Ciencia y Tecnología
Secretaría de Ciencia, Humanidades, Tecnología e Innovación

Graph theory-based automated quantum algorithm for efficient querying of acyclic and multiloop causal configurations

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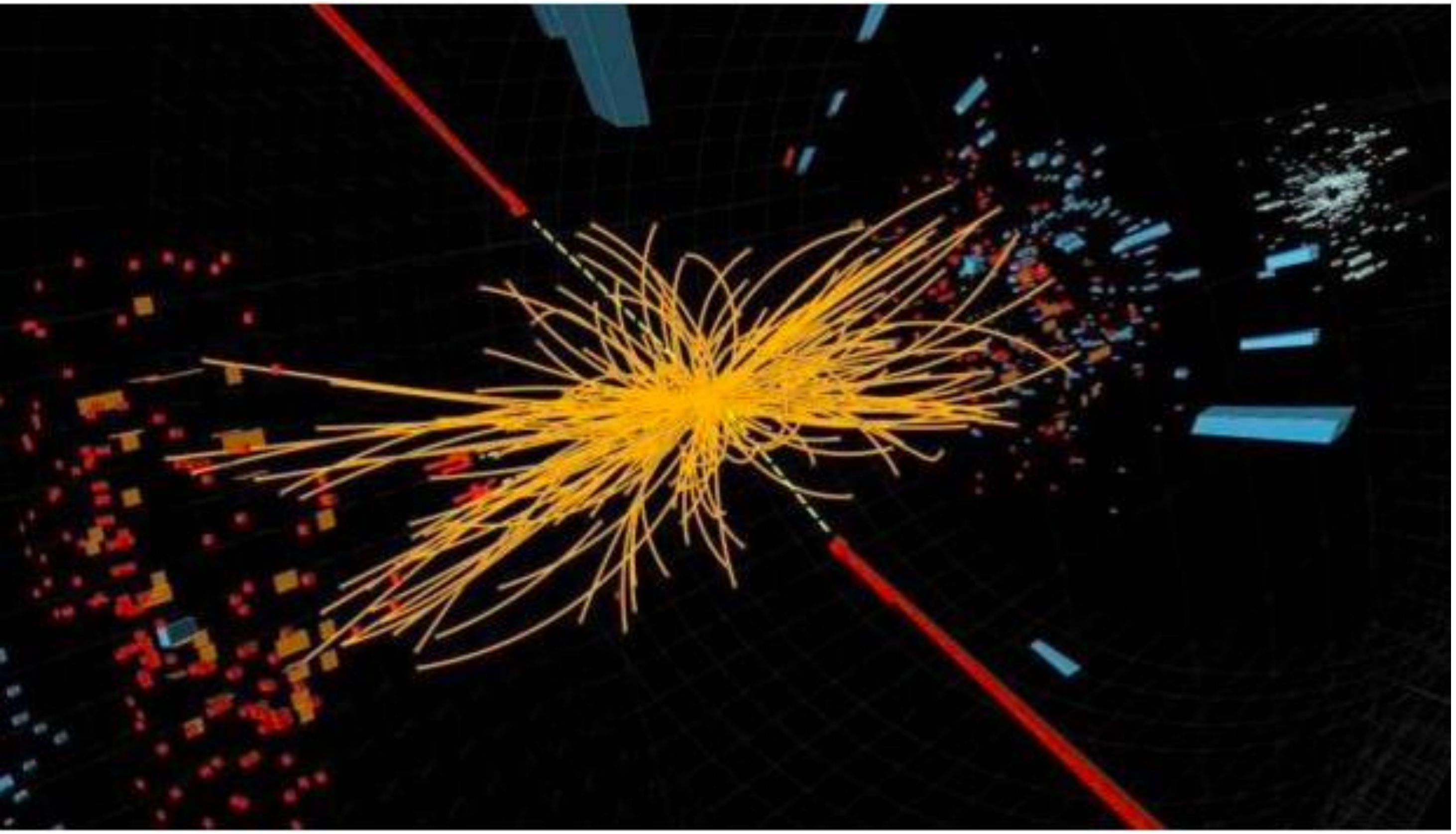
1. Motivation
2. Causality from the Loop-Tree Duality
3. Quantum algorithms for querying causality of multiloop graphs
4. The Minimum Clique-optimised quantum algorithm
5. Implementation of the MCA quantum algorithm to multiloop topologies
6. Conclusions

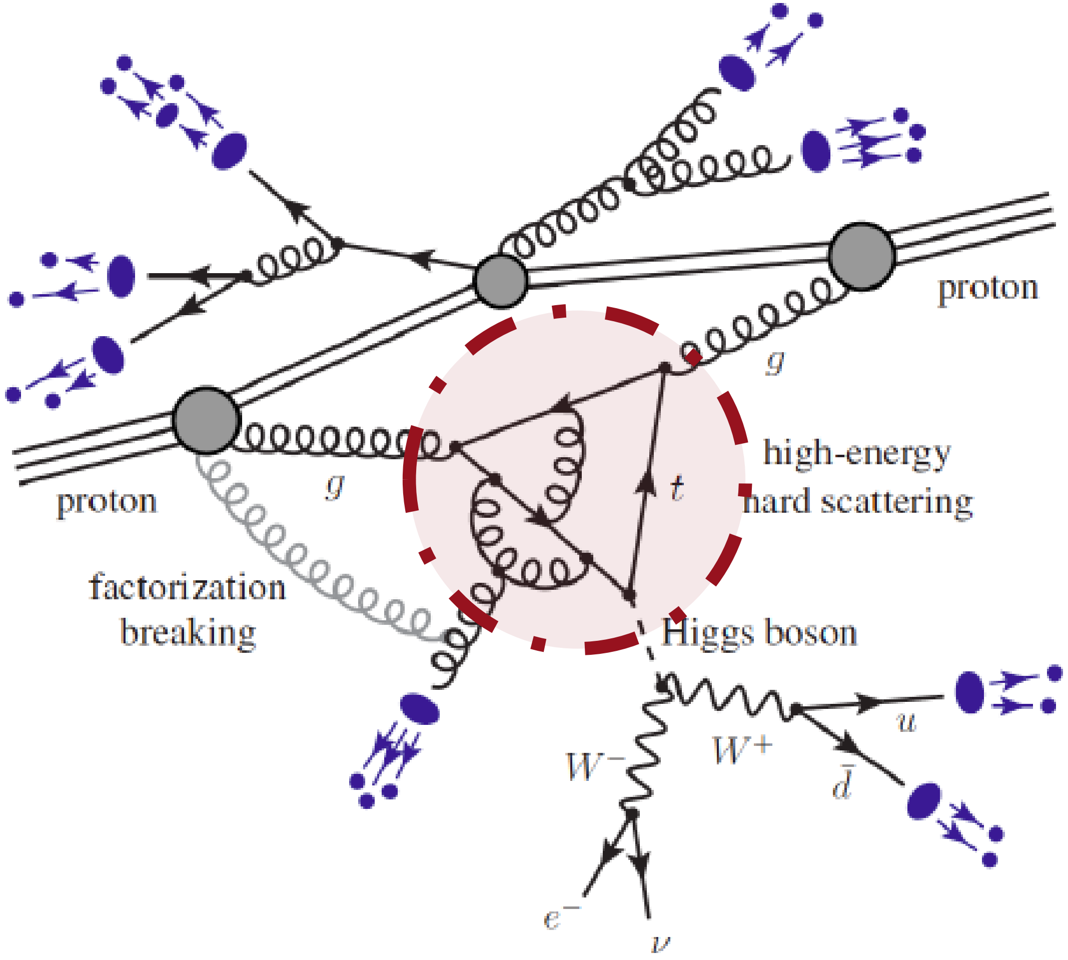
A FACT:

Challenging **demands** are pushing the **precision frontier** towards more accurate theoretical predictions beyond the current state of the art.

Motivate exploring **new technologies**

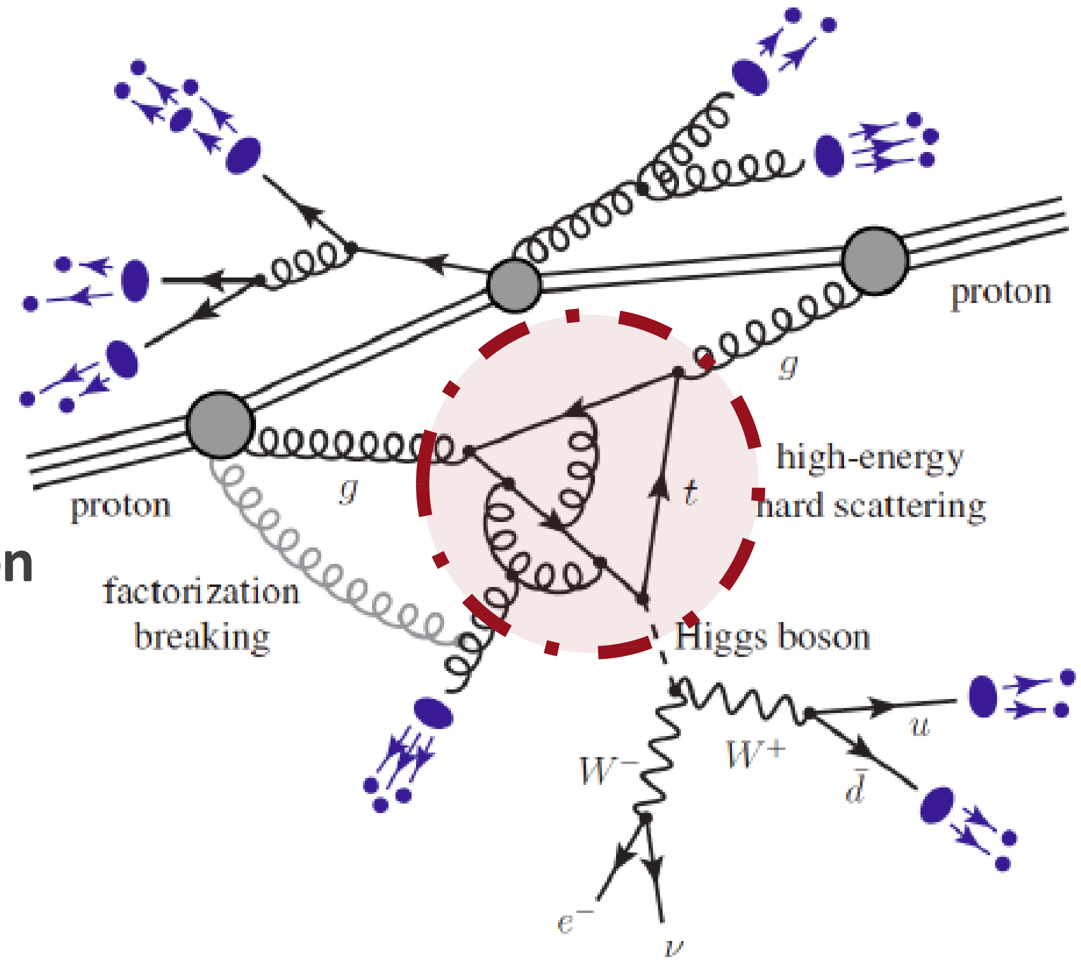
Theorist view





- **Factorization** into short distance (hard scattering) and long distance (initial and final state)
- **QFT** = quantum mechanics + space-time + creation-annhilation of particles

- Track reconstruction
- Parton densities
- Parton showers
- Quantum machine learning
- Monte Carlo integration
- Tree-level helicity amplitudes
- Jets in a medium
- Jet clustering



- ❑ **Multiloop scattering amplitudes**

Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
Clemente, Crippa, Jansen, Ramírez, Rentería, GR, Sborlini, Vale Silva, 2210.13240

Bootstrap the LTD causal representation.

Scattering amplitude in the Feynman representation

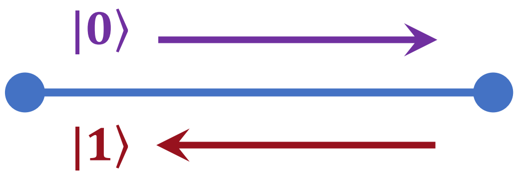
$$\mathcal{A}_{\mathrm{F}}^{(L)} = \int_{\ell_1 \ldots \ell_L} \mathcal{N}\left(\{\ell_s\}_L, \{k_j\}_P\right) \prod_{i=1}^{n} G_{\mathrm{F}}(q_i)$$

- L loop momenta
- P external particles
- n Feynman propagators

$$G_F(q_i) = \frac{1}{\left(q_{i,0} - q_{i,0}^{(+)}\right)\left(q_{i,0} + q_{i,0}^{(+)}\right)}$$

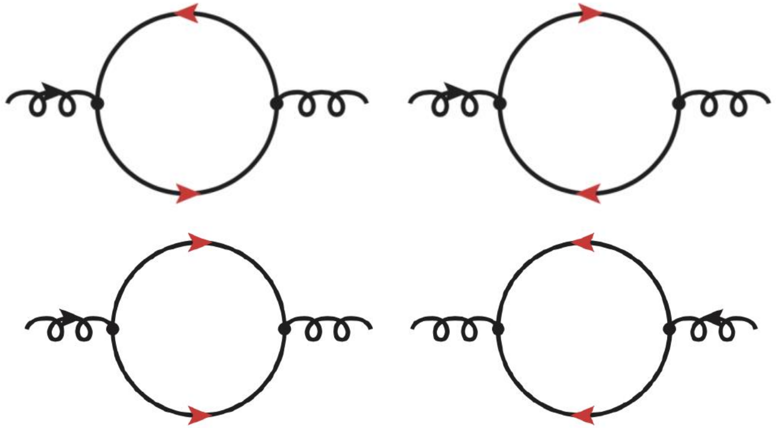
- $q_i = \ell + p_i$
- $q_{i,0}^{(+)} = \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$

Feynman propagators encode the propagation of a particle in either directions.



$$G_F(q_i) \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

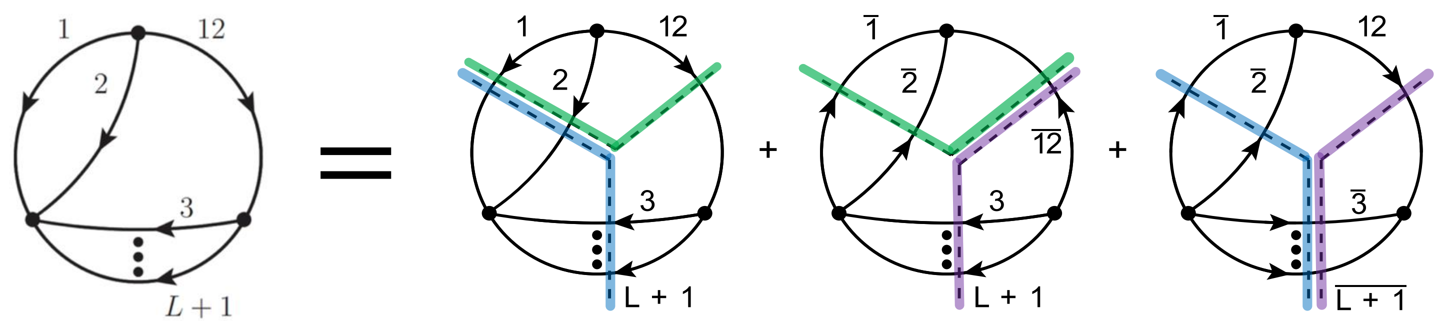
A multiloop Feynman diagram can be considered as a quantum superposition of 2^n states



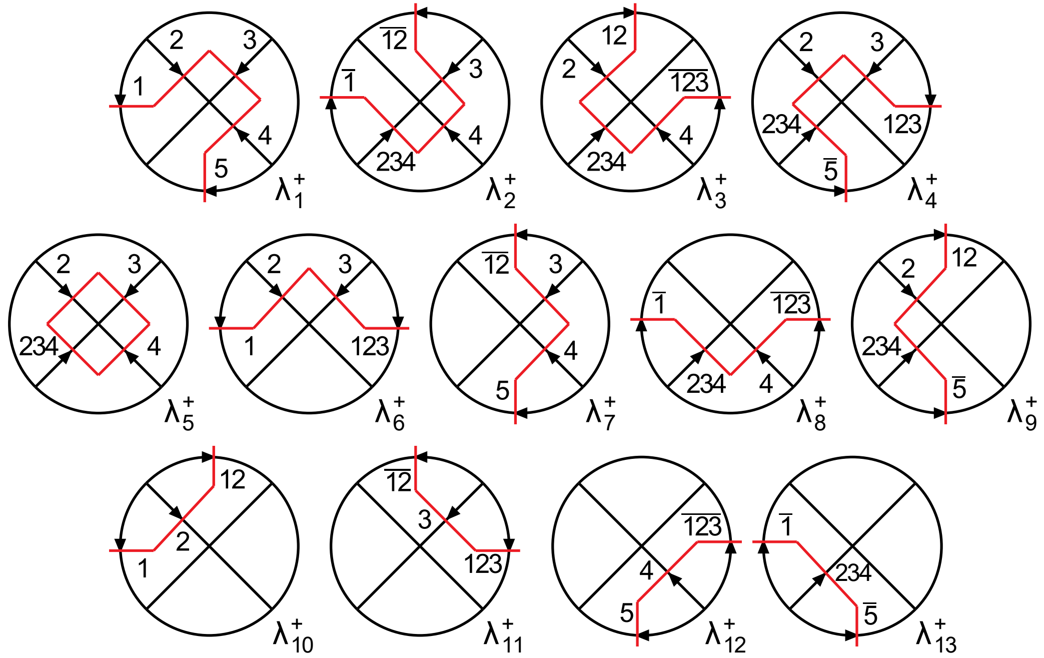
Cyclic state

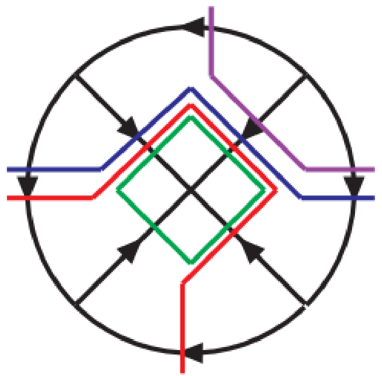
Acyclic state

The LTD causal representation

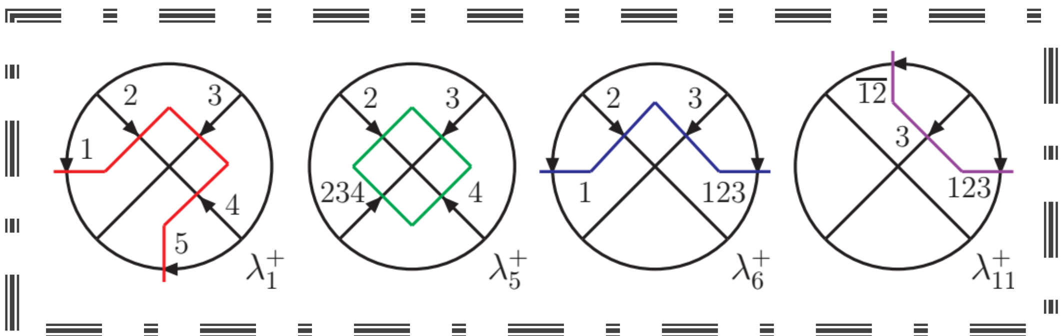


Manifestly free of unphysical singularities





Compatibility among **26 thresholds**

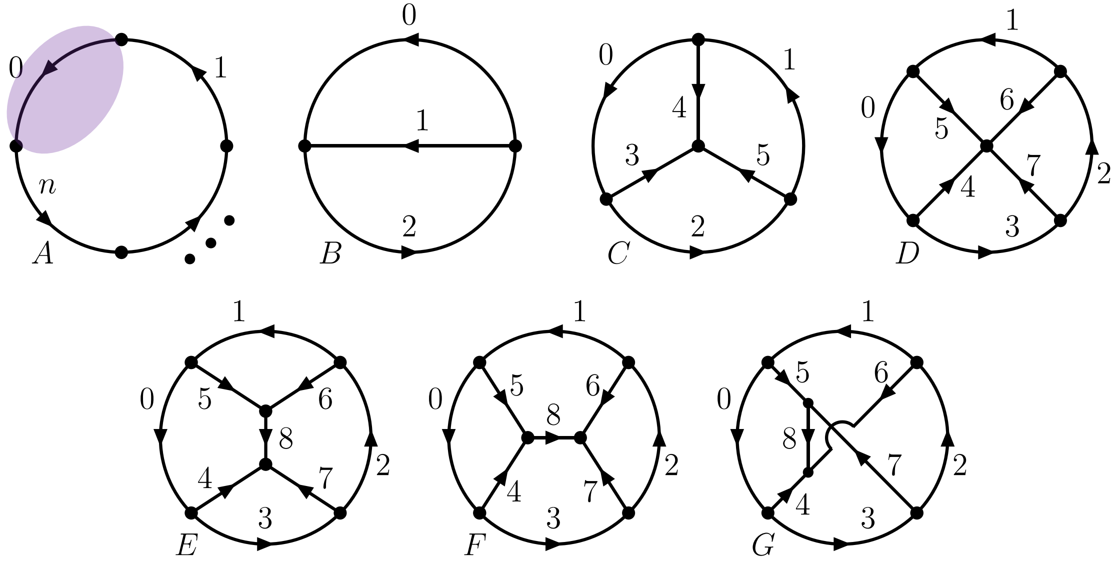


Quantum computing approach

Bootstrap the LTD causal representation of representative multiloop topologies.

○ S. Ramírez-Uribe, A. E. Rentería-Olivo, G. Rodrigo, G. F. R. Sborlini, and L. Vale Silva, "Quantum algorithm for Feynman loop integrals" , JHEP 05, 100 (2022), arXiv:2105.08703 [hep-ph].

Two possible on-shell states that we identify with the quantum sates |1⟩ and |0⟩



Causal representation is closely related to graph theory:

causal configurations are directed acyclic graphs

Quantum query algorithm based on Grover's algorithm

i Encode

ii Initialize

iii Oracle

iv Diffusion

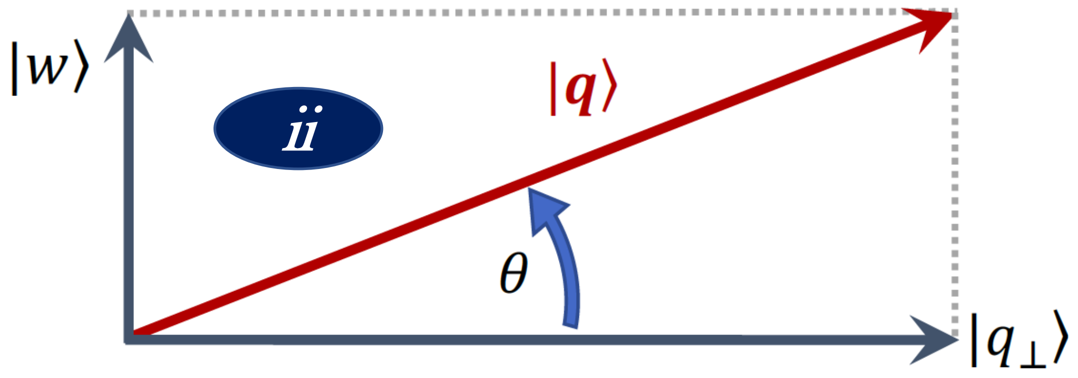
v Measure

Quantum query algorithm based on Grover's algorithm

i Encode

ii Initialize

$|\boldsymbol{q}\rangle = \cos\theta\,|q_\perp\rangle + \sin\theta\,|w\rangle$



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Quantum query algorithm based on Grover's algorithm

i Encode

ii Initialize

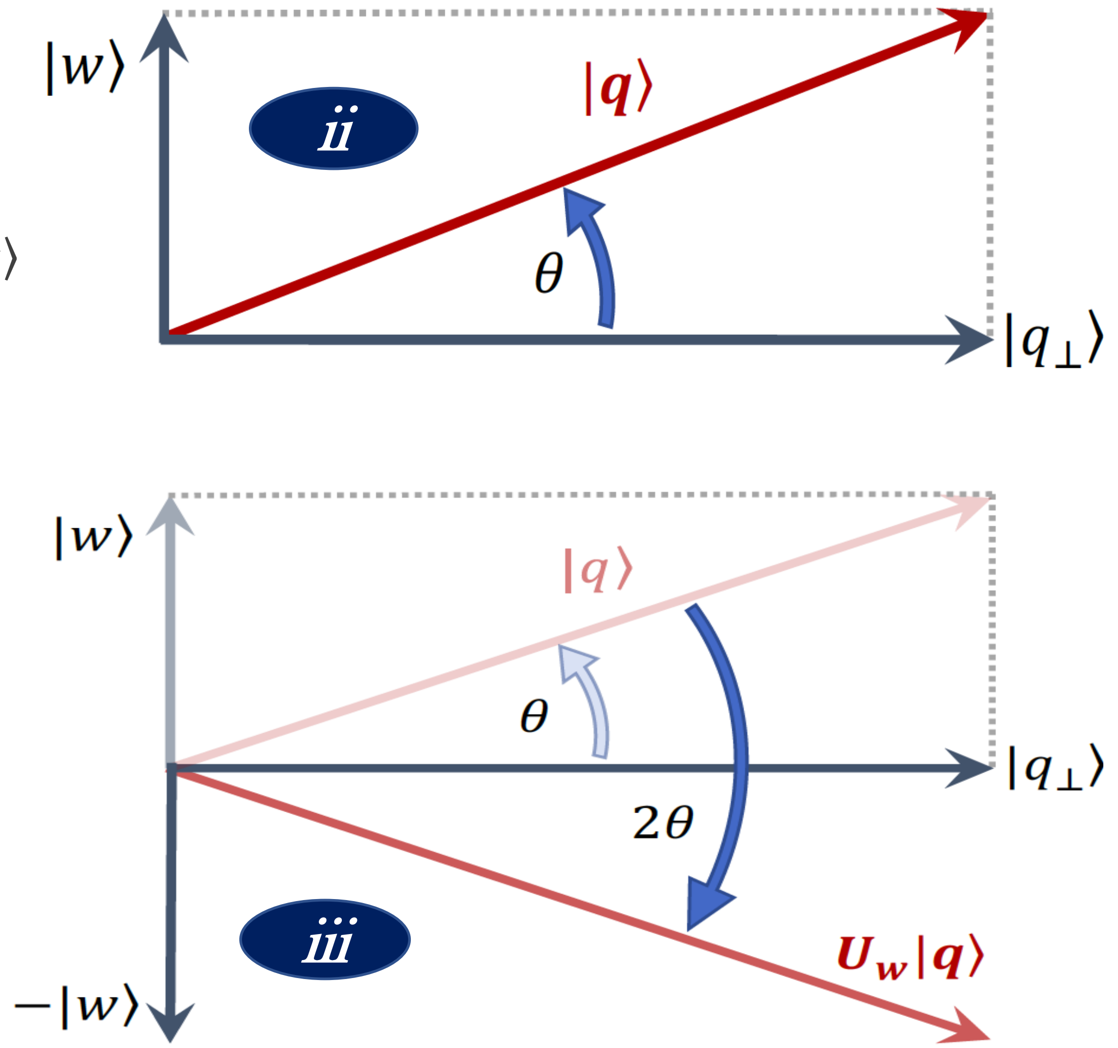
$|\boldsymbol{q}\rangle = \cos\theta\, |q_\perp\rangle + \sin\theta\, |w\rangle$

iii Oracle

$$U_w|x\rangle = \begin{cases} -|x\rangle, if\ x \in w \\ |x\rangle, if\ x \notin w \end{cases}$$

iv Diffusion

v Measure



Quantum query algorithm based on Grover's algorithm

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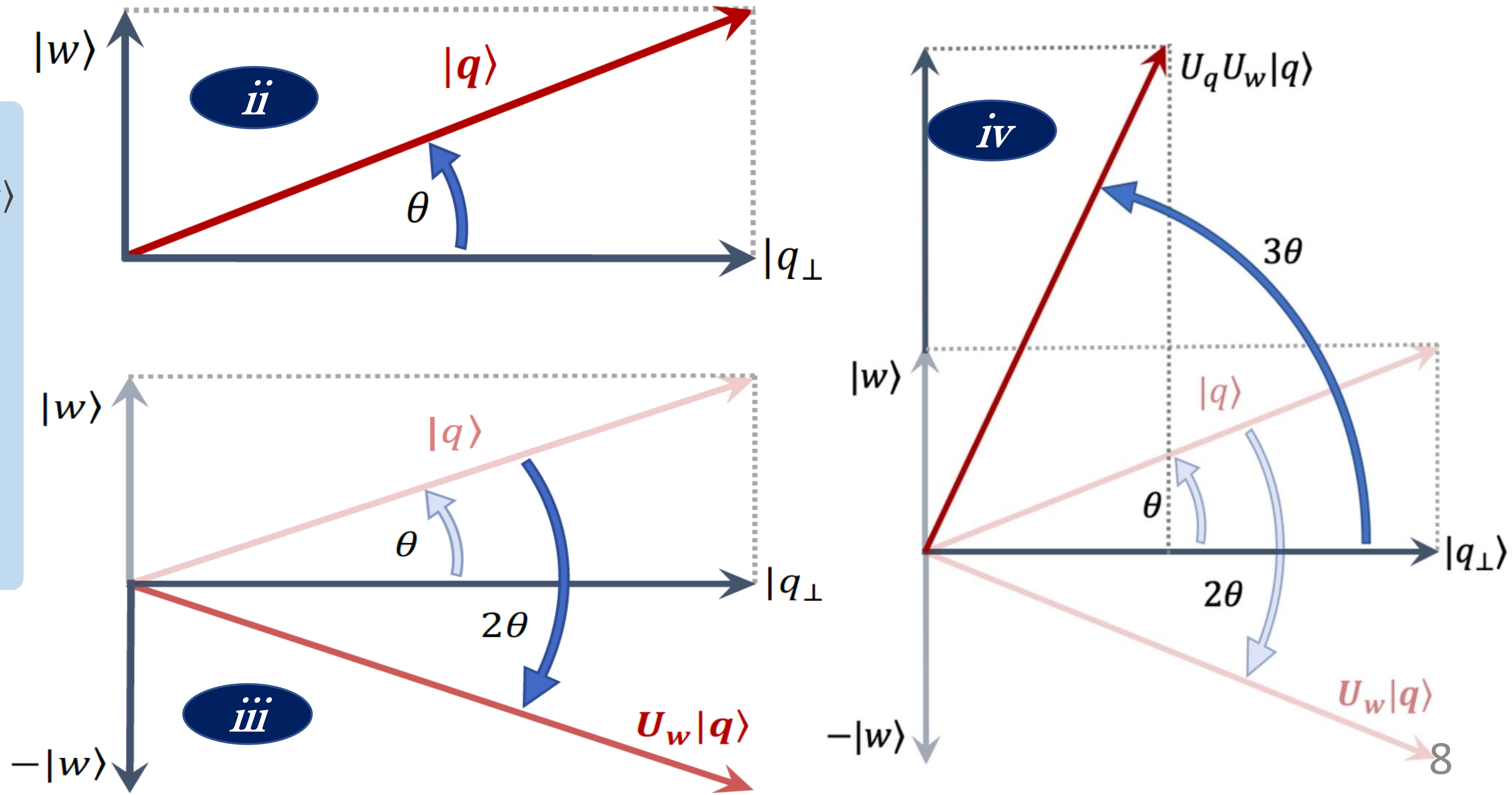
iv Diffusion

$U_q = 2|q\rangle\langle q| - \boldsymbol{I}$

v Measure



Quantum query algorithm based on Grover's algorithm

The iterative application of the U_w and U_q

$$\left(U_q U_w\right)^t |q\rangle = \cos\theta_t \, |q_\perp\rangle + \sin\theta_t \, |w\rangle \, ,$$
$$\theta_t = (2t+1)\theta$$

θ_t has to be chosen according to

Probability of a noncausal solution $\frac{\cos^2\theta_t}{N-r} \ll \frac{\sin^2\theta_t}{r}$ Probability of a causal solution

To obtain a desired performance

$$\theta \leq \pi/6 \; (r \leq N/4)$$

Quantum query algorithm based on Grover's algorithm

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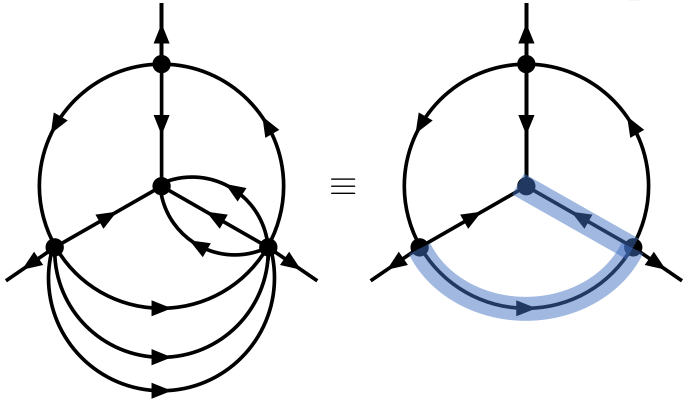
For Feynman loop integrals, given a causal solution (directed acyclic configuration) the mirror state is also a causal solution.

- Halve the total number of solutions. → Single out one of the edges

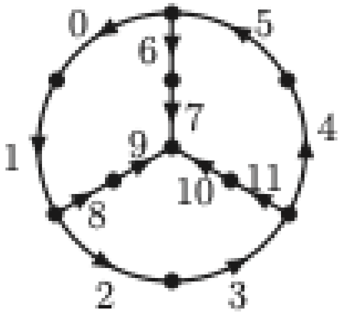
Boolean functions

- An edge is defined as the union of propagators connecting two interaction vertices.
- Eloops are the loops associated to a reduced graph

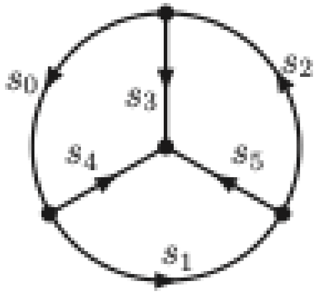
Three-eloop



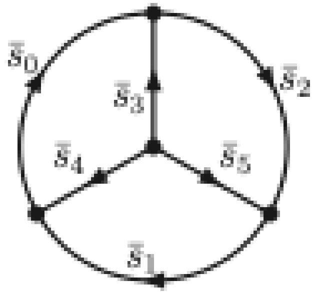
$e_i \equiv i$ $\qquad$ $s_j \equiv \bigwedge_{i \in z_j} e_i$ $\qquad$ $\bar{s}_j \equiv \bigwedge_{i \in z_j} \bar{e}_i$



A



B



C

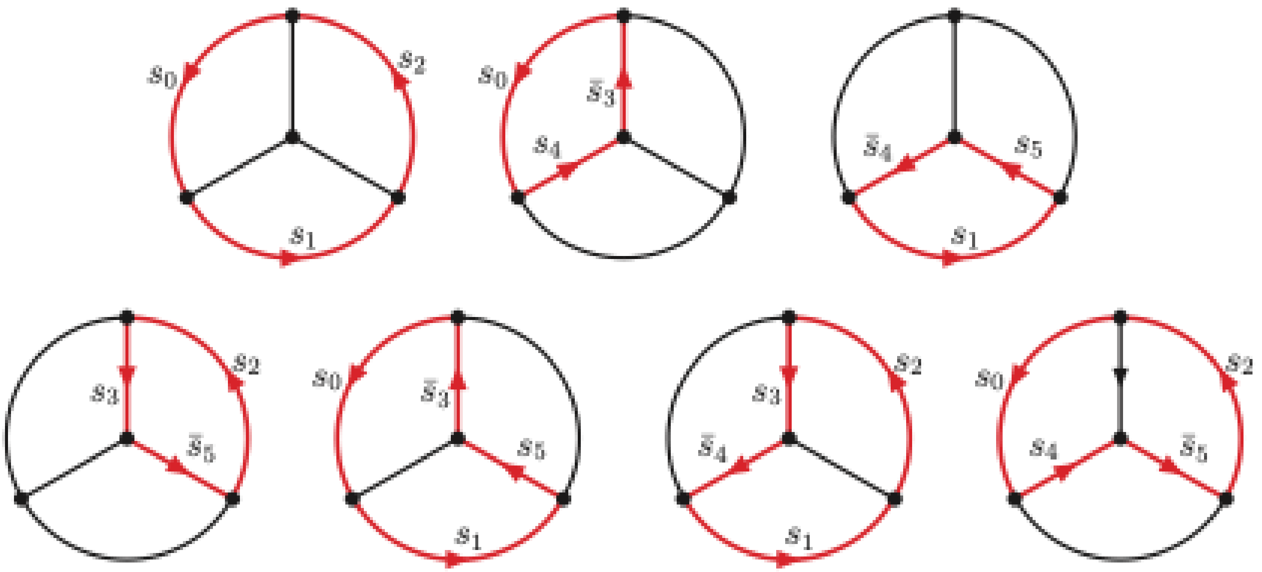
Boolean functions associated to A

$s_0 = 0 \wedge 1,\quad s_1 = 2 \wedge 3,\quad s_2 = 4 \wedge 5,$

$s_3 = 6 \wedge 7,\quad s_4 = 8 \wedge 9,\quad s_5 = 10 \wedge 11$

Eloop clauses

- Directed cyclic conditions are store in eloop clauses.



$$c_0 = s_0 \wedge s_1 \wedge s_2, \qquad c_1 = s_0 \wedge \bar{s}_3 \wedge s_4,$$
$$c_2 = s_1 \wedge \bar{s}_4 \wedge s_5, \qquad c_3 = s_2 \wedge s_3 \wedge \bar{s}_5,$$
$$c_4 = s_0 \wedge s_1 \wedge \bar{s}_3 \wedge s_5, \qquad c_5 = s_1 \wedge s_2 \wedge s_3 \wedge \bar{s}_4,$$
$$c_6 = s_0 \wedge s_2 \wedge s_4 \wedge \bar{s}_5, \qquad c_{7+k} = \bar{c}_k.$$

After tagging e_0:

$$c_0 = s_0 \wedge s_1 \wedge s_2, \qquad c_1 = s_0 \wedge \bar{s}_3 \wedge s_4,$$
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$$c_4 = s_0 \wedge s_1 \wedge \bar{s}_3 \wedge s_5, \; c_5 = s_1 \wedge s_2 \wedge s_3 \wedge \bar{s}_4,$$
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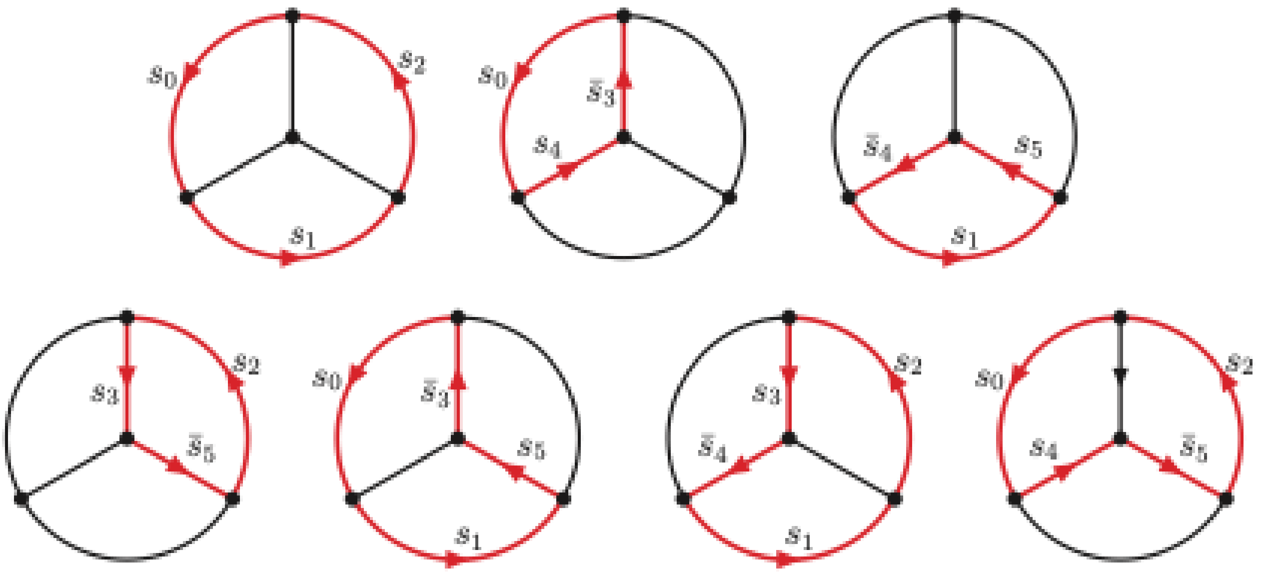

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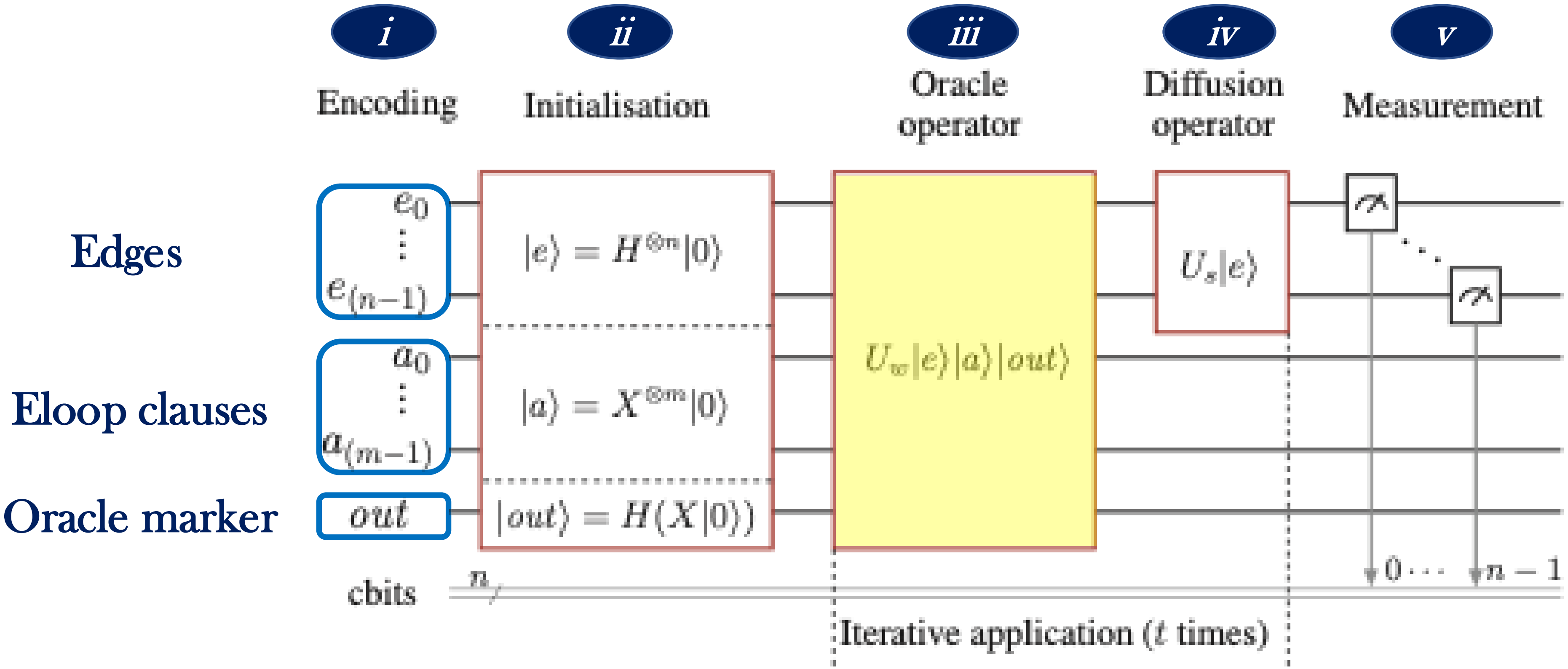
- Causal configurations are identified by suppressing the cyclic configurations.

$$\mathcal{A}c \equiv \bigwedge_j \neg c_j$$

Quantum circuit model of a quantum query algorithm



The oracle opeator design

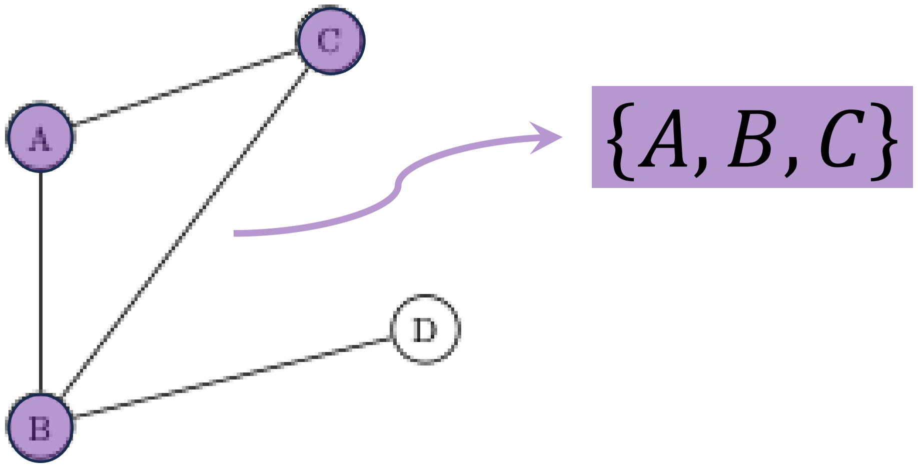
How do we determine the number of ancillary qubits and the gate order?

Process the graph structure more efficiently by applying concepts from graph theory

Data structure

- Clicque (complete subgraph).

Subset of vertices from a specific graph in which each pair of vertices is directly connected by an edge.



From a Feynman diagram perspective

Data structure

◎ **Clicque (complete subgraph).**

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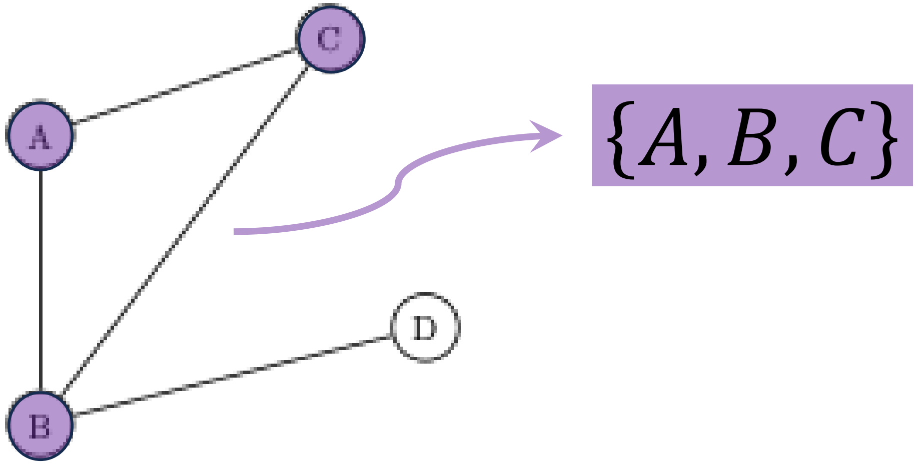


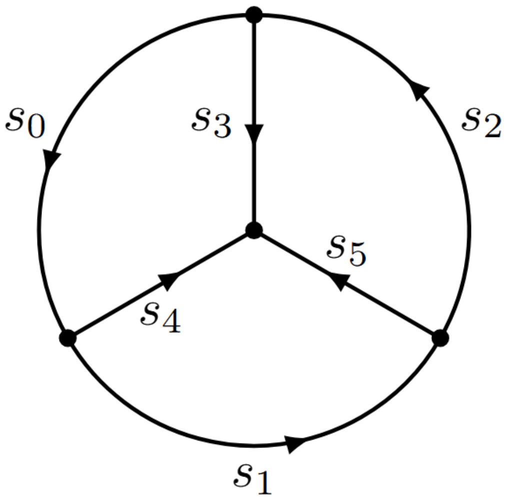
From a Feynman diagram perspective

A clique can be understood as a subset of interaction vertices that form non-causal configurations independently of the rest of the diagram.

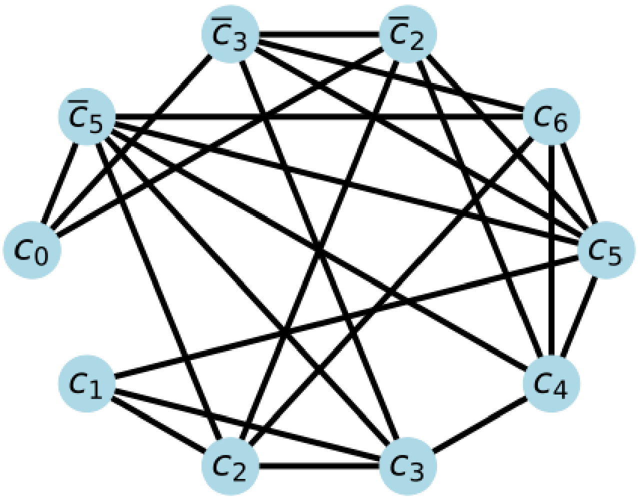
Determine the minimum number of cliques that generate a graph.

→ **Minimum Clique Partition (MCP)**

Ancillary qubits optimisation



Graph representing the adjacency matrix of mutually exclusive clauses



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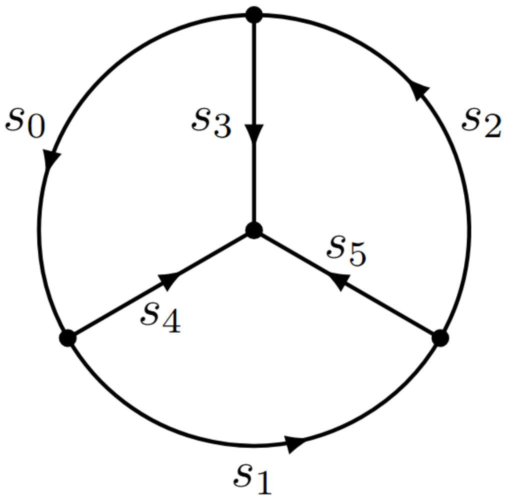

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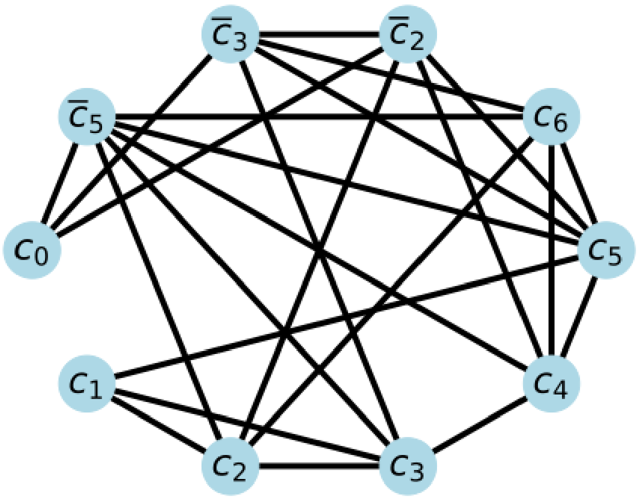
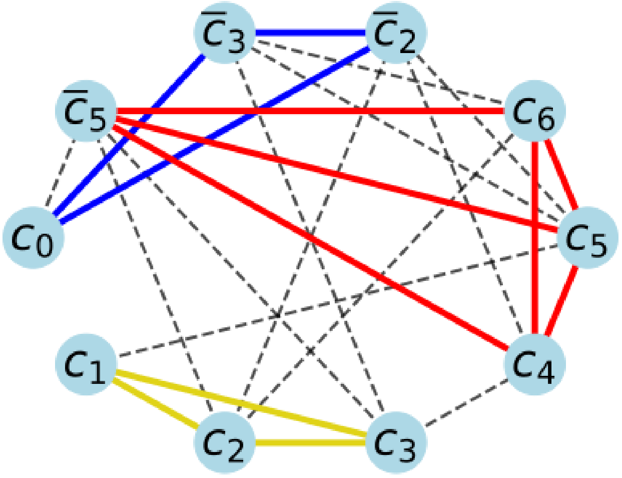


Minimum number of cliques in a graph



We obtain $\mathtt{MAUXC}^{(\mathrm{L},e)}$, indicating the clauses stored in the same ancillary qubit.

Oracle design automation

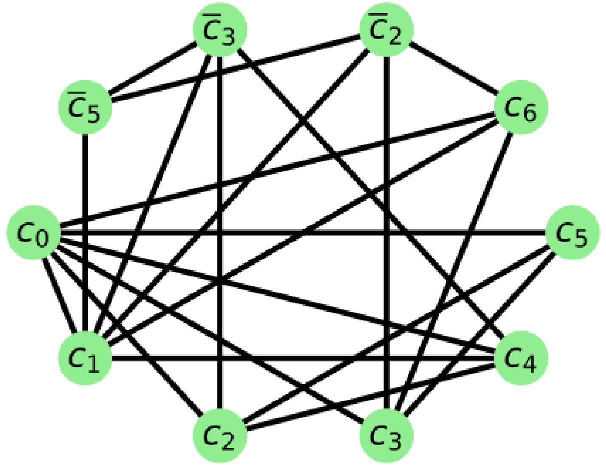
IDEA

Grouping the eloop clauses from $\texttt{MAUXc}^{(\mathrm{L},e)}$ into sets.

Optimise the order in which the sets are implemented.

1 **Clustering conditions:**

a) Disjunctive clauses.
b) Clauses matching at least in one state.
c) The outer clause is taken as an independet set.



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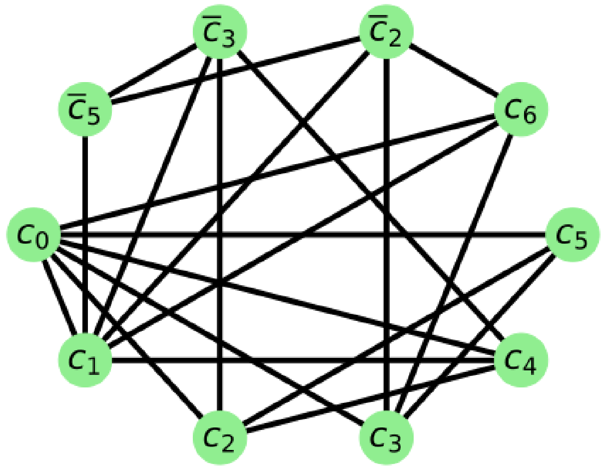
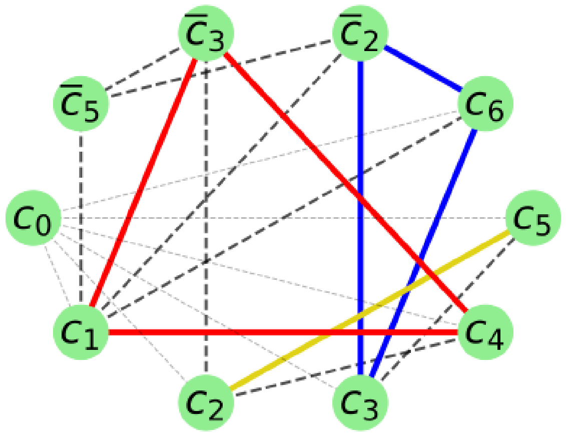


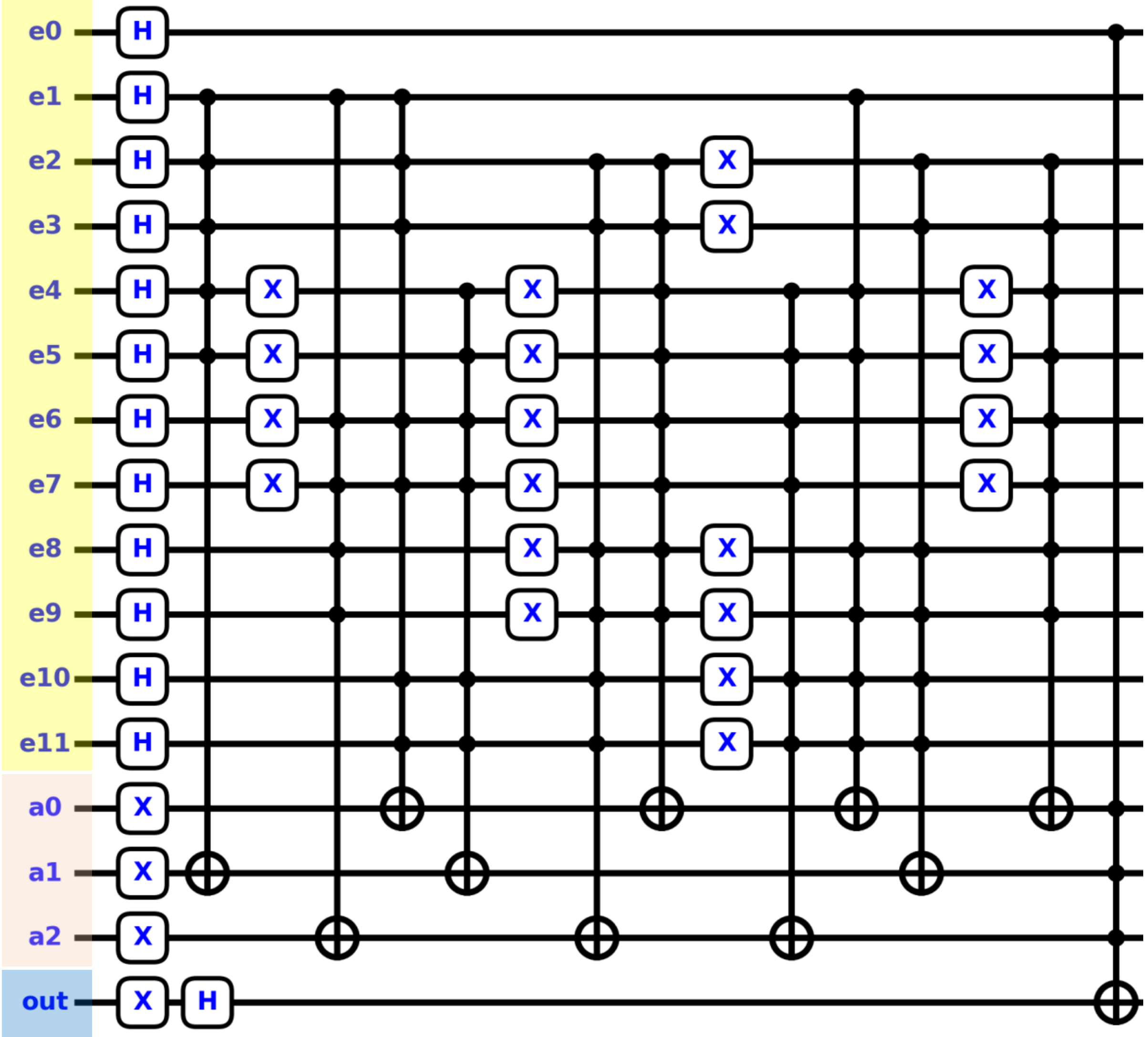
2 Grouping blocks

Reduces the number of grouping blocks of clauses and optimise the application order.



We obtain $\mathtt{OMUTc}^{(\mathrm{L},e)}$, the optimal order for implementing the eloop clauses.

Quantum circuit of the MCA quantum algorithm



i **Encoding the information**

Edges → $|e\rangle$

Eloop clauses → $|a\rangle$

Oracle marker → $|\text{out}\rangle$

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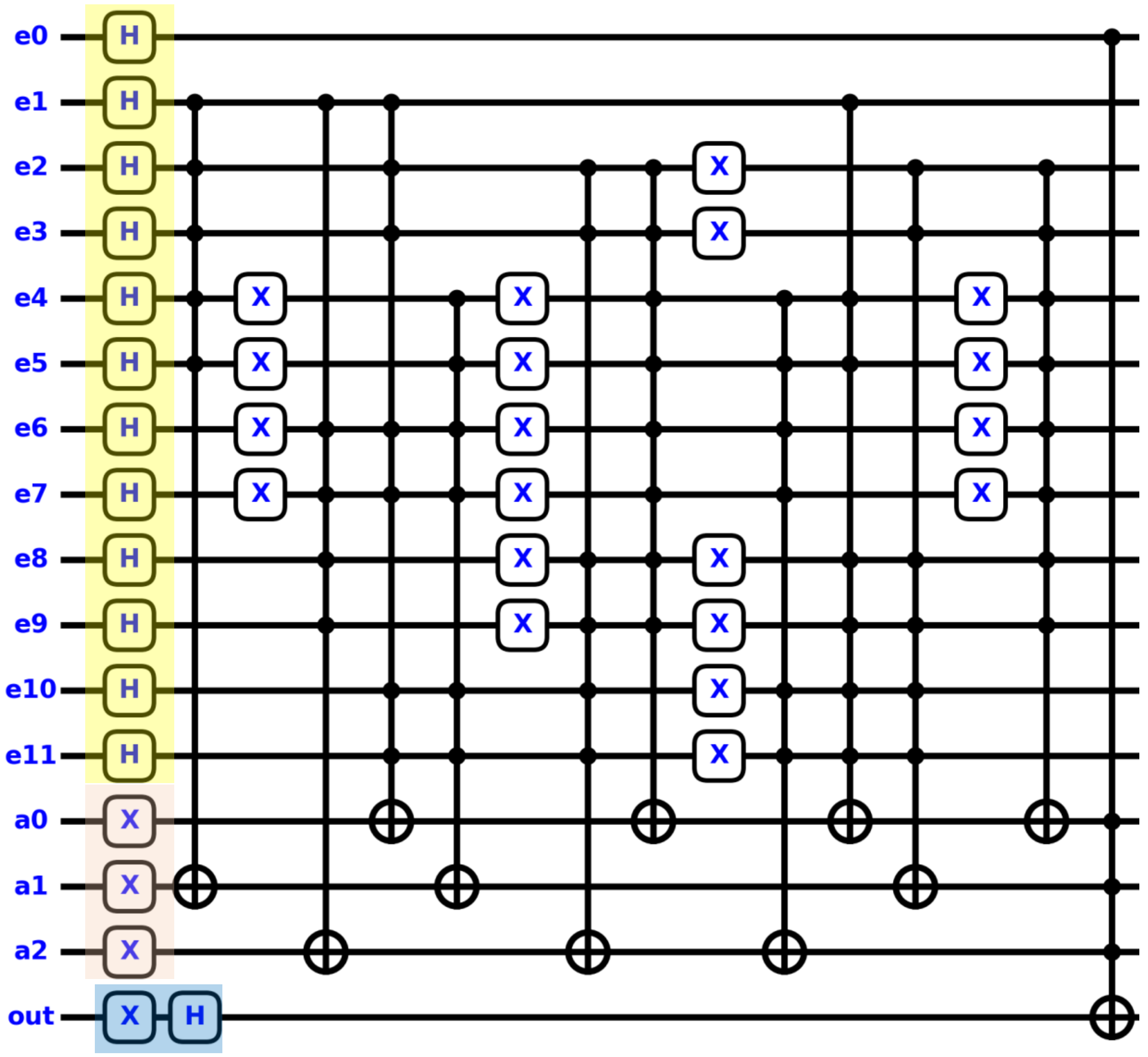

i **Encoding the information**

Edges → $|e\rangle$

Eloop clauses → $|a\rangle$

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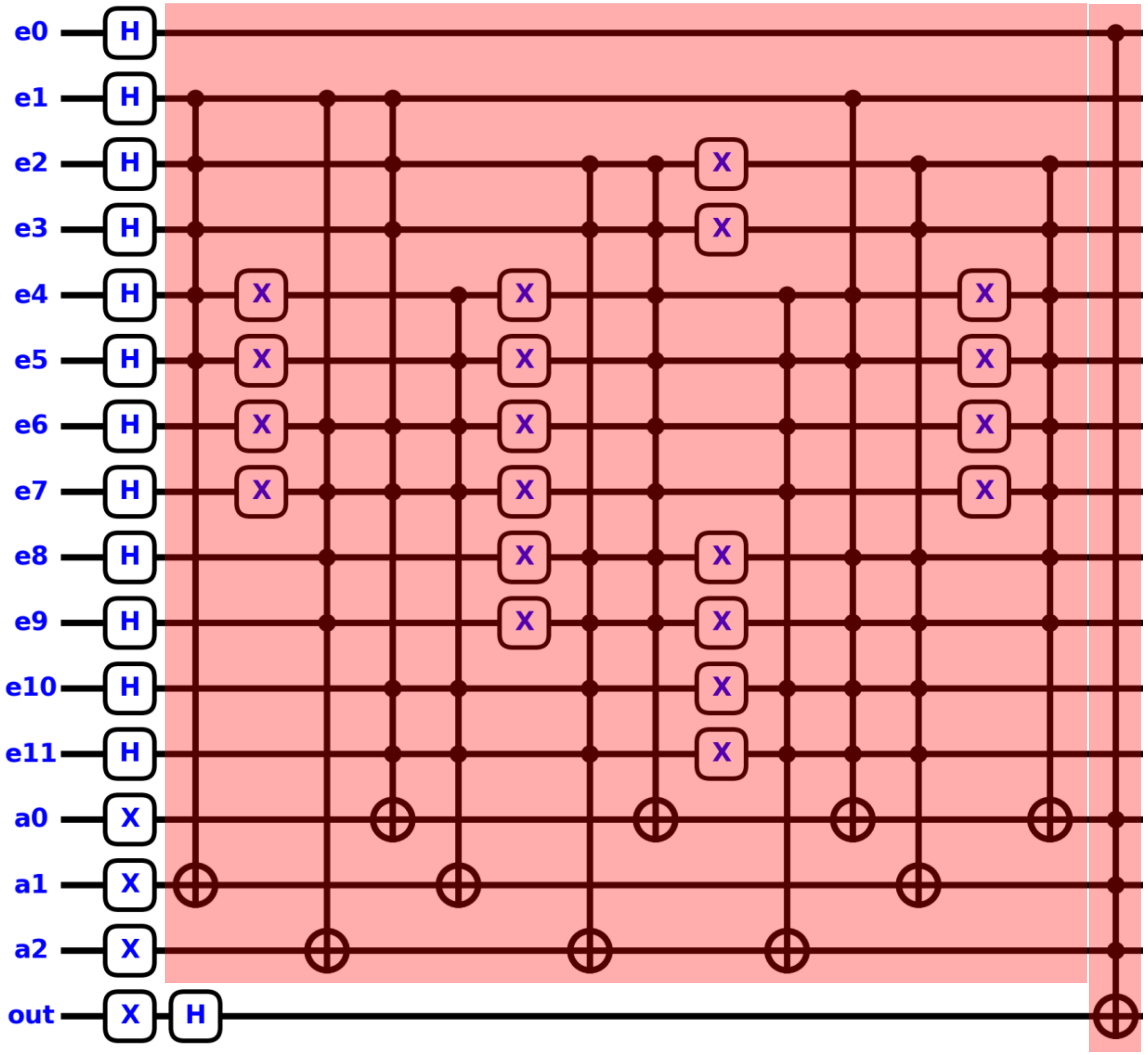
ii **Initialisation**

$|e\rangle \rightarrow \text{H}^{\otimes 12}|0\rangle = |+\rangle^{\otimes 12}$

$|a\rangle \rightarrow \text{X}^{\otimes 3}|0\rangle = |1\rangle^{\otimes 3}$

$|\text{out}\rangle \rightarrow \text{H}(\text{X}|0\rangle) = |-\rangle$

Quantum circuit of the MCA quantum algorithm



iii Oracle operator

Each eloop clause is mapped to a multi-controlled Toffoli and XNOT gates.

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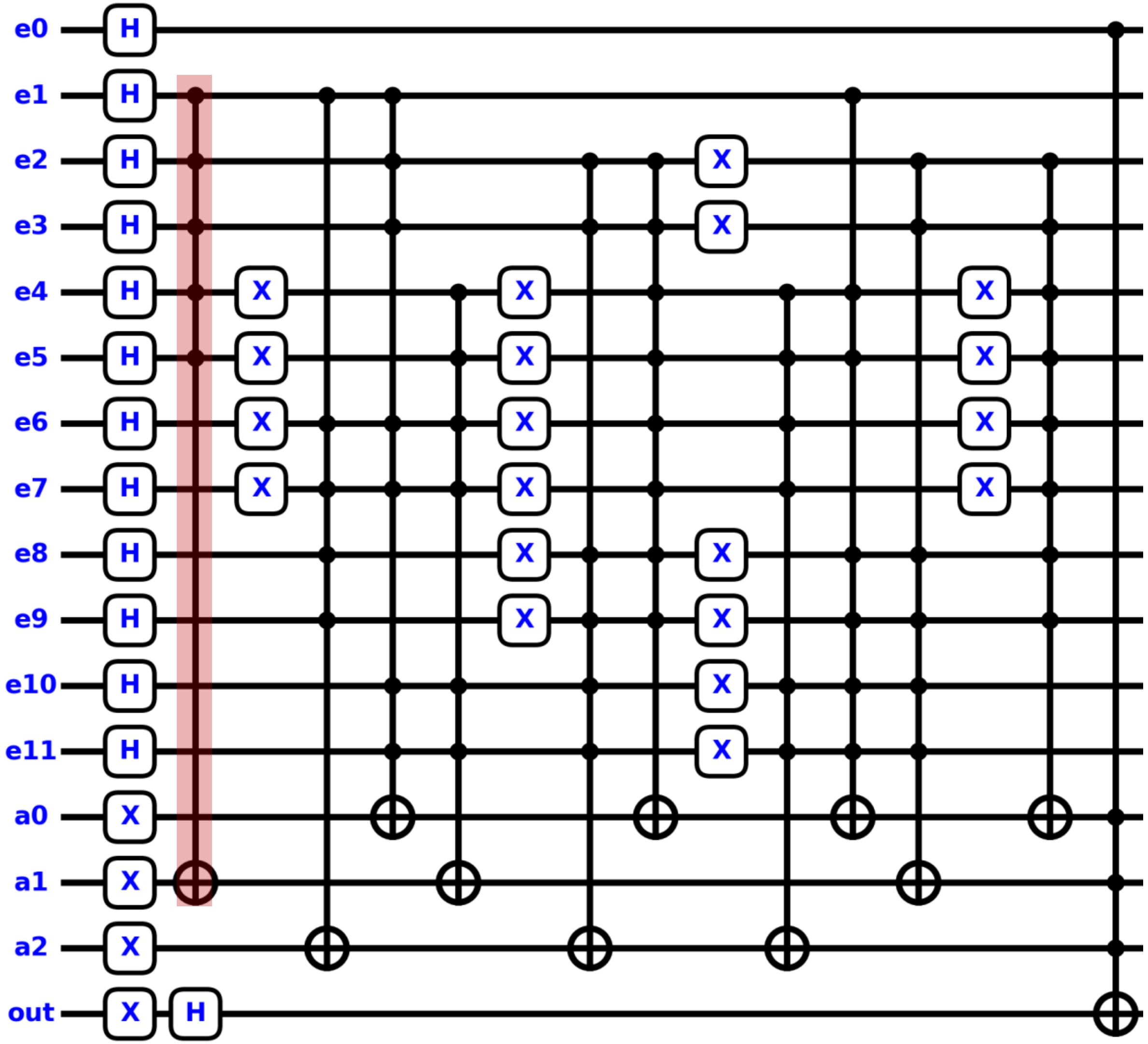

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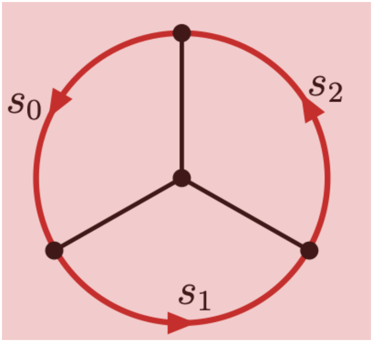

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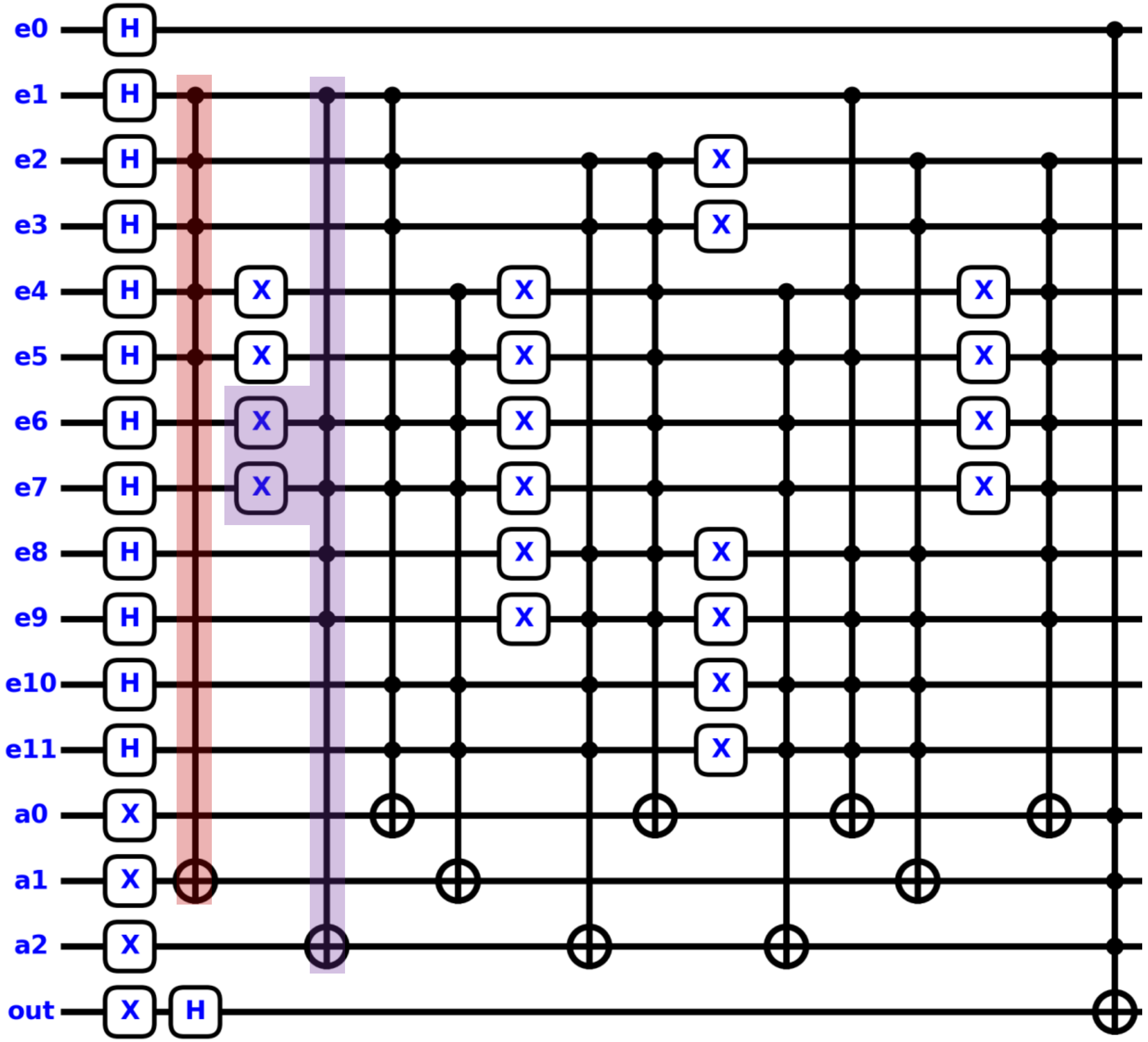

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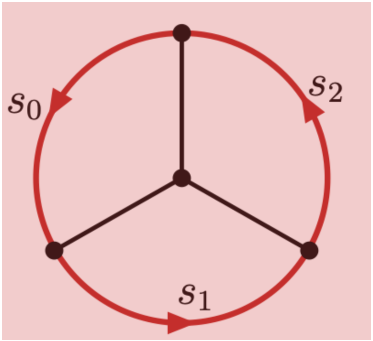
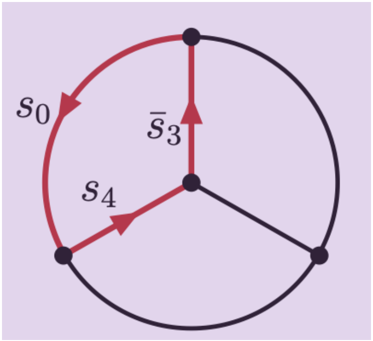
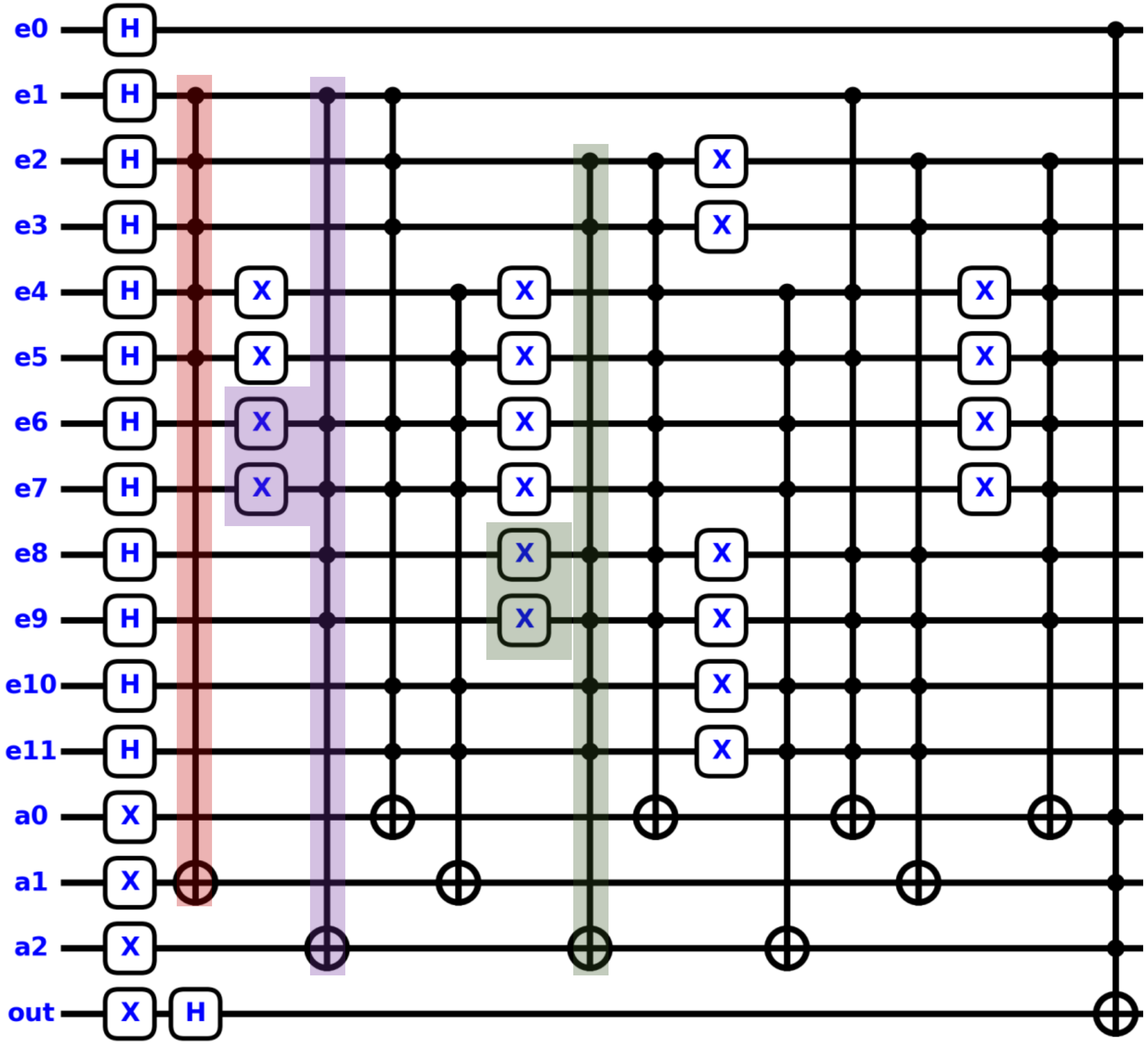




Quantum circuit of the MCA quantum algorithm



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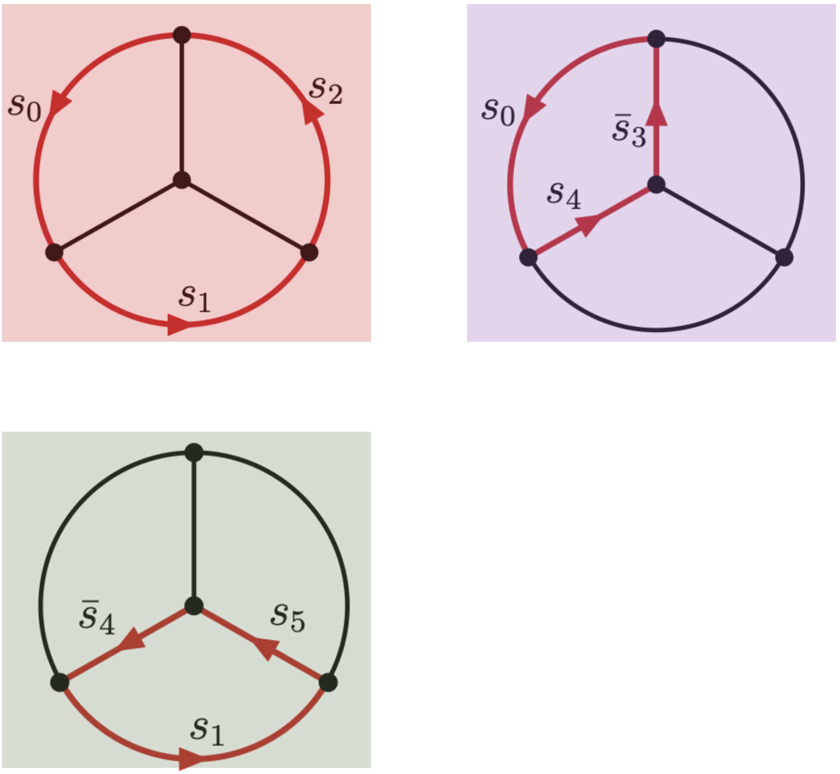

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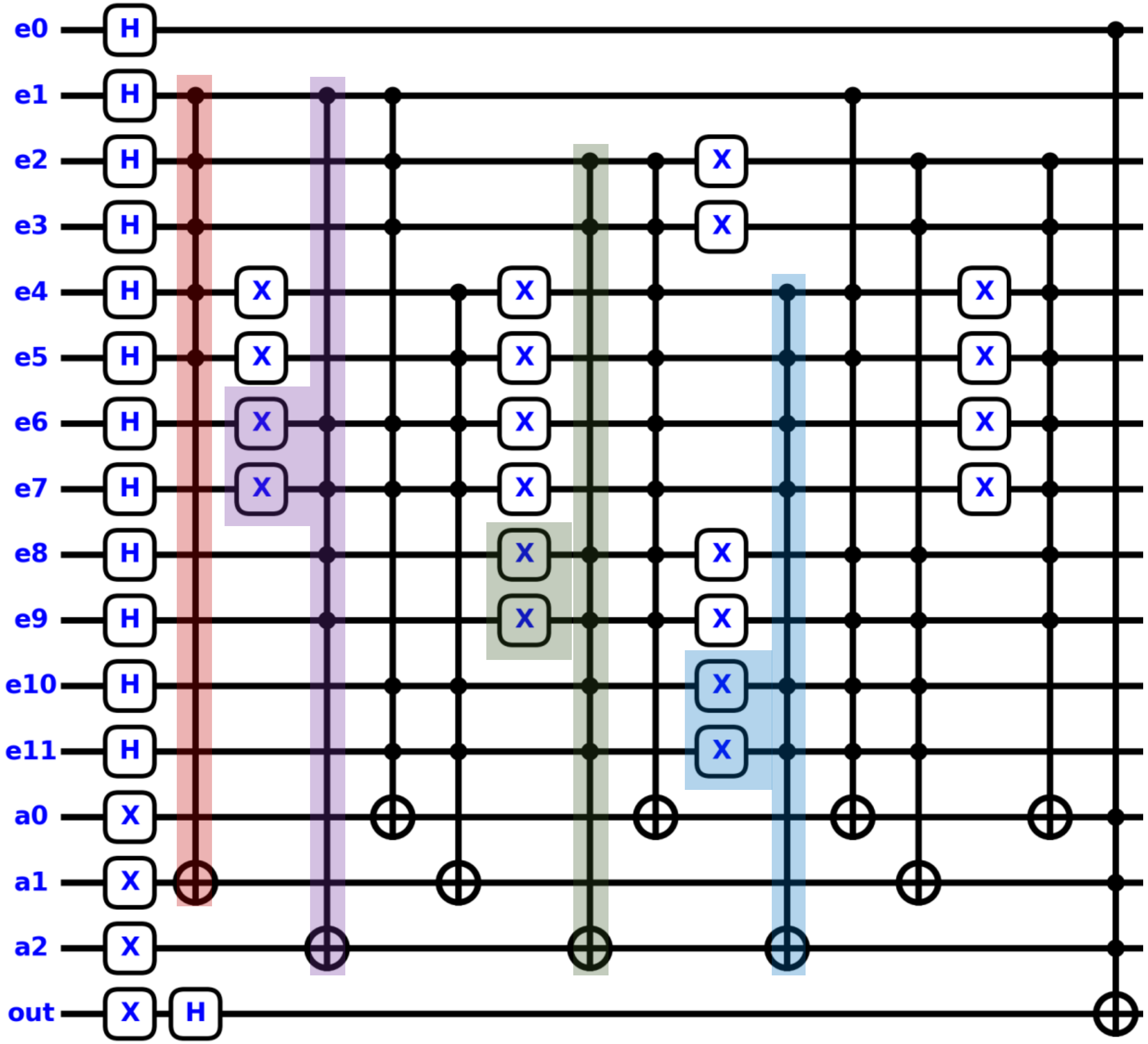

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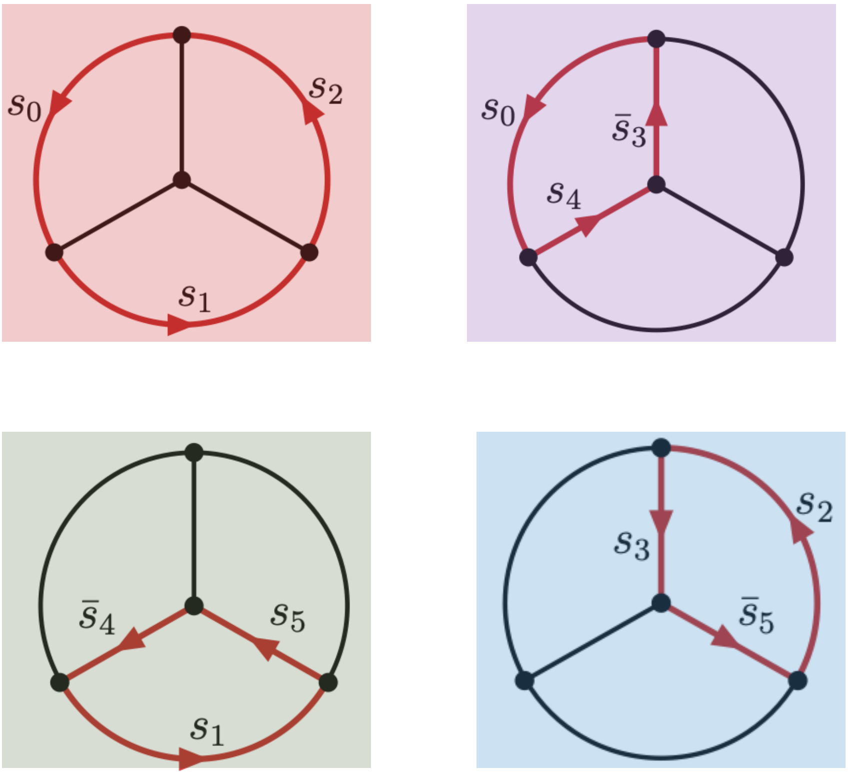
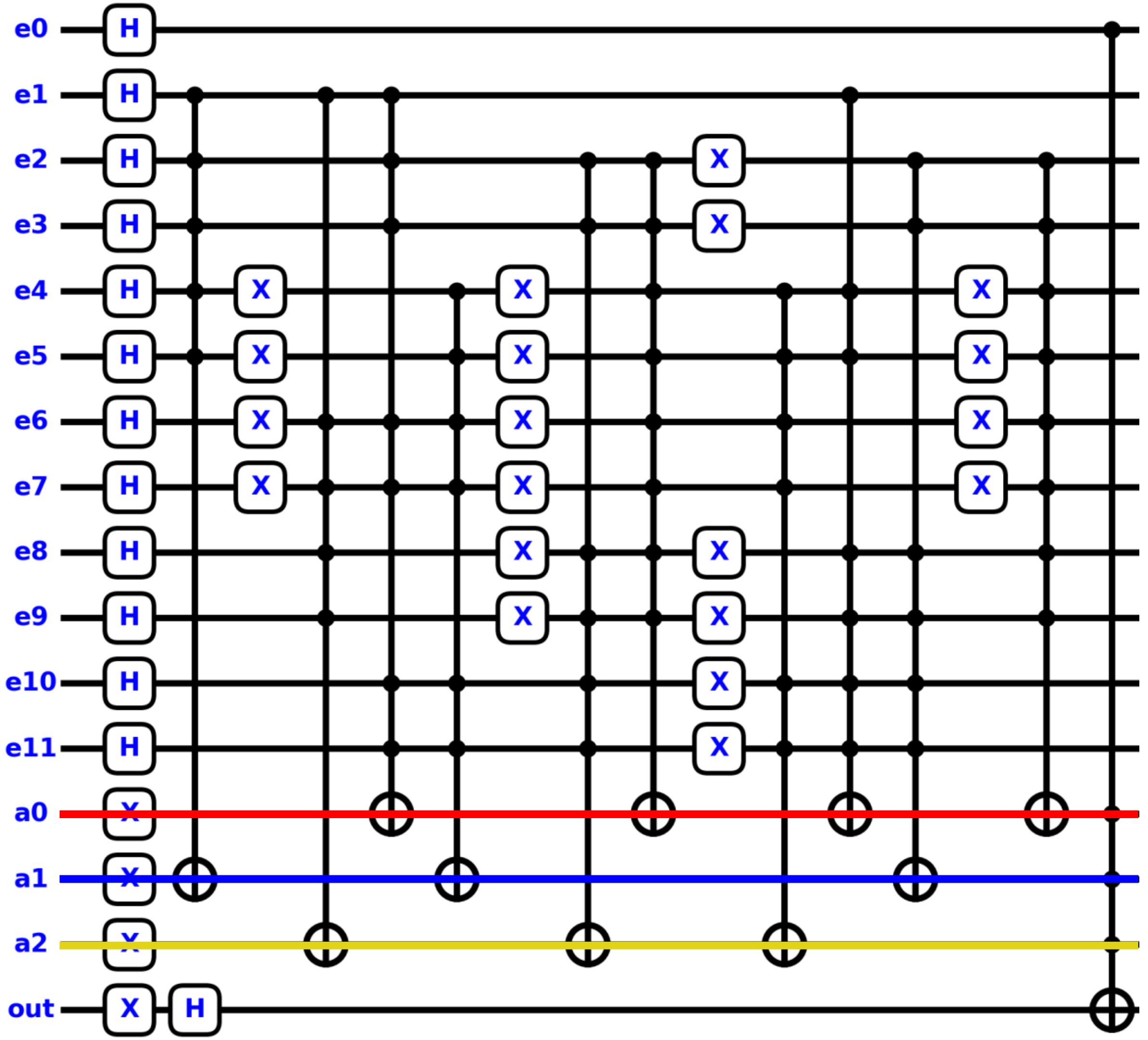
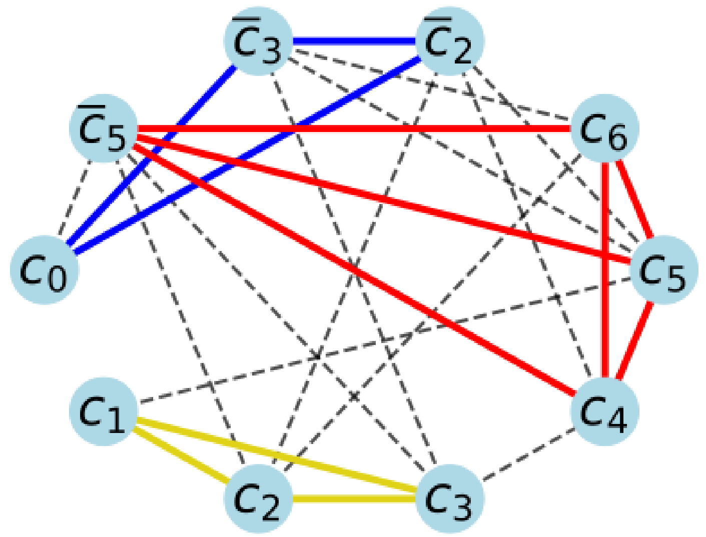


Quantum circuit of the MCA quantum algorithm



iii **Oracle operator**

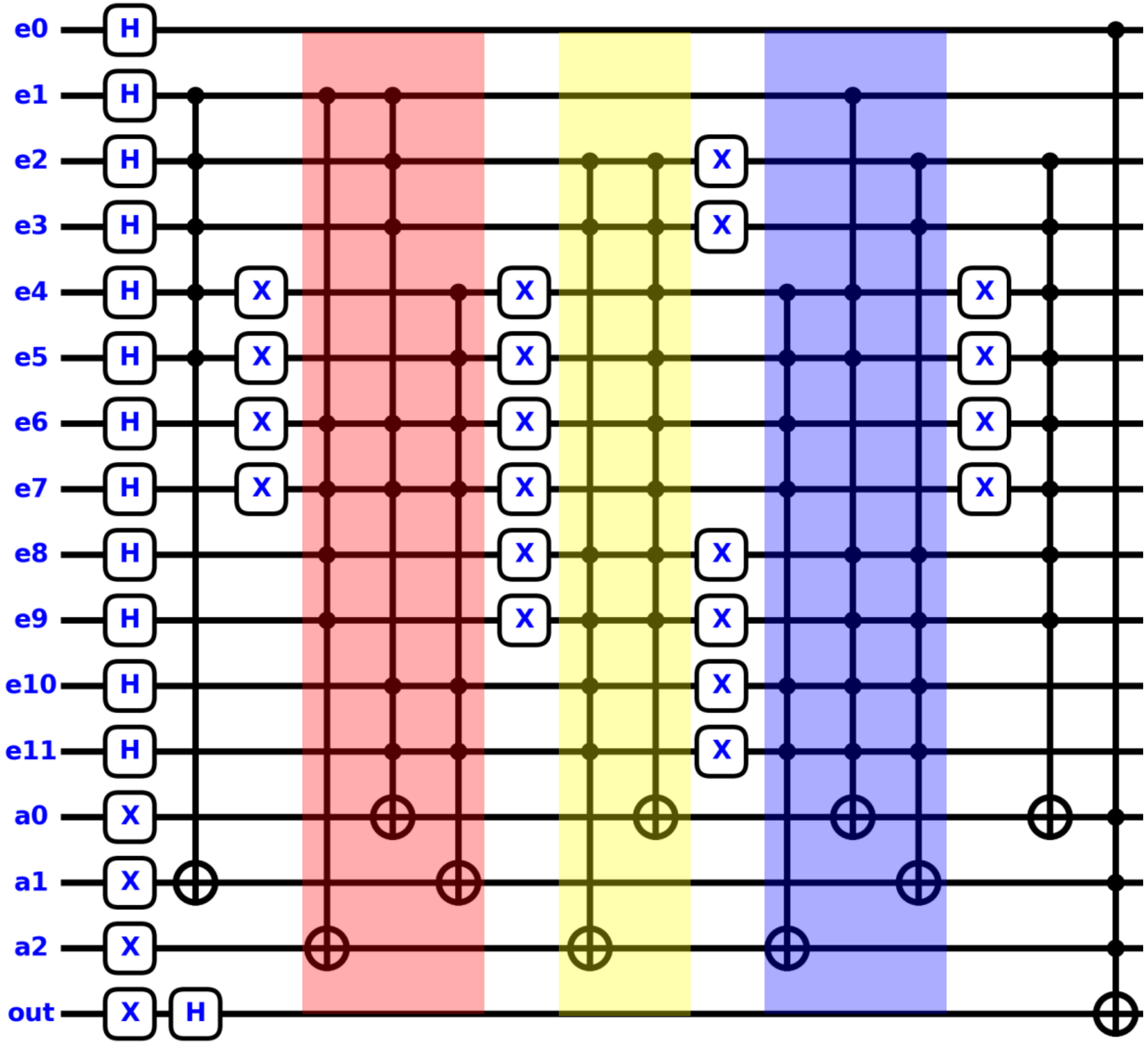
Ancillary qubits optimisation



An algorithm is developed to solve the Minimum Clique Partition problem:

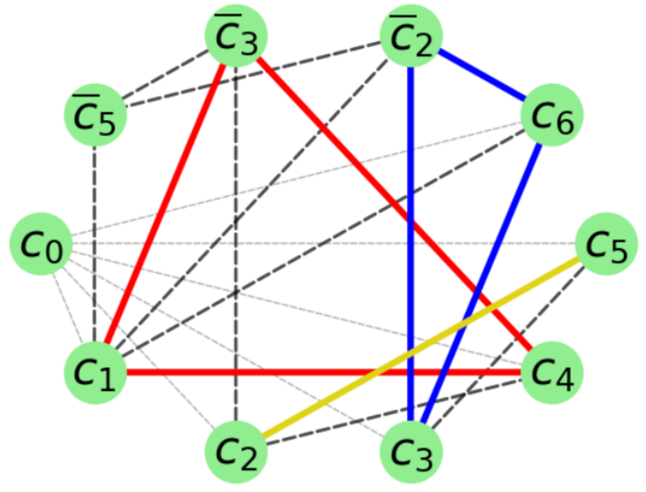
$$\mathrm{MAUXC}^{(3,12)} = \{\{c_4, c_5, \bar{c}_5, c_6\}, \{c_0, \bar{c}_2, \bar{c}_3\}, \{c_1, c_2, c_3\}\}$$

Quantum circuit of the MCA quantum algorithm



iii **Oracle operator**

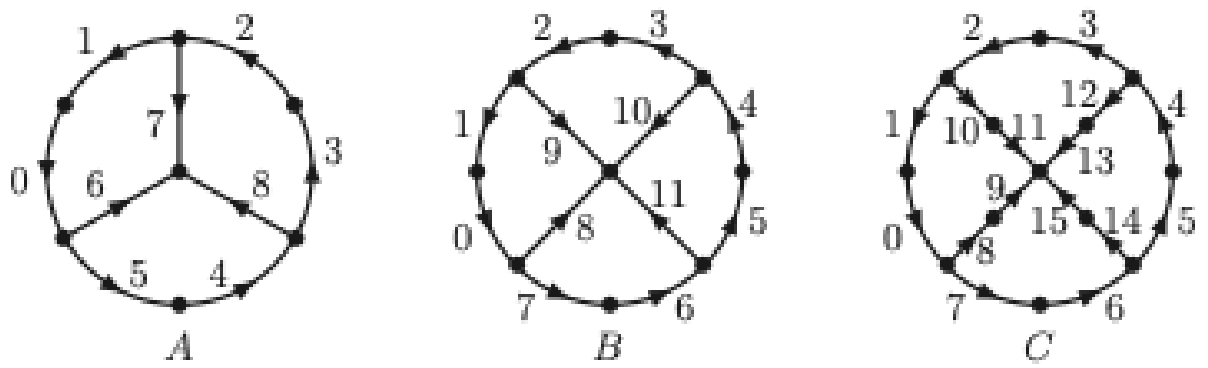
Oracle design automation



We obtain the gate blocks and the optimization of the implementation order

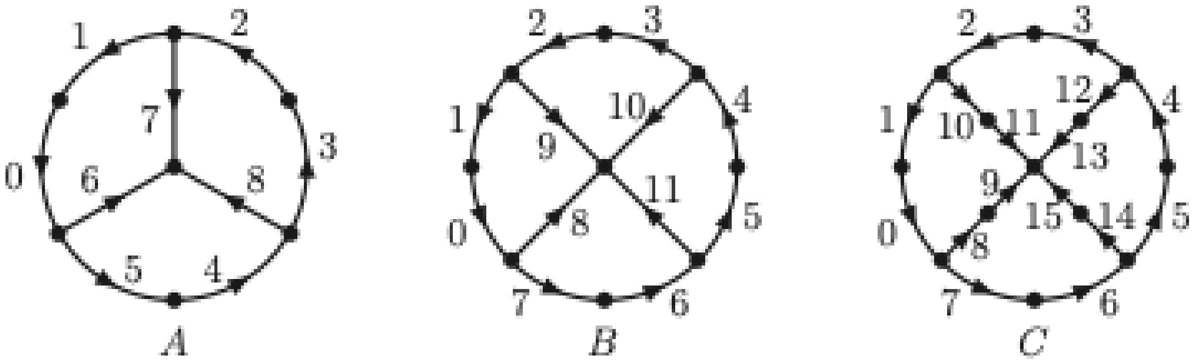
$$\mathrm{OMUTC}^{(3,12)} = \{\{c_0\},\{c_1,c_4,\bar{c}_3\},\{c_2,c_5\},\{c_3,c_6,\bar{c}_2\},\{\bar{c}_5\}\}$$

The implementation of the MCA quantum algorithm



eloops (edges)	$\|e\rangle$	$\|a\rangle$	Total Qubits	Quantum Depth	Total states
three (9)	9	2 | 4	12 | 14	15 | 17	512
three (12)	12	3 | 7	16 | 21	23 | 31	8192
four$^{(c)}$ (12)	12	4 | 5	17 | 18	15 | 15	4096
four$^{(c)}$ (16)	16 + 1	6 | 13	24 | 31	39 | 45	131072

The implementation of the MCA quantum algorithm



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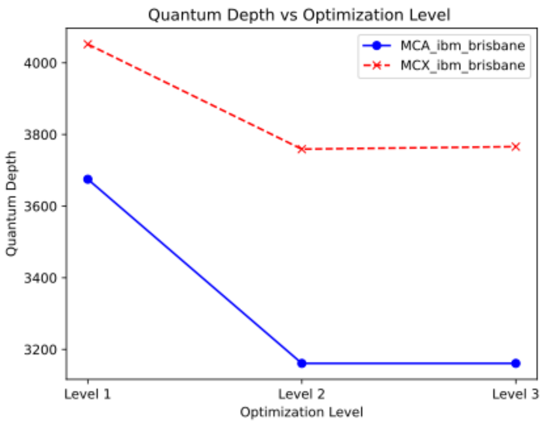
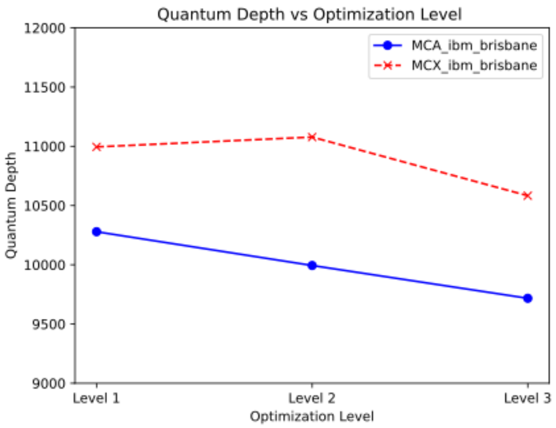
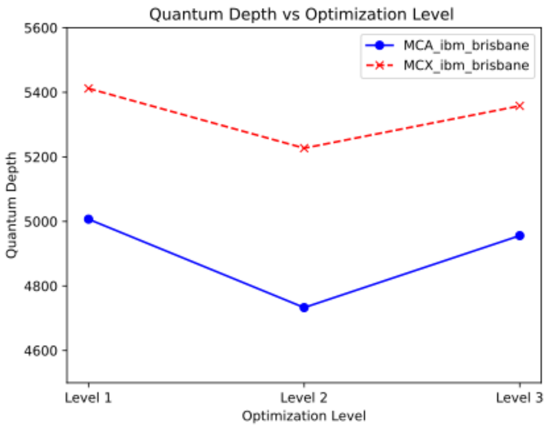
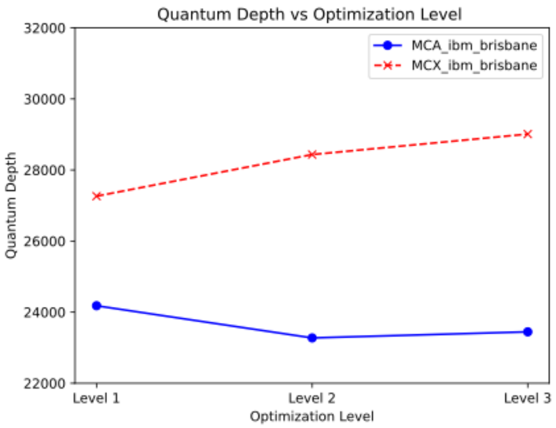
- **Transpilation,** the process of compiling a given quantum circuit to match the specific topology and native gate set of a particular quantum device hardware.

Transpilation behaviour

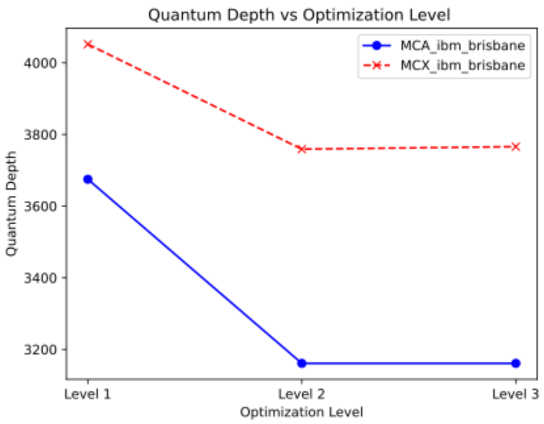


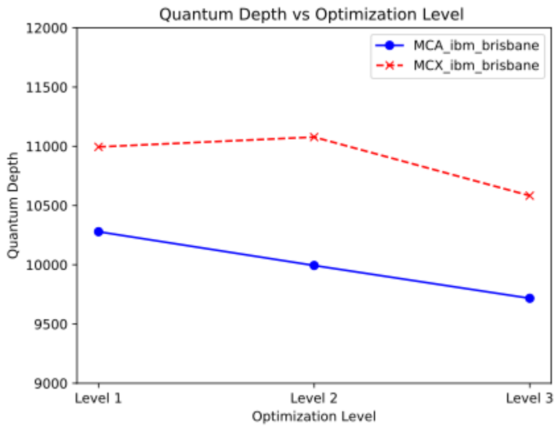


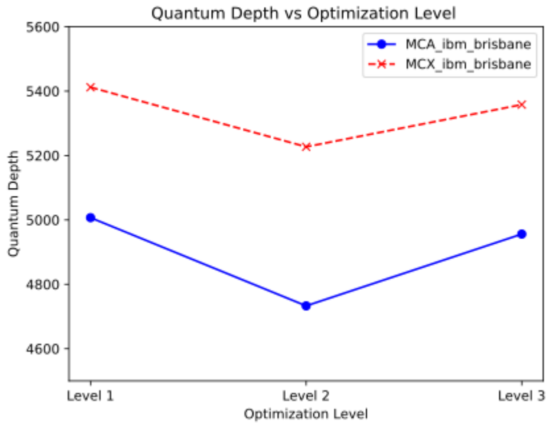


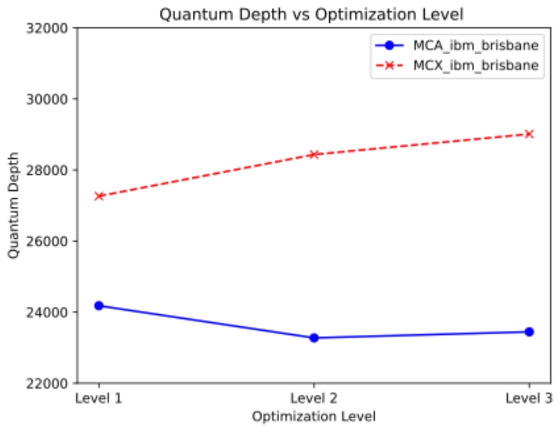


Transpilation behaviour



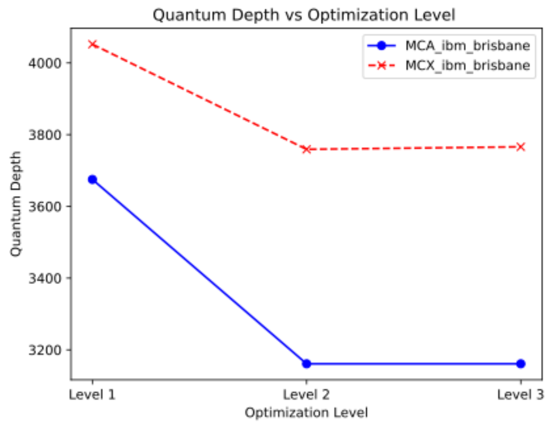


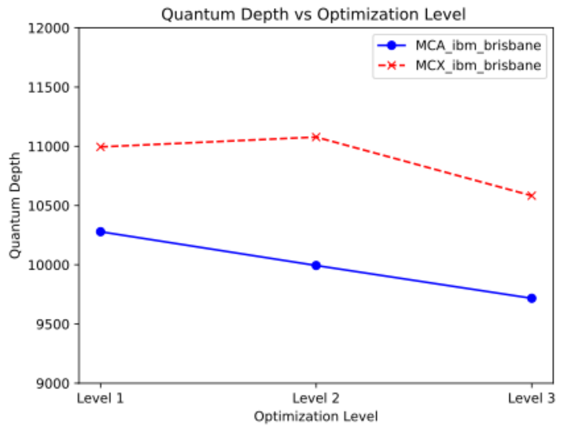


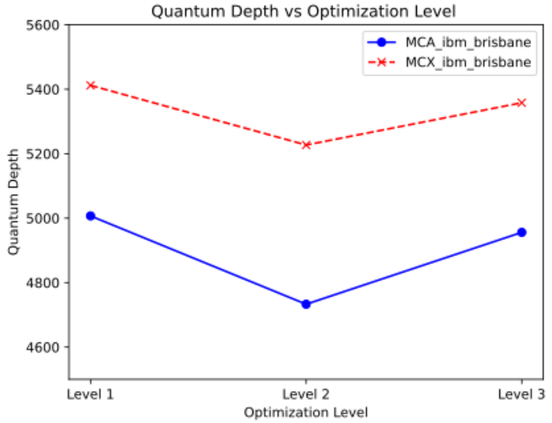


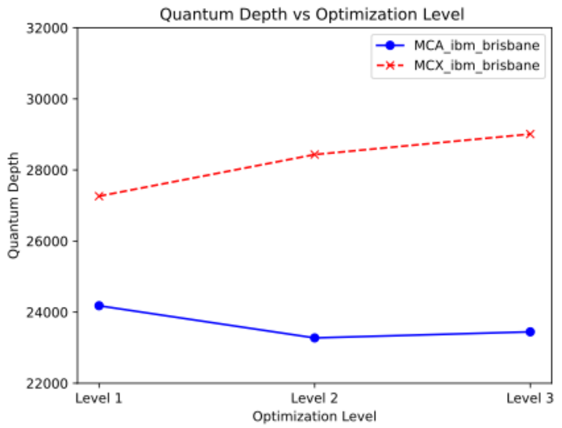
- The quantum **circuit area is defined by the product of the transpiled quantum circuit depth and the number of qubits required in the transpilation.**

Transpilation behaviour









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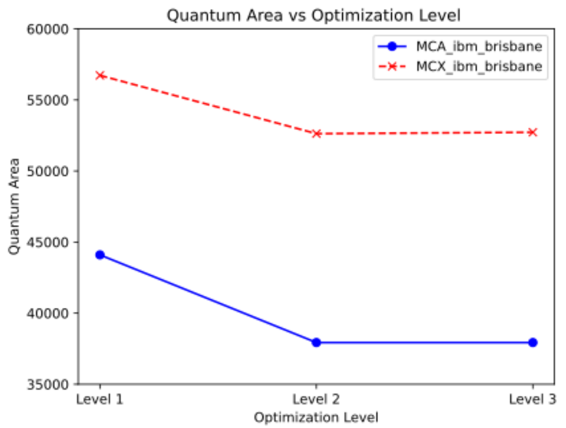
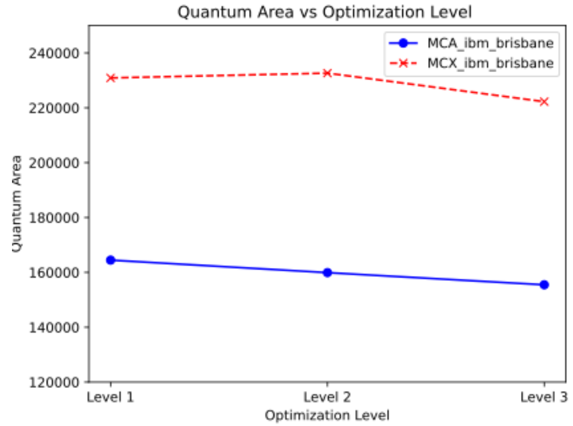
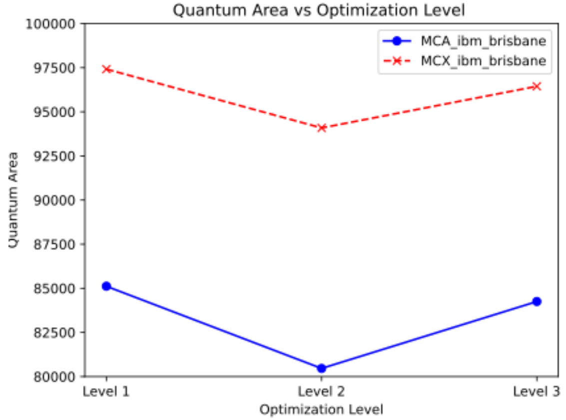
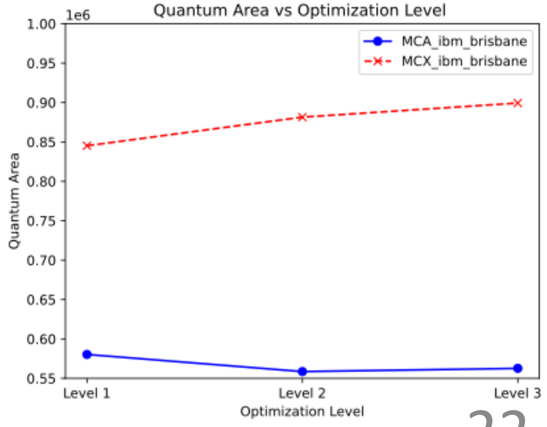


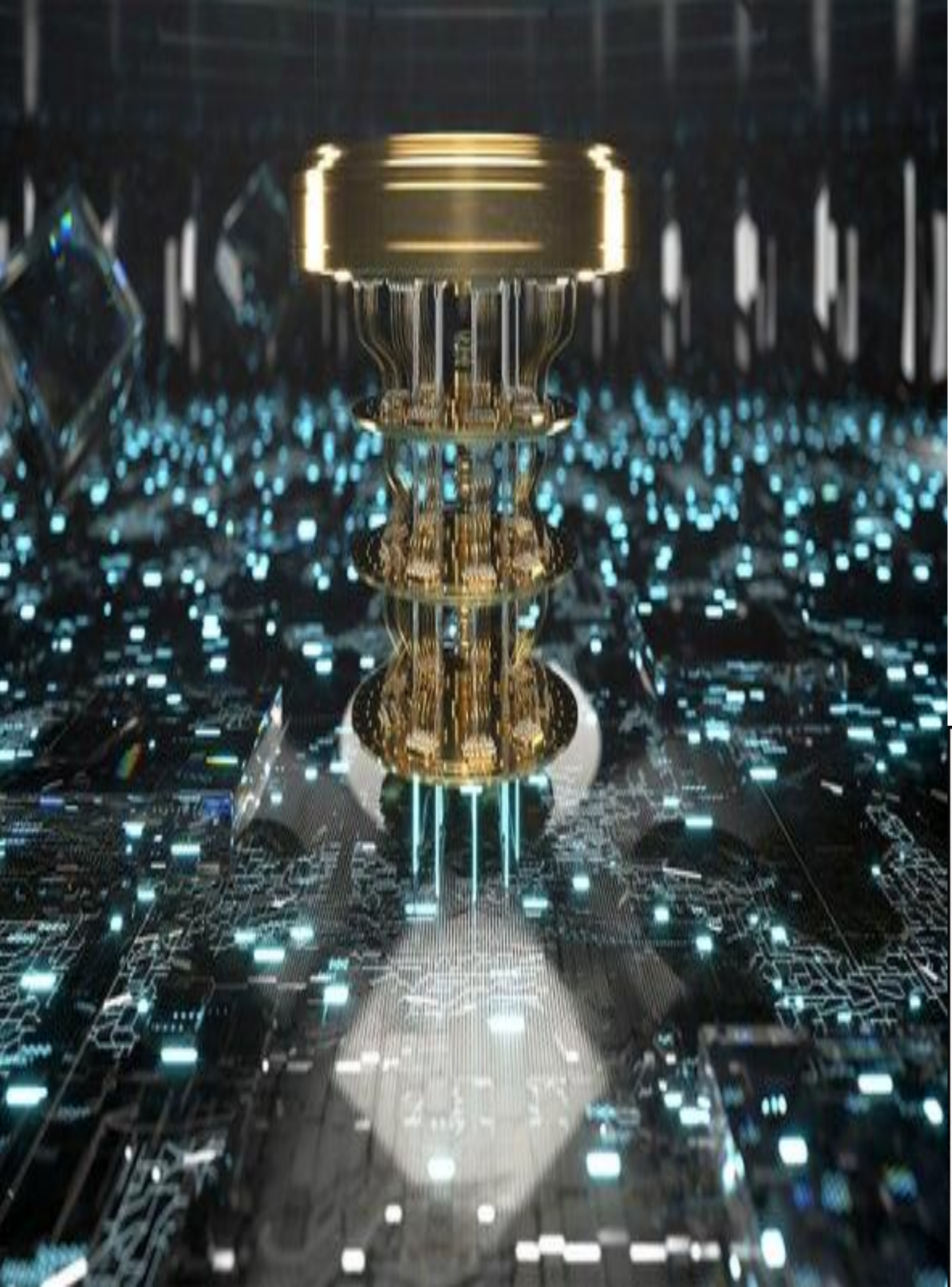




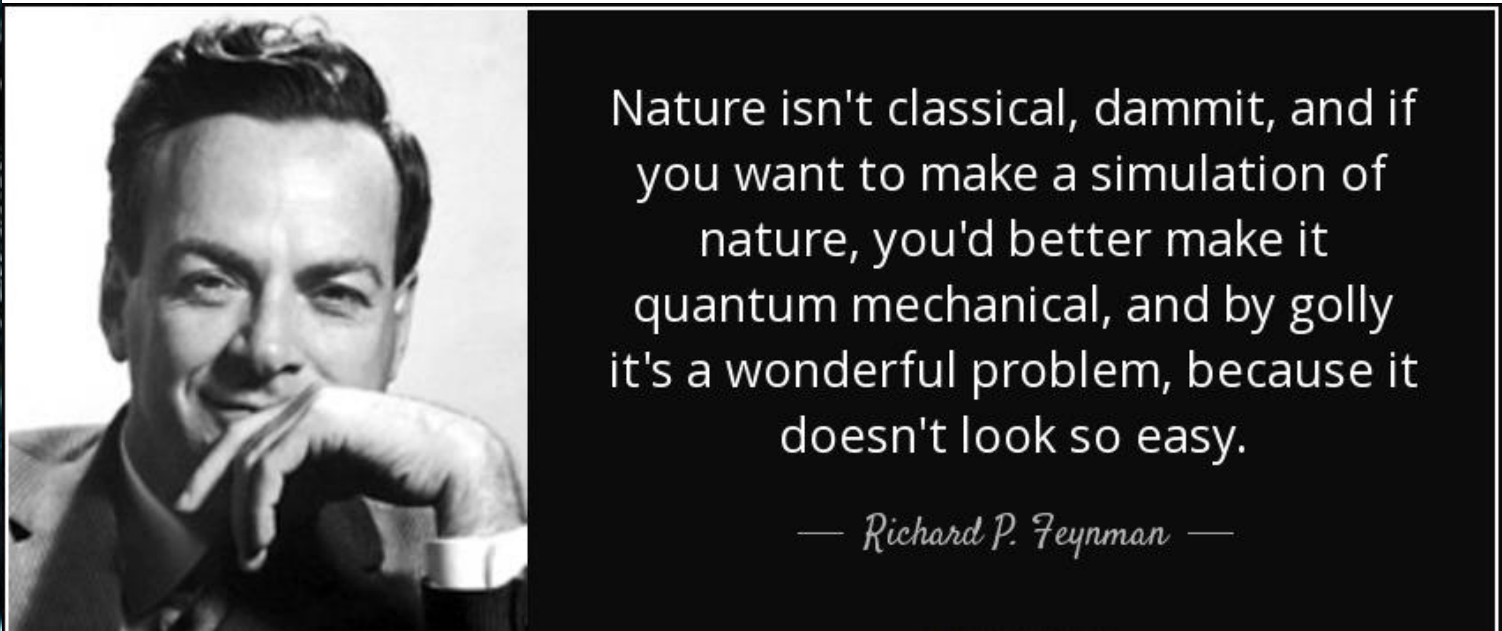


Conclusions

- We have disegned an automated query quantum algorithm, the Minimun Clique-optimised quantum Algorithm (MCA), for the identification of DAG configurations of multiloop graphs.
- We have evaluated the performance of quantum circuits generated by the MCA algorithm after transpilation
- The MCA algorithm can be extended beyond particle physics, establishing a use case on quantum optimization for graph theory problems and any other clause-satisfiability applications.



¡Gracias!


Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.
— Richard P. Feynman —