



UNIVERSIDAD NACIONAL
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One-Loop Effects in Low-Scale Seesaw

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Introduction

- Low-scale Type-I Seesaw viability via radiative stability of m_ν
- 1-loop self-energy & Casas-Ibarra Parametrization
- Prediction of the $Br(\mu \rightarrow e\gamma)$ and constraint

Type-1 Seesaw Model

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \overline{N}_R^c \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

$$M_{\text{tot}} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

THRESHOLD !

under seesaw assumption

$$M_D \ll M_R$$

$$m_\nu^{(0)} = -M_D M_R^{-1} M_D^T$$



Casas-Ibarra Parametrization

$$M_D = i U_{PMNS} \sqrt{m_\nu} R^T \sqrt{M_R}$$



$$R = \begin{pmatrix} c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\ c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\ s_2 & s_1 c_2 & c_1 c_2 \end{pmatrix},$$

$$c_i = \cos \theta_i, \quad s_i = \sin \theta_i, \quad \theta_i = x_i + i y_i, \quad i = 1, 2, 3$$

$$\cosh y_i, \sinh y_i \sim \frac{e^{y_i}}{2}$$

$$U_{PMNS}^T m_\nu^{(0)} U_{PMNS} = m_\nu^{(\text{diag})}$$

$$R_{ij} \sim e^{\pm \text{Im}(\theta_k)}$$

Entries Cancellation

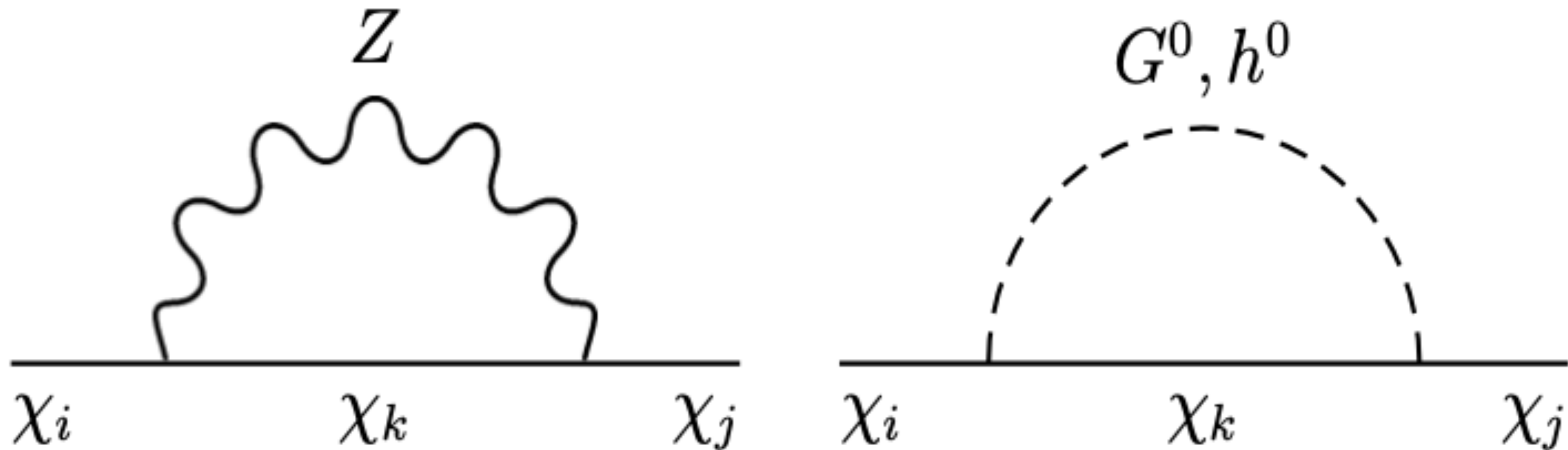
$$m_\nu^{(\text{diag})} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \sqrt{m_1^2 + \Delta m_{sol}^2} & 0 \\ 0 & 0 & \sqrt{m_1^2 + \Delta m_{atm}^2} \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \text{diag}(e^{-i\alpha_1}, e^{-i\alpha_2}, 1)$$

1-loop correction

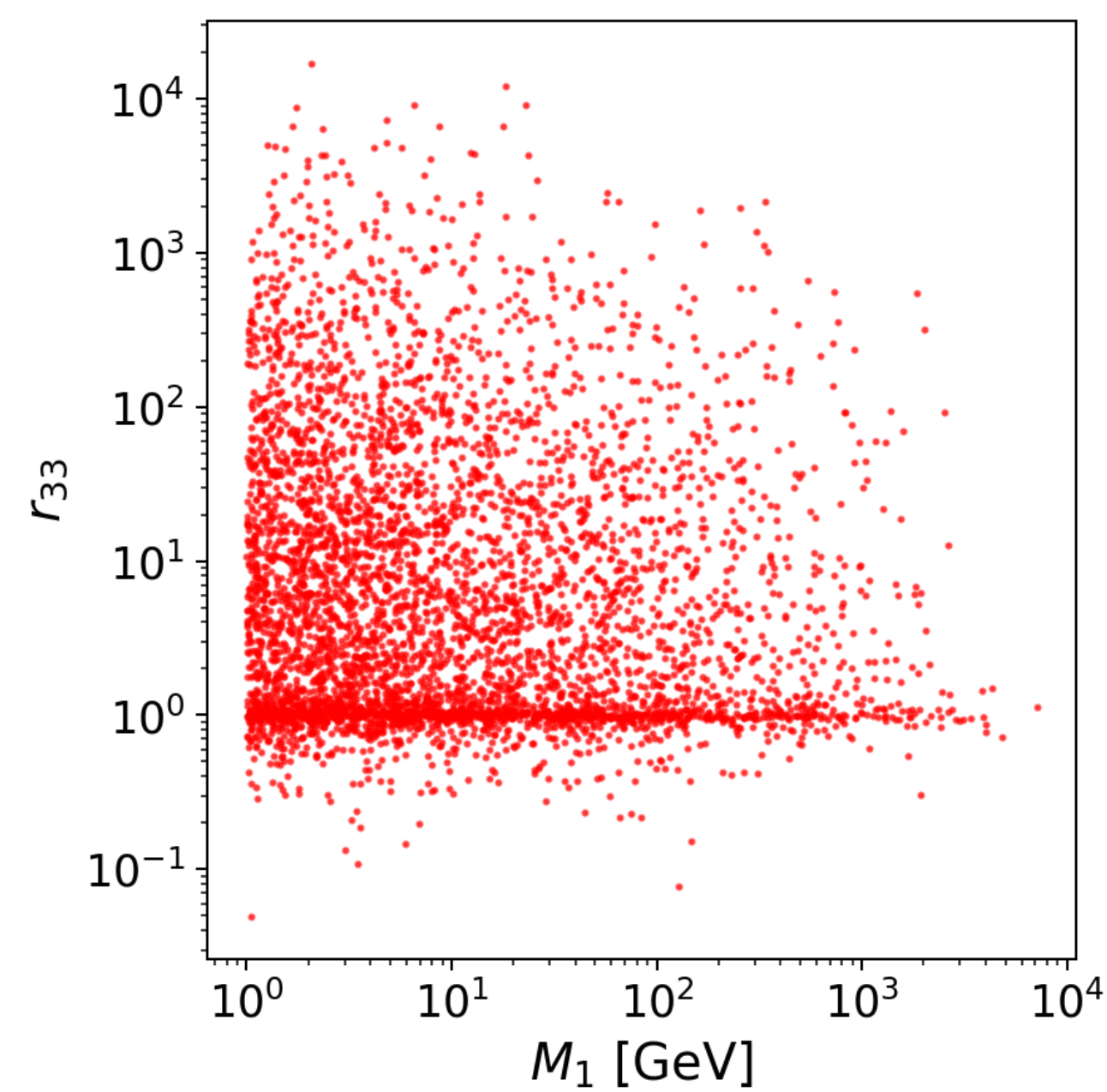
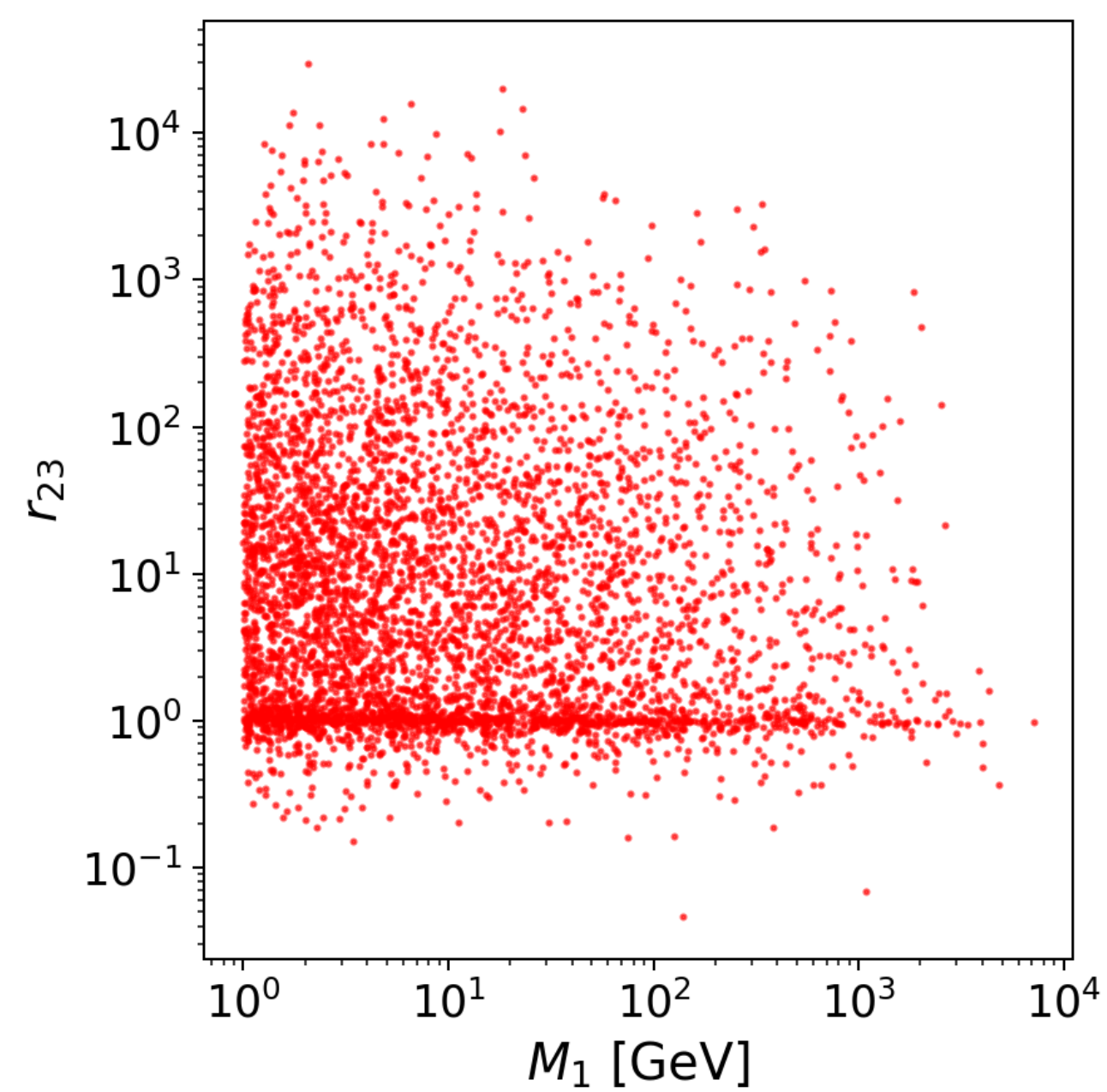
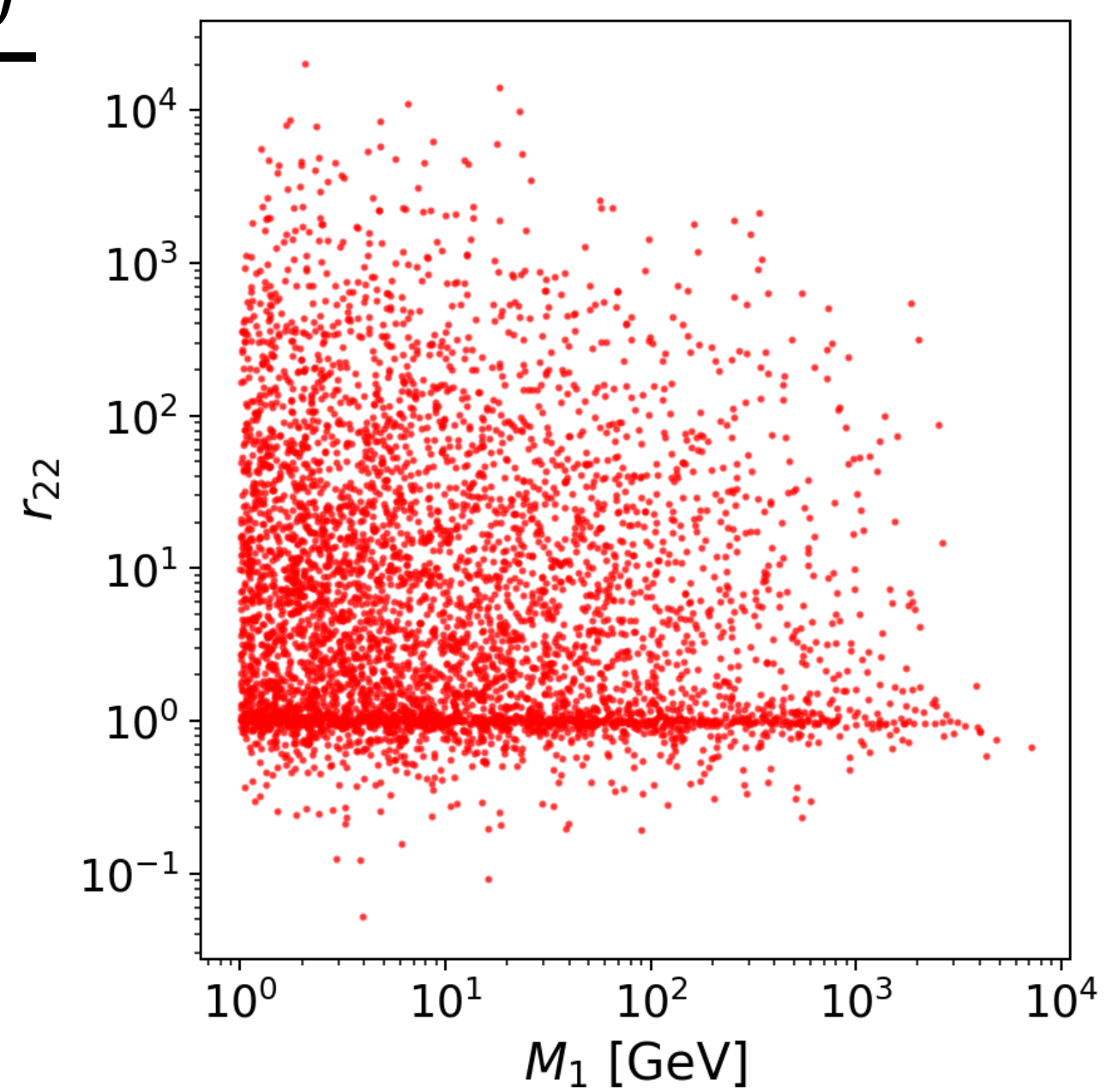
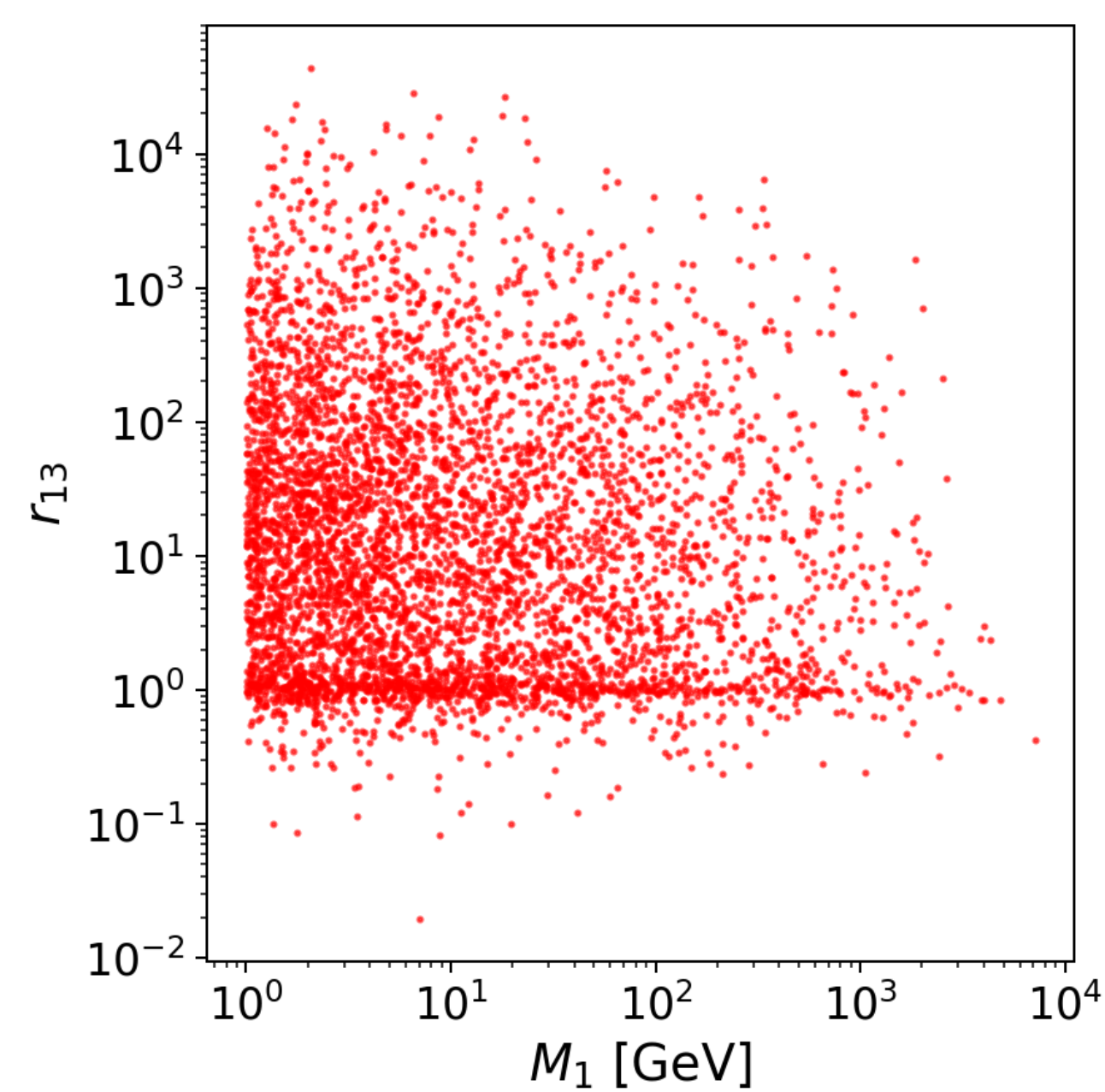
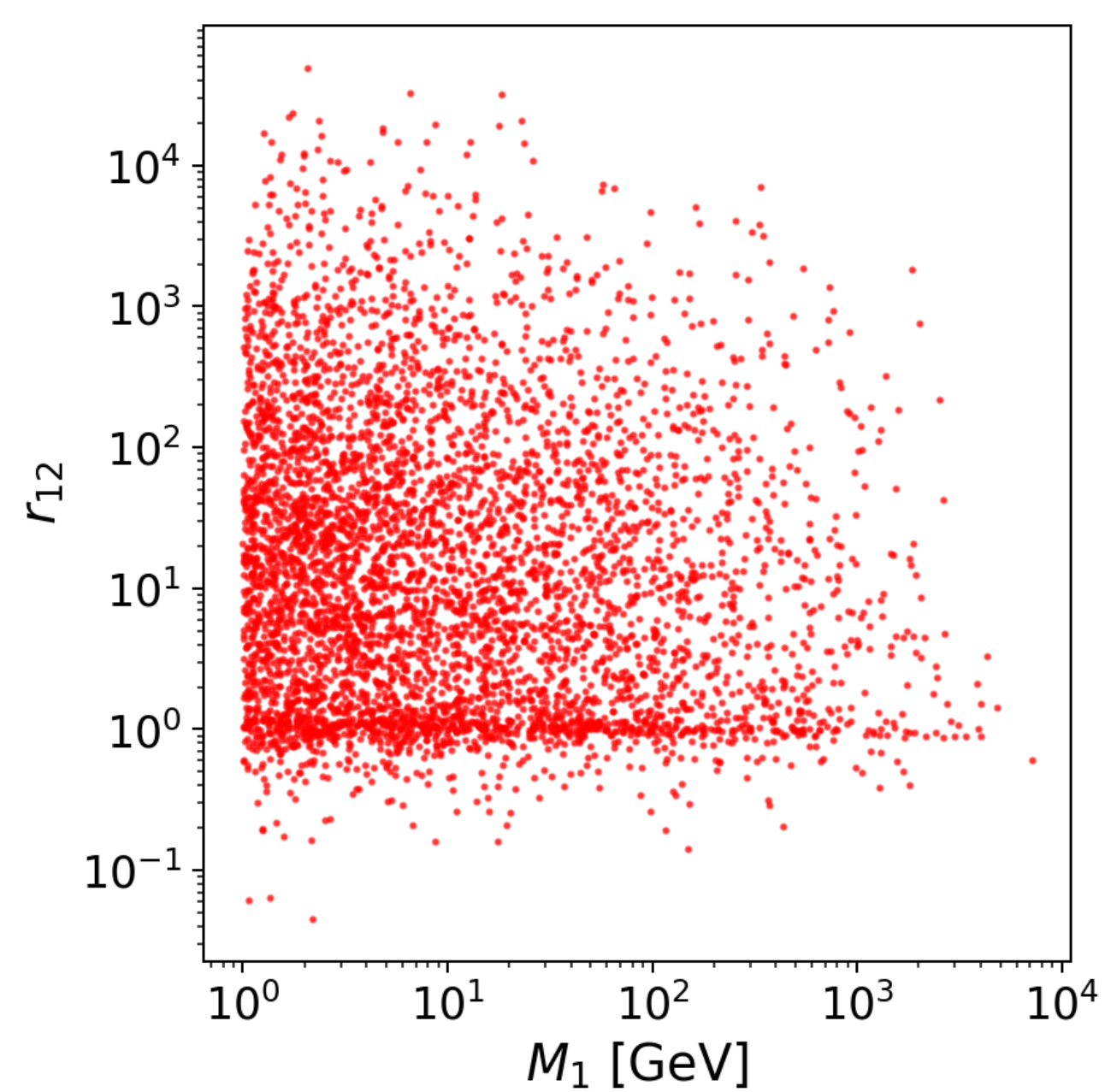
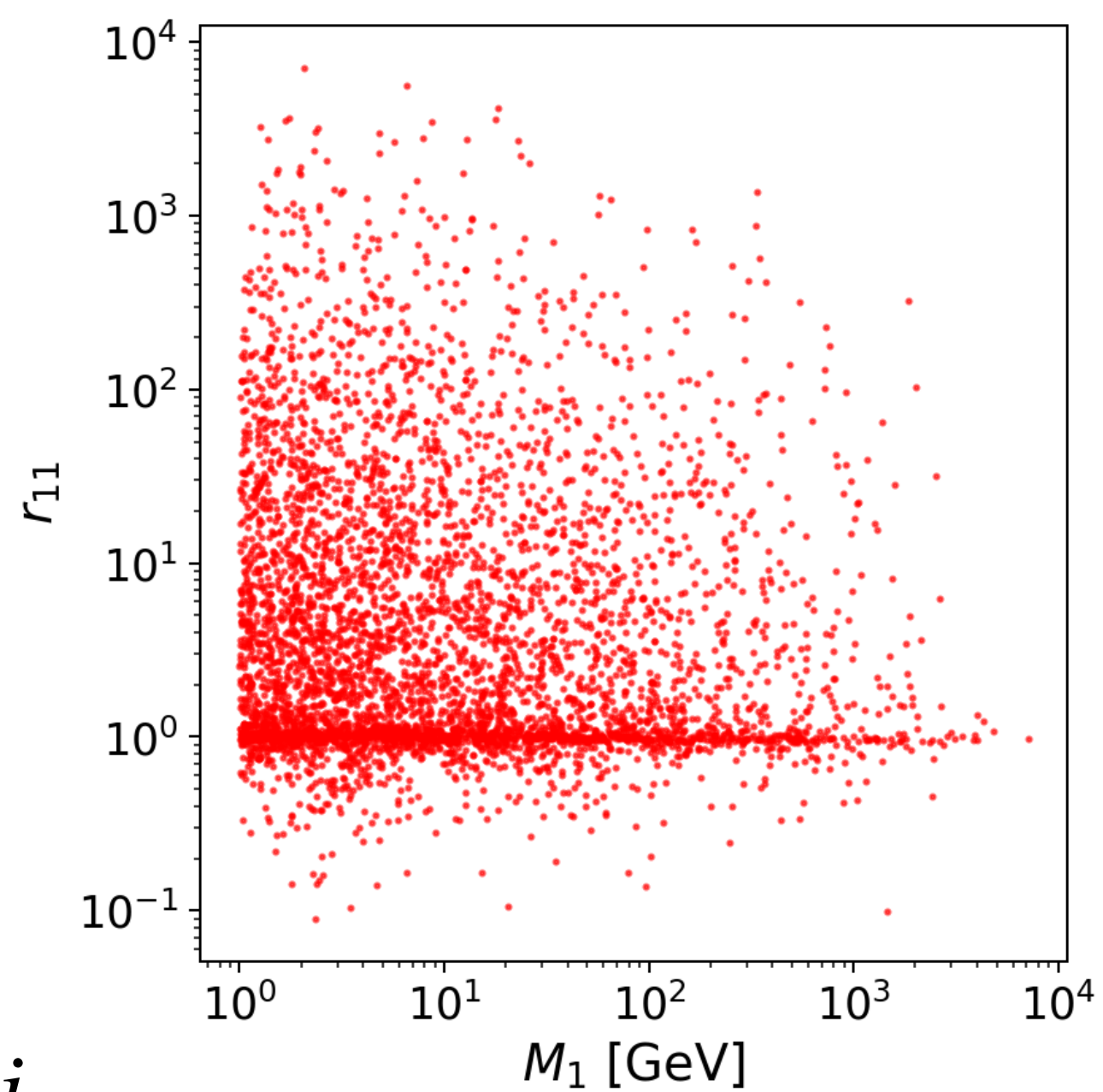
$$\delta M = M_D^T M_R^{-1} \Sigma_1 M_D = m_\nu^{(1)}$$

$$\Sigma_1 = \frac{g^2}{64\pi^2 M_W^2} \left[m_H^2 \ln\left(\frac{M_R^2}{m_H^2}\right) + 3 m_Z^2 \ln\left(\frac{M_R^2}{m_Z^2}\right) \right].$$



W. Grimus and L. Lavoura , 2002

$$r_{ij} \equiv \frac{(m_\nu^{(0)} + m_\nu^{(1)})_{ij}}{(m_\nu^{(0)})_{ij}}$$



Usual Type-1 Seesaw

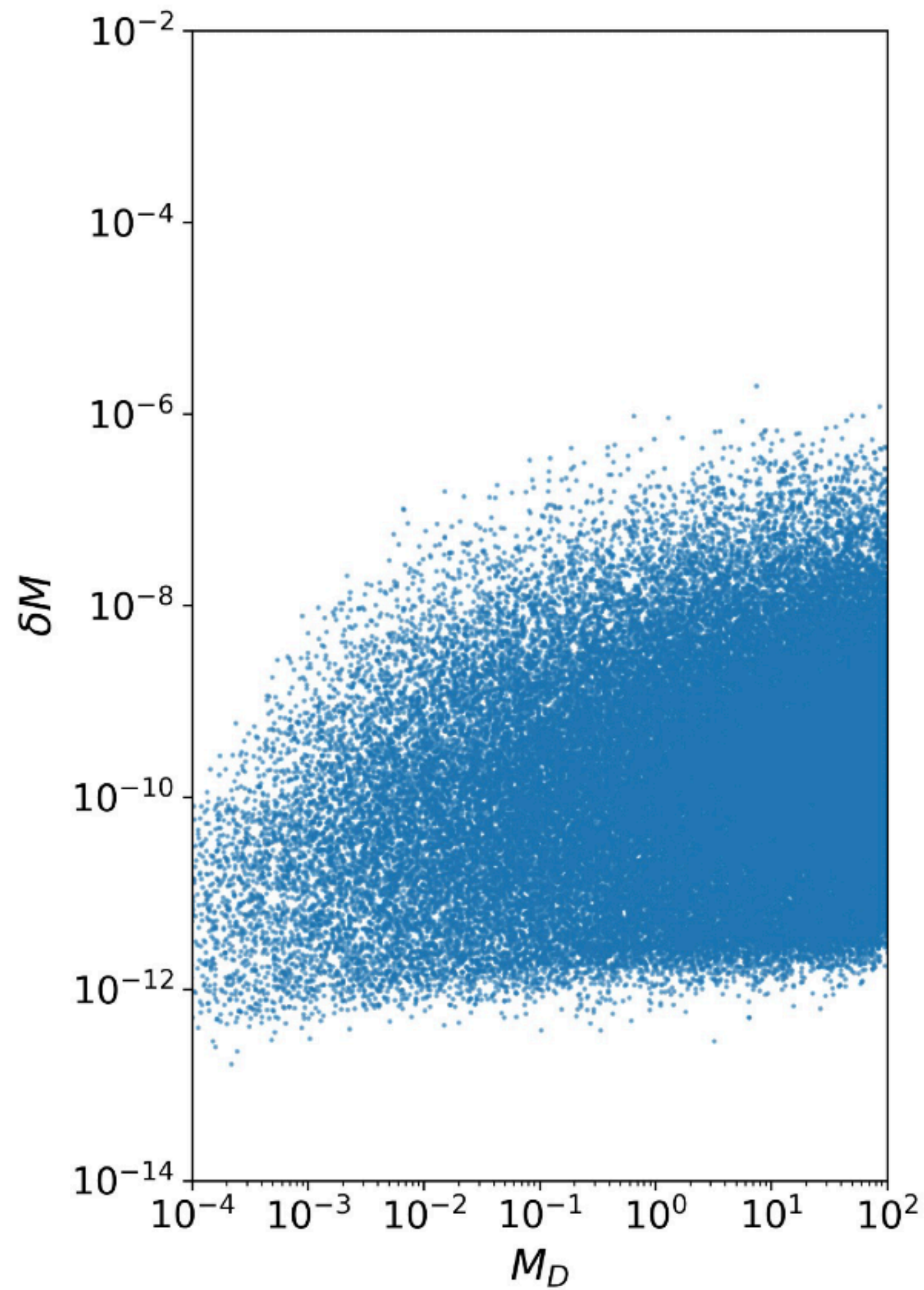
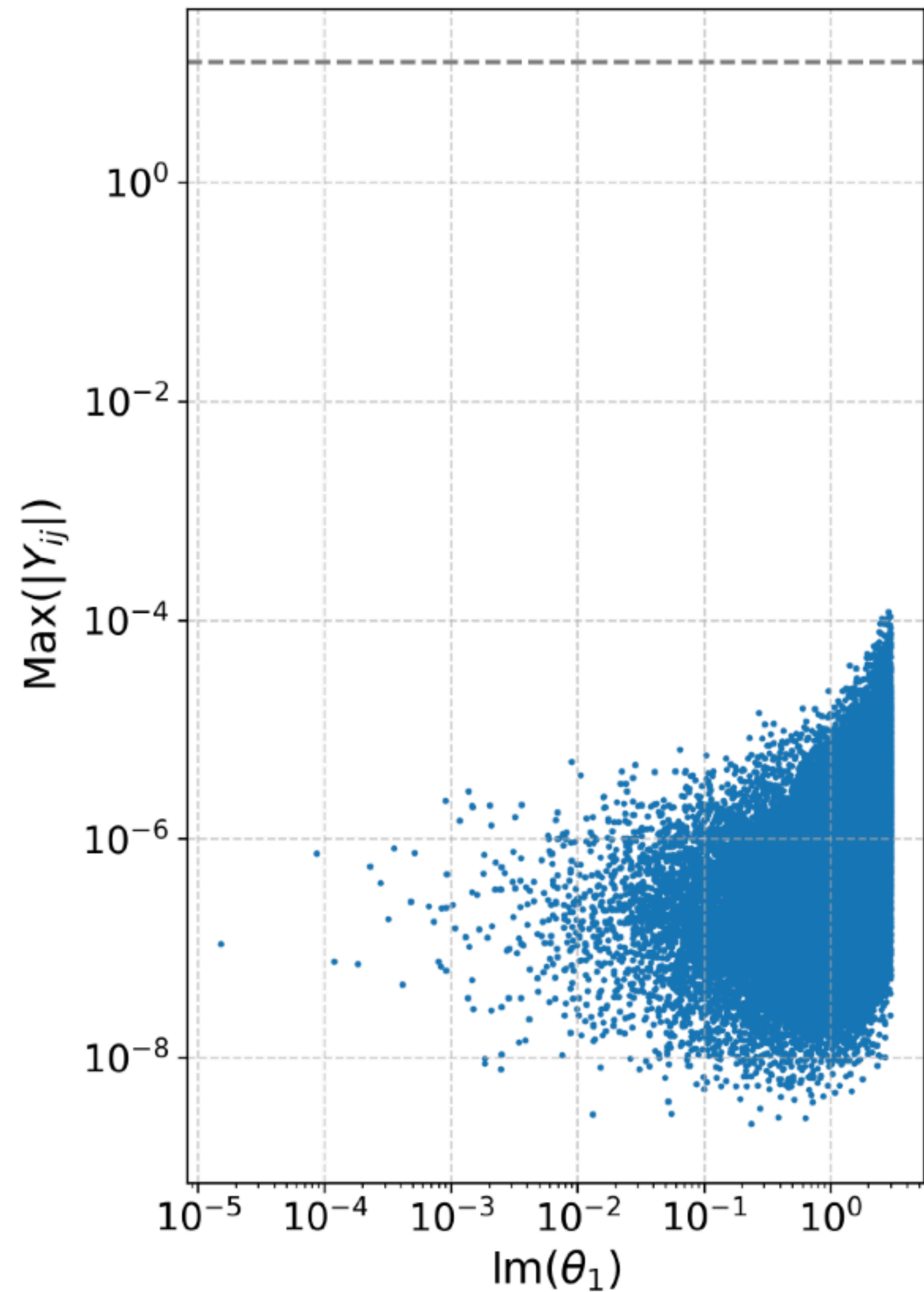
$$m_\nu^{(0)} = -M_D M_R^{-1} M_D^T \quad \longrightarrow \quad M_D = Y \frac{V}{\sqrt{2}} \quad \longrightarrow \quad m_\nu \simeq \frac{Y_\nu^2 v^2}{M_R}$$

$$m_\nu \simeq 0.05 \text{ eV}, \quad V = 246 \text{ GeV}$$

$$M_R = 10^{15} \text{ GeV} \Rightarrow Y_\nu \sim 1$$

$$M_R = 1 \text{ GeV} \Rightarrow Y_\nu \sim 10^{-8}$$

Case 1: $z_1 = z_2 = z_3 = 3$



$$z_i \equiv \left| \max \left(\text{Im}(\theta_i) \right) \right|$$

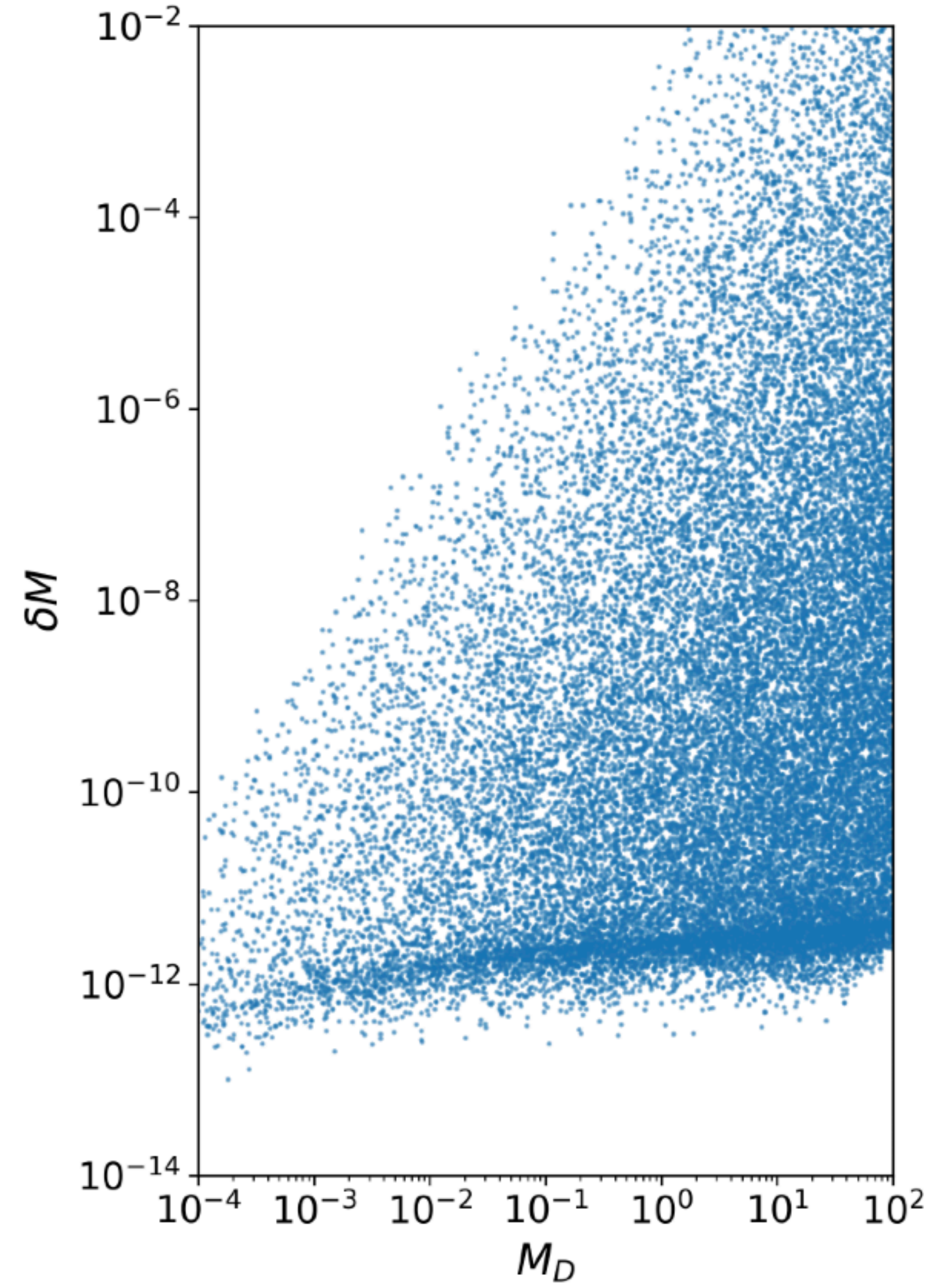
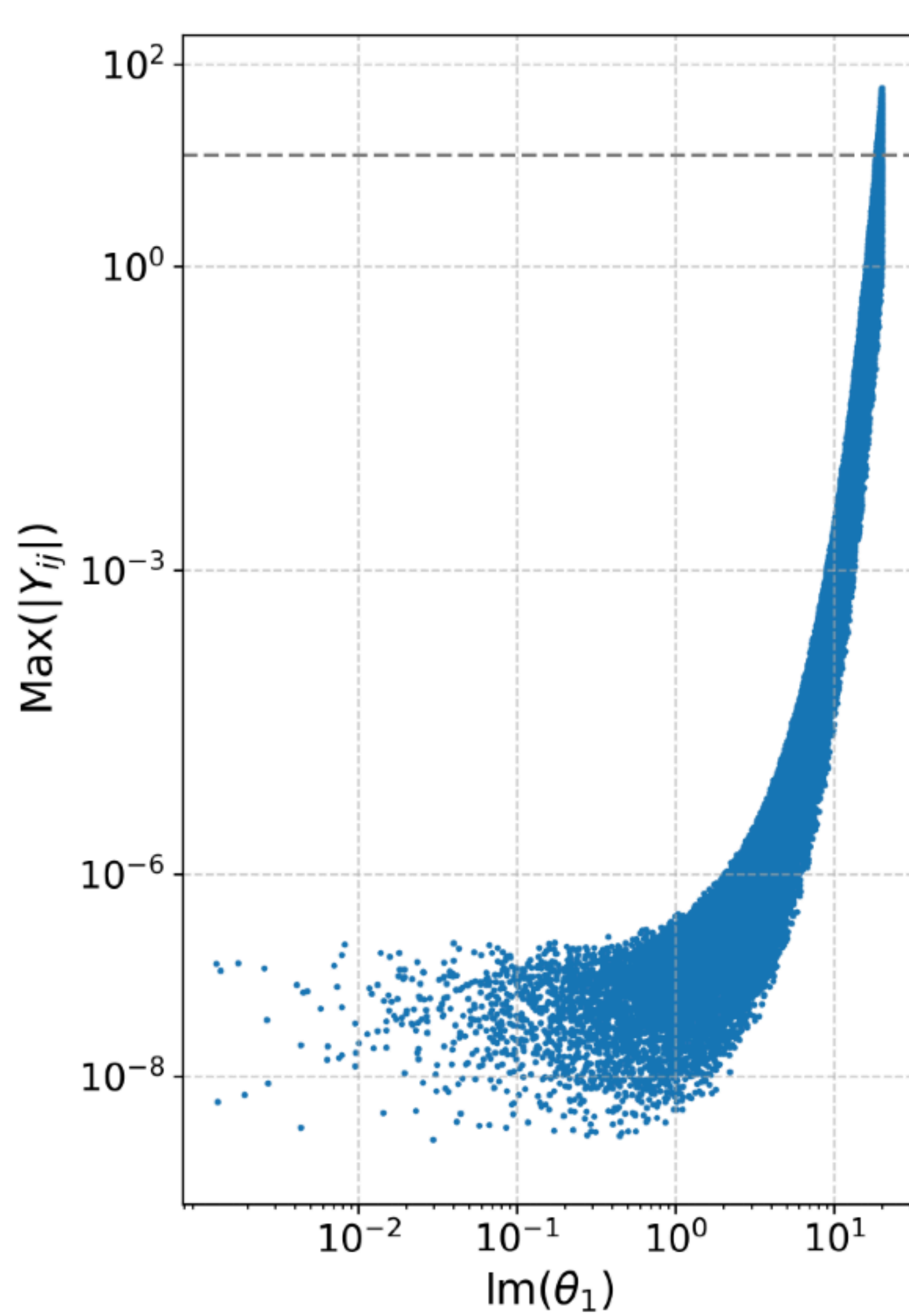
$$M_D = Y \frac{V}{\sqrt{2}}$$

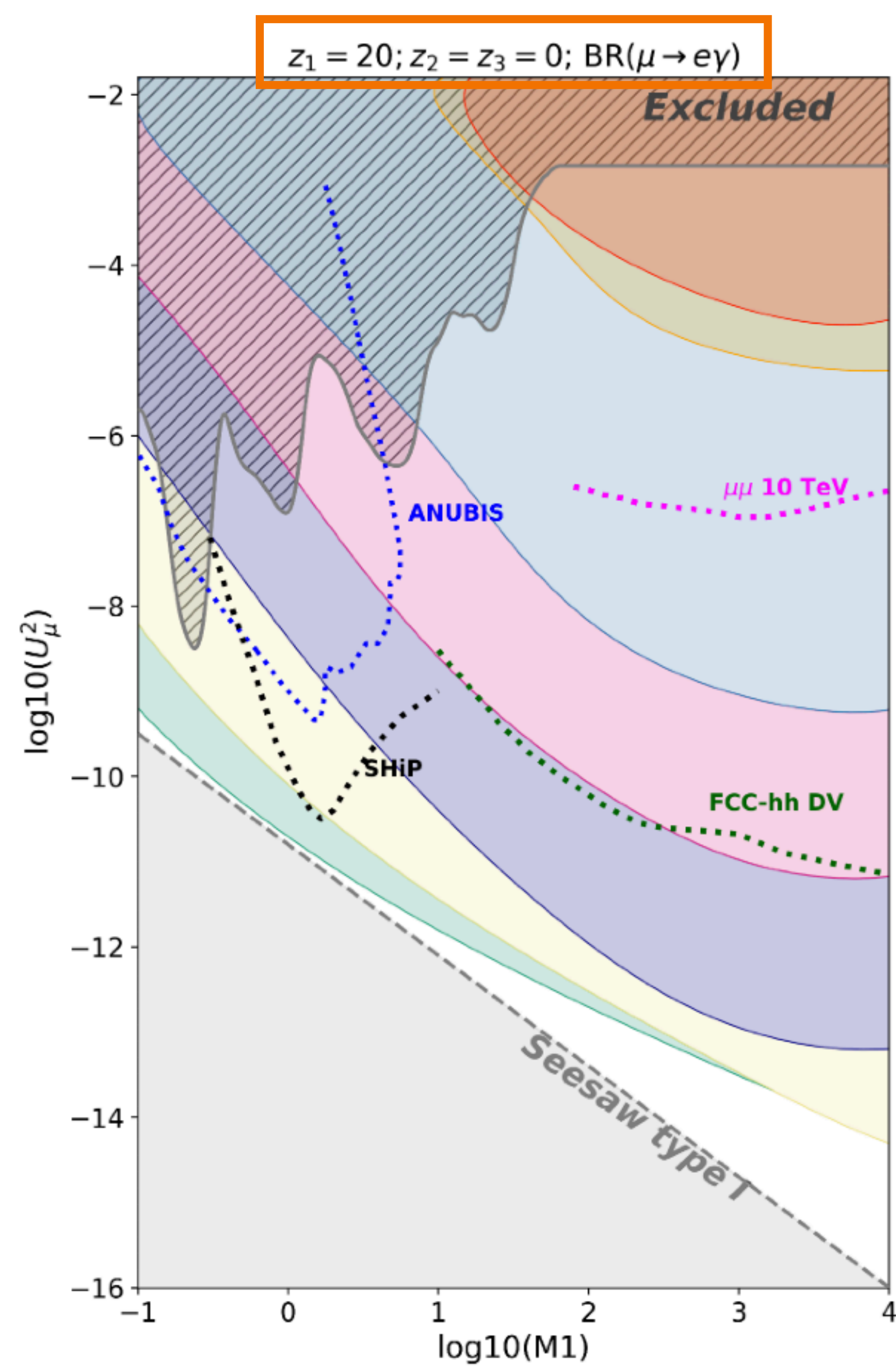
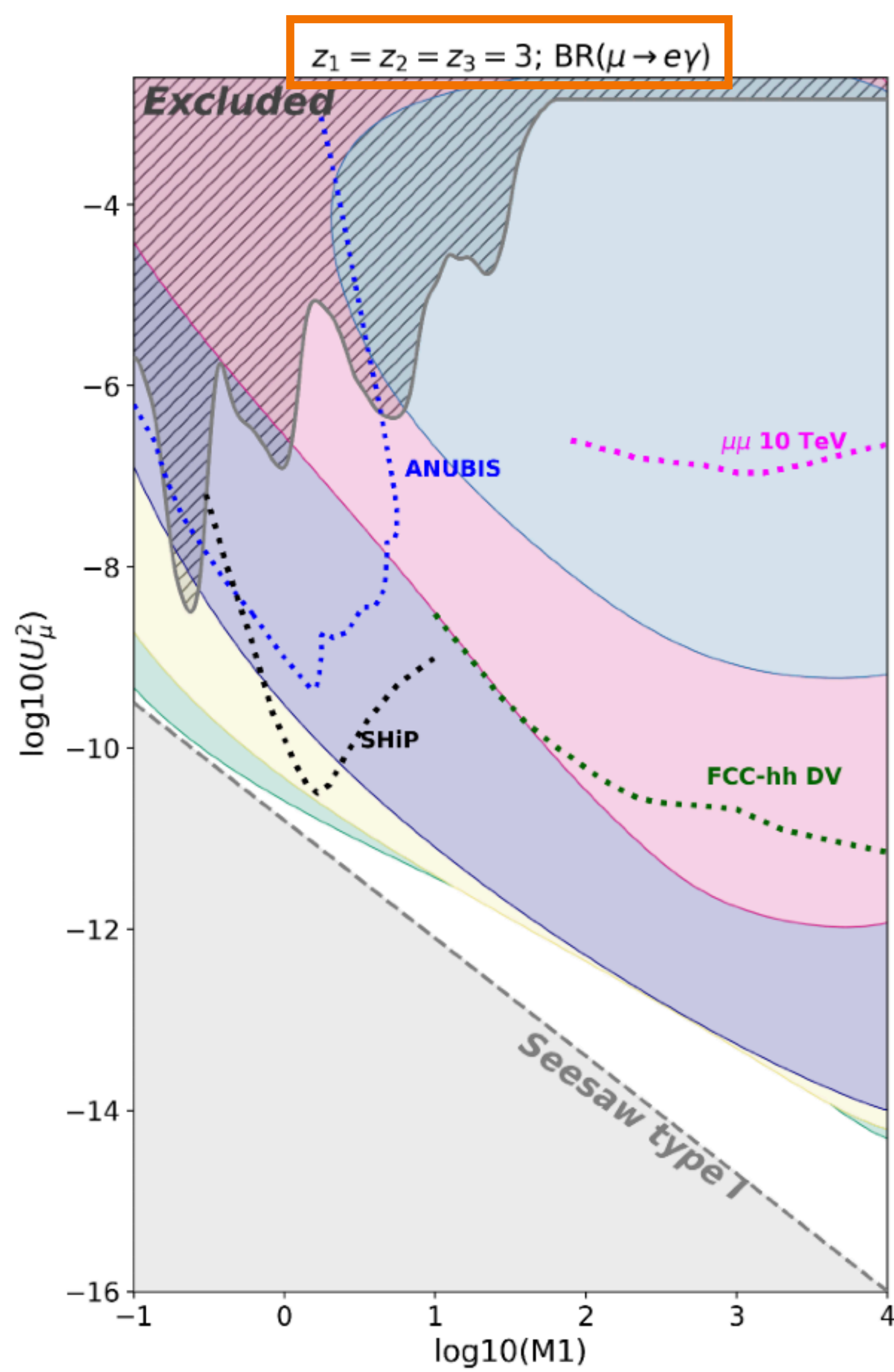
$$R \propto e^{\text{Im}(\theta)}$$

$$M_D \uparrow \Rightarrow Y_\nu \uparrow$$

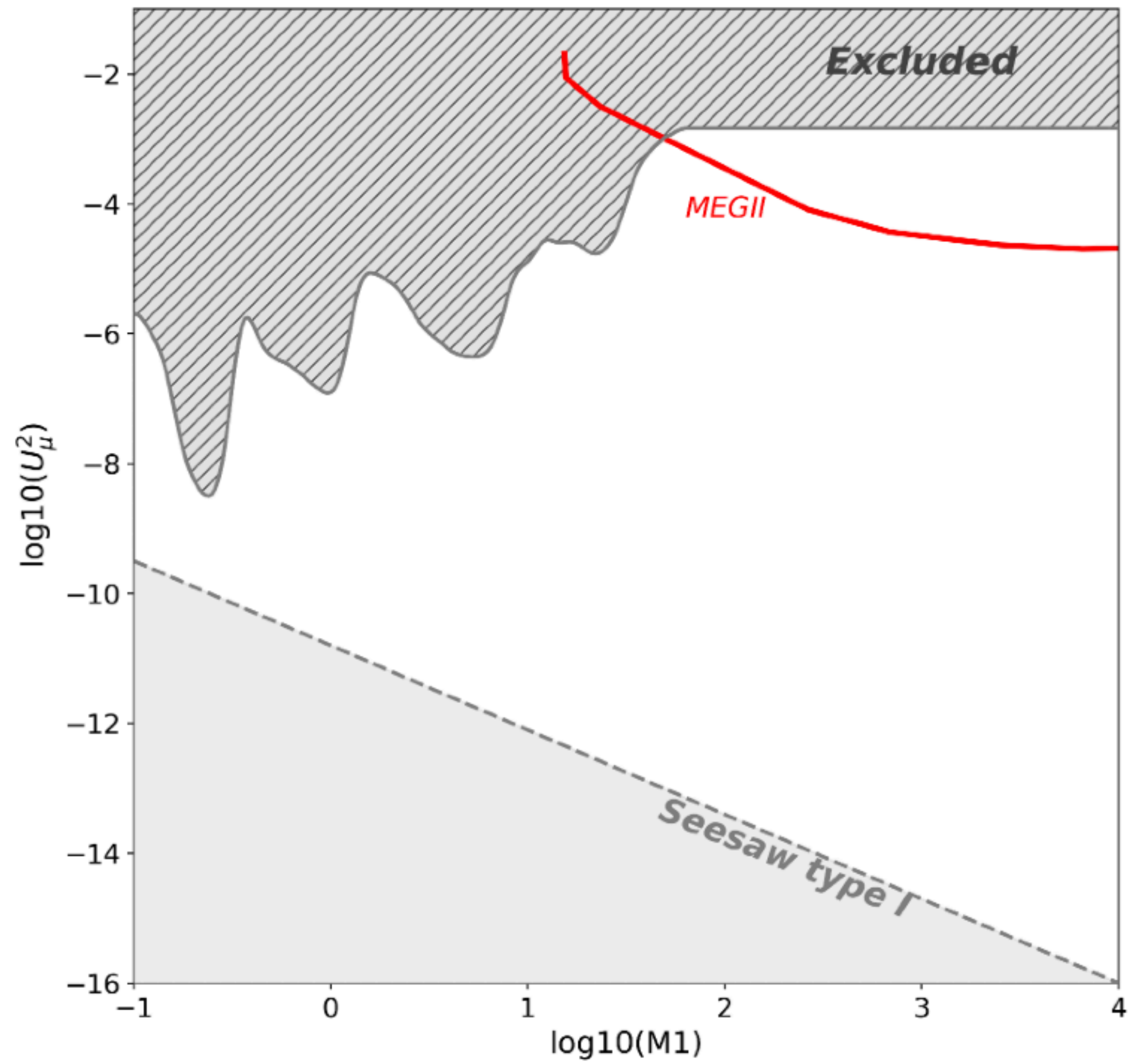
$$U^2 \sim \frac{M_D^2}{M_R^2} \uparrow$$

Case 2: $z_1 = 20; z_2 = z_3 = 0$

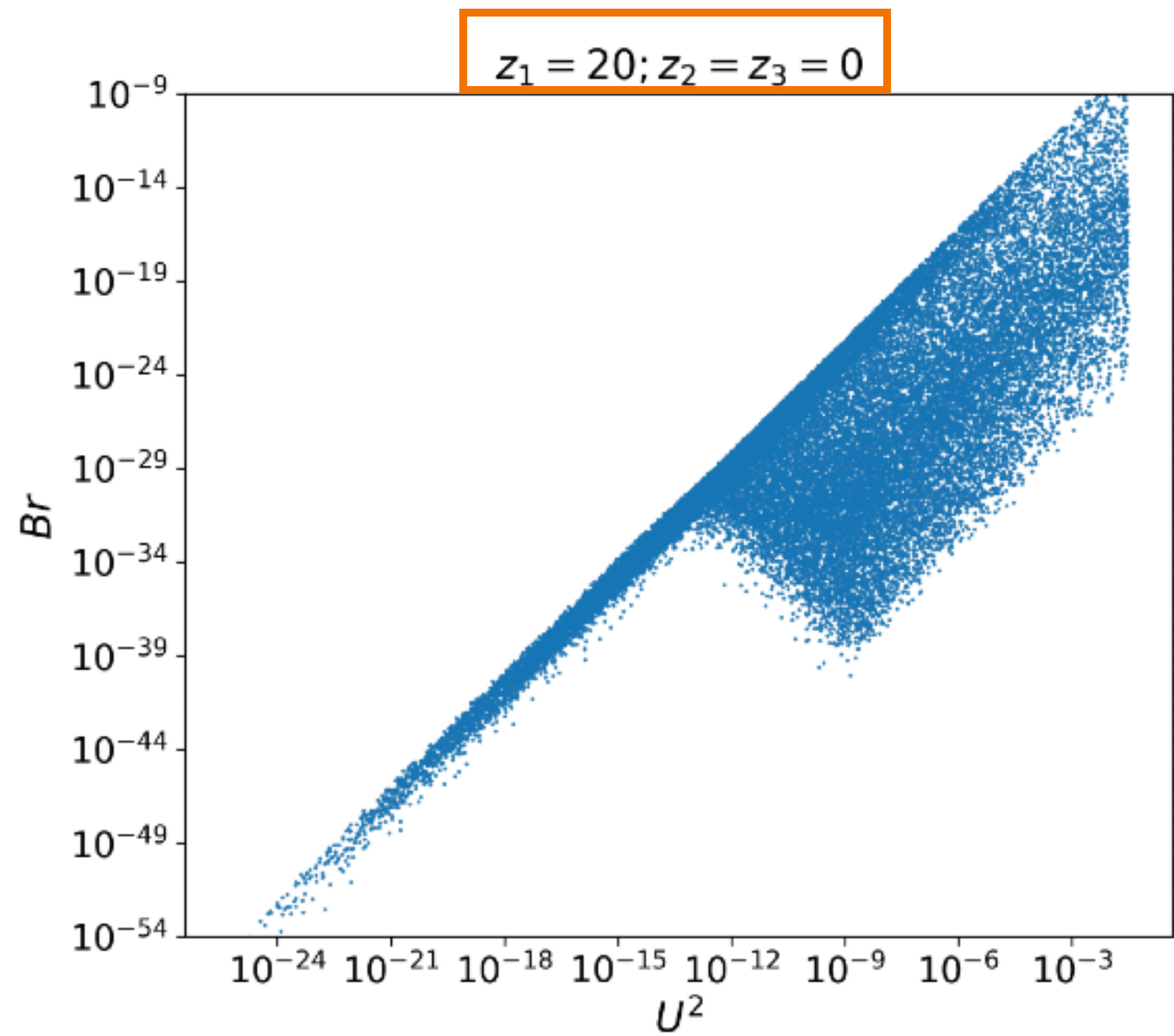
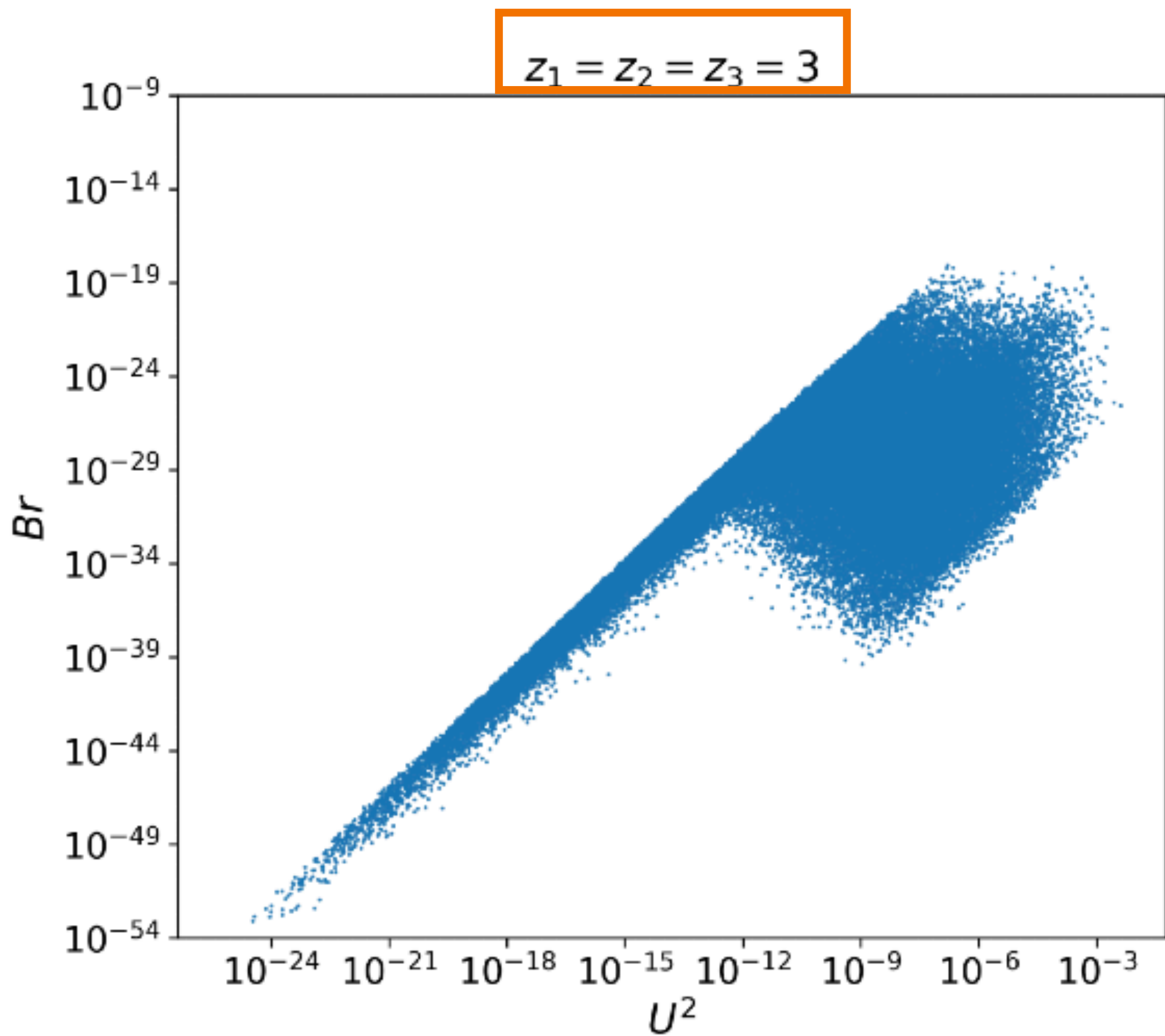




- $Br > 10^{-35}$
- $Br > 10^{-32}$
- $Br > 10^{-30}$
- $Br > 10^{-26}$
- $Br > 10^{-22}$
- $Br > 10^{-14}$
- $Br > 10^{-13}$



$$U_2 \equiv \sum_{\alpha=e,\mu,\tau} \sum_{i=4}^6 |U_{\alpha i}|^2$$



$$m_\nu = m_\nu^{(0)} + \delta M \equiv - M_D^{(1)T} \frac{1}{M'_R} M_D^{(1)}$$

$$\frac{1}{M'_R} = \frac{1}{M_R} (1 - \Sigma_1)$$

$$M_D^{(1)} = i\sqrt{M'_R} R \sqrt{m_\nu} U_{PMNS}$$

$$\delta M' = M_D^{(1)T} \Sigma_1 M_D^{(1)}$$

$$M'_{tot} = \begin{pmatrix} \delta M' & M_D^{(1)T} \\ M_D^{(1)} & M_R \end{pmatrix} \longrightarrow U$$

$$R^T \Sigma_1 R \sim \sinh^2(\text{Im } \theta_i)$$

Loop corrections break
cancellation $\rightarrow R$ entries
directly affect m_ν structure

Mass Splitting

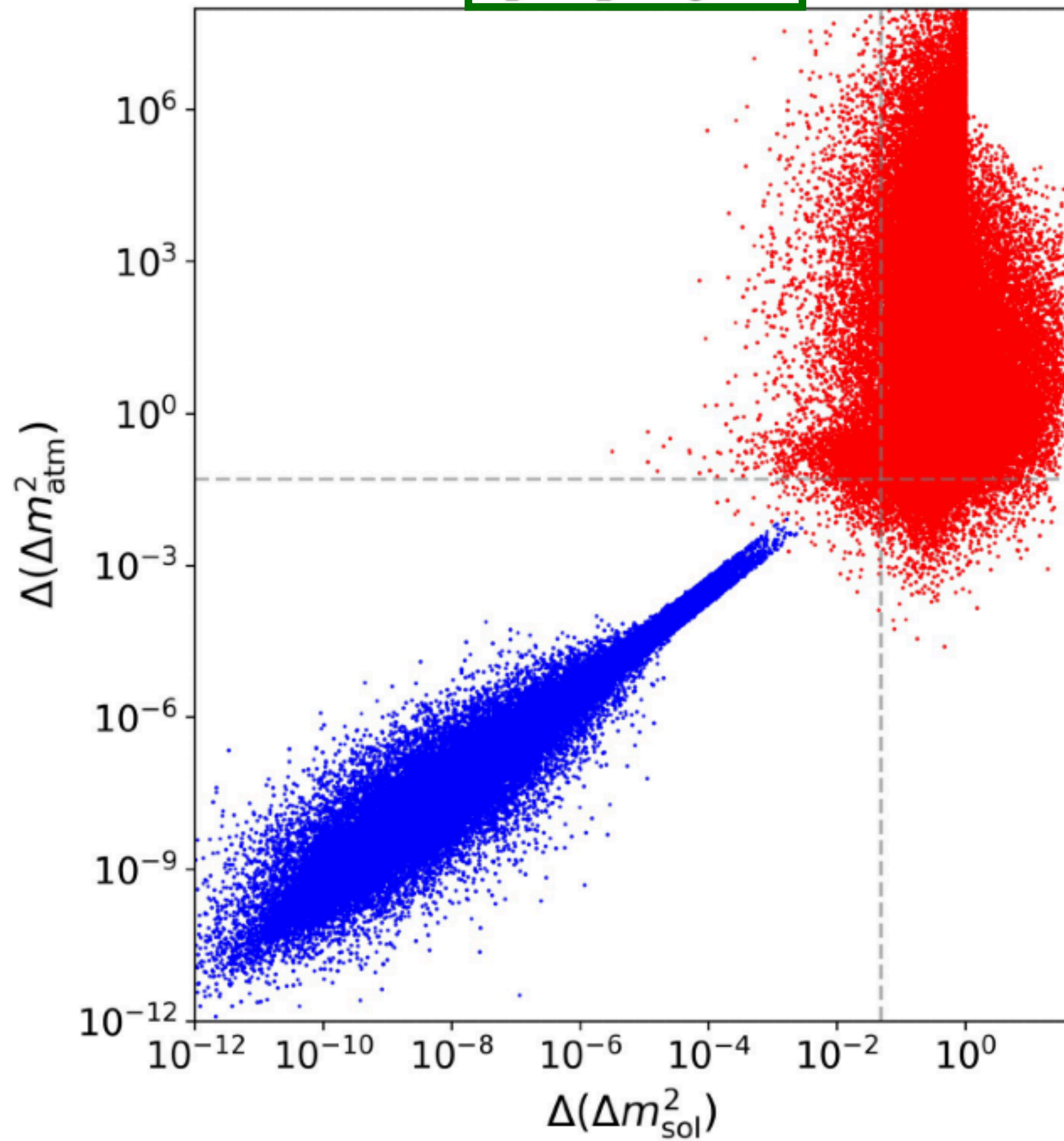
$$U^T M_{tot}'^\dagger M_{tot}' U = \text{diag}(m_1^2, m_2^2, m_3^2, M_4^2, M_5^2, M_6^2)$$

$$\Delta m_{sol}^2 = m_2^2 - m_1^2, \quad \Delta m_{atm}^2 = m_3^2 - m_1^2$$

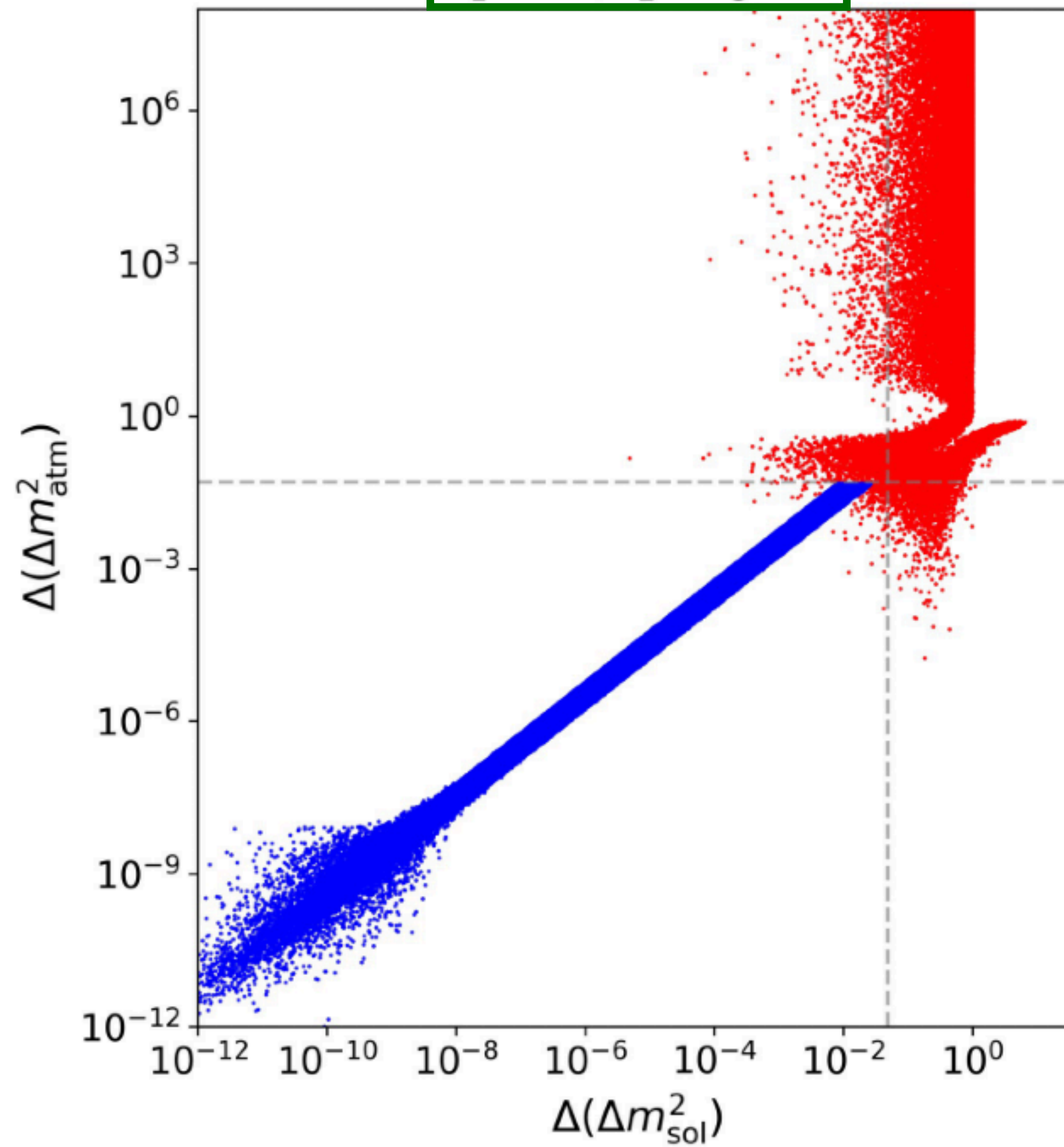
$$\Delta(\Delta m_{sol}^2) = \frac{\Delta m_{sol, \text{rec}}^2 - \Delta m_{sol, \text{exp}}^2}{\Delta m_{sol, \text{exp}}^2},$$

$$\Delta(\Delta m_{atm}^2) = \frac{\Delta m_{atm, \text{rec}}^2 - \Delta m_{atm, \text{exp}}^2}{\Delta m_{atm, \text{exp}}^2}$$

$$z_1 = z_2 = z_3 = 3$$



$$z_1 = 20; z_2 = z_3 = 0$$



Mixing Angles

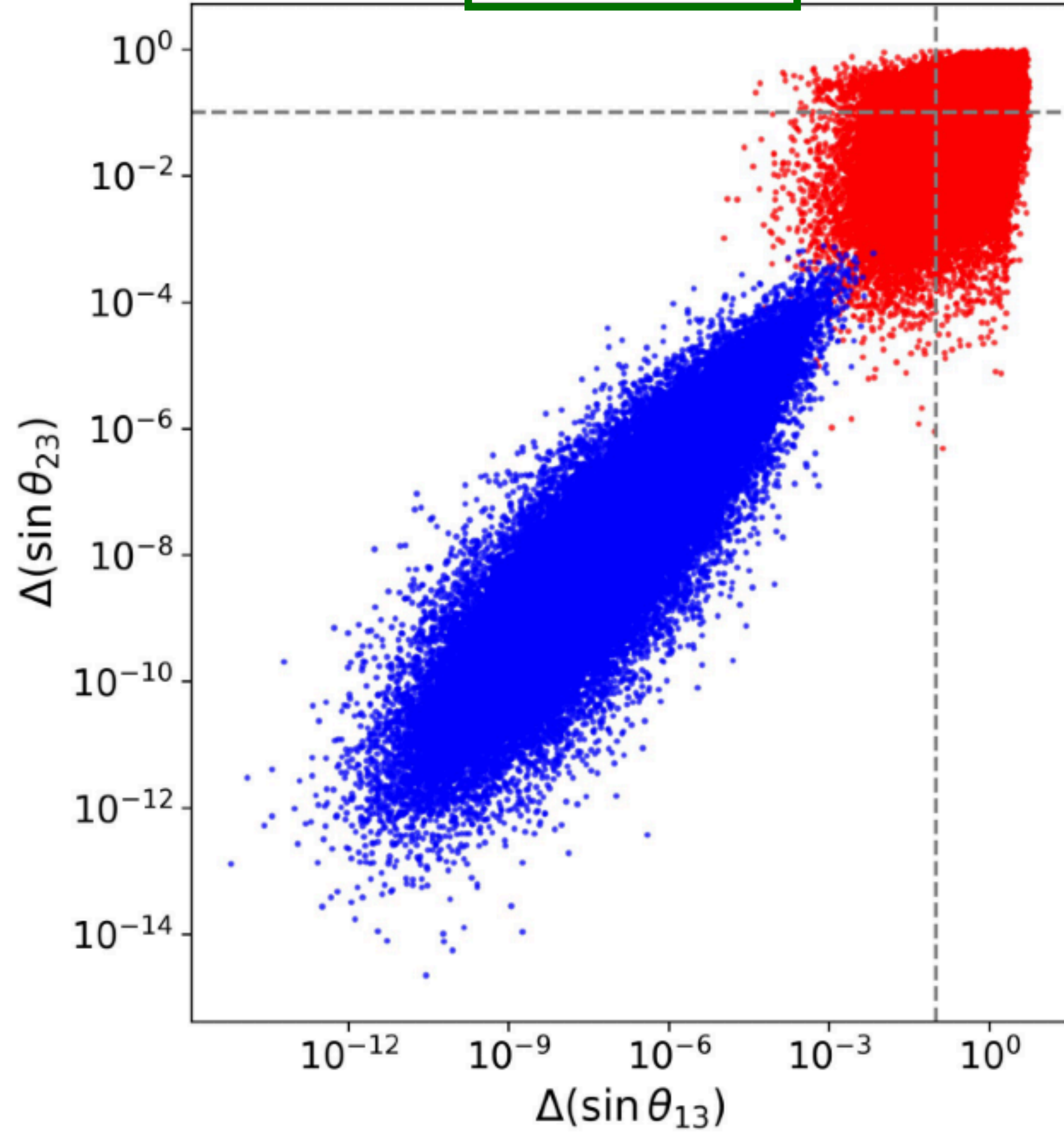
$$M'_{\text{tot}} \rightarrow U \rightarrow \sin \theta_{ij}^{\text{rec}} \rightarrow \Delta(\sin \theta_{ij})$$

$$\Delta(\sin \theta_{12}) = \frac{\sin \theta_{12,\text{rec}} - \sin \theta_{12,\text{exp}}}{\sin \theta_{12,\text{exp}}}$$

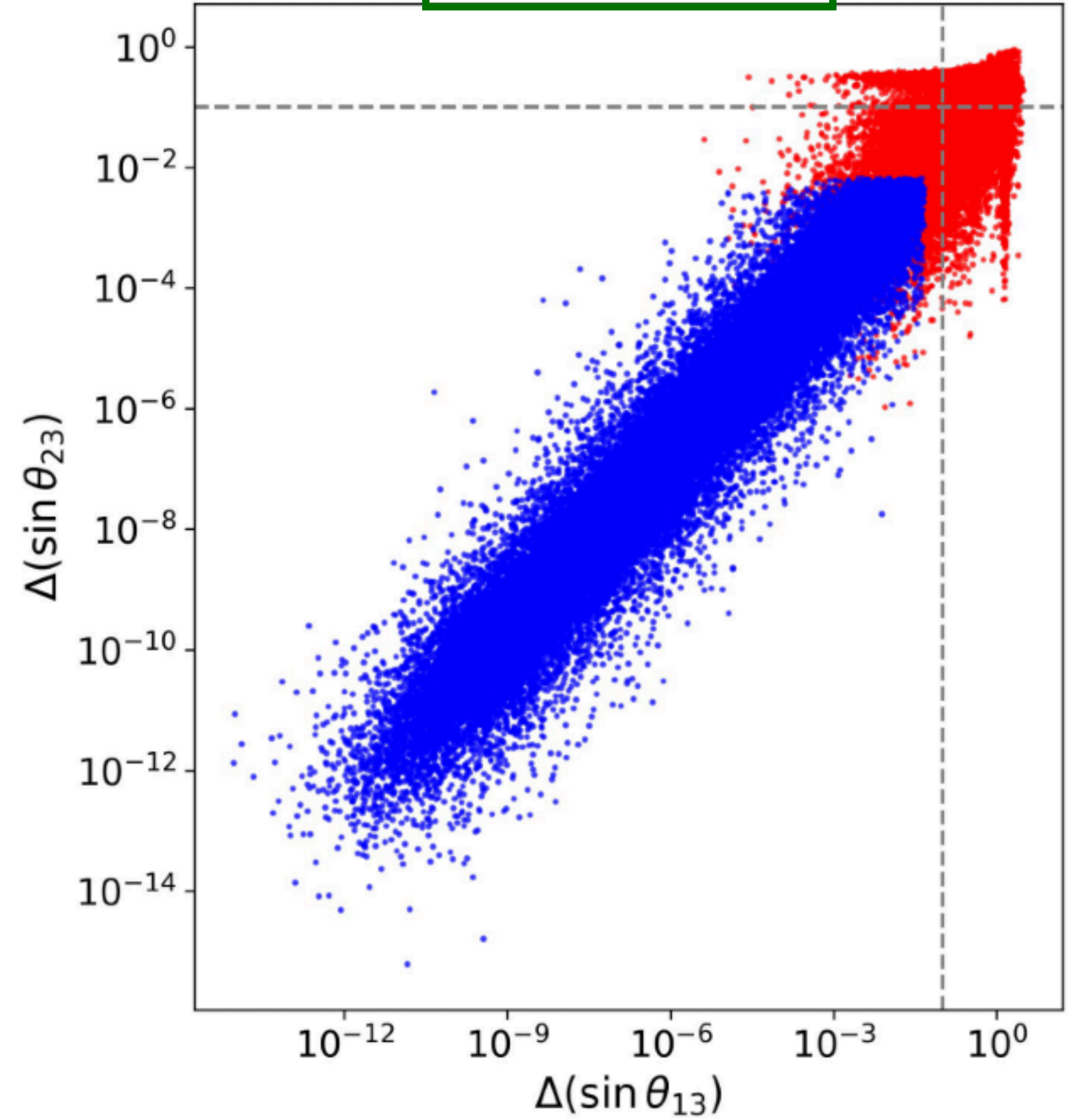
$$\Delta(\sin \theta_{13}) = \frac{\sin \theta_{13,\text{rec}} - \sin \theta_{13,\text{exp}}}{\sin \theta_{13,\text{exp}}}$$

$$\Delta(\sin \theta_{23}) = \frac{\sin \theta_{23,\text{rec}} - \sin \theta_{23,\text{exp}}}{\sin \theta_{23,\text{exp}}}$$

$$z_1 = z_2 = z_3 = 3$$



$$z_1 = 20; z_2 = z_3 = 0$$



Conclusions

- Low-energy models are viable alternatives to the standard scale.
- MEGII limits on $Br(\mu \rightarrow e\gamma)$ provide competitive constraints for $M_R > 100$ GeV.
- The complex CI is incorrect under loop effects.

Thanks!

Questions?

Effect of higher order corrections

$$\Sigma_2 = \frac{1}{16\pi^2} \Sigma_1$$
$$\Sigma_3 = \frac{1}{(16\pi^2)^2} \Sigma_1$$

