Non minimal $SU(5)$ modular flavor model

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Antonio Carlos Samaniego Flores

Institute of physics
Universidad Nacional Autónoma de México

Instituto de Física
UNAM

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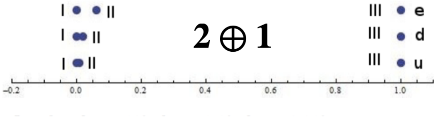
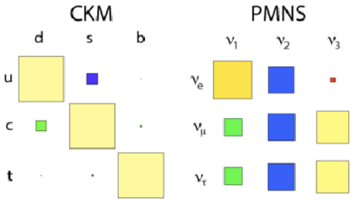
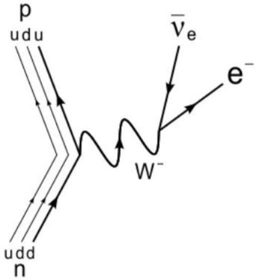
Content

Introduction



$\Delta m^2_{21} \approx 7.49 \times 10^{-5}\ \mathrm{eV}^2,\quad \Delta m^2_{32} \approx 2.51 \times 10^{-3}\ \mathrm{eV}^2.$





The goal is to find a consistent theory of flavor!

$$\text{SUSY}\ \ SU(5) \times \text{Modular} - S_3\ \ \&\ \ \text{6HDM}\ \ +\ \ \text{RHNs}$$

Why do we use discrete symmetries?

- We don’t have to deal with Goldstone bosons.
- It is possible to generate natural hierarchical structures.
- It is possible to construct more predictive models by reducing the parameter space.

Why make it modular?

- We don’t need to introduce additional mechanisms like flavon fields.
- Modular symmetries appear naturally in more fundamental theories such as string theory.
- It is possible to obtain a richer and more elegant mathematical formulation due to the geometrical origin.
- Some modular groups are related with discrete symmetry groups.

Flavor symmetry

The modular group, $\Gamma = SL(2,\mathbb{Z})$, can be defined as

$$\gamma(\tau) \to \frac{a\tau + b}{c\tau + d}, \quad \tau \in \mathbb{H}, \quad T : \tau \to \tau + 1, \quad S : \tau \to -\frac{1}{\tau}.$$

Modular forms of weight k are functions that transform as

$$Y(\tau) \to (c\tau + d)^k \rho_{\mathrm{N}}(\gamma) Y(\tau); \quad \rho_{\mathrm{N}}(\gamma) \in \Gamma_{\mathrm{N}}.$$

Finite modular groups are related with discrete symmetries:
$\Gamma_{\mathrm{N}} \simeq S_3, A_4, S_4, A_5$ for $N = 2, \ldots, 5$ respectively. S_3 has three irreps $\mathbf{2}$, $\mathbf{1}$, $\mathbf{1}'$.

We will assume that fields are transformed as modular forms (Feruglio 2017)

$$\Phi \to (c\tau + d)^{k_\Phi} \rho_\Phi(\gamma) \Phi.$$

In order to have modular invariance, the Yukawa couplings must be modular forms of certain fixed modular weight.

Model building

The matter content is included in the irreducible representations of the gauge group as follows (T. Kobayashi et. al. 2020):

$$F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L , \quad H^u = \begin{pmatrix} \mathbf{H}_1^u \\ \mathbf{H}_2^u \\ \mathbf{H}_3^u \\ h^{+u} \\ h^{0u} \end{pmatrix} , \quad T = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_L$$

Field	$SU(5)$	$\Gamma_2 \simeq S_3$	k	Field	$SU(5)$	$\Gamma_2 \simeq S_3$	k
(H_1^d, H_2^d)	$\bar{\mathbf{5}}$	2	0	(T_1, T_2)	$\mathbf{10}$	2	-2
H_3^d	$\bar{\mathbf{5}}$	1	0	T_3	$\mathbf{10}$	$1'$	0
(H_1^u, H_2^u)	$\mathbf{5}$	2	0	$Y_{\mathbf{2}}^{(2)}(\tau)$	$\mathbf{1}$	2	2
H_3^u	$\mathbf{5}$	1	0	$Y_{\mathbf{2}}^{(4)}(\tau)$	$\mathbf{1}$	2	4
(F_1, F_2)	$\bar{\mathbf{5}}$	2	-2	$Y_{\mathbf{1}}^{(4)}(\tau)$	$\mathbf{1}$	1	4
F_3	$\bar{\mathbf{5}}$	$1'$	0	$H_{\overline{45}}$	$\overline{\mathbf{45}}$	1	0
(N_1^c, N_2^c)	$\mathbf{1}$	2	-2	H_{45}	$\mathbf{45}$	1	0
N_3^c	$\mathbf{1}$	$1'$	0				

In GUT models, the relation $\mathbf{Y}_e = \mathbf{Y}_d^T$ arises. The Yukawa potential is given by

$$W \supseteq W_u + W_{d,l} + W_\nu ,$$

for the up-type quark sector, we have

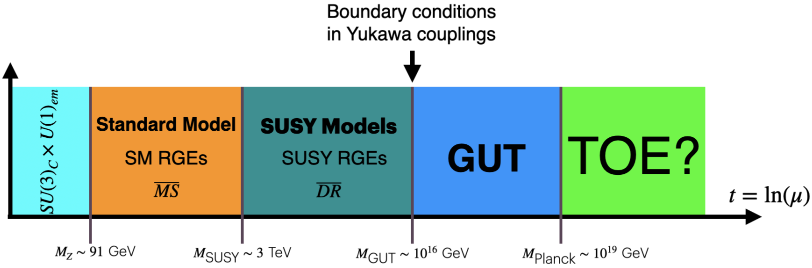
$$W_u \supseteq \frac{1}{4}y_1^u Y_{\mathbf{1}}^{(4)}\left(\bar{T}_1T_1+\bar{T}_2T_2\right)H_3^u+\frac{1}{4}y_2^u\left[Y_b\left(\bar{T}_1T_2+\bar{T}_2T_1\right)+Y_c\left(\bar{T}_1T_1-\bar{T}_2T_2\right)\right]H_3^u$$
$$+\frac{1}{4}y_3^u\left[\left(Y_1\bar{T}_1T_3-Y_2\bar{T}_2T_3\right)H_1^u-\left(Y_1\bar{T}_2T_3+Y_2\bar{T}_1T_3\right)H_2^u\right]+\frac{1}{4}y_4^u\left[\left(-Y_2\bar{T}_3T_2+Y_1\bar{T}_3T_1\right)H_1^u\right.$$
$$\left.-\left(Y_2\bar{T}_3T_1+Y_1\bar{T}_3T_2\right)H_2^u\right]+\frac{1}{4}y_5^u\bar{T}_3T_3H_3^u+\frac{1}{4}\tilde{\mathcal{Y}}_1Y_{\mathbf{1}}^{(4)}\left(\bar{T}_1T_1+\bar{T}_2T_2\right)H_{45}$$
$$+\frac{1}{4}\tilde{\mathcal{Y}}_2\left[Y_b\left(\bar{T}_1T_2+\bar{T}_2T_1\right)+Y_c\left(\bar{T}_1T_1-\bar{T}_2T_2\right)\right]H_{45}+\frac{1}{4}\tilde{\mathcal{Y}}_3\bar{T}_3T_3H_{45},$$

for down-type quark sector, charged leptons and neutrinos

$$W_{d,l} \supseteq \sqrt{2}y_1^d Y_{\mathbf{1}}^{(4)}\left(\bar{F}_1T_1+\bar{F}_2T_2\right)H_3^d+\sqrt{2}y_2^d\left[Y_b\left(\bar{F}_1T_2+\bar{F}_2T_1\right)+Y_c\left(\bar{F}_1T_1-\bar{F}_2T_2\right)\right]H_3^d$$
$$+\sqrt{2}y_3^d\left[\left(Y_1\bar{F}_1T_3-Y_2\bar{F}_2T_3\right)H_1^d-\left(Y_1\bar{F}_2T_3+Y_2\bar{F}_1T_3\right)H_2^d\right]+\sqrt{2}y_4^d\left[\left(-Y_2\bar{F}_3T_2+Y_1\bar{F}_3T_1\right)H_1^d\right.$$
$$\left.-\left(Y_2\bar{F}_3T_1+Y_1\bar{F}_3T_2\right)H_2^d\right]+\sqrt{2}y_5^d\bar{F}_3T_3H_3^d+\sqrt{2}\mathcal{Y}_1Y_{\mathbf{1}}^{(4)}\left(\bar{F}_1T_1+\bar{F}_2T_2\right)H_{\overline{45}}$$
$$+\sqrt{2}\mathcal{Y}_2\left[Y_b\left(\bar{F}_1T_2+\bar{F}_2T_1\right)+Y_c\left(\bar{F}_1T_1-\bar{F}_2T_2\right)\right]H_{\overline{45}}+\sqrt{2}\mathcal{Y}_3\bar{F}_3T_3H_{\overline{45}},$$

$$W_\nu \supseteq y_1^n\left[\left(Y_b\bar{N}_2^c+Y_c\bar{N}_1^c\right)F_1+\left(Y_b\bar{N}_1^c-Y_c\bar{N}_2^c\right)F_2\right]H_3^u+y_2^nY_a\left(\bar{N}_1^cF_1+\bar{N}_2^cF_2\right)H_3^u$$
$$+y_3^n\left[-\left(Y_1\bar{N}_1^c-Y_2\bar{N}_2^c\right)F_3H_1^u+\left(Y_1\bar{N}_2^c+Y_2\bar{N}_1^c\right)F_3H_2^u\right]+y_4^n\left[-Y_2\bar{N}_3^c\left(F_1H_2^u+F_2H_1^u\right)\right.$$
$$\left.+Y_1\bar{N}_3^c\left(F_1H_1^u-F_2H_2^u\right)\right]+y_5^n\bar{N}_3^cF_3H_3^u+\frac{1}{2}\mathcal{Y}_\nu Y_{\mathbf{1}}^{(4)}\left(\bar{N}_1^cN_1^c+\bar{N}_2^cN_2^c\right)+\frac{1}{2}M_3\bar{N}_3^cN^c.$$

RGE evolution



From the potential, we construct the Yukawa matrices

$$W \supseteq \bar{u}_R \mathbf{Y}_i^u Q_L \mathcal{H}_i^u + \bar{d}_R \mathbf{Y}_i^d Q_L \mathcal{H}_i^d + \bar{e}_R \mathbf{Y}_i^e L_L \mathcal{H}_i^d + \bar{N}_R \mathbf{Y}_i^\nu L_L \mathcal{H}_i^u \quad i = 1, 2, 3.$$
$$+ \frac{1}{2} \bar{N}^c M_R N^c + 2\bar{d}_R \mathbf{Y}_{\bar{45}} Q_L \mathcal{H}_{\bar{45}} - 2\bar{e}_R \mathbf{Y}_{\bar{45}} L_L \mathcal{H}_{\bar{45}}.$$

If we make a rotation between the basis

$$\mathcal{H}_i^k = \frac{h_i^{0k}}{v_k} \mathcal{H}^k, \quad v_k = \sqrt{\sum_i (h_i^{0k})^2}, \quad k = u, d,$$

it is possible to establish the relation

$$\mathbf{Y}_u^{\mathrm{MSSM}} = \sum_i \frac{h_i^{0u}}{v_u} \mathbf{Y}_i^u, \qquad \mathbf{Y}_d^{\mathrm{MSSM}} = \cos\Omega \sum_i \frac{h_i^{0d}}{v_d} \mathbf{Y}_i^d + 2\sin\Omega \mathbf{Y}_{\bar{45}},$$
$$\mathbf{Y}_\nu^{\mathrm{MSSM}} = \sum_i \frac{h_i^{0u}}{v_u} \mathbf{Y}_i^\nu, \qquad \mathbf{Y}_e^{\mathrm{MSSM}} = \cos\Omega \sum_i \frac{h_i^{0d}}{v_d} \left(\mathbf{Y}_i^d\right)^T - 2\sin\Omega \mathbf{Y}_{\bar{45}},$$

with Ω, the mixing angle between the Higgs doublets content in $\bar{\mathbf{5}}_i$ and $\overline{\mathbf{45}}$. It connects the model with the MSSM in M_{GUT}. The potential takes the form

$$W \supseteq \bar{u}_R \mathbf{Y}_u^{\mathrm{MSSM}} Q_L \mathcal{H}^u + \bar{d}_R \mathbf{Y}_d^{\mathrm{MSSM}} Q_L \mathcal{H}_l^d + \bar{e}_R \mathbf{Y}_e^{\mathrm{MSSM}} L_L \mathcal{H}_l^d + \bar{N}_R \mathbf{Y}_\nu^{\mathrm{MSSM}} L_L \mathcal{H}^u + \frac{1}{2} \bar{N}_R M_R N_R.$$

The Yukawa matrices for quarks and charged leptons are given by

$$\mathbf{Y}_u^{\mathrm{MSSM}} = \begin{pmatrix} Y_a\mu_1^u + Y_c\mu_2^u & Y_b\mu_2^u & Y_1\bar{\mu}_1^u - Y_2\bar{\mu}_2^u \\ Y_b\mu_2^u & Y_a\mu_1^u - Y_c\mu_2^u & -Y_1\bar{\mu}_2^u - Y_2\bar{\mu}_1^u \\ Y_1\bar{\mu}_1^u - Y_2\bar{\mu}_2^u & -Y_1\bar{\mu}_2^u - Y_2\bar{\mu}_1^u & \mu_5^u \end{pmatrix},$$

$$\begin{aligned}
\mathbf{Y}_d^{\mathrm{MSSM}} &= \cos\Omega \begin{pmatrix} Y_a\mu_1^d + Y_c\mu_2^d & Y_b\mu_2^d & Y_1\mu_3^d - Y_2\mu_4^d \\ Y_b\mu_2^d & Y_a\mu_1^d - Y_c\mu_2^d & -Y_2\mu_3^d - Y_1\mu_4^d \\ Y_1\mu_5^d - Y_2\mu_6^d & -Y_2\mu_5^d - Y_1\mu_6^d & \mu_7^d \end{pmatrix} \\
&\quad +2\sin\Omega \begin{pmatrix} Y_a\mathcal{Y}_1 + Y_c\mathcal{Y}_2 & Y_b\mathcal{Y}_2 & 0 \\ Y_b\mathcal{Y}_2 & Y_a\mathcal{Y}_1 - Y_c\mathcal{Y}_2 & 0 \\ 0 & 0 & \mathcal{Y}_3 \end{pmatrix}, \\
\mathbf{Y}_e^{\mathrm{MSSM}} &= \cos\Omega \begin{pmatrix} Y_a\mu_1^d + Y_c\mu_2^d & Y_b\mu_2^d & Y_1\mu_5^d - Y_2\mu_6^d \\ Y_b\mu_2^d & Y_a\mu_1^d - Y_c\mu_2^d & -Y_2\mu_5^d - Y_1\mu_6^d \\ Y_1\mu_3^d - Y_2\mu_4^d & -Y_2\mu_3^d - Y_1\mu_4^d & \mu_7^d \end{pmatrix} \\
&\quad -2\sin\Omega \begin{pmatrix} Y_a\mathcal{Y}_1 + Y_c\mathcal{Y}_2 & Y_b\mathcal{Y}_2 & 0 \\ Y_b\mathcal{Y}_2 & Y_a\mathcal{Y}_1 - Y_c\mathcal{Y}_2 & 0 \\ 0 & 0 & \mathcal{Y}_3 \end{pmatrix}.
\end{aligned}$$

For neutrinos, we have the following matrices

$$\mathbf{Y}_\nu^{\mathrm{MSSM}} = \begin{pmatrix} Y_c\mu_1^n + Y_a\mu_2^n & Y_b\mu_1^n & Y_2\mu_4^n - Y_1\mu_3^n \\ Y_b\mu_1^n & Y_a\mu_2^n - Y_c\mu_1^n & Y_2\mu_3^n + Y_1\mu_4^n \\ Y_1\mu_5^n - Y_2\mu_6^n & -Y_2\mu_5^n - Y_1\mu_6^n & \mu_7^n \end{pmatrix}, \qquad M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M_3 \end{pmatrix}.$$

Matching and SUSY TC

The RGE are given by

$$\frac{d\mathbf{Y}_k^{\text{MSSM}}}{dt} = \mathbf{Y}_k^{\text{MSSM}} \left(\frac{1}{16\pi^2} \beta_k^{(1)} + \frac{1}{(16\pi^2)^2} \beta_k^{(2)} \right), \quad k = u, d, e, \nu,$$

$$\frac{d}{dt}\kappa = \kappa \left(\frac{1}{16\pi^2} \beta_\kappa^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\kappa^{(2)} \right), \quad \frac{d}{dt}M = M \left(\frac{1}{16\pi^2} \beta_M^{(1)} + \frac{1}{(16\pi^2)^2} \beta_M^{(2)} \right).$$

With β_k corresponding to the MSSM (Castano, Piard, and Ramond 1994; Antusch, Kersten, Lindner, Ratz, and Schmidt 2005). When SUSY is broken, it is important to take the corrections into account. The corrections to the Yukawa matrices, in the basis where $\mathbf{Y}_u$ is diagonal, look like (Antusch and Maurer 2013)

$$\mathbf{Y}_u^{\text{SM}} \simeq \mathbf{Y}_u^{\text{MSSM}} \sin\bar{\beta}, \qquad \mathbf{Y}_d^{\text{SM}} \simeq (1 + \text{diag}(\bar{\eta}_q, \bar{\eta}_q, \bar{\eta}_b))\mathbf{Y}_d^{\text{MSSM}} \cos\bar{\beta},$$

$$\mathbf{Y}_e^{\text{SM}} \simeq (1 + \text{diag}(\bar{\eta}_l, \bar{\eta}_l, 1)\mathbf{Y}_e^{\text{MSSM}} \cos\bar{\beta}, \quad \mathbf{Y}_\nu^{\text{SM}} \simeq (1 + \text{diag}(\bar{\eta}_\nu, \bar{\eta}_\nu, 1))\mathbf{Y}_\nu^{\text{MSSM}} \cos\bar{\beta}.$$

With the objective of performing the running, the package REAP V.1.11.5 was used (Antusch, Kersten, Lindner, Ratz, and Schmidt 2005). As initial conditions, in M_{GUT} we start by studying simpler cases at the symmetry point $\tau = i$ and taking into account the data of (M. C. Gonzalez-Garcia et. al. 2024)

$$M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV}, \quad M_{\text{SUSY}} = 3 \text{ TeV}, \quad \tan\bar{\beta} = 5, \quad \Omega = \pi/4$$

$$\mathbf{Y}_u^{\text{MSSM}} \approx \begin{pmatrix} \hat{\mu}_1^u & 0 & 0 \\ 0 & \hat{\mu}_1^u & 0 \\ 0 & 0 & \hat{\mu}_5^u \end{pmatrix}, \quad \mathbf{Y}_\nu^{\text{MSSM}} \approx \begin{pmatrix} \hat{\mu}_2^n & 0 & 0 \\ 0 & \hat{\mu}_2^n & 0 \\ 0 & 0 & \hat{\mu}_7^n \end{pmatrix},$$

in addition

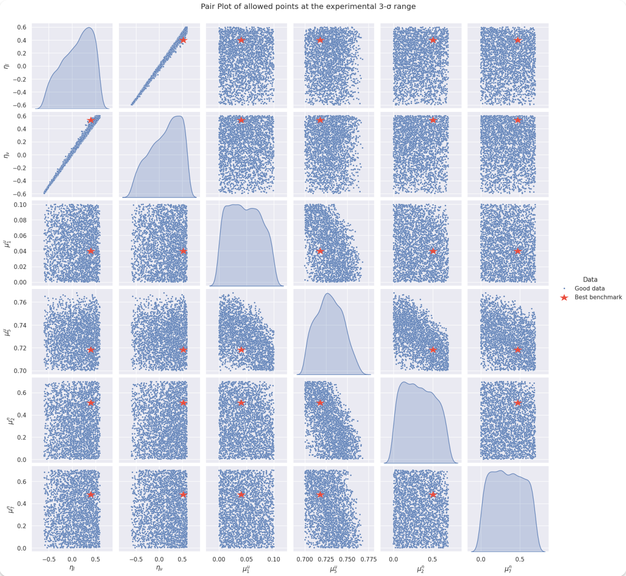
$$\mathbf{Y}_d^{\text{MSSM}} \approx \frac{\sqrt{2}}{2} \begin{pmatrix} \hat{\mu}_1^d + 2\hat{\mathcal{Y}}_1 & c_1 & c_2 \\ c_1 & \hat{\mu}_1^d + 2\hat{\mathcal{Y}}_1 & c_3 \\ c_4 & c_5 & \hat{\mu}_7^d \end{pmatrix}, \quad \mathbf{Y}_e^{\text{MSSM}} \approx \frac{\sqrt{2}}{2} \begin{pmatrix} \hat{\mu}_1^d - 2\hat{\mathcal{Y}}_1 & c_1 & c_4 \\ c_1 & \hat{\mu}_1^d - 2\hat{\mathcal{Y}}_1 & c_5 \\ c_2 & c_3 & \hat{\mu}_7^d \end{pmatrix},$$

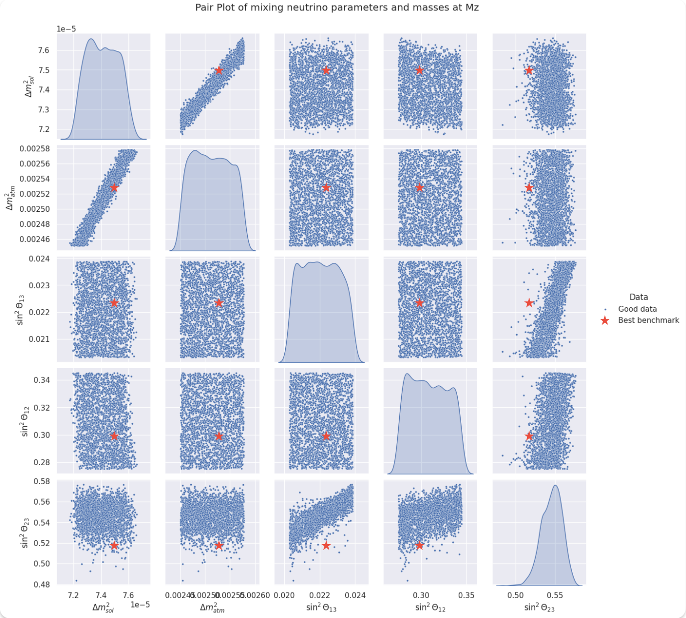
where

$$c_1 = 0.1558 \times 10^{-3} + 0.22 \times 10^{-5} i, \quad c_2 = 0.2393 \times 10^{-4} + 0.6778 \times 10^{-4} i,$$
$$c_3 = 0.8913 \times 10^{-3} - 0.4 \times 10^{-6} i, \quad c_4 = 0.239 \times 10^{-4} - 0.6775 \times 10^{-4} i,$$
$$c_5 = 0.8915 \times 10^{-3} + 0.4 \times 10^{-6} i.$$

Some results

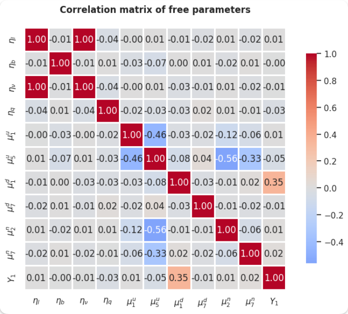
By performing a random scan using REAP with the initial conditions

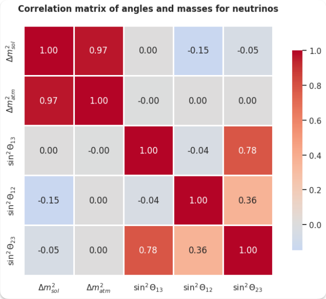




Pair Plot of mixing neutrino parameters and masses at Mz
1e−5
Δm^2_{sol}
Δm^2_{atm}
$\sin^2\Theta_{13}$
$\sin^2\Theta_{12}$
$\sin^2\Theta_{23}$
Data
Good data
Best benchmark

In quantitative terms, the possible correlations between the free parameters can be appreciated in the following correlation matrices



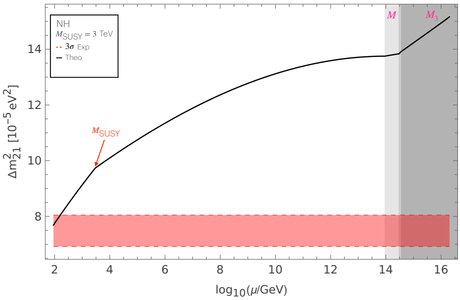


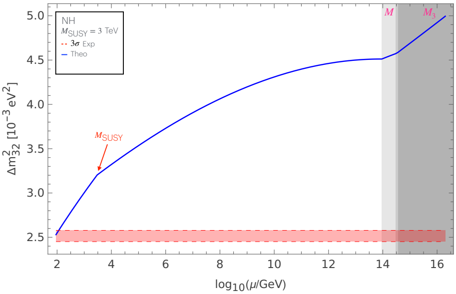
Conclusions and perspectives

- The flavor symmetry plays an important role in the structure and quantity of the terms in the potential.
- In modular models, an additional mechanism is not necessary. It reduces the number of free parameters, maintaining simplicity and thus increasing predictive power.
- According to the partial scan of the parameter space, it is possible to establish some relation between the free parameters. It will be helpful to reduce the parameter space.
- Exploring methods to reduce the parameter space by using the underlying properties of the symmetries.
- Extending the analysis to the SSB sector will require some dynamical mechanism to break SUSY (Gravity, AMSB?).

References

1. Feruglio F. Are neutrino masses modular forms? In: *From My Vast Repertoire ...: Guido Altarelli's Legacy*. Ed. by Levy A, Forte S, and Ridolfi G. 2017:227–66. DOI: 10.1142/9789813238053_0012. arXiv: 1706.08749 [hep-ph].
2. T. Kobayashi et. al. Modular S_3-invariant flavor model in SU(5) grand unified theory. PTEP 2020;2020:053B05.
3. Castano DJ, Piard EJ, and Ramond P. Renormalization group study of the Standard Model and its extensions. 2. The Minimal supersymmetric Standard Model. Phys. Rev. D 1994;49:4882–901.
4. Antusch S, Kersten J, Lindner M, Ratz M, and Schmidt MA. Running neutrino mass parameters in see-saw scenarios. JHEP 2005;03:024.
5. Antusch S and Maurer V. Running quark and lepton parameters at various scales. JHEP 2013;11:115.
6. M. C. Gonzalez-Garcia et. al. NuFit-6.0: Updated global analysis of three-flavor neutrino oscillations. JHEP 2024;12:arXiv:2410.05380.
7. Blazek T, Raby S, and Pokorski S. Finite supersymmetric threshold corrections to CKM matrix elements in the large tan Beta regime. Phys. Rev. D 1995;52:4151–8.


NH
$M_{SUSY} = 3$ TeV
3σ Exp
Theo
M
M_3
M_{SUSY}
Δm^2_{21} [10^{-5} eV^2]
$\log_{10}(\mu/\mathrm{GeV})$


NH
$M_{SUSY} = 3$ TeV
3σ Exp
Theo
M
M_3
M_{SUSY}
Δm^2_{32} [10^{-3} eV^2]
$\log_{10}(\mu/\mathrm{GeV})$

Grupo modular:

El grupo modular se define como

$$\Gamma = SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} |a,b,c,d \in \mathbb{Z}, \mathrm{ad} - \mathrm{bc} = 1 \right\} \text{ ó } \gamma(\tau) \to \frac{a\tau + b}{c\tau + d},$$

con $\tau \in \mathbb{C}; Im(\tau) > 0$. Los generadores satisfacen $S^2 = 1$, $(ST)^3 = 1$ y son dados por

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Esto está asociado a las transformaciones

$$T : \tau \to \tau + 1, \qquad S : \tau \to -\frac{1}{\tau}.$$

Se define al grupo modular finito como

$$\Gamma_{\mathrm{N}} = \bar{\Gamma}/\bar{\Gamma}(N), \text{ con } \bar{\Gamma}, \bar{\Gamma}(N) = \Gamma, \Gamma(N)/\{1_{2\times 2}, -1_{2\times 2}\},$$

cuyos generadores satisfacen que $S^2 = (ST)^3 = T^N = 1$. En donde $N \in \mathbb{N}$ y se conoce como nivel, además de que

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod\ N}) \right\}.$$

Se puede mostrar que hay una representación, $\rho_{\mathrm{N}}(\gamma)$, de Γ_{N} tal que para $\mathrm{N} = 2$, $a = \rho_{\mathrm{N}}(T)$ y $b = \rho_{\mathrm{N}}(S)$, entonces $\Gamma_2 \simeq S_3$.

Formas Modulares

Existen ciertas funciones de τ las cuales, bajo Γ_{N}, transforman como

$$f(\tau) \to (c\tau + d)^k \rho_{\mathrm{N}}(\gamma) f(\tau),$$

con k un número entero, positivo y par. Una forma modular de peso dos es la siguiente

$$Y_{\mathbf{2}}^{(2)} = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}, \quad \text{en donde} \quad Y_1(\tau) = \frac{\sqrt{3}i}{2\pi} Y(1,-1,0|\tau),\; Y_2(\tau) = \frac{i}{2\pi} Y(1,1,-2|\tau).$$

Además de que

$$Y(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}|\tau) = \frac{d}{d\tau}\left(\tilde{\alpha} \log \eta\left(\frac{\tau}{2}\right) + \tilde{\beta} \log \eta\left(\frac{\tau+1}{2}\right) + \tilde{\gamma} \log \eta(2\tau)\right),$$

con $\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} = 0$ y $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$, $q = \mathrm{e}^{2\pi i \tau}$. Adicionalmente, se puede obtener que

$$Y_{\mathbf{1}}^{(4)} = Y_1^2 + Y_2^2, \quad Y_{\mathbf{2}}^{(4)} = \begin{pmatrix} Y_1^{(4)}(\tau) \\ Y_2^{(4)}(\tau) \end{pmatrix}, \quad Y_1^{(4)} = 2Y_1 Y_2, \quad Y_2^{(4)} = Y_1^2 - Y_2^2.$$

S_3 tiene tres irreps $\mathbf{2}$, $\mathbf{1}$, $\mathbf{1}'$ y operan como (Kronecker products)

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}, \qquad \mathbf{2} \otimes \mathbf{1}' = \mathbf{2}, \qquad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}, \qquad \mathbf{1} \otimes \mathbf{1}' = \mathbf{1}'.$$

$\{1, T, TS, ST^{-1}S, S, ST, STS, T^{-1}S, T^{-1}\}$ of the group G.

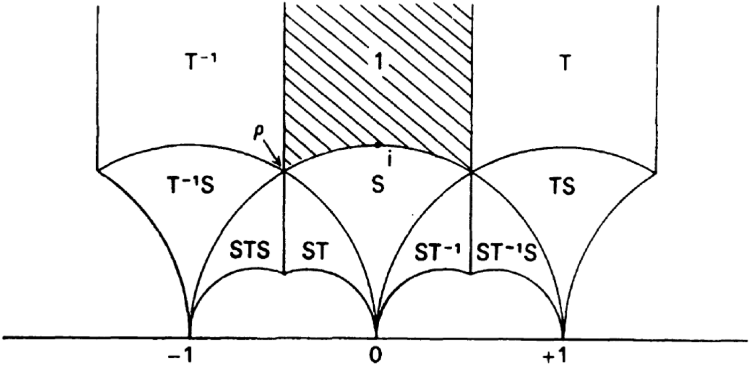
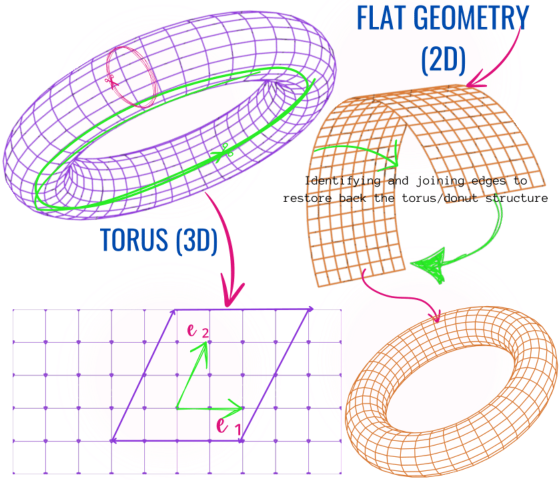
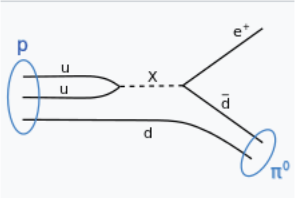


Figure 1: Fundamental Domain D of $\mathcal{H}$ under action of $\mathrm{SL}_2(\mathbb{Z})$.


FLAT GEOMETRY
(2D)
Identifying and joining edges to
restore back the torus/donut structure
TORUS (3D)
e 2
e 1

Proton decay

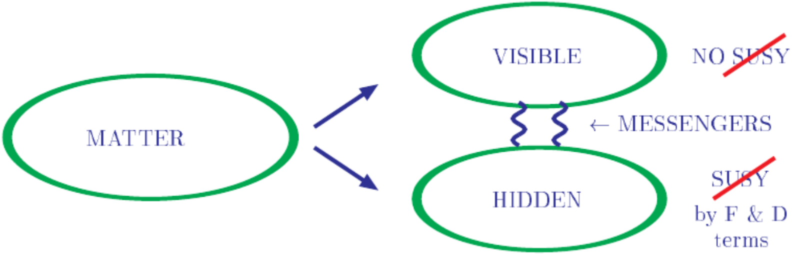
Se presenta la fuente más común del decaimiento del protón en $SU(5)$[1]. Dos quarks up se aniquilan dando lugar a un bosón X^+, el cual decae a un positrón y un antiquark down.



En donde el tiempo de vida media es

$$\tau_p \approx \frac{M_X^4}{\alpha_5^2 m_p^5} > 5 \times 10^{32} \text{ yrs} \Rightarrow M_X \geq 10^{15} \text{ GeV}.$$

[1]W. de Boer, "Grand unified theories and supersymmetry in particle physics and cosmology," Prog. Part. Nucl. Phys. **33** (1994), 201-302 doi:10.1016/0146-6410(94)90045-0 [arXiv:hep-ph/9402266 [hep-ph]].



- Mediación gravitacional
- Mediación a través de interacciones de norma
- Mediación por efectos de anomalía conforme

Sector de rompimiento suave de SUSY

Una teoría con $\mathcal{N} = 1$ Supersimetría global tiene la acción

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \int d^4x d^2\theta\, W(\Phi) + \text{h.c.}$$

en donde $\Phi = (\tau, \varphi^I)$. Bajo transformaciones de Γ_N, la invarianza de la acción requiere que

$$\begin{cases} \tau \to \frac{a\tau+b}{c\tau+d} \\ \varphi^I \to (c\tau+d)^{-k_I} \rho_N^I(\gamma) \varphi^I \end{cases} \quad \Rightarrow \quad \begin{cases} W(\Phi) \to W(\Phi) \\ K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi}) \end{cases}$$

Una propuesta sería

$$K(\Phi, \bar{\Phi}) = -h \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^I|^2$$

El siguiente paso es caracterizar el mecanismo de ruptura de SUSY bajo la simetría modular (e. g. Supergravedad)

$$G(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi}) + \log\ W(\Phi) + \log\ W(\bar{\Phi})$$

Se analiza el potencial escalar asociado, se introduce un mediador cuyo término F adquiere un VEV que rompe SUSY e induce términos SSB.

Condiciones de empatamiento entre modelos con RHN

Se implementan las condiciones de empatamiento de acuerdo al siguiente esquema[2]

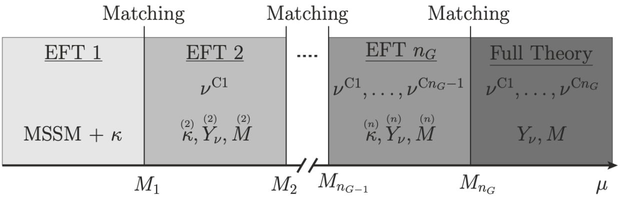


Figure 1. Illustration of the effective theories for the RG evolution in seesaw models with non-degenerate masses M_n of the right-handed neutrinos. At the threshold $\mu = M_n$, the right-handed neutrino ν^{Cn} is integrated out of the theory and the "EFT (n+1)" and "EFT n" are matched.

---

[2] S. Antusch and E. Cazzato, "One-Loop Right-Handed Neutrino Threshold Corrections for Two-Loop Running in Supersymmetric Type I Seesaw Models," JHEP **12**, 066 (2015) doi:10.1007/JHEP12(2015)066 [arXiv:1509.05604 [hep-ph]].

Corecciones supersimétricas a un lazo

En la base en la que $\mathbf{Y}_u$ es diagonal (Blazek, Raby, and Pokorski 1995)

$$
\begin{aligned}
\mathbf{Y}_u^{\text{SM}} &\simeq \mathbf{Y}_u^{\text{MSSM}} \sin\beta, \\
\mathbf{Y}_d^{\text{SM}} &\simeq (1 + \Delta_d + \Delta_u)\mathbf{Y}_d^{\text{MSSM}} \cos\beta,
\end{aligned}
$$

en donde $16\pi^2 \Delta_d^* = V_{CKM} \Gamma_d V_{CKM}^\dagger \tan\beta$ y $16\pi^2 \Delta_u^* = \Gamma_u \tan\beta$, con $\tan\beta = v_u/v_d$.

$$
\begin{aligned}
\mathbf{\Gamma}_{d\,ij} &= \frac{8}{3} g_3^2 \, (\Gamma_{dL}^\dagger)_{i\alpha} \int dk \, \frac{M_{\tilde{g}}}{(k^2 + M_{\tilde{g}}^2)(k^2 + m_{\tilde{d}_\alpha}^2)} \, (\Gamma_{dR})_{\alpha l} \, \frac{(\mathbf{m}_d^{0\,Diag})_{lj}^{-1}}{\tan\beta} \, , \\
\mathbf{\Gamma}_{u\,ij} &= -\lambda_{u_i}^{0Diag} \, (\Gamma_{uR}^\dagger)_{i\alpha} \int dk \, \frac{U_{2A}^{*\,\dagger} \, m_{\chi_A} \, V_{A2}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} \, (\Gamma_{uL})_{\alpha j} \, (v_u)^{-1} \\
&\quad + g_2 \, (\Gamma_{uL}^\dagger)_{i\alpha} \int dk \, \frac{U_{2A}^{*\,\dagger} \, m_{\chi_A} \, V_{A1}}{(k^2 + m_{\chi_A}^2)(k^2 + m_{\tilde{u}_\alpha}^2)} \, (\Gamma_{uL})_{\alpha j} \, (v_u)^{-1} \quad .
\end{aligned}
$$