master Formula factories the time dependence from the particle production mechanism

CONCLUSIONS

- The gravitational spectrum of a scalar fields minimally coupled to gravity is $\Omega_{GW}(f) \sim f^5$, on super horizon scales
- The computed signals are not inside of any sensibility curve
- This motivates the search of HFGWs with new experiments

THE END...



REFERENCES

- Baumann, D. (2022). Cosmology. Cambridge: Cambridge University Press.
- Birrell, N. D., & Davies, P. C. W. (1982). Quantum Fields in Curved Space. Cambridge: Cambridge University Press.
- Domènech, G. (2021). Scalar induced gravitational waves review. Universe, 7(11), 398. https://doi.org/10.3390/universe7110398
- Ellis, J., Garcia, M. A. G., Nanopoulos, D. V., Olive, K. A., & Verner, S. (2022). BICEP/Keck constraints on attractor models of inflation and reheating. Physical Review D, 105(4), 043504. https://doi.org/10.1103/PhysRevD.105.043504
- Kallosh, R., & Linde, A. (2013). Universality class in conformal inflation. Journal of Cosmology and Astroparticle Physics, 2013(07), 002. https://doi.org/10.1088/1475-7516/2013/07/002

REFERENCES

- Kolb, E. W., & Long, A. J. (2024). Cosmological gravitational particle production and its implications for cosmological relics. Reviews of Modern Physics, 96(4), 045005. https://doi.org/10.1103/RevModPhys.96.045005

- Maggiore, M. (2007). Gravitational waves: Vol. 1. Theory and experiments. Oxford University Press.

- Mukhanov, V. (2005). Physical Foundations of Cosmology. Cambridge: Cambridge University Press.

- Parker, L., & Toms, D. (2009). Quantum field theory in curved spacetime: Quantized fields and gravity. Cambridge University Press.

DIAPOSITIVAS DE APOYO

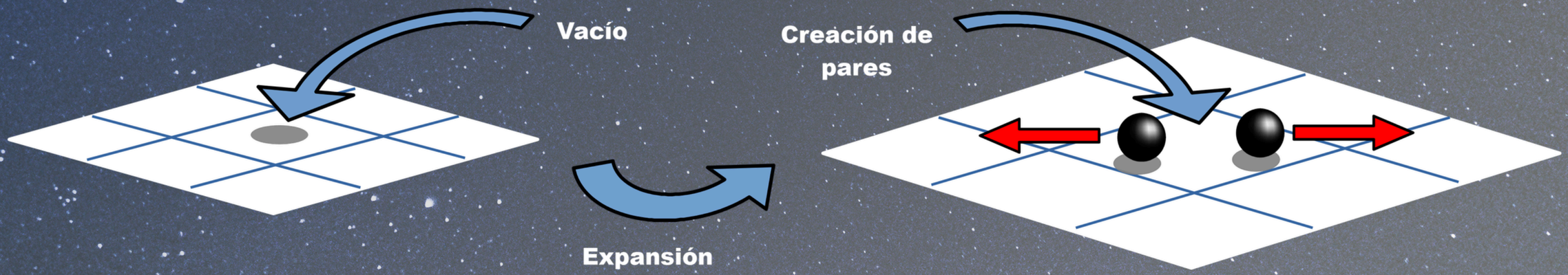
INFLACIÓN

Las longitudes de onda son estiradas **fuera del horizonte** causal

PRODUCCIÓN TEMPRANA DE PARTÍCULAS

Producción Gravitacional

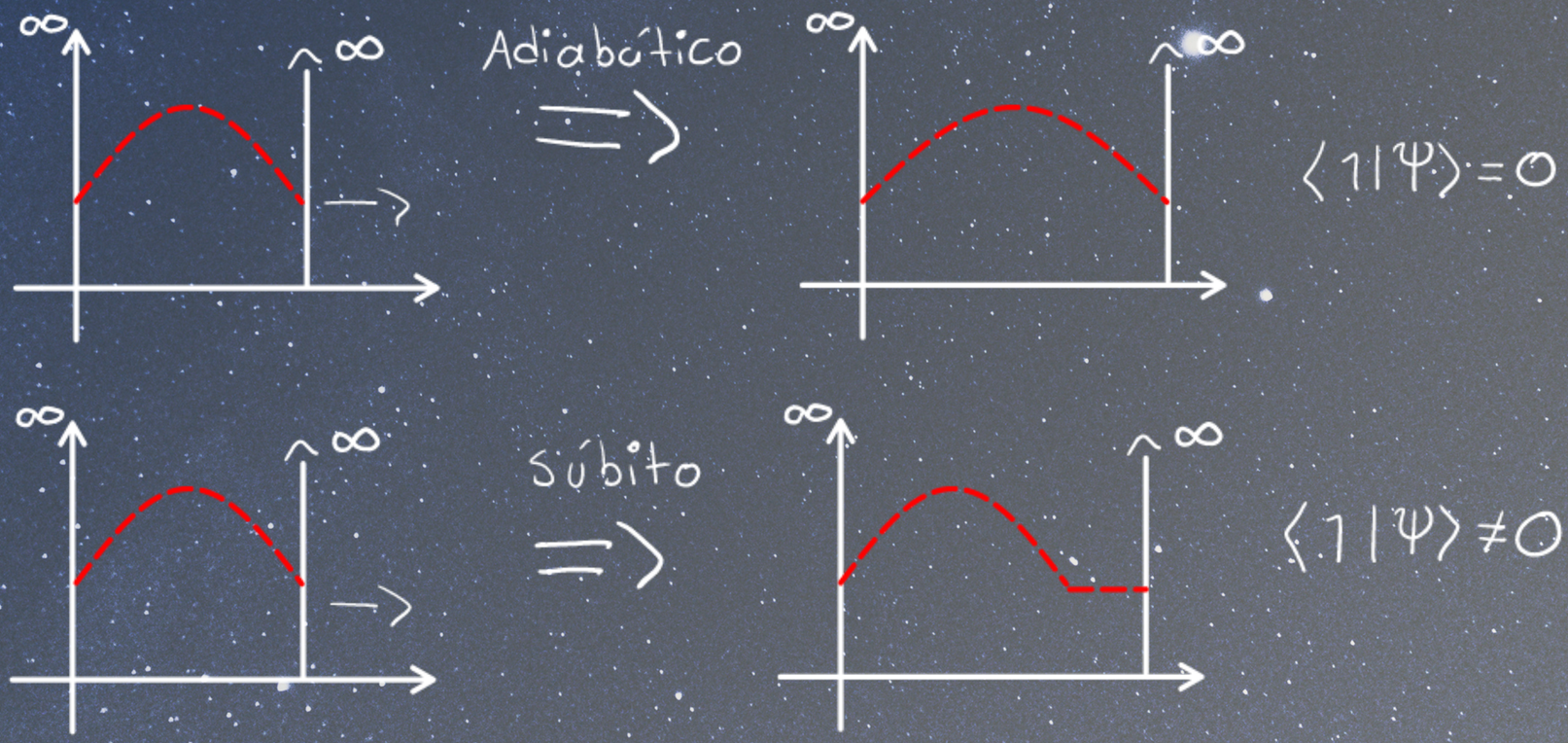
Excitación del vacío a estados de multipartículas en el marco de una teoría cuántica de campos en espacios curvos



PRODUCCIÓN TEMPRANA DE PARTÍCULAS

Precalentamiento

Etapa no perturbativa después del final de la inflación.

Consiste una producción eficiente de partículas que sruge por la amplificación de excitaciones impulsada por acoplamientos entre el inflatón y otros campos

PRODUCCIÓN TEMPRANA DE PARTÍCULAS