Generalized susceptibilities near the QCD critical point

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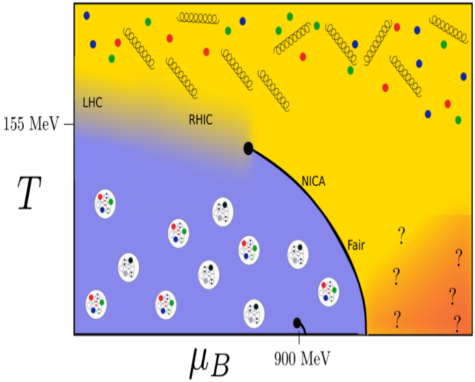
Outline

1. Motivation: QCD phase diagram
2. Generalized susceptibilities
3. Study with Hadron Resonance Gas

Motivation: QCD phase diagram

- Calculations indicate a possible critical point in the QCD phase diagram.
- The critical point can be studied from the derivatives of the partition function.
- Experimentally, heavy-ion collisions are used.



Generalized susceptibilities

Assuming the system is described by a grand canonical ensemble, with partition function

$$Z(T, \mu_q, V). \quad (1)$$

The generalized susceptibility of order n associated with the conserved quantity q is the n-th derivative of the pressure with respect to μ_q/T:

$$\chi_q^{(n)} = \frac{1}{VT^3} \frac{\partial^n \log Z}{\partial(\mu_q/T)^n} = \frac{\partial^n p(T, \mu_q)}{\partial(\mu_q/T)^n}. \quad (2)$$

These quantities are sensitive to the critical point. They related to the correlation length ξ [1]

$$\chi_q^{(2)} \sim \xi^2, \quad \chi_q^{(3)} \sim \xi^{4,5}, \quad \chi_q^{(4)} \sim \xi^7. \quad (3)$$

In the thermodynamic limit, ξ diverges at the critical point[2].

[1] M.A. Stephanov, Phys.Rev.Lett.102:032301,2009

[2] Frithiof Karsch et al., Phys.Lett.B695:136-142,2011

How do we measure susceptibilities?

Susceptibilities are accessible experimentally via event-by-event fluctuations of conserved charges. Let ΔN be the net charge in a given event and $\delta N = \Delta N - \langle \Delta N \rangle$ its deviation. The statistical cumulants are defined as:

$$C_1 = \langle \Delta N \rangle, \quad C_2 = \langle (\delta N)^2 \rangle, \quad C_3 = \langle (\delta N)^3 \rangle, \qquad (4)$$
$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2.$$

Cumulants are related to the generalized susceptibilities[3]

$$C_n = VT^3 \chi_q^{(n)} \qquad (5)$$

[3] M.A. Stephanov, Phys.Rev.Lett.102:032301,2009

Phase diagram using the Hadron Resonance Gas

Hadron Resonance Gas model

The system is modeled as a non-interacting gas of hadrons and resonances, described by the partition function:

$$\log Z^{\mathrm{HRG}}(T,V,\mu) = \sum_{i\in \mathrm{PDG}} \pm \frac{V g_i}{2\pi^2} \int_0^\infty dp\, p^2 \log(1 \pm e^{-(E_i-\mu_j)/T}) \tag{6}$$

with $+$ or $-$ for a boson or a fermion. With chemical potential

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S \tag{7}$$

The ideal HRG model lacks a critical point. However, it can be phenomenologically introduced by incorporating interactions in the gas [4].

[4]V. Vovhenko et al., Comput. Phys. Commun. 244, 295 (2019)

HRG, Quantum Van der Waals

The simplest way to include interactions and a critical point is by generalizing the van der Waals gas:

$$p(T,\mu) = \sum_i p_i^{\text{ideal}}(T,\mu_i^*) - \sum_{ij} a_{ij} n_i n_j, \tag{8}$$

with

$$\mu_i^* + \sum_j b_{ij} p_j^* - \sum_j (a_{ij} + a_{ji}) n_j = \mu_i. \tag{9}$$

Using experimental data, the a_{ij} and b_{ij} can be determined. THERMAL-FIST package [5], to calculate the susceptibilities.

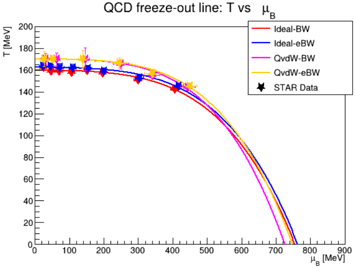
[5] V. Vovhenko et al., Comput. Phys. Commun. 244, 295 (2019)

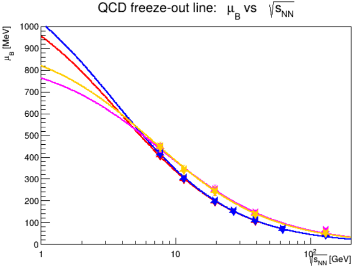
Freeze-out Line

Experimentally, measurements are restricted to the chemical freeze-out line. Using STAR data, and the parametrization[6]:

$$\mu_B(\sqrt{s_{NN}}) = \frac{a}{1 + b\sqrt{s_{NN}}}, \quad T(\mu_B) = c - d\mu_B^2 - e\mu_B^4, \qquad (10)$$

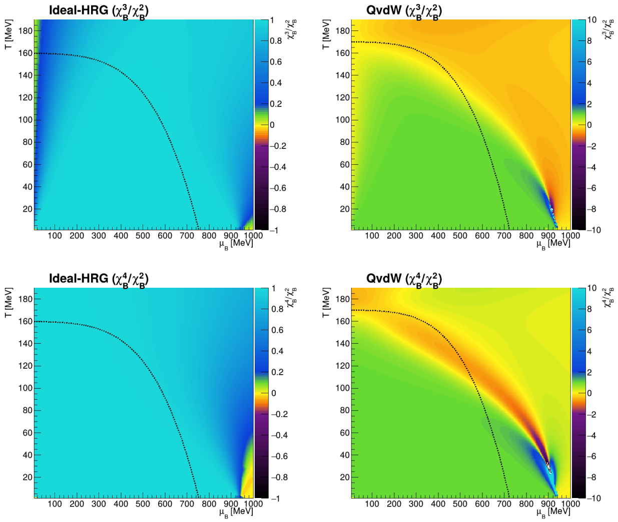
Model	a [GeV]	b [GeV^{-1}]	c[GeV^{-3}]	d[GeV]	e [GeV^{-1}]
I-HRG	0.160 ± 0.001	0.035 ± 0.04	0.430 ± 0.28	1.201 ± 0.20	0.25 ± 0.05
QvdW	0.170 ± 0.001	0.030 ± 0.04	0.559 ± 0.20	0.858 ± 0.05	0.12 ± 0.01



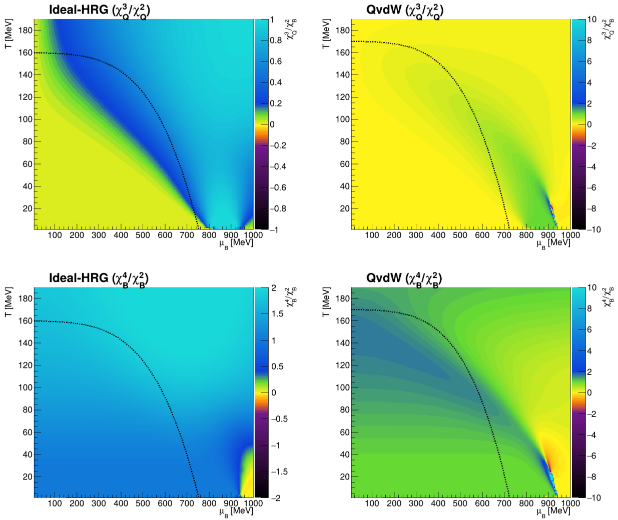


[6]F. Reyes and E. Cuautle, eprint arXiv:2606.14018

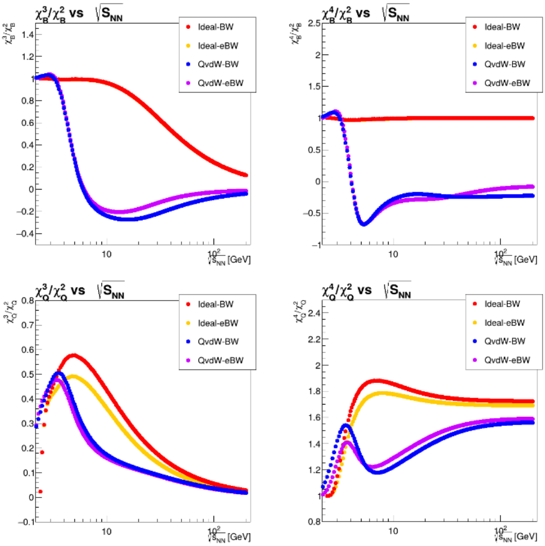
Ideal vs QvdW Susceptibilities (Baryon number) (T_c, μ_c)=(19.5 MeV, 15 MeV)



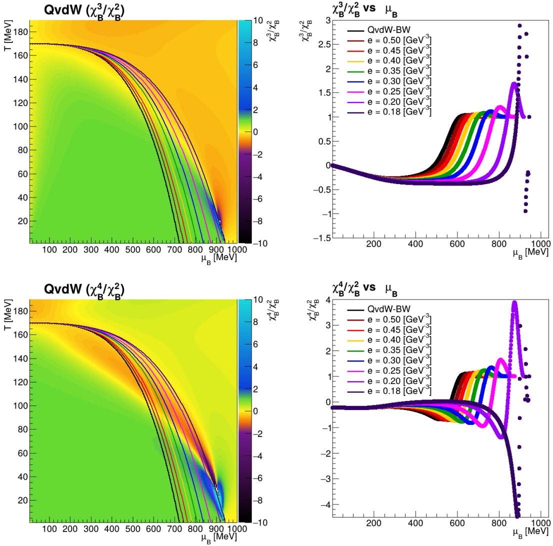
Ideal vs QvdW Susceptibilities (Electric Charge)



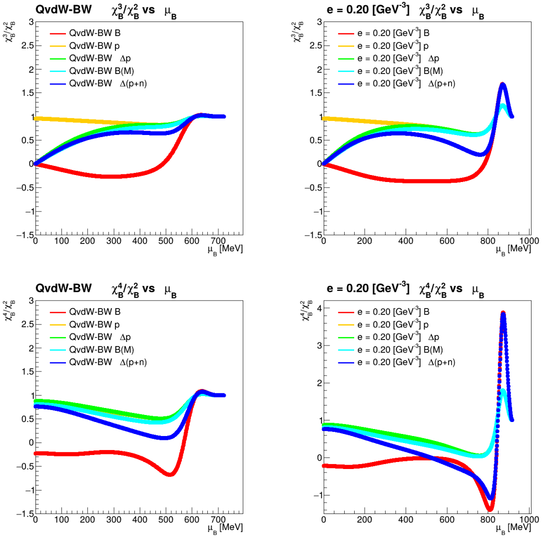
Susceptibilities in the freeze-out line



Susceptibilities in the freeze-out line



Susceptibilities of baryon number proxy



Summary

- Generalized susceptibilities are important observables in the search for the critical point.
- The HRG model allows estimating the phase diagram and comparing scenarios with and without a critical point against experimental data.
- Susceptibility ratios strongly depend on the proximity of the chemical freeze-out line to the critical point.
- The susceptibility depends on the proxy for baryon number. The neutrons measurements are very important to reach higher better behavior of susceptibilities and the critical end point.