Ciencia y Tecnología
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Recent advances in Spinning Black Hole dynamics from Scattering Amplitudes

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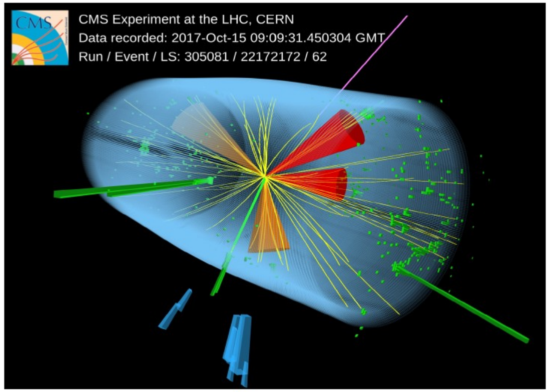
UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO
POR MI RAZA HABLARÁ EL ESPÍRITU

How can Particle Physics Theory help to understand classical Motion of astrophysical objects?

Bound Orbit

General Relativity

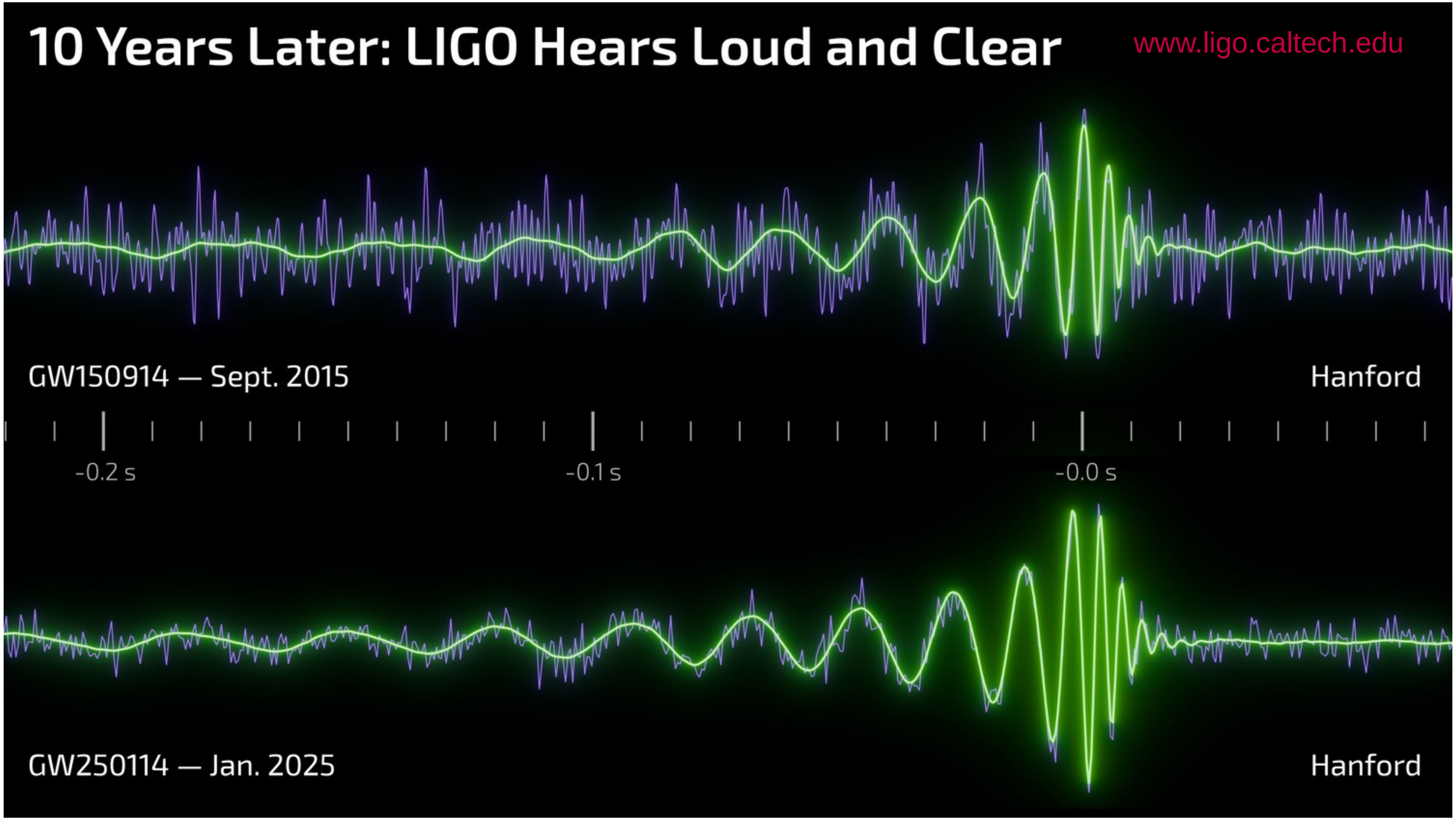
Unbound Orbit



Quantum Field Theory

Introduction

Gravitational Waves enter the Precision Era

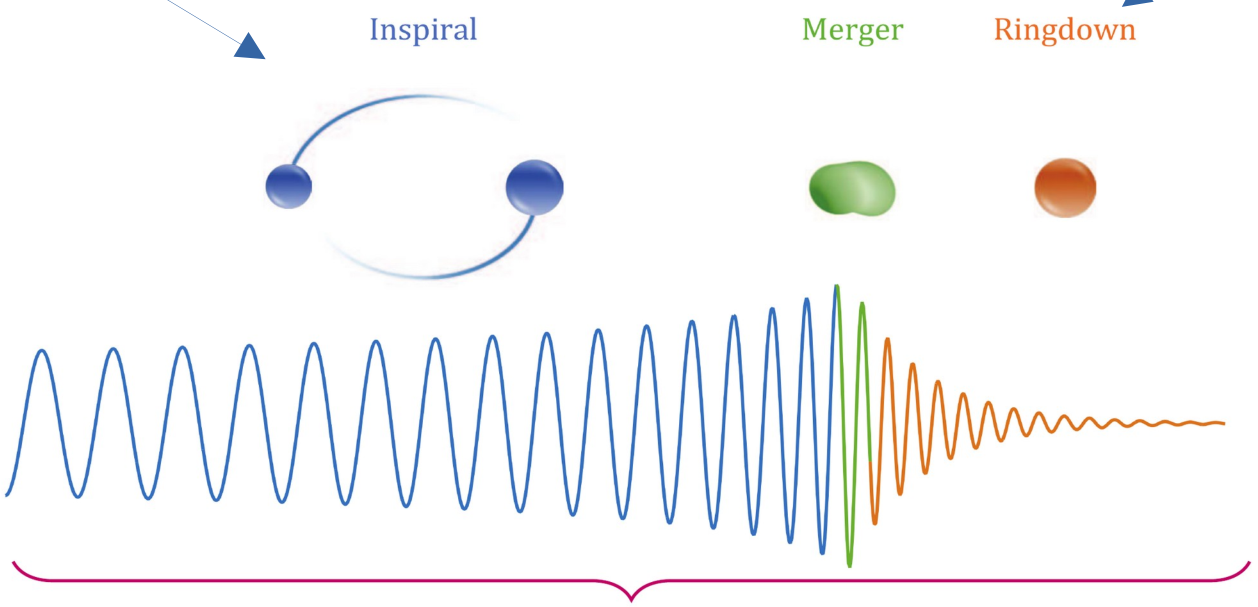


Stages of a Binary Merger

Perturbation Theory

Numerical GR

BHPT



Waveform

[Antelis, Moreno, EPJP 132 (2017) 1,10]

Future Gravitational Wave Observatories

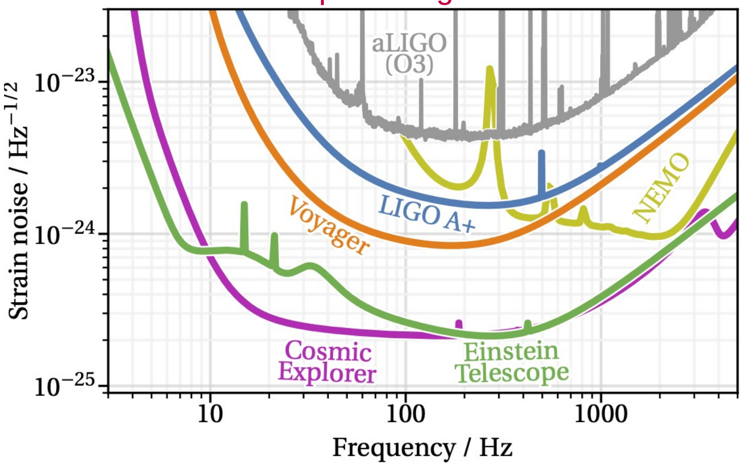
Future ground based Observatories
→ Advanced LIGO
→ Einstein Telescope
→ Cosmic Explorer

Future space based Observatories
→ LISA
→ TianQin
→ Taiji

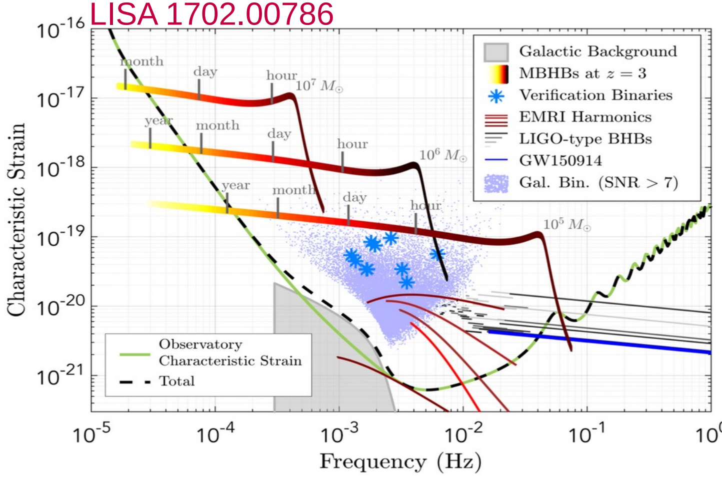
Physics goals:
→ Tests of General Relativity
→ Black Hole Formation
→ Neutron Star Equation of State

Requires theoretical modeling of perturbative inspiral phase up to $\mathcal{O}(G^7)$

www.cosmicexplorer.org



LISA 1702.00786



Non-Spinning Black Holes

General Relativity as a QFT

- The Action

$$S[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\sum_{i=1}^{2}\left(g^{\mu\nu}(\partial_\mu\phi_i)(\partial_\nu\phi_i) - m_i^2\phi_i^2\right)\right]$$

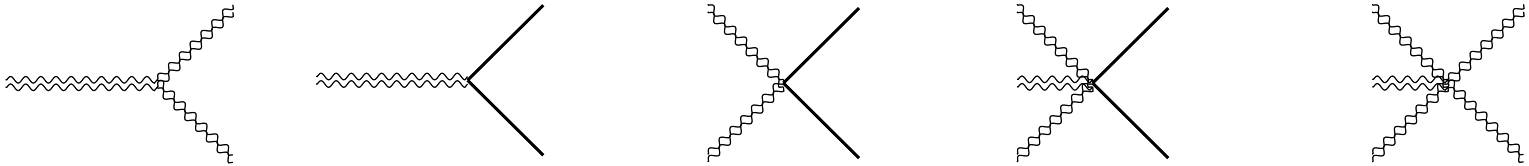
- Expand spacetime around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu} \qquad\qquad \kappa = \sqrt{32\pi G}$$

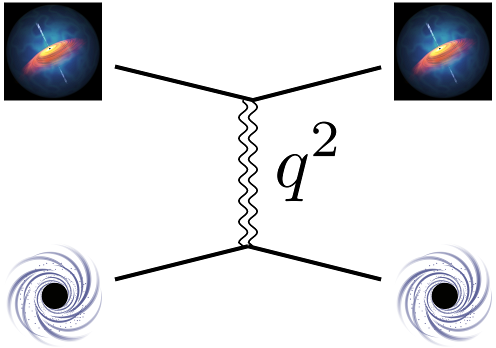
- General Relativity is non-linear

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa\, h^{\mu\nu} + \kappa^2\, h^{\mu\lambda}h^{\nu}{}_{\lambda} + \ldots$$

- Generates infinite many Feynman Rules



Quantum Field Theory at work - I



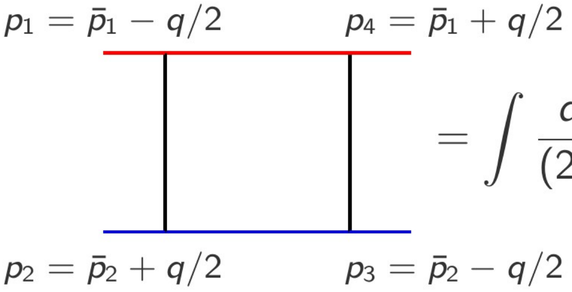
$$A^{\text{tree}} = \frac{4\pi G}{E_1 E_2} \frac{m_1^2 m_2^2 (1 - 2\sigma^2)}{q^2} + \cdots$$

The classical potential is given by

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} A^{\text{tree}} = -\frac{G m_1 m_2}{r} \left(\frac{m_1 m_2}{E_1 E_2} (2\sigma^2 - 1) \right)$$
$$= -\frac{G m_1 m_2}{r} + \mathcal{O}(v^2)$$

- $q \sim \hbar \to 0$ reveals the classical physics
- Amplitudes provide full special relativistic corrections
- Loop-Amplitudes provide higher-order $\mathcal{O}(G^n)$ corrections

Quantum Field Theory at work - II



$$= \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2(\ell-q)^2[(\ell+p_1)^2 - m_1^2][(\ell-p_2)^2 - m_2^2]}$$

$$= \frac{\hbar^4}{\hbar^4} \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2(\ell-q)^2} \left[\frac{1}{\hbar^2} \frac{1}{[2(\ell\cdot\bar{p}_1) + i\epsilon][-2(\ell\cdot\bar{p}_2) + i\epsilon]} + \mathcal{O}(\hbar) \right]$$

$\ell \sim q \sim \hbar$

High-Rank Tensor integrals can be dropped

$$\int \frac{d^4\ell}{(2\pi)^4} \frac{\ell^\mu \ell^\nu \ell^\alpha \ell^\beta}{\ell^2(\ell-q)^2[(\ell+p_1)^2 - m_1^2][(\ell-p_2)^2 - m_2^2]} \sim \mathcal{O}\left(\frac{\hbar^4}{\hbar^2}\right) = \mathcal{O}(\hbar^2)$$

[Parra-Martinez, Ruf, Zeng arXiv:2005.04236]

Spinning Black Holes

Amplitude Approaches [Slide stolen from Mao Zeng]

- Massive spinor helicity amplitudes (spin exponentiation) [Arkani-Hamed, Huang, Huang, ‘17 …] High ranks for tensor integrals, difficult IBP reduction beyond one-loop

- Non-transverse higher-spin fields [Bern, Luna, Roiban, Shen, Zeng ’20, Alaverdian, Bern, Kosmopoulos, Luna, Roiban, Scheopner, Teng ‘24] Needs separation between spin-magnitude changing effects and conservative dynamics; not yet achieved beyone one-loop

- **Finite-spin massive particles** [Holstein, Ross ‘07, Vaidya ‘14, Maybee, O’Connell, Vines ‘19, Damgaard, Haddad, Helset ‘19, Aoude, Haddad, Helset ‘20]
  - → Straightforward option for > 1-loop calculations
  - → Spin-j accesses $\mathcal{O}(S^{2j})$ perturbative multipole moments
  - → Plagued by Casimir ambiguity, resolved by spin-interpolation

- Beyond amplitudes: PMEFT with spin [Liu, Porto, Yang], WQFT [Jakobsen, Mogull, Plefka, Steinhoff ‘21, Haddad, Jakobsen, Mogull, Plefka ‘24], Generalized Wilson Lines [Bonocore, Kulesza, Pirsch ‘25]

State-of-the-art for Spin effects

- High orders for spin at 1-loop [Bern, Kosmopoulos, Luna, Roiban, Teng ‘22; Aoude, Haddad, Helset ‘22,’23; Bohnenblust, Cangemi, Johannson, Pichini ‘24]
- Gravitational Waveform at $\mathcal{O}(G^3S^2)$ [Bohnenblust, Ita, MK, Schlenk ‘23,’25]
- Conservative + dissipative effects in WQFT at $\mathcal{O}(G^3S^2)$ [Jakobsen, Mogull, ‘22]
- All-order-in-spin radiation reaction [Alessio, Di Vecchia, ‘22]
- Energy loss from PMEFT [Riva, Vernizzi, Wong ‘22]
- Three-loop in WQFT $\mathcal{O}(G^4S^1)$ [Jakobsen, Mogull, Plefka, Sauer, Xu ‘23]
- Conservative dynamics at $\mathcal{O}(G^3S^2)$ from amplitudes [Febres Cordero, MK, Lin, Ruf, Zeng ‘22; Akpinar, Febres Cordero, MK, Ruf, Zeng ‘24]
- Unprecedented results at $\mathcal{O}(G^3S^4)$ [Akpinar, Febres Cordero, MK, Smirnov, Zeng ‘25]

Spin of a Spinning Black Hole?

- Soft expansion performed as in the case of non-spinning BHs

- Form-factor decomposition, e.g

$$\mathcal{A}(\phi V \to \phi V) = \sum_{n=1}^{4} M_n \epsilon_\mu(p_1,\lambda_1) T_n^{\mu\nu} \epsilon_\nu^\star(p_4,\lambda_4$$

$$T_1 = (\epsilon_1 \cdot \epsilon_4^\star) \qquad T_2 = (\epsilon_1 \cdot q)(\epsilon_4^\star \cdot q)$$

$$T_3 = q^2(\epsilon_1 \cdot \bar{p}_2)(\epsilon_4^\star \cdot \bar{p}_2) \qquad T_4 = (\epsilon_1 \cdot \bar{p}_2)(\epsilon_4^\star \cdot q) - (\epsilon_1 \cdot q)(\epsilon_4^\star \cdot \bar{p}_2)$$

- Spin dependence obtained from

$$\bar{\varepsilon}^{\star\mu}\bar{\varepsilon}^{\nu} = -\left(\eta^{\mu\nu} - \frac{\bar{p}_1'^{\mu}\bar{p}_1'^{\nu}}{m_1^2}\right) + \frac{i}{2m_1}\epsilon^{\mu\nu\alpha\beta}\bar{p}'_{1\alpha}S_\beta - \frac{1}{2}(S^\mu S^\nu + S^\nu S^\mu)\,,$$

- Complicated expression at higher spin

Spin Universality and Ambiguity

- Fixed spin-j amplitudes have access to $\mathcal{O}(S^{2j})$ multipoles

$$\mathcal{A}(\phi_1\phi_2 \to \phi_1\phi_2) \sim \mathbb{1}, \mathcal{A}(\phi\psi \to \phi\psi) \sim \{\mathbb{1}, S\cdot l\},$$
$$\mathcal{A}(\phi V \to \phi V) \sim \{\mathbb{1}, S\cdot l, (q\cdot S)^2, (p\cdot S)^2\}$$

- Spin Universality: (conjecture!)
  Classical multipole moments do not depend on the nature of the test particle

- For arbitrary spin there should be a fifth spin-operator

$$O_5 = q^2 S^2 \sim \frac{\hbar^2}{\hbar^2} = 1\ , \qquad O_5^{\text{spin-j}} = q^2 S^2 = q^2 j(j+1)\mathbb{1}$$

- 1-Loop example: Classical physics lives at 1/|q| and quantum physics at |q|

$$\mathcal{A} = \frac{1}{|q|}\left[c_0\mathbb{1} + c_5(q^2S^2) + \ldots\right] = \frac{1}{|q|}\left[c_0 + c_5 q^2 j(j+1)\right]\mathbb{1}$$

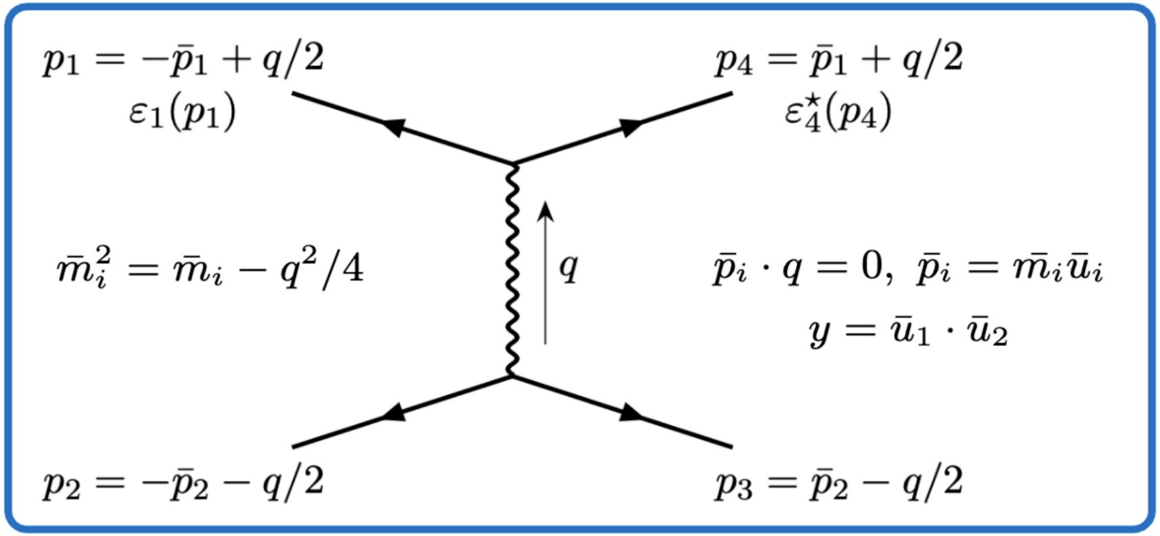
- Casimir operator indistinguishable from Quantum physics!

→ Spin Interpolation!

A Tower of Spin Theories

- Consider the following Scattering Processes

$$X(p_1,\epsilon_1)+\phi(p_2)\to\phi(p_3)+X(p_4,\epsilon_4) \qquad X=\{\Phi, V^\mu H^{\mu\nu}\}$$



$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} &= \frac{2R}{\kappa^2}\\ &+\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)-\frac{1}{2}m_2^2\phi^2\\ &+\mathcal{L}_{m_1}\end{aligned}$$

A Tower of Spin Theories

[Abreu, Dormans, Febres Cordero, Ita, Kraus, Page, Pascual, S. Ruf, Sotnikov, '10]…..

Caravel

Feynman rules from $\mathcal{L}$

Numerical 3-point, 4-point and 5-point tree level amplitudes

Numerical agreement with analytical results obtained with double copy

Generalised Unitarity

Potential region $\longrightarrow$ no radiation!

Numerical tree-level amplitude + 1-loop and 2-loop integrands

Agreement with analytic tree-level and 1-loop amplitudes

Soft expansion + private FIRE6 code

Numerical tree-level, 1-loop and 2-loop amplitudes

4 form factors for spin-1
9 form factors for spin-2

Form factor decomposition of spinning amplitude

$$\mathcal{M} = \sum_n M_n \, \varepsilon^\star_{4\mu\nu\ldots} T_n^{\mu\nu\alpha\beta\ldots} \varepsilon_{1\alpha\beta\ldots}$$

Theory dependent information

Spin information

Spin in Quantum Field Theory

- Spin operator for massive spin-s boson

$$\mathbb{S}^{\mu}_{ij} = \frac{(-1)^s}{2m} \epsilon^{\mu\lambda\alpha\beta} p_\lambda \epsilon^{\star}_{iI(s)}(p) \left(\Sigma_{\alpha\beta}\right)^{I(s)}{}_{J(s)} \, \epsilon^{J(s)}_{j}(p)$$

- With the Lorentz-Generators in the spin-s representation

$$\left(\Sigma_{\alpha\beta}\right)^{I(s)}{}_{J(s)} = 2is\delta^{(I1}_{[\alpha}\eta_{\beta](J1}\delta^{I_2}_{J_2}\cdots\delta^{I_s)}_{J_s)}$$

- The classical spin vector can be seen as the expectation value

$$S^\mu = \sum_{i,j} \omega_i^\star \mathbb{S}^\mu_{ij}\omega_j \qquad \sum_i |\omega_i|^2 = 1$$

- Products of classical spin vectors → Symmetric products of spin operators

$$S^\mu S^\rho \cdots S^\sigma S^\nu = \sum_{i,k,\ldots} \omega_i^\star \mathbb{S}^{(\mu}_{ik}\mathbb{S}^{\rho}_{kj} \ldots \mathbb{S}^{\sigma}_{rm}\mathbb{S}^{\nu)}_{mj}\omega_j$$

Spin Interpolation - I

- Write Ansatz for the amplitude valid for generic spin representation

$$\{1, i(n\cdot S), (q\cdot S)^2, q^2 S^2, q^2(\bar{p}_2\cdot S)^2\} \quad n^\mu = \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{p}_{1\alpha} \bar{p}_{2\beta}$$

- Ansatz for the finite part of the amplitude reads

$$\mathcal{M}^{(L)}_{fin} = \frac{\kappa^{2+2L}}{(-q^2)^{1-L/2+L\epsilon}} \sum_{n=0}^{4} \sum_{m} \alpha^{(L,n,m)} \mathcal{O}^{(n,m)}$$

- Spin Universality
  $\rightarrow$ spin coefficients (classical AND quantum) are independent of spin representation!

- Spin structures $\mathcal{O}^{(n,m)}$ organized in terms of

$$\Omega_\pm = (q\cdot S)^2 \pm q^2 S^2$$

Spin Interpolation - II

- To fix the coefficients we evaluate the Ansatz for $s=0,1,2$ and expand up to $\mathcal{O}(q^4)$

  → 14 classical coefficients, given by the leading q contribution of each structure
  → 6 quantum suppressed coefficients

- For each representation, project onto the appropriate form factor basis

$$\mathcal{A} = \sum_n M_n \epsilon^\star_{4,\mu\nu,\dots} T_n^{\mu\nu\alpha\beta\dots} \epsilon_{1\alpha\beta\dots}$$

- Solve a linear system of equations by demanding equality with the corresponding spinning amplitude

  → 31 linear relations, of which 11 are linearly dependent
  → 20 linearly independent relations fixes the 1+5+14=20 spin coefficients

Obtained for the first time results at $\mathcal{O}(G^3S^4)$

Spin-Shift Symmetry

- Spin-Shift Symmetry states the Amplitude is invariant under

$$S^\mu \to S^\mu + \xi q^\mu$$

One-loop conjecture
[Bern, Kosmopoulos, Luna, Roiban, Teng '22]
[Aoude, Haddad, Helset '22]

- The only possible way to organize Spin multipoles

$$c_+\Omega_+ = c_+\left[(q\cdot S)^2 + q^2 S^2\right] \Rightarrow c_+ = 0$$
$$c_-\Omega_- = c_-\left[(q\cdot S)^2 - q^2 S^2\right] \Rightarrow c_- \neq 0$$

- The two-loop quartic in spin amplitude organizes itself as

$$\alpha^{(2,n,m)} = \alpha^{(2,n,m)}_{\text{probe}} + m_1^3 m_2^3 \alpha^{(2,n,m)}_{\text{1SF}}$$

Spin-Shift symmetric

Spin-Shift Symmetry breaking

First time spin-shift symmetry observed up to $\mathcal{O}(G^3 S^4)$

The radial action and Observables

- Extract the radial action from the finite part of the Amplitude:

$$I_r = \int \frac{d^4q}{(2\pi)^2}\delta(p_1\cdot q)\delta(-p_2\cdot q)e^{ib\cdot q}\mathcal{M}_{fin}$$

Agreement with [Gonzo, Shi, '24] in the limit of a spinless probe in a Kerr background to quartic order in spin

- Dirac Brackets to compute observables

Following [Gonzo,Shi '24]

$$\Delta\lambda^\mu = \sum_j \frac{1}{j!}\underbrace{\{I_r,\{I_r,\cdots,\dots,\{I_r,\lambda^\mu\}\dots\}}_{j\ \text{times}} \quad \text{for} \quad \lambda^\mu = \{u_1^\mu, u_2^\mu, a_1^\mu, a_2^\mu\}$$

$$a_i = S_i/m_i$$

$$\{b^\mu, u_i^\nu\} = -\operatorname{sgn}_i \frac{(y^2-1)b^\mu b^\nu + l^\mu l^\nu}{m_i b^2(y^2-1)}\,,\ \{b^\mu, a_i^\nu\} = \operatorname{sgn}_i \frac{u_i^\nu((y^2-1)b^\mu(b\cdot a_i) + l^\mu(l\cdot a_i))}{m_i b^2(y^2-1)}$$

$$\{b^\mu, b^\nu\} = \frac{(b^2(y u_2^\mu - u_1^\mu) - l^\mu(a_1\cdot u_2))b^\nu - (\mu\leftrightarrow\nu)}{m_1 b^2(y^2-1)} + (1\leftrightarrow 2, b\to -b)\,,\ \{a_i^\mu, a_i^\nu\} = \frac{\epsilon^{\mu\nu}{}_{\rho\sigma}u_i^\rho a_i^\sigma}{m_i}$$

Validation of our results

- Agreement with the aligned-spin scattering angle in **post-Newtonian results** [Bautista, Khalil, Sergola, Kavanagh, Vines, '24] through $\mathcal{O}(G^3S^4)$

- Conservative observables from Dirac Brackets agree with our previous $\mathcal{O}(G^3S^2)$ results from **KMOC** [Febres Cordero, MK, Lin, Ruf, Zeng '22] and with **WQFT** results [Jakobsen, Mogull '22]

- Using all-order-in-spin radiation reaction amplitudes [Alessio, Di Vecchia '22] and acting with Dirac Brackets, obtain **cancellation of high-energy divergences** in observables through $\mathcal{O}(G^3S^4)$, previously observed through $\mathcal{O}(G^3S^2)$ using WQFT [Jakobsen, Mogull '22]

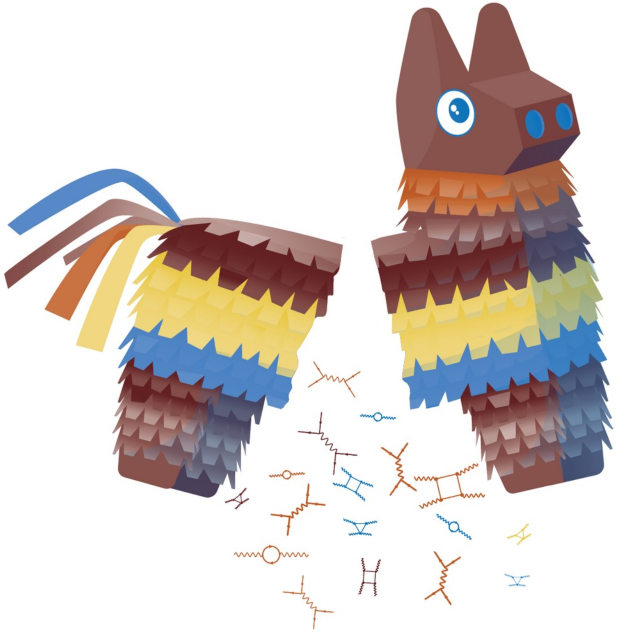
- Strong and highly non-trivial checks provide confidence in our partly conjectural setup!

Conclusions & Outlook

Conclusions & Outlook

- First calculation of $\mathcal{O}(G^3 S^4)$ conservative dynamics of binary Kerr Black Hole using scattering amplitudes from two-loop numerical unitarity!

What is next?
→ Disspative effects?
→ Higher-loop orders?
→ Higher spin?



- What if we are interested in Neutron stars?

→ Higher-dimensional Operators
→ How to do Spin interpolation?

- What is the origin of the Spin-Shift symmetry?!