Photon propagation in a charged Bose–Einstein condensate model

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- Introduction
- Scalar BEC Model and SSB
- Electromagnetic Field interacting with charged scalar BEC
- Eigen Modes and Dispersion Relations
- Dispersion Relations under different conditions
- Summary and Outlook

- Particle propagation has interesting consequences in thermal backgrounds
- Extension of the Standard Model
- Additional scalar particle (Neutral and Charged) interacting particularly with Neutrino and Photons have implications in Nuclear Physics, Astrophysics, Cosmological Plasma, additional contribution to the effective potential.
- Propagation of fermion in a thermal background that contains a scalar BEC.
- Problem of Dark Matter can also be addressed through the self-interactiong scalar fields forming DM.
- In Condensed Matter Physics, study of photon propagation in atomic BEC
- Reduction of Light velocity (c) to 17 m/s
- Useful for Quantum Technology

Scalar BEC Model & SSB

The Lagrangian for this model is

$$d_\mu \equiv \partial_\mu - i\mu u_\mu \,,$$

$$L_\phi = (d^\mu \phi)^* (d_\mu \phi) - V_\phi \,,$$

$$V_\phi = m^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \,.$$

μ → Chemical Potential

Φ→ Charged Scalar

u_μ=(1,0), in the medium rest-frame

$v_\mu = \mu \, u_\mu$

$$L_\phi = (\partial^\mu \phi)^* (\partial_\mu \phi) + i[\phi^* (v \cdot \partial \phi) - (v \cdot \partial \phi^*)\phi] - U(\phi) \,,$$

$$U \equiv V_\phi - \mu^2 \phi^* \phi = -(\mu^2 - m^2)\phi^* \phi + \lambda (\phi^* \phi)^2 \,.$$

Φ=0 is the minimum

If $m^2 > \mu^2$, U corresponds to standard massive complex scalar field with mass $m^2 - \mu^2$.

Let us consider the case when a $\mu^2 > m^2$, Φ=0 is not the minimum, it develops a non-zero vacuum expectation value and U(1) symmetry is broken.

We can write

$$\phi = \frac{1}{\sqrt{2}}(\phi_0 + \phi_1 + i\phi_2) \ , \quad \text{with} \quad \langle\phi\rangle \equiv \frac{1}{\sqrt{2}}\phi_0 \ , \quad \phi_0^2 = \frac{\mu^2 - m^2}{\lambda} \ .$$

Then the Lagrangian density is given by

$$L_\phi = \frac{1}{2}\left[(\partial^\mu\phi_1)^2 + (\partial^\mu\phi_2)^2\right] + \phi_2 v\cdot\partial\phi_1 - \phi_1 v\cdot\partial\phi_2 - U(\phi)$$

$$U(\phi) = -\frac{1}{2}(\mu^2 - m^2)[(\phi_0+\phi_1)^2 + \phi_2^2] + \frac{1}{4}\lambda[(\phi_0+\phi_1)^2 + \phi_2^2]^2 \ .$$

Mass of ϕ_1

$$m_1^2 = 2(\mu^2 - m^2) \ ,$$

And ϕ_2 is massless

Mixed through v

To find the modes which have definite dispersion relations

Expressing as Matrix

$$\hat{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

Lagrangian density in momentum space will be

$$L_\phi^{(2)}(k) = \frac{1}{2}\hat{\phi}^*(k)\Delta_\phi^{-1}(k)\hat{\phi}(k),$$

where

$$\Delta_\phi^{-1}(k) = \begin{pmatrix} k^2 - m_1^2 & 2i v \cdot k \\ -2i v \cdot k & k^2 \end{pmatrix}.$$

Determinant (D) of Δ_ϕ^{-1},

D=0, gives the dispersion relation

$$\omega_\pm^2(\kappa) = \kappa^2 + \frac{1}{2}m_\rho^2 \pm \sqrt{\frac{1}{4}m_\rho^4 + 4\mu^2\kappa^2},$$

$$m_\rho^2 = m_1^2 + 4\mu^2,$$

$$\omega = u \cdot k,$$
$$\kappa = \sqrt{\omega^2 - k^2}.$$

$$\omega \neq \kappa,$$

In the rest frame of the medium

$$k^\mu = (\omega, \vec{\kappa}),$$

$$\omega_+(0) = m_\rho,$$
$$\omega_-(0) = 0.$$

Goldstone mode

The combinations of $\phi_{1,2}$ that have the definite dispersion relations $\omega_\pm$ (

EM field interacting with the charged scalar BEC

$$L = -\frac{1}{4}F^2 + L_{\phi A},$$ where $F_{\mu\nu}$ is the electromagnetic field tensor and

$$L_{\phi A} = (\hat{D}^\mu \phi)^* (\hat{D}_\mu \phi) - V_\phi, \qquad \hat{D}_\mu = d_\mu + iqA_\mu = \partial_\mu - i v_\mu + iqA_\mu,$$

Spectrum of the Model

To determine the spectrum of the model we adopt the unitary gauge, which is the convenient one to use for this purpose.

Thus we parametrize ϕ in the form

$$\phi = \frac{1}{\sqrt{2}}(\phi_0 + \rho)e^{i\theta/\phi_0}.$$

The field θ does not appear in V_ϕ, and by a gauge transformation it disappears also from the kinetic term. To be clear that we are employing the unitary gauge we will denote by V_μ the transformed vector potential

$$V_\mu = A_\mu + \frac{1}{q\phi_0}\partial_\mu\theta.$$

$$L_{\phi A} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(v - qV)^\mu(v - qV)_\mu(\phi_0 + \rho)^2 - V_\phi,$$

By expanding it

$$L_{\phi A} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(q\phi_0)^2V^2$$

$$+\frac{1}{2}q^2V^2\rho^2 + q^2\phi_0\rho V^2 - qv \cdot V(\phi_0 + \rho)^2 - U_\phi,$$

$$U_\phi = -\frac{1}{2}(\mu^2 - m^2)(\phi_0 + \rho)^2 + \frac{1}{4}(\phi_0 + \rho)^4.$$

Again we consider $\mu^2 > m^2$,

and choose ϕ_0 such that U_ϕ has the minimum at $\rho = 0$, thus,

$$\phi_0^2 = \frac{\mu^2 - m^2}{\lambda}.$$

Bilinear part of L (including the EM Tensor)

$$L^{(2)} = -\frac{1}{4}F^2 + \frac{1}{2}m_V^2 V^2 + \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}m_1^2\rho^2 - 2m_V(v \cdot V)\rho\,,$$

where

$$m_V = q\phi_0,$$
$$m_1^2 = 2(\mu^2 - m^2).$$

The physical picture that emerges is this: the field θ becomes the longitudinal component of V, and we end up with two fields, V and ρ. The longitudinal component of V is mixed with the scalar field ρ (the term $\rho v \cdot V$). As we show, the consequence is that the propagating modes with definite dispersion relations involve a superposition of the longitudinal component of V and the ρ.

Eigenmodes and the Dispersion relations

In momentum space different terms of $L^{(2)}$ have the form

$$-\frac{1}{4}F^2 \to -V^{*\mu}k^2\tilde{g}_{\mu\nu}V^\nu, \quad \frac{1}{2}(\partial\rho)^2 \to k^2\rho^*\rho, \quad \frac{1}{2}m_V^2V^2 \to m_V^2V^{*\mu}V_\mu,$$

$$\frac{1}{2}m_1^2\rho^2 \to m_1^2\rho^*\rho, \qquad (u\cdot V)\rho \to (u\cdot V)^*\rho + c.c,$$

and therefore

$$L^{(2)}(k) = -V^{*\mu}[k^2\tilde{g}_{\mu\nu} - m_V^2 g_{\mu\nu}]V^\nu + \rho^*(k^2 - m_1^2)\rho$$
$$-2\mu m_V[(u\cdot V)^*\rho + c.c],$$

where

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}.$$

Mixes V with ρ

We decompose V^μ in the form

$$V^\mu = V_T^\mu + V_L e_3^\mu + \frac{k^\mu}{\sqrt{k^2}} V_k,$$

where

$$V_T^\mu = \sum_{i=1,2} V_i e_i^\mu \qquad e_{1,2}^\mu = (0, \vec{e}_{1,2}), \quad e_3^\mu = \frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}},$$

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu = u_\mu - \frac{(k \cdot u) k_\mu}{k^2}, \qquad \sqrt{-\tilde{u}^2} = \frac{\kappa}{\sqrt{k^2}}.$$

Thus the quadratic part of the Lagrangian density is

Decoupled

No K.E., not a dynamical variable

$$L^{(2)}(k) = \boxed{-(k^2 - m_V^2) V_T^* \cdot V_T} + (k^2 - m_V^2) V_L^* V_L + m_V^2 V_k^* V_k + (k^2 - m_1^2)\rho^* \rho$$
$$+ 2\mu m_V \sqrt{-\tilde{u}^2} \left[V_L^* \rho + c.c. \right] - \frac{2\mu m_V (k \cdot u)}{\sqrt{k^2}} \left[V_k^* \rho + c.c \right].$$

Eliminate this term using Lagrange equation

$$m_V^2 V_k - \frac{2\mu m_V (k \cdot u)}{\sqrt{k^2}} \rho = 0, \quad \longrightarrow \quad V_k = \frac{2\mu (k \cdot u)}{m_V \sqrt{k^2}} \rho.$$

Finally we get

$$L^{(2)}(k) = -(k^2 - m_V^2)V_T^* \cdot V_T + (k^2 - m_V^2)V_L^* V_L + \left(k^2 - m_1^2 - \frac{4\mu^2(k\cdot u)^2}{k^2}\right)\rho^*\rho$$
$$+2\mu m_V\sqrt{-\tilde{u}^2}\left[V_L^*\rho + c.c.\right].$$

Equations of motion

$$(k^2 - m_V^2)V_L + 2\mu m_V\sqrt{-\tilde{u}^2}\rho = 0,$$
$$2\mu m_V\sqrt{-\tilde{u}^2}V_L + \left[k^2 - m_1^2 - \frac{4\mu^2(k\cdot u)^2}{k^2}\right]\rho = 0,$$

For $\kappa = 0$ V_L and ρ decouple

$$(\omega^2 - m_V^2)V_L = 0,$$
$$(\omega^2 - m_\rho^2)\rho = 0,$$

$$m_\rho^2 = m_1^2 + 4\mu^2.$$

For $\kappa \neq 0$ the dispersion relations are

$$\omega^2 = \kappa^2 + \frac{1}{2}(m_\rho^2 + m_V^2) \pm \left[\frac{1}{4}(m_\rho^2 - m_V^2)^2 + 4\mu^2\kappa^2\right]^{\frac{1}{2}}.$$

Interpreting the Dispersion Relations under different conditions

Final Dispersion relations

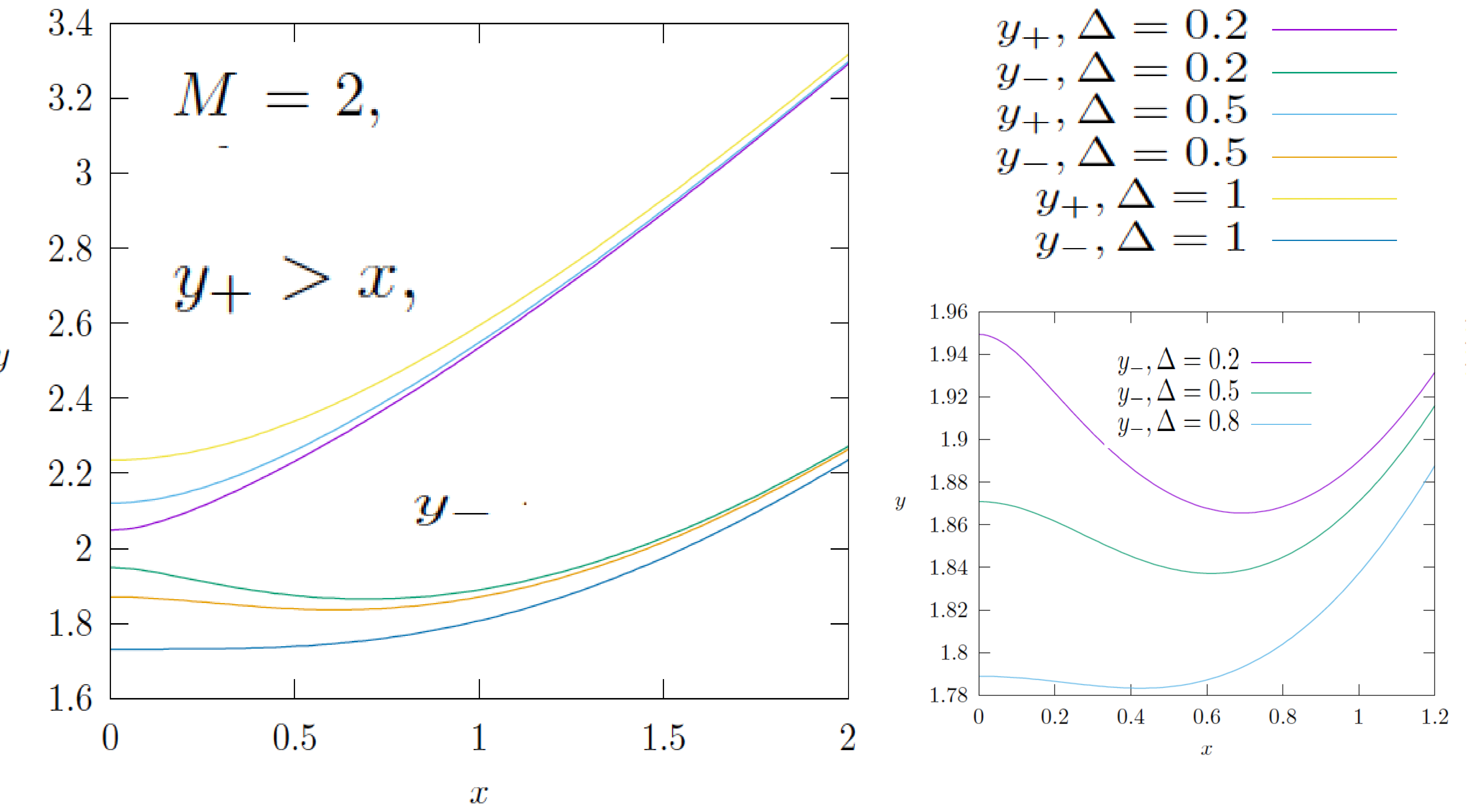
$$\omega^2 = \kappa^2 + \frac{1}{2}(m_\rho^2 + m_V^2) \pm \left[\frac{1}{4}(m_\rho^2 - m_V^2)^2 + 4\mu^2\kappa^2\right]^{\frac{1}{2}}.$$

This can be written in a normalized form and study under different conditions

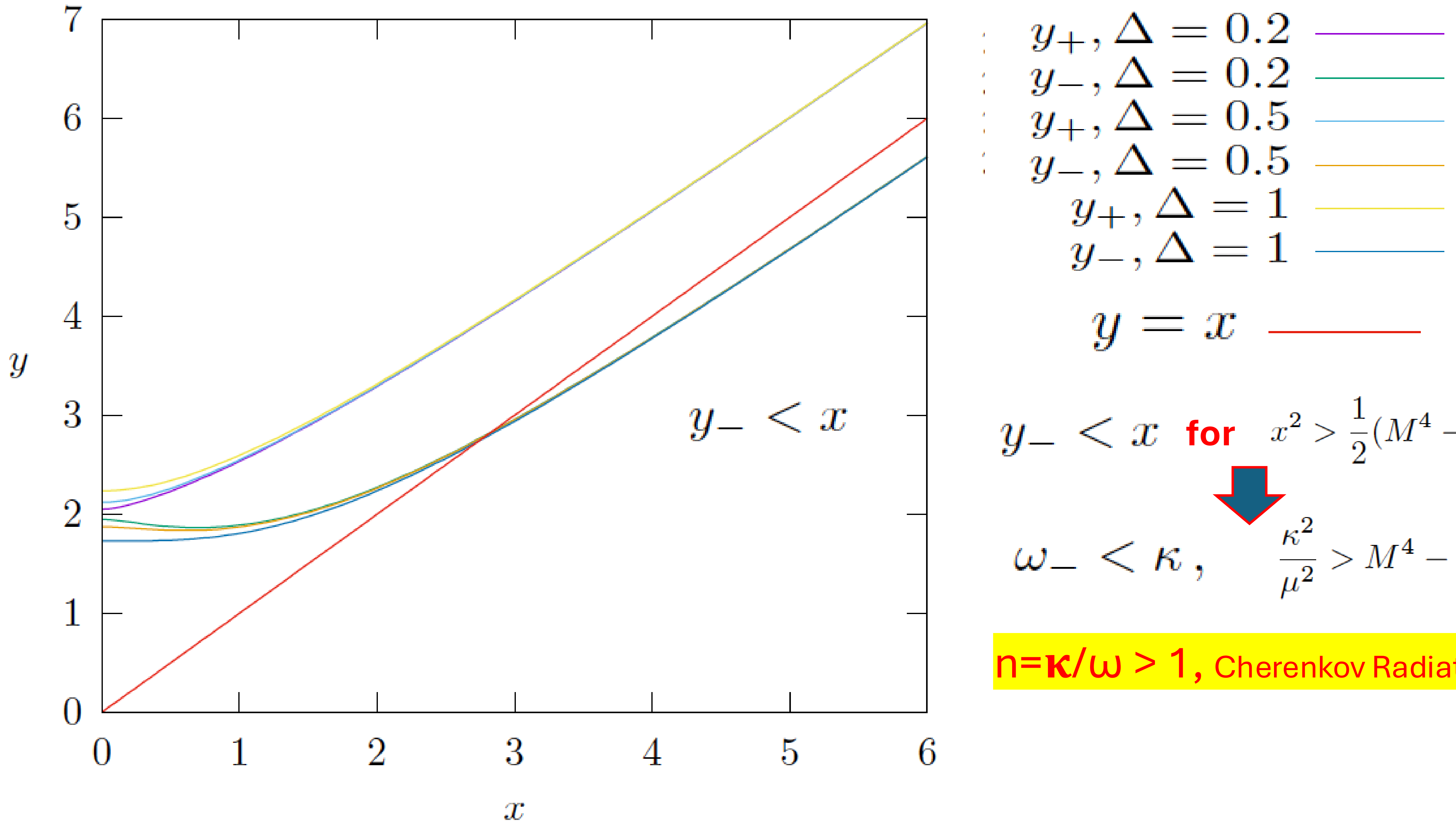
$$x^2 = \frac{\kappa^2}{2\mu^2}, \qquad M^2 = \frac{m_+^2 + m_V^2}{4\mu^2}, \qquad \Delta < M^2,$$

$$y^2 = \frac{\omega^2}{2\mu^2}, \qquad \Delta = \frac{|m_+^2 - m_V^2|}{4\mu^2}. \qquad M^2 > 1.$$

$$y_\pm^2 = x^2 + M^2 \pm \sqrt{\Delta^2 + 2x^2}.$$



M = 2,
$y_+ > x$,
y_-
$y_+, \Delta = 0.2$
$y_-, \Delta = 0.2$
$y_+, \Delta = 0.5$
$y_-, \Delta = 0.5$
$y_+, \Delta = 1$
$y_-, \Delta = 1$
$y_-, \Delta = 0.2$
$y_-, \Delta = 0.5$
$y_-, \Delta = 0.8$
x
y



$y_- < x$ **for** $x^2 > \frac{1}{2}(M^4 - \Delta^2)\,.$

$$\omega_- < \kappa\,, \qquad \frac{\kappa^2}{\mu^2} > M^4 - \Delta^2\,,$$

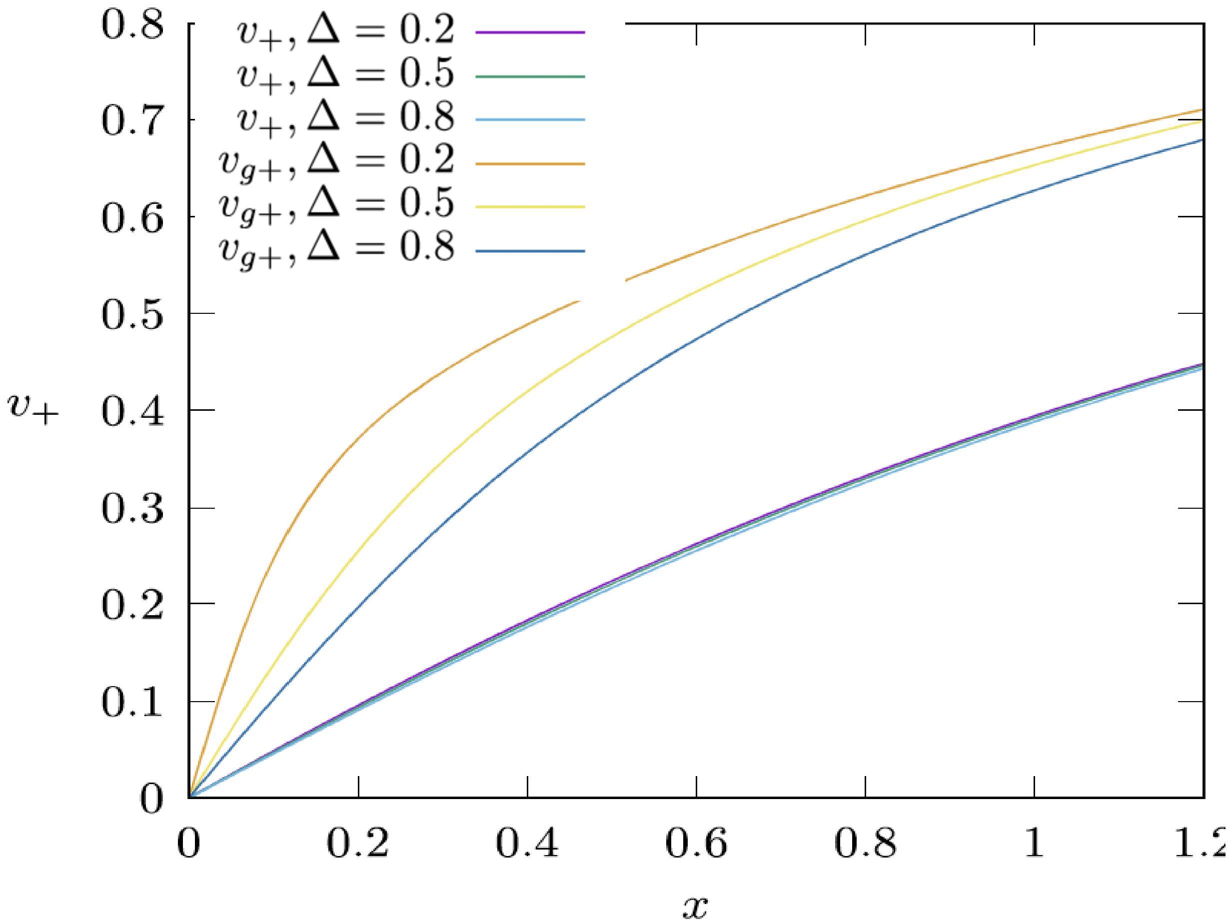
n=κ/ω > 1, Cherenkov Radiation

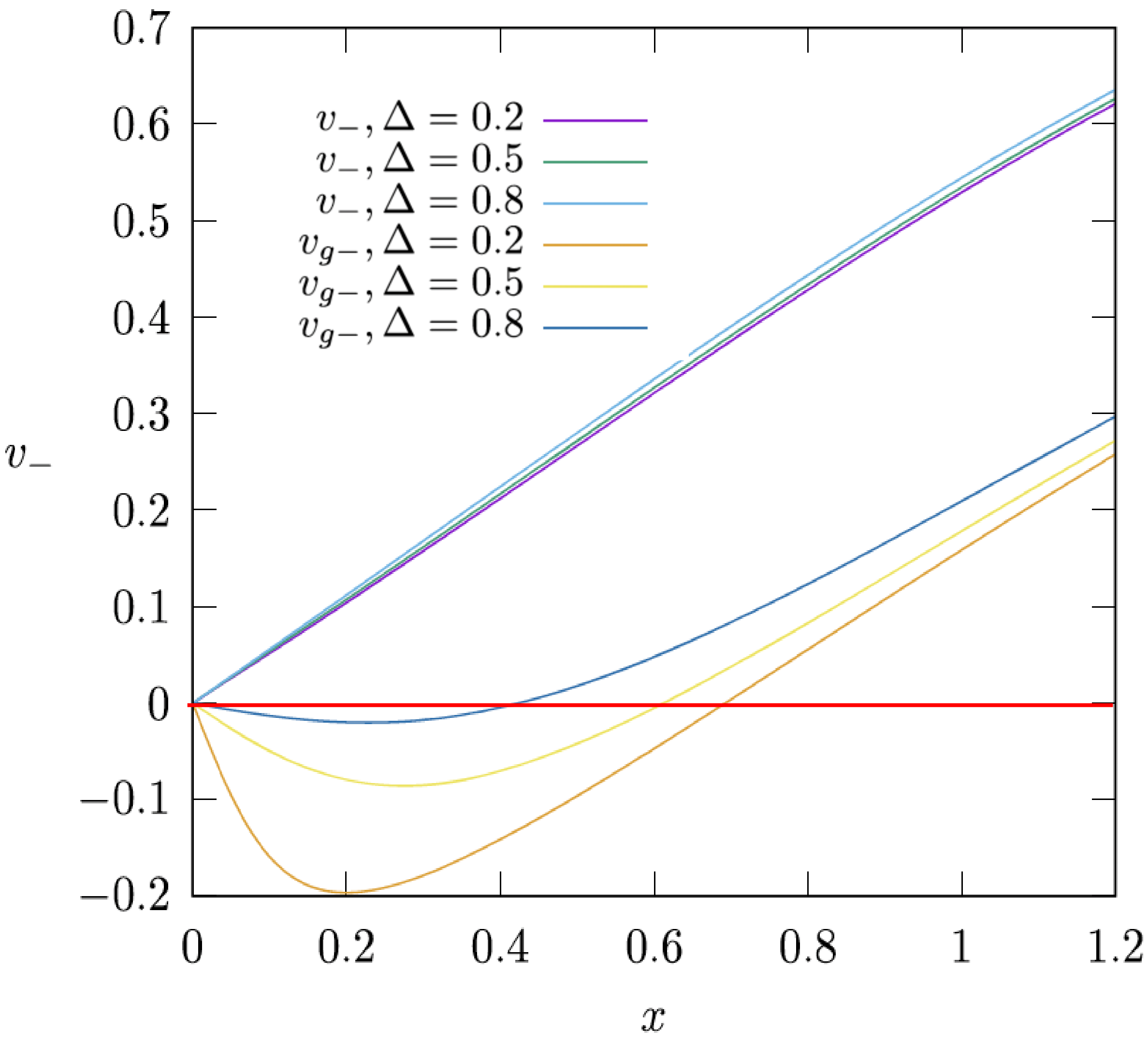
The group velocity for each branch is

Phase velocity

$$v_\pm \equiv \kappa/\omega_\pm$$

$$v_g = \frac{\partial\omega}{\partial\kappa} = \frac{x}{y}\left[1 \pm \frac{1}{\sqrt{\Delta^2 + 2x^2}}\right].$$





Summary & Outlook

- Studied Photon propagation in a BEC of charged scalar.
- Derived the dispersion relations for the Collective Modes in BEC
- For Photons: Two modes are usual → Transverse Photons
- Additional Two modes (+) and (--) are combination of Longitudianl Photon and massive Scalar field.
- Their dispersion relations are very different.
- The (+) increases steadly as momentum increases and always $\omega > \kappa$
- The(--) has very interesting feature: Slowing down of Light in BEC, Cherenkov Radiation
- Nontrivial Optical properties

Thank You