



# Átomos gravitacionales Nobles

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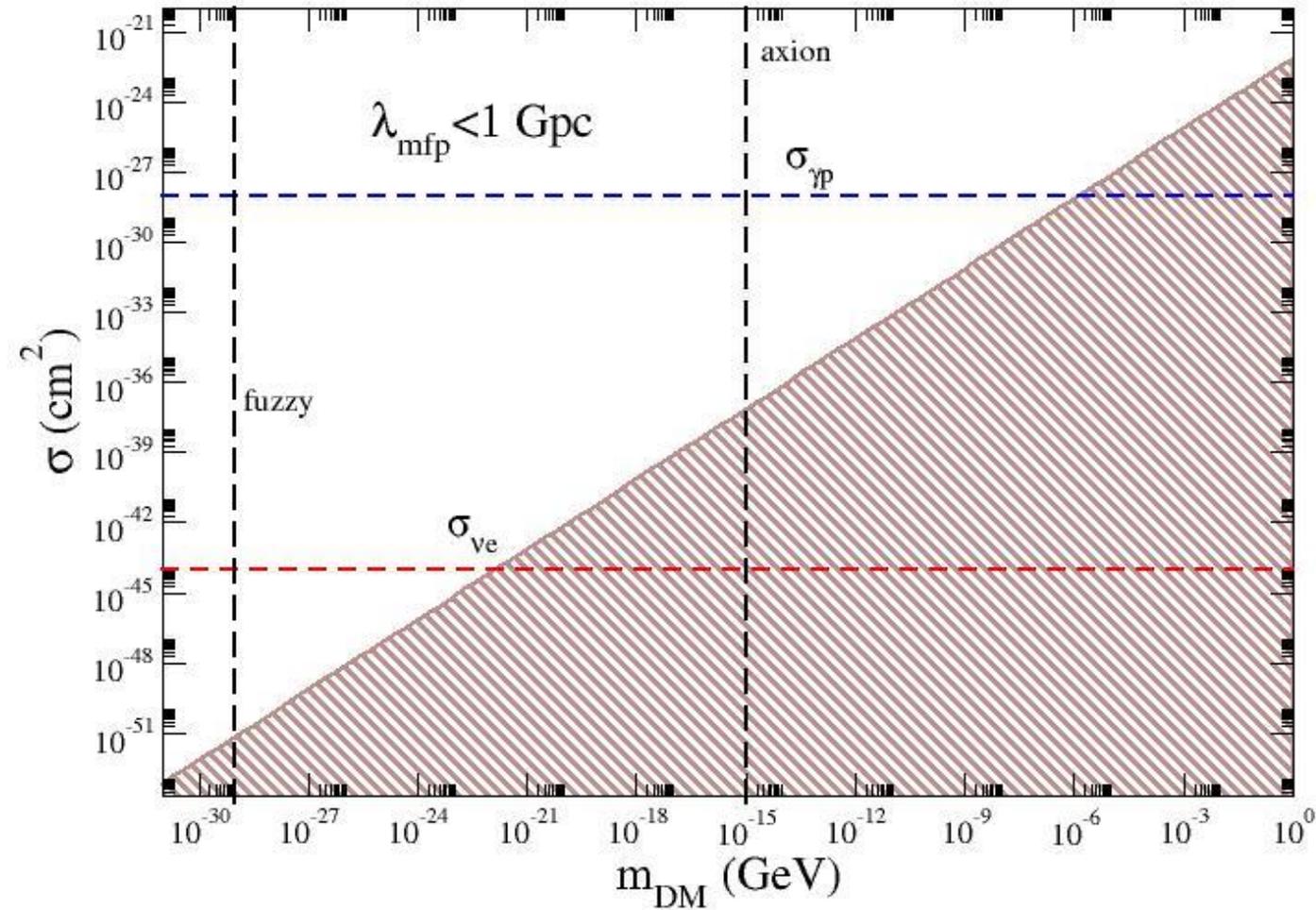


# Outline

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- 1. Lightness is the new darkness
- 2. Ultra-light dark matter (at galactic scales)
- 3. Semiclassical description of the Einstein-Klein-Gordon system
- 4. Boson stars and their relatives
  - Multi-state boson stars
  - L-Boson stars
- 5. What about the supermassive black hole at the center of galaxies?
- 6. Noble gravitational atoms: self-gravitating black hole scalar wigs with angular momentum number

# Lightness is the new darkness



# The darkest scenario:

- Dark matter interacts only through gravitational interactions:
- 1) Forget how to detect it on Earth
- 2) Main properties: The mass and the spin
- 3) Consider the case of a bosonic particle that interact only through gravitation

# Scalar field as dark matter?

- A different approach: The Scalar Field Dark Matter model (SFDM)  
The Dark Matter is modeled by a scalar field with a ultra-light associated particle. ( $m \sim 10^{-23} \text{eV}$ )
  - At cosmological scales it behaves as cold dark matter  
T. Matos, L.A. Urena-Lopez, Class. Quant. Grav. 17 L75 (2000),  
V. Sahni and L.M. Wang, Phys. Rev D 62, 103517 (2000).
  - At galactic scales, it does not have its problems: neither a cuspy profile, nor a over-density of satellite galaxies.

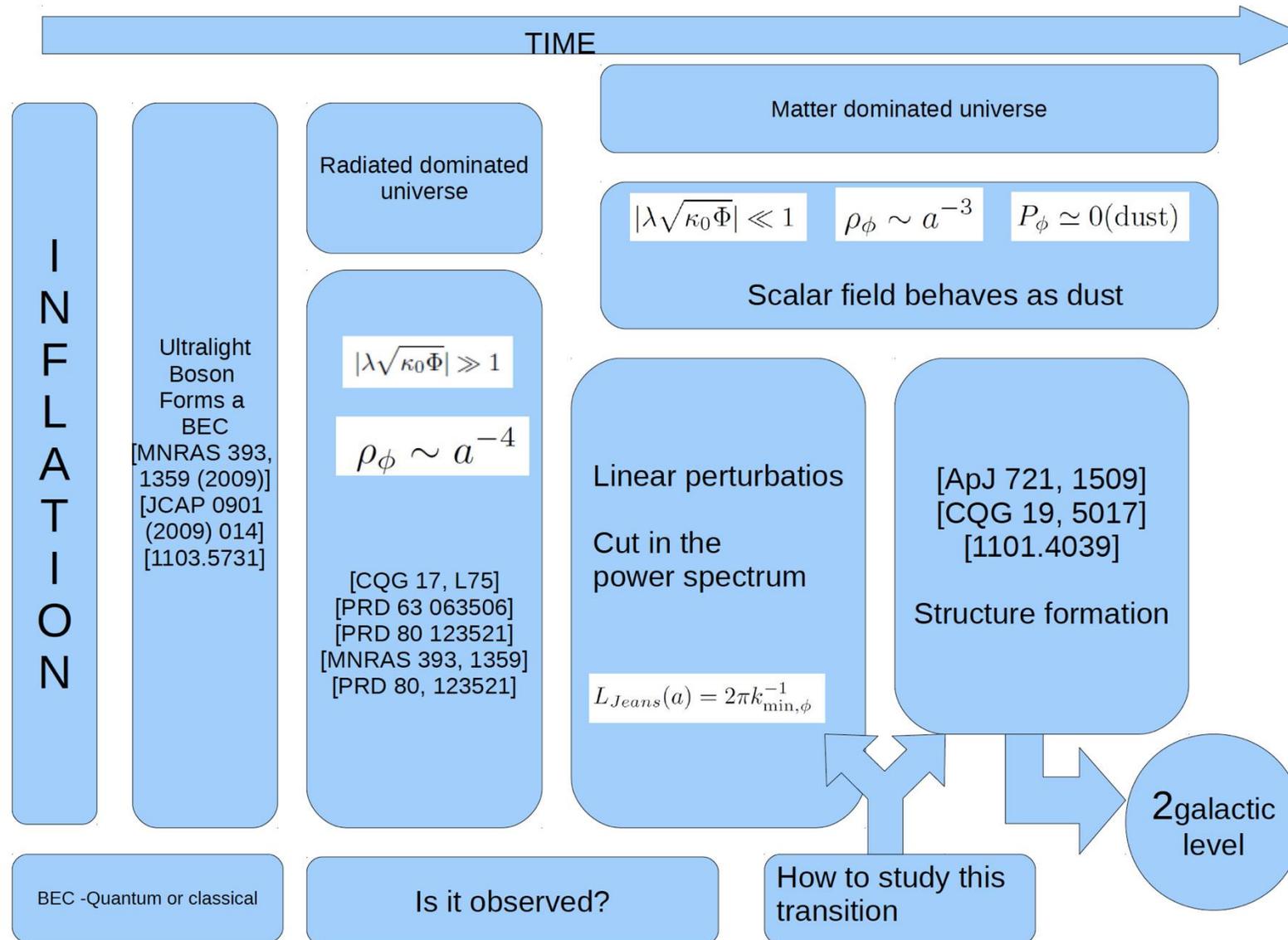
## Ultralight scalars as cosmological dark matter

Lam Hui Jeremiah P. Ostriker ,Scott Tremaine, Edward Witten  
**Phys.Rev. D95 (2017) no.4, 043541**

$$\mathcal{L}_{\text{L-SFDM}} = \mathcal{L}_{\text{GR}} + \mathcal{L}_B + \mathcal{L}_\Lambda - \sqrt{-g}[\Phi'^{\mu}\Phi_{,\mu} + 2V(\Phi)]$$

# An ultra-light boson as dark matter?

$$\mathcal{L}_{L-SFDM} = \mathcal{L}_{GR} + \mathcal{L}_B + \mathcal{L}_\Lambda - \sqrt{-g}[\Phi^{,\mu}\Phi_{,\mu} + 2V(\Phi)]$$



Can the dark matter halo be a self-gravitating object made of ultralight spin-zero bosons?

DM properties are known by particle physicist  
(Lagrangian, EOS...)



What kind of astrophysical object can they form?

# Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State\*

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and

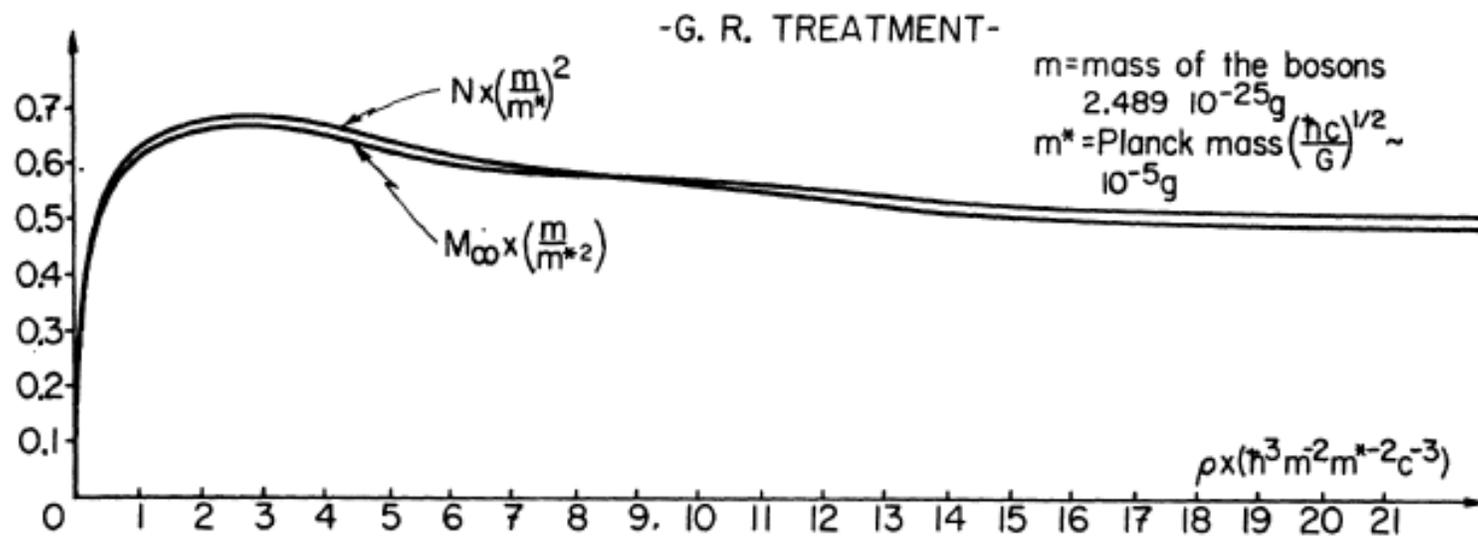
*Institute for Advanced Study, Princeton, New Jersey 08540*

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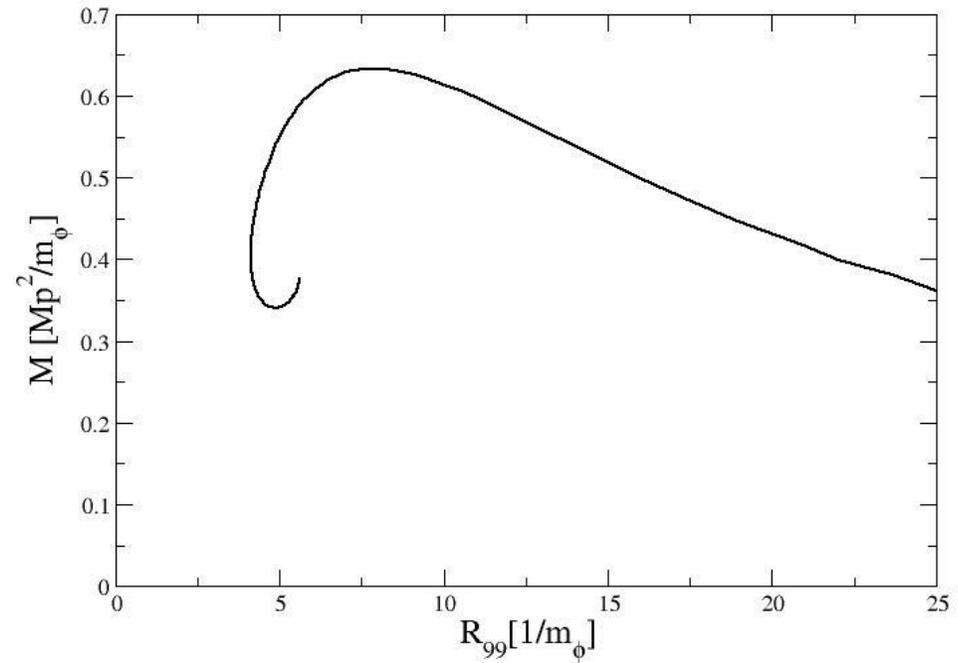
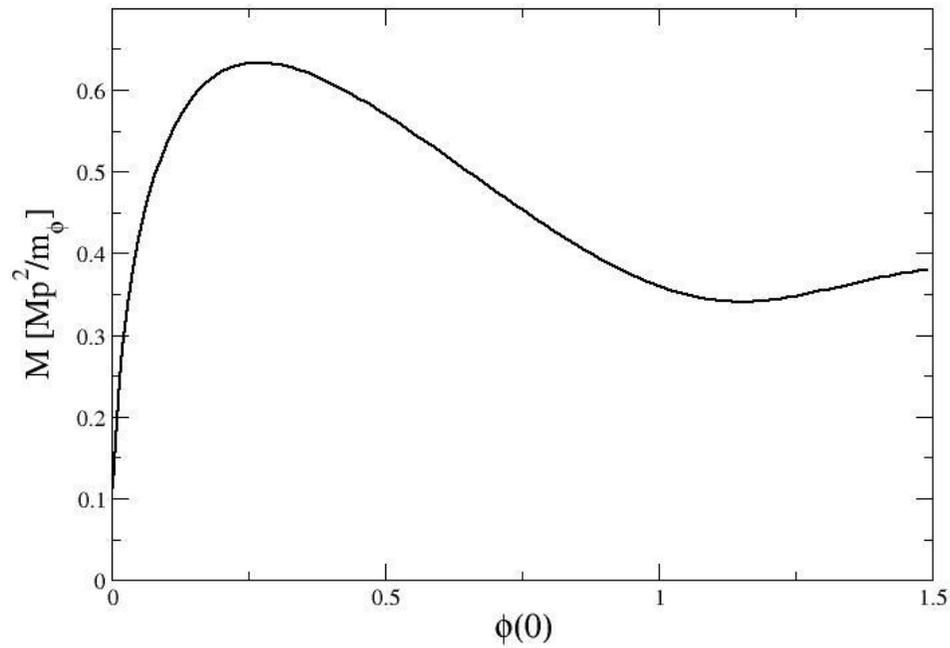
SILVANO BONAZZOLA‡

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(Received 4 February 1969)

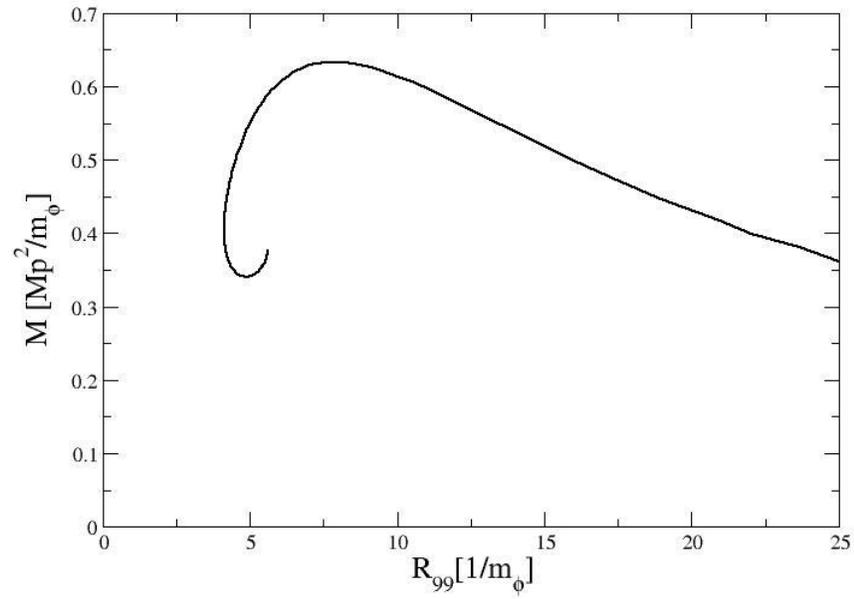
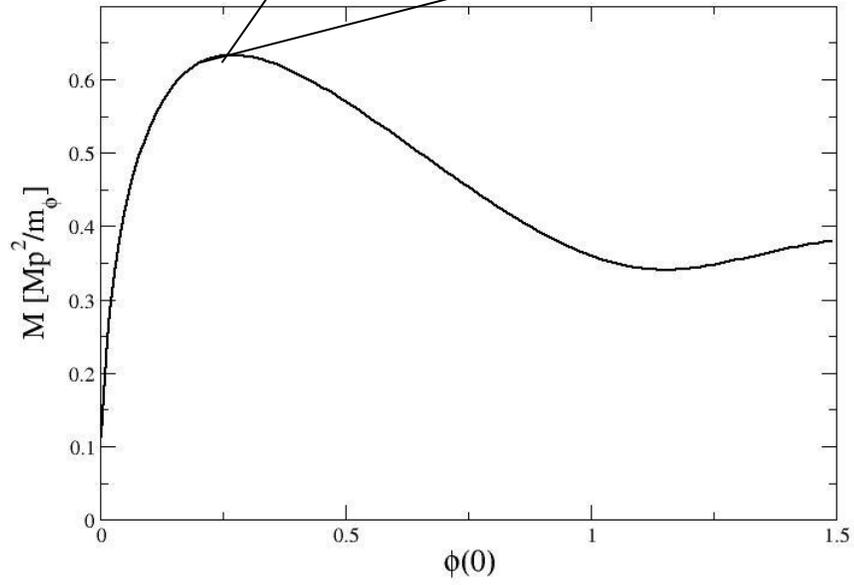
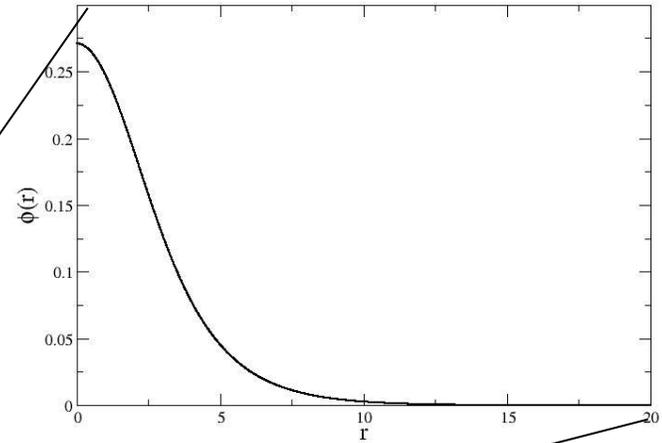


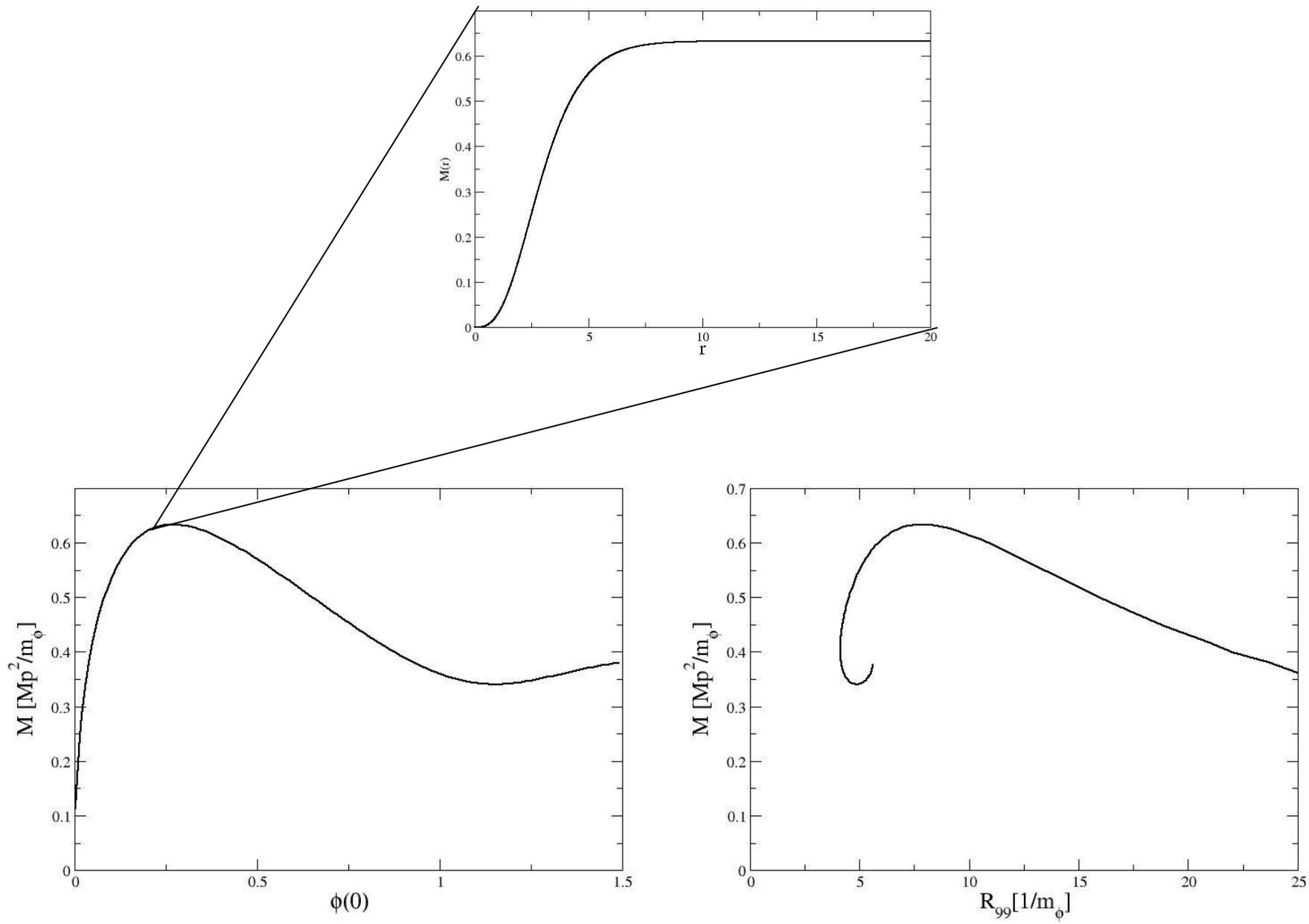
# Possible boson star configurations



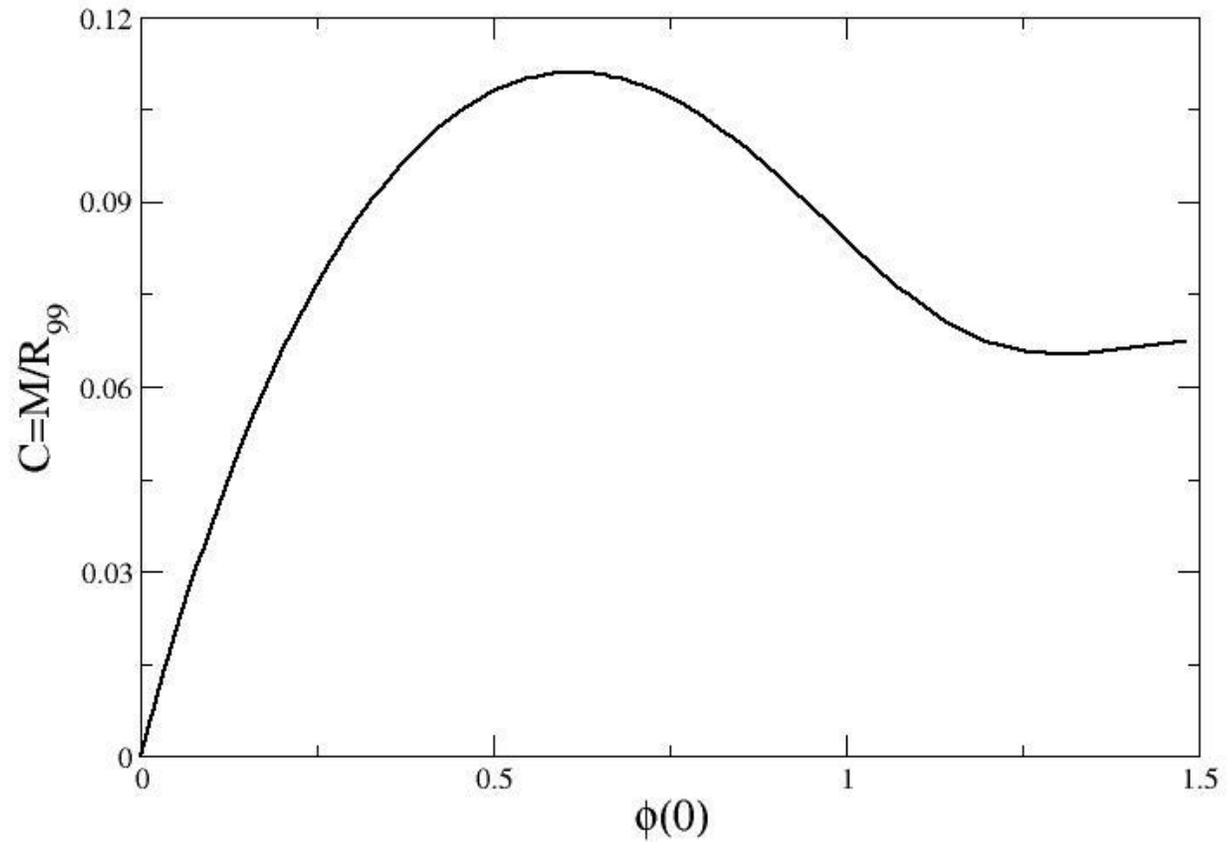
Possible bos

ations





# Compactness



Typical compactness:

Sun= .00001

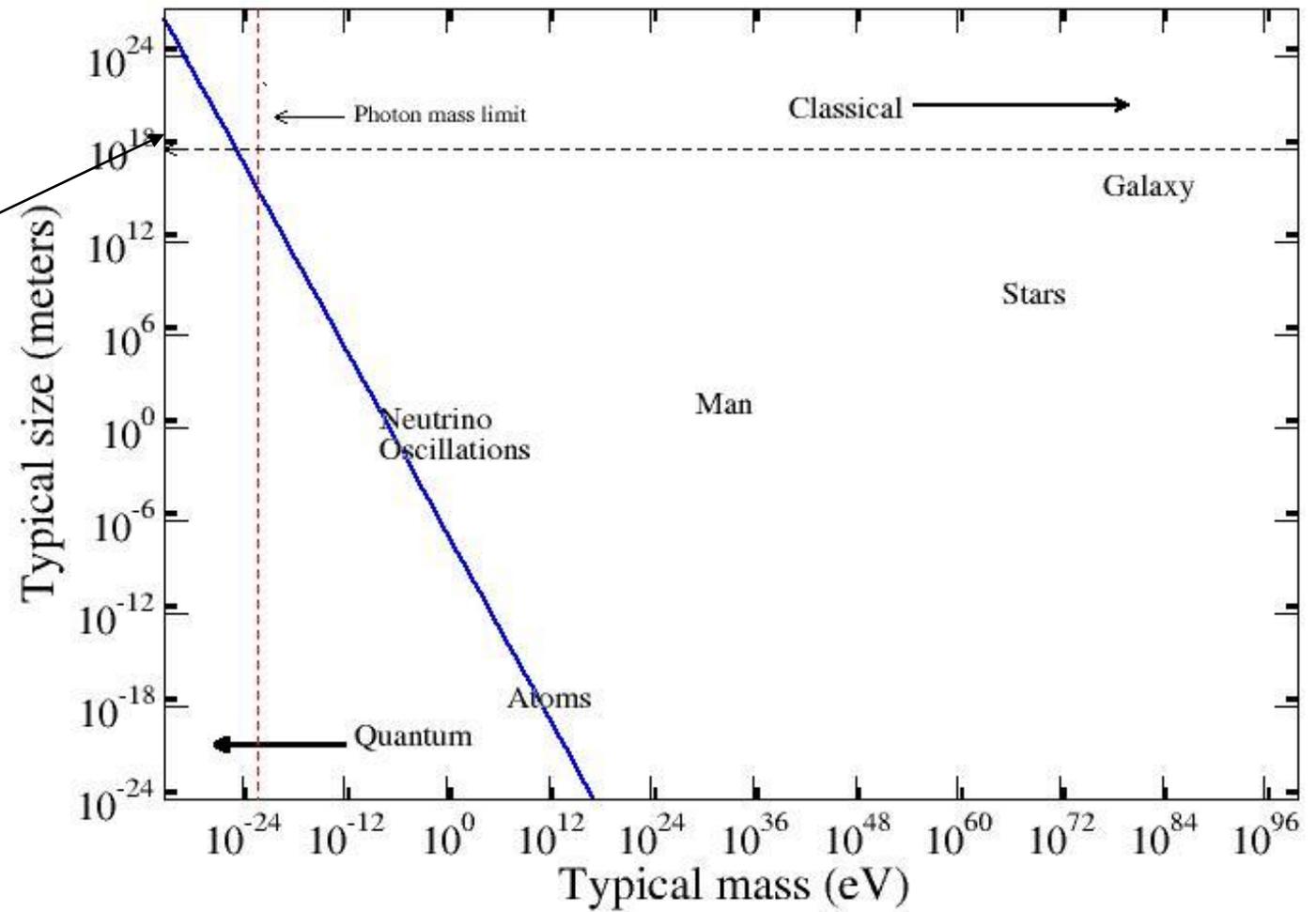
Dark matter halo=.0001

Neutron Star=0.2

# Back to the quantum

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

For ultralight Particles, the Quantum effects are visible at Astrophysical scales



# Semiclassical description of the Einstein-Klein-Gordon system

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

# The program:

- Step 1:
  - (i) Consider a stationary, globally hyperbolic background spacetime on which the free quantum fields are defined. The assumption of stationarity allows one to introduce a preferred space of “positive-norm” solutions of the matter field equations, hence a preferred vacuum state. This in turn provides a well-defined theory for the quantum fields, which we describe in terms of a Fock space representation. In particular, field operators can be written (formally) as linear combinations of creation and annihilation operators, wherein the “coefficients” are mode functions  $f_I(x)$  that solve the *classical complex* field equations.

# Paso 1: Espacio de Hilbert

Inner product

$$(\phi_1, \phi_2) := \int_{\Sigma_t} j^\mu n_\mu d\gamma = -i \int_{\Sigma_t} [\phi_1 (\mathfrak{L}_n \phi_2^*) - (\mathfrak{L}_n \phi_1) \phi_2^*] d\gamma.$$

Commutation relations

$$[\hat{\phi}(t, \vec{x}), \hat{\pi}(t, \vec{y})] = i\delta^{(3)}(\vec{x} - \vec{y}),$$
$$[\hat{\phi}(t, \vec{x}), \hat{\phi}(t, \vec{y})] = [\hat{\pi}(t, \vec{x}), \hat{\pi}(t, \vec{y})] = 0,$$

A single real quantum scalar field

$$\hat{\phi}(x) = \sum_I [\hat{a}_I f_I(x) + \hat{a}_I^\dagger f_I^*(x)],$$

Hilbert's space

$$|N_1, N_2, \dots\rangle = \frac{(\hat{a}_1^\dagger)^{N_1}}{\sqrt{N_1!}} \frac{(\hat{a}_2^\dagger)^{N_2}}{\sqrt{N_2!}} \dots |0\rangle,$$

- Step 2: (ii) Compute the expectation value  $\langle T_{\mu\nu} \rangle$  of the stress energy-momentum tensor operator with respect to a given state in the Fock space. In order to do so, we need a regularization and renormalization prescription that removes the ill-defined ultraviolet behavior of the theory, leading to sensible finite outcomes. To achieve this, in this work, we impose normal ordering. More sophisticated approaches include, e.g., adiabatic subtraction [69] and Pauli-Villars renormalization [70–72], although we expect the differences between such methods and ours to be suppressed in the limit of large occupation numbers, as we consider in our configurations, which we also assume to be far from the Planck scale. More generally, we can also compute a statistical average by tracing  $\hat{T}_{\mu\nu}$  with a density operator. This offers the interesting possibility of considering, for instance, thermal states with a given temperature.

## Step 2: Semiclasical gravity

Semiclassical

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle.$$

Normal ordering

$$:\hat{a}_I \hat{a}_I^\dagger: = \hat{a}_I^\dagger \hat{a}_I$$

Second quantization

$$\hat{T}_{\mu\nu} = \frac{1}{2} \sum_{I,J} [\hat{a}_I \hat{a}_J T_{\mu\nu}(f_I, f_J) + \hat{a}_I^\dagger \hat{a}_J T_{\mu\nu}(f_I^*, f_J) + \text{H.c.}].$$

Averages

$$\langle N_1, N_2, \dots | \hat{T}_{\mu\nu} | N_1, N_2, \dots \rangle = \sum_I N_I T_{\mu\nu}(f_I, f_I^*),$$

- Step 3:

(iii) Solve the semiclassical Einstein equations  $G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$  sourced by the expectation value (or statistical average) of the (renormalized) stress energy-momentum tensor. This step takes into account the backreaction of the quantum fields on the classical geometry.

# Static space-time

$$ds^2 = -\alpha^2(\vec{x})dt^2 + \gamma_{ij}(\vec{x})dx^i dx^j;$$

$$\partial_t^2 \phi - \alpha D^i (\alpha D_i \phi) + \alpha^2 m_0^2 \phi = 0,$$

$$f_I(t, \vec{x}) = \frac{1}{\sqrt{2\omega_I}} e^{-i\omega_I t} u_I(\vec{x}),$$

$$\langle u_1, u_2 \rangle := \int_{\Sigma} u_1^*(\vec{x}) u_2(\vec{x}) \frac{d\gamma}{\alpha(\vec{x})}, \quad u_1, u_2 \in Y.$$

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle.$$

$$R^{(3)} = 16\pi G\rho,$$

$$R_{ij}^{(3)} - \frac{1}{\alpha} D_i D_j \alpha = 4\pi G [\gamma_{ij}(\rho - S) + 2S_{ij}],$$

where:

$$\rho = \sum_I \frac{N_I}{2\omega_I} \left[ |Du_I|^2 + \left( \frac{\omega_I^2}{\alpha^2} + m_0^2 \right) |u_I|^2 \right],$$

$$j_k = \sum_I \frac{N_I}{2} \frac{i}{\alpha} [(D_k u_I) u_I^* - u_I (D_k u_I^*)],$$

$$S_{ij} = \sum_I \frac{N_I}{2\omega_I} \left\{ (D_i u_I)(D_j u_I^*) + (D_j u_I)(D_i u_I^*) - \gamma_{ij} \left[ |D_i u_I|^2 - \left( \frac{\omega_I^2}{\alpha^2} - m_0^2 \right) |u_I|^2 \right] \right\},$$

# Static-spherically symmetric

$$\gamma_{ij}dx^i dx^j = \gamma^2 dr^2 + r^2 d\Omega^2, \quad \gamma = \left(1 - \frac{2GM}{r}\right)^{-1/2},$$

Harmonic  
expansion

$$u_I(\vec{x}) = v_{n\ell}(r) Y^{\ell m}(\vartheta, \varphi), \quad I = (n\ell m),$$

Inner product

$$\int_0^\infty v_{n\ell}(r) v_{n'\ell}^*(r) \frac{\gamma(r)}{\alpha(r)} r^2 dr = \delta_{nn'}.$$

$$-\frac{\alpha}{\gamma r^2} \left( \frac{\alpha r^2}{\gamma} v'_{n\ell} \right)' + \alpha^2 \left[ \frac{\ell(\ell+1)}{r^2} + m_0^2 \right] v_{n\ell} = (\omega_{n\ell})^2 v_{n\ell}$$

# Einstein-Klein-Gordon system

$$\frac{2GM'}{r^2} = \sum_{n\ell} \frac{\kappa_\ell N_{n\ell m}}{\omega_{n\ell}} \left[ \frac{|v'_{n\ell}|^2}{\gamma^2} + \left( \frac{(\omega_{n\ell})^2}{\alpha^2} + m_0^2 + \frac{\ell(\ell+1)}{r^2} \right) |v_{n\ell}|^2 \right], \quad (44a)$$

$$\frac{1}{\alpha\gamma r^2} \left( \frac{r^2 \alpha'}{\gamma} \right)' = \sum_{n\ell} \frac{\kappa_\ell N_{n\ell m}}{\omega_{n\ell}} \left[ \left( 2 \frac{(\omega_{n\ell})^2}{\alpha^2} - m_0^2 \right) |v_{n\ell}|^2 \right], \quad (44b)$$

$$\frac{(\alpha\gamma)'}{r\alpha\gamma^3} = \sum_{n\ell} \frac{\kappa_\ell N_{n\ell m}}{\omega_{n\ell}} \left[ \frac{|v'_{n\ell}|^2}{\gamma^2} + \frac{(\omega_{n\ell})^2}{\alpha^2} |v_{n\ell}|^2 \right], \quad (44c)$$

Normalized functions:  $\psi_{n\ell} = \sqrt{\frac{N_{n\ell m}}{\omega_{n\ell}}} v_{n\ell}.$

$$\psi''_{n\ell} = - \left[ \gamma^2 + 1 - (2\ell + 1)r^2\gamma^2 \left( \frac{\ell(\ell + 1)}{r^2} + m_0^2 \right) (\psi_{n\ell})^2 \right] \frac{\psi'_{n\ell}}{r} - \left( \frac{(\omega_{n\ell})^2}{\alpha^2} - \frac{\ell(\ell + 1)}{r^2} - m_0^2 \right) \gamma^2 \psi_{n\ell},$$

$$\gamma' = \sum_{n\ell} \frac{2\ell + 1}{2} r\gamma \left[ \left( \frac{(\omega_{n\ell})^2}{\alpha^2} + \frac{\ell(\ell + 1)}{r^2} + m_0^2 \right) \gamma^2 (\psi_{n\ell})^2 + (\psi'_{n\ell})^2 \right] - \left( \frac{\gamma^2 - 1}{2r} \right) \gamma,$$

$$\alpha' = \sum_{n\ell} \frac{2\ell + 1}{2} r\alpha \left[ \left( \frac{(\omega_{n\ell})^2}{\alpha^2} - \frac{\ell(\ell + 1)}{r^2} - m_0^2 \right) \gamma^2 (\psi_{n\ell})^2 + (\psi'_{n\ell})^2 \right] + \left( \frac{\gamma^2 - 1}{2r} \right) \alpha,$$

This leads to:

$$\psi''_{nl} = - \left[ \gamma^2 + 1 - (2l + 1)r^2\gamma^2 \left( \frac{l(l+1)}{r^2} + m_0^2 \right) \psi_{nl}^2 \right] \frac{\psi'_{nl}}{r} - \left( \frac{\omega_{nl}^2}{\alpha^2} - \frac{l(l+1)}{r^2} - m_0^2 \right) \gamma^2 \psi_{nl}$$

$$\frac{d\gamma}{dr} = \sum_{nl} \frac{2l+1}{2} r \gamma \left[ \left( \frac{\omega_{nl}^2}{\alpha^2} + \frac{l(l+1)}{r^2} + m_0^2 \right) \gamma^2 \psi_{nl}^2 + \psi_{nl}'^2 \right] - \left( \frac{\gamma^2 - 1}{2r} \right) \gamma,$$

$$\frac{d\alpha}{dr} = \sum_{nl} \frac{2l+1}{2} r \alpha \left[ \left( \frac{\omega_{nl}^2}{\alpha^2} - \frac{l(l+1)}{r^2} - m_0^2 \right) \gamma^2 \psi_{nl}^2 + \psi_{nl}'^2 \right] + \left( \frac{\gamma^2 - 1}{2r} \right) \alpha,$$

$$n = 1, l = 0$$

$$n = 1, 2, 3 \dots, l = 0$$

$$n = 1, l = 1, 2, 3 \dots$$

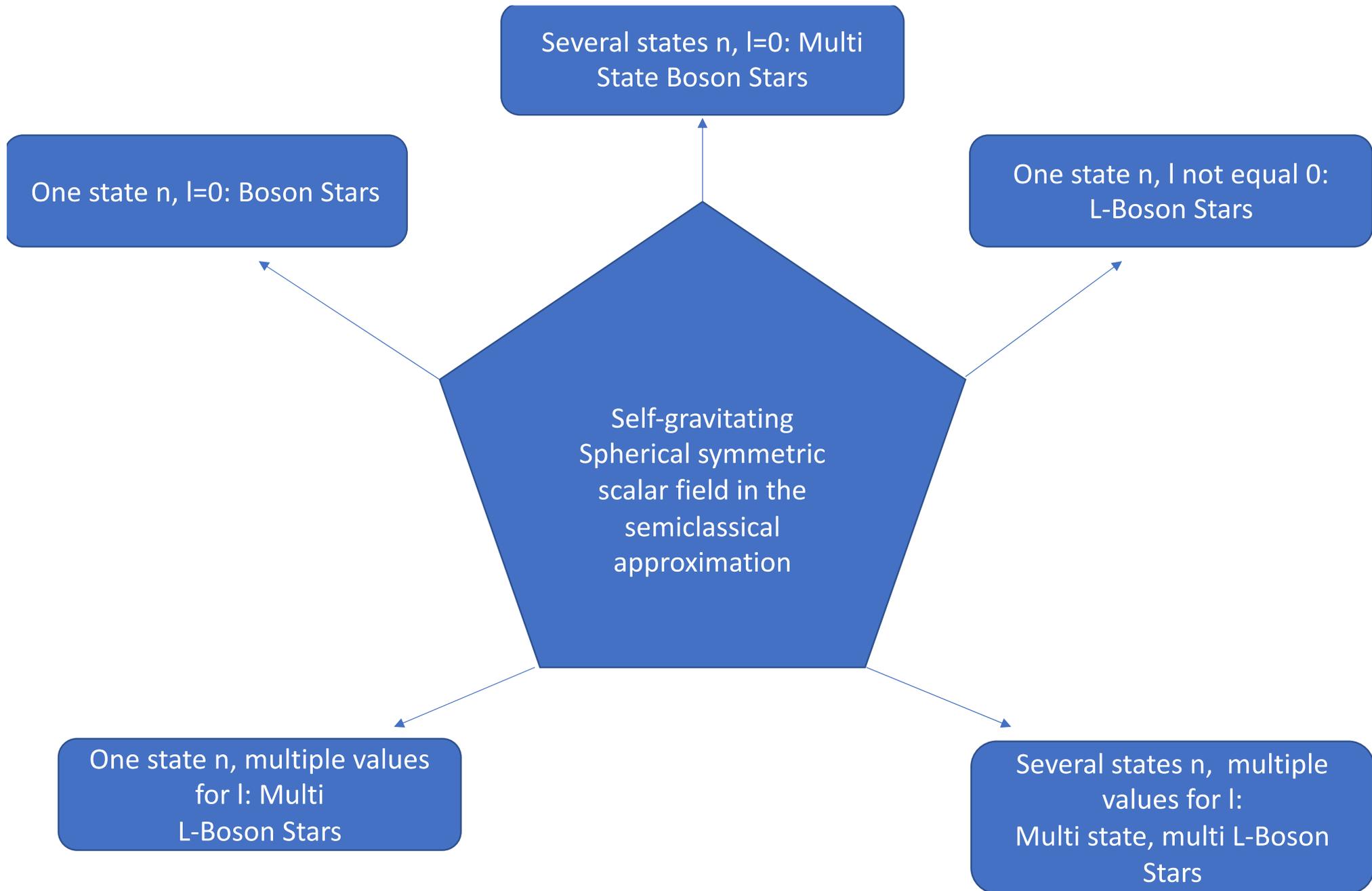
Boson stars

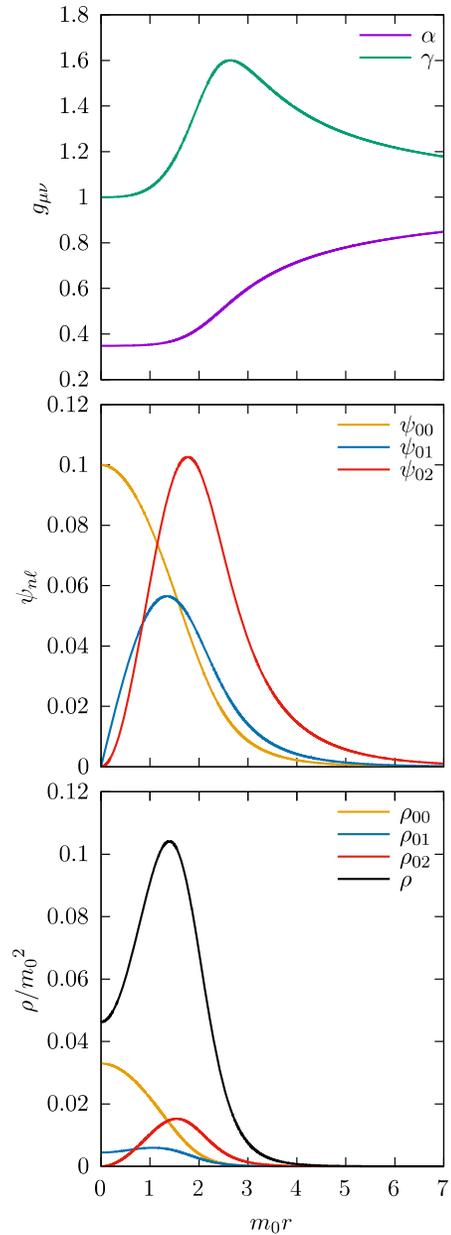
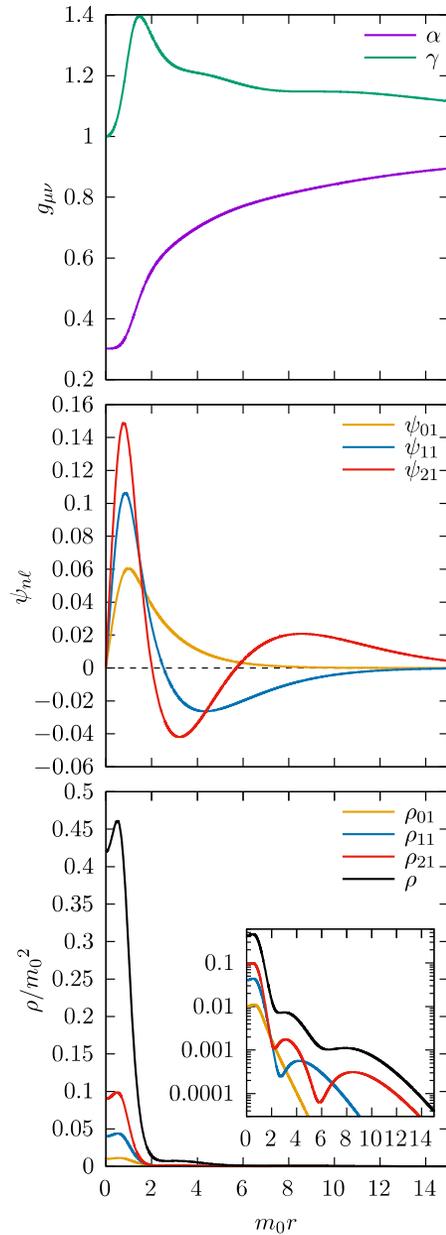
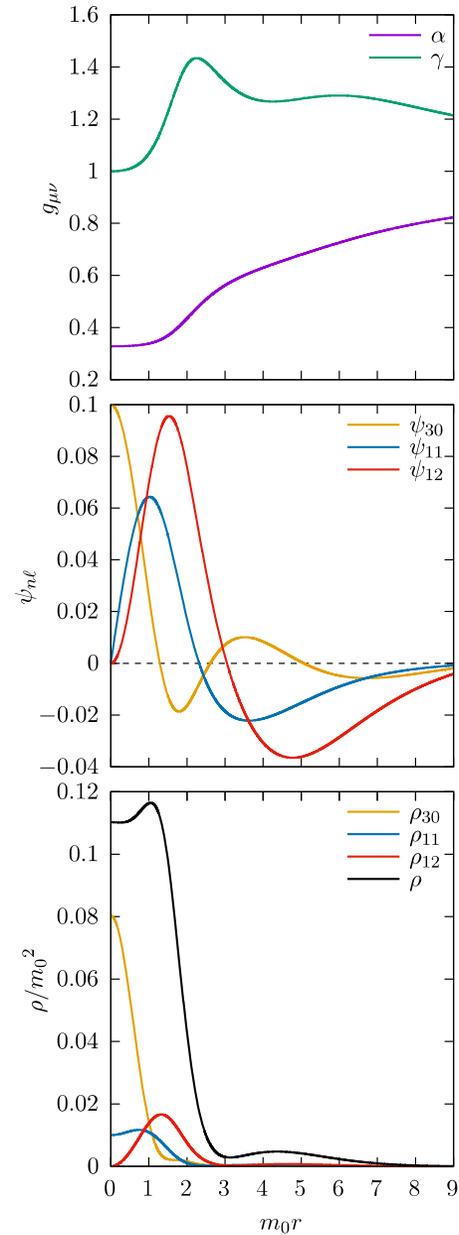
Multistate Boson stars

$l$ -Boson Star



## Boson stars relatives




 (a) Multi- $\ell$  boson star

 (b) Multi-state  $\ell$ -boson star

 (c) Multi- $\ell$  multi-state boson star

Name	$n$	$\ell$	$\psi_{n\ell}^0/m_0^\ell$	$\omega_{n\ell}/m_0$	$m_0^2 N_{nl}$	$m_0^2 N$	Fig.
Multi- $\ell$ boson star	0	0	0.1	0.5278	0.0195	0.8134	1(a)
	0	1	0.2	0.6453	0.0243		
	0	2	0.4	0.7736	0.1442		
Multistate $\ell$ -boson star	0	1	0.3	0.7438	0.0289	1.3884	1(b)
	1	1	0.6	0.8235	0.1150		
	2	1	0.9	0.8792	0.3189		
Multi- $\ell$ multistate boson star	3	0	0.1	0.8679	0.0133	1.2745	1(c)
	1	1	0.3	0.7497	0.0439		
	1	2	0.5	0.8247	0.2259		

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### Boson stars and their relatives in semiclassical gravity

Miguel Alcubierre<sup>1</sup>, Juan Barranco<sup>2</sup>, Argelia Bernal<sup>2</sup>, Juan Carlos Degollado<sup>3</sup>, Alberto Diez-Tejedor<sup>2</sup>,  
Miguel Megevand<sup>4</sup>, Darío Núñez<sup>1</sup>, and Olivier Sarbach<sup>5</sup>

Some properties of boson  
star relatives

# Multi-State Boson stars (MSBS)

$$\hat{\Phi} = \sum_{nlm} \hat{b}_{nlm} \Phi_{nlm}(t, \mathbf{x}) + \hat{b}_{nlm}^\dagger \Phi_{nlm}^*(t, \mathbf{x})$$

$$\hat{T}_{ab} = \partial_a \hat{\Phi} \partial_b \hat{\Phi} - \frac{1}{2} g_{ab} (g^{cd} \partial_c \hat{\Phi} \partial_d \hat{\Phi} + \mu^2 |\hat{\Phi}|^2)$$

$$G_{ab} = 8\pi \langle Q | \hat{T}_{ab} | Q \rangle$$

$$ds^2 = -\alpha^2(r) dt^2 + a^2(r) dr^2 + r^2 d\Omega.$$

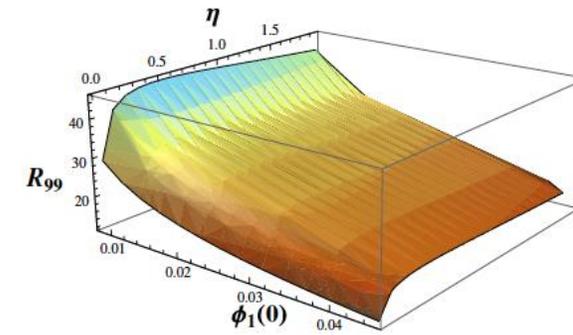
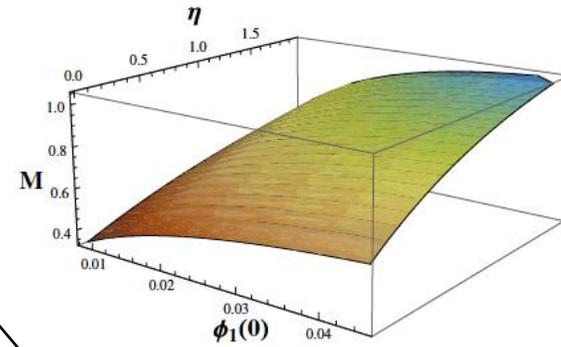
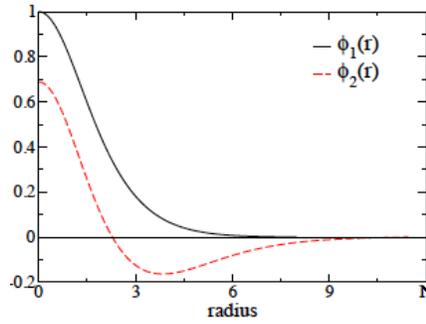
$$\partial_r a = \frac{a}{2} \left\{ -\frac{a^2 - 1}{r} + 4\pi r \sum_{n=1}^{\mathcal{J}} \left[ \left( \frac{\omega_n^2}{\alpha^2} + m^2 \right) a^2 \phi_n^2 + \Phi_n^2 \right] \right\},$$

$$\partial_r \alpha = \frac{\alpha}{2} \left\{ \frac{a^2 - 1}{r} + 4\pi r \sum_{n=1}^{\mathcal{J}} \left[ \left( \frac{\omega_n^2}{\alpha^2} - m^2 \right) a^2 \phi_n^2 + \Phi_n^2 \right] \right\},$$

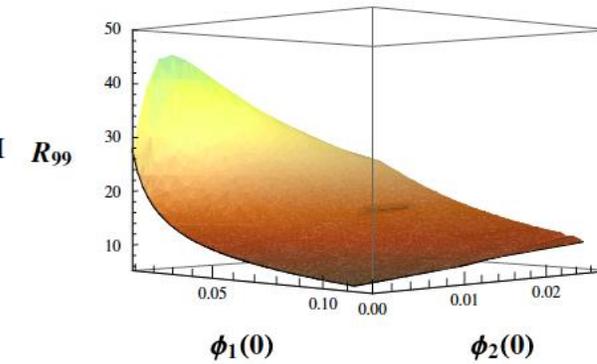
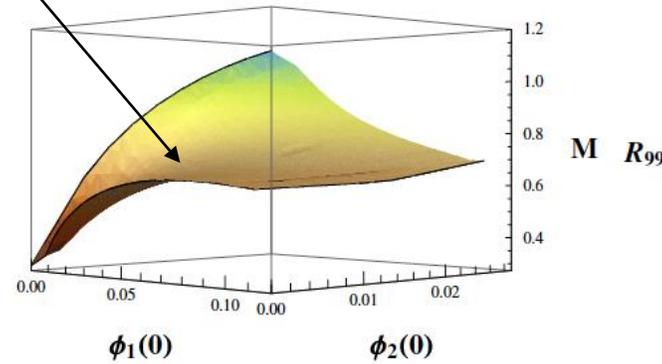
$$\partial_r \phi_n = \Phi_n,$$

$$\partial_r \Phi_n = - \left\{ 1 + a^2 - 4\pi r^2 a^2 m^2 \left( \sum_{s=1}^{\mathcal{J}} \phi_s^2 \right) \right\} \frac{\Phi_n}{r} - \left( \frac{\omega_n^2}{\alpha^2} - m^2 \right) \phi_n a^2.$$

$n=2$



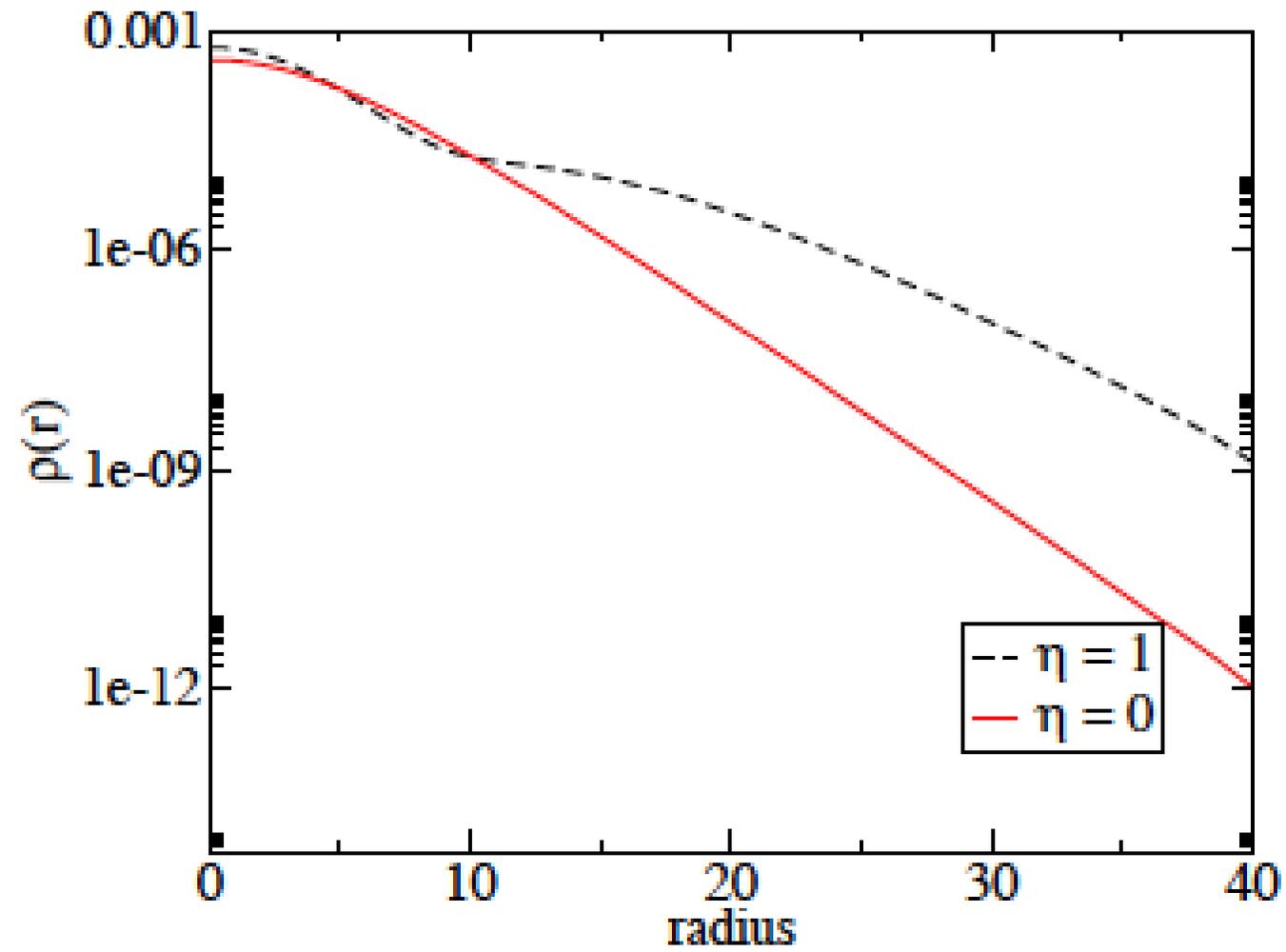
$$\eta = \frac{N^{(2)}}{N^{(1)}}$$



PHYSICAL REVIEW D 81, 044031 (2010)

## Multistate boson stars

A. Bernal,<sup>1</sup> J. Barranco,<sup>1</sup> D. Alic,<sup>1,2</sup> and C. Palenzuela<sup>1,3</sup>



# $\ell$ -Boson Star

$$\sum_{m=-\ell}^{\ell} |Y^{\ell m}(\vartheta, \varphi)|^2 = \frac{2\ell + 1}{4\pi}$$

$$T_{\mu\nu} = \frac{1}{2} \sum_i [\nabla_\mu \Phi_i^* \nabla_\nu \Phi_i + \nabla_\mu \Phi_i \nabla_\nu \Phi_i^* - g_{\mu\nu} (\nabla_\alpha \Phi_i^* \nabla^\alpha \Phi_i + \mu^2 \Phi_i^* \Phi_i)]$$

$$\Phi_{\ell m}(t, r, \vartheta, \varphi) = \phi_\ell(t, r) Y^{\ell m}(\vartheta, \varphi)$$

$$ds^2 = -\alpha^2 dt^2 + \gamma^2 dr^2 + r^2 d\Omega^2, \quad \gamma^2 := \frac{1}{1 - \frac{2M}{r}},$$

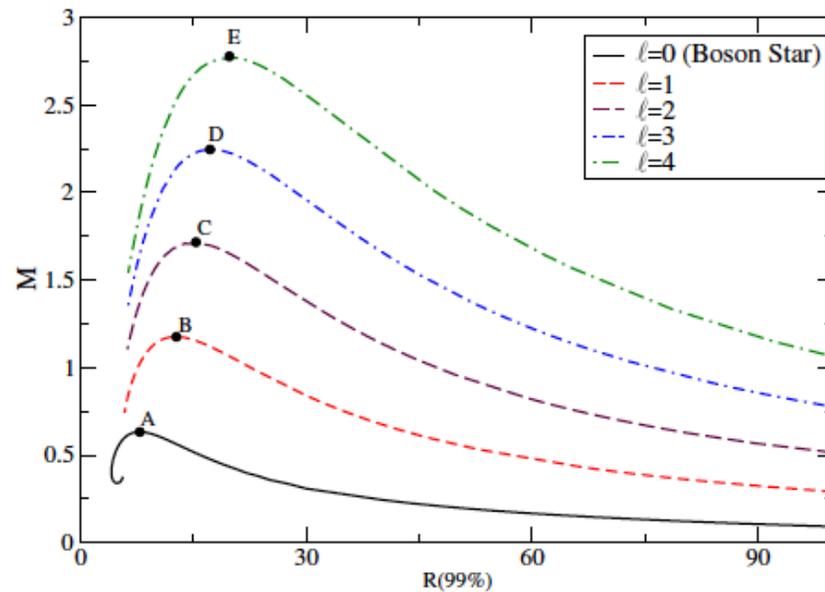
$$u_\ell := \psi_\ell / r^\ell$$

# $\ell$ -Boson Star

$$\gamma' = \frac{2\ell + 1}{2} r \gamma \left[ \left( \frac{\omega^2}{\alpha^2} + \frac{\ell(\ell + 1)}{r^2} + \mu^2 \right) \gamma^2 u_\ell^2 r^{2\ell} + (u_\ell' r^\ell + \ell u_\ell r^{\ell-1})^2 \right] - \left( \frac{\gamma^2 - 1}{2r} \right) \gamma,$$

$$\alpha' = \frac{2\ell + 1}{2} r \alpha \left[ \left( \frac{\omega^2}{\alpha^2} - \frac{\ell(\ell + 1)}{r^2} - \mu^2 \right) \gamma^2 u_\ell^2 r^{2\ell} + (u_\ell' r^\ell + \ell u_\ell r^{\ell-1})^2 \right] + \left( \frac{\gamma^2 - 1}{2r} \right) \alpha,$$

$$u_\ell'' = \left( \mu^2 - \frac{\omega^2}{\alpha^2} \right) \gamma^2 u_\ell - (\gamma^2 + 2\ell + 1) \frac{u_\ell'}{r} + \ell^2 (\gamma^2 - 1) \frac{u_\ell}{r^2} + (2\ell + 1) \left( \mu^2 + \frac{\ell(\ell + 1)}{r^2} \right) \gamma^2 (r u_\ell' + \ell u_\ell) u_\ell^2 r^{2\ell},$$



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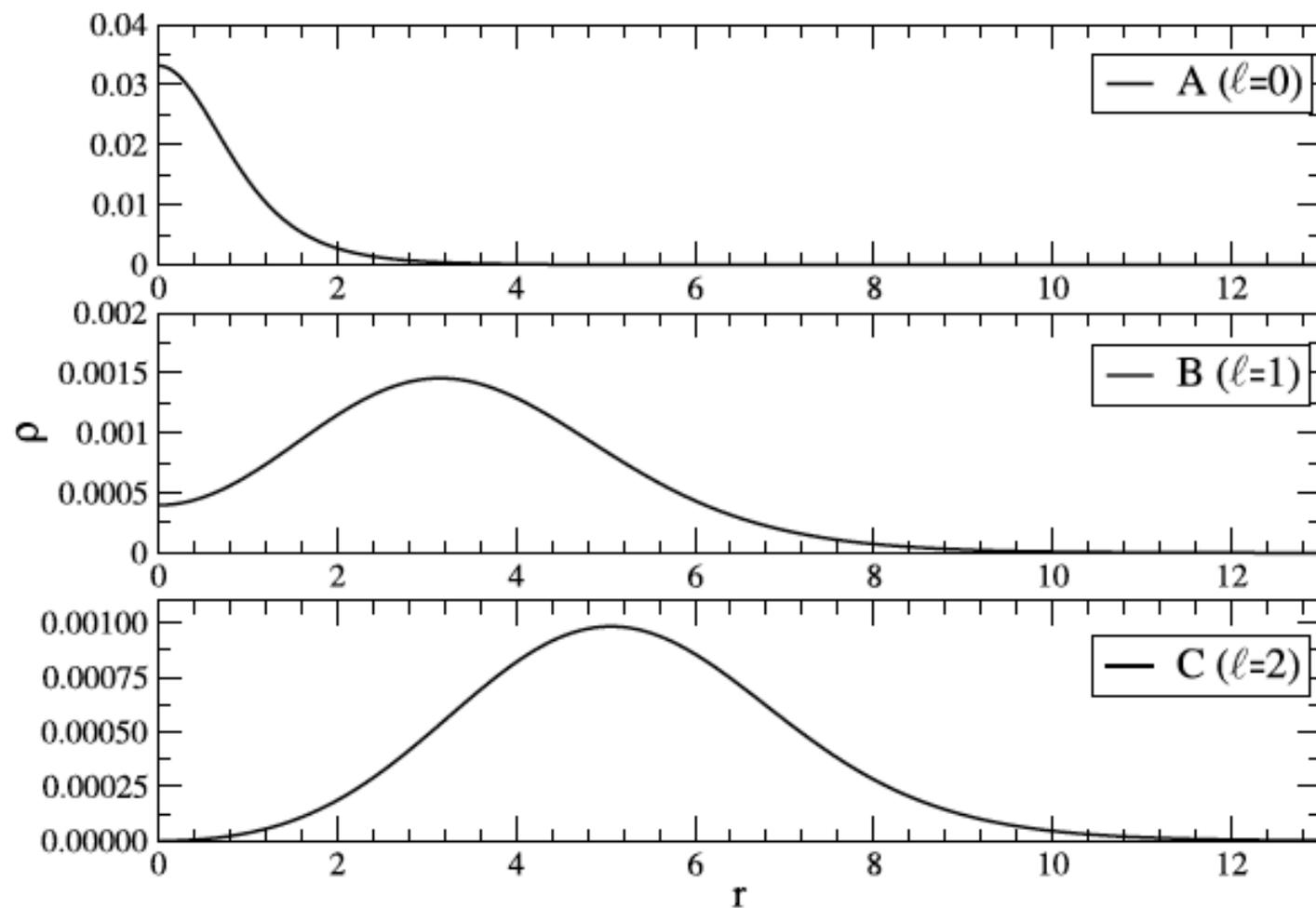
Classical and Quantum Gravity

<https://doi.org/10.1088/1361-6382/aadc6e>

Letter

## $\ell$ -boson stars

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 Juan Carlos Degollado<sup>3</sup>, Alberto Diez-Tejedor<sup>2</sup>,  
 Miguel Megevand<sup>4</sup>, Darío Núñez<sup>1</sup> and Olivier Sarbach<sup>5,6</sup>



Configuration	$M$	$R(99\%)$	$\omega$	$M/R(99\%)$
A ( $\ell = 0$ )	0.63	7.89	0.854	0.08
B ( $\ell = 1$ )	1.18	12.75	0.836	0.09
C ( $\ell = 2$ )	1.72	15.35	0.832	0.11
D ( $\ell = 3$ )	2.25	17.22	0.820	0.13
E ( $\ell = 4$ )	2.78	19.80	0.819	0.14

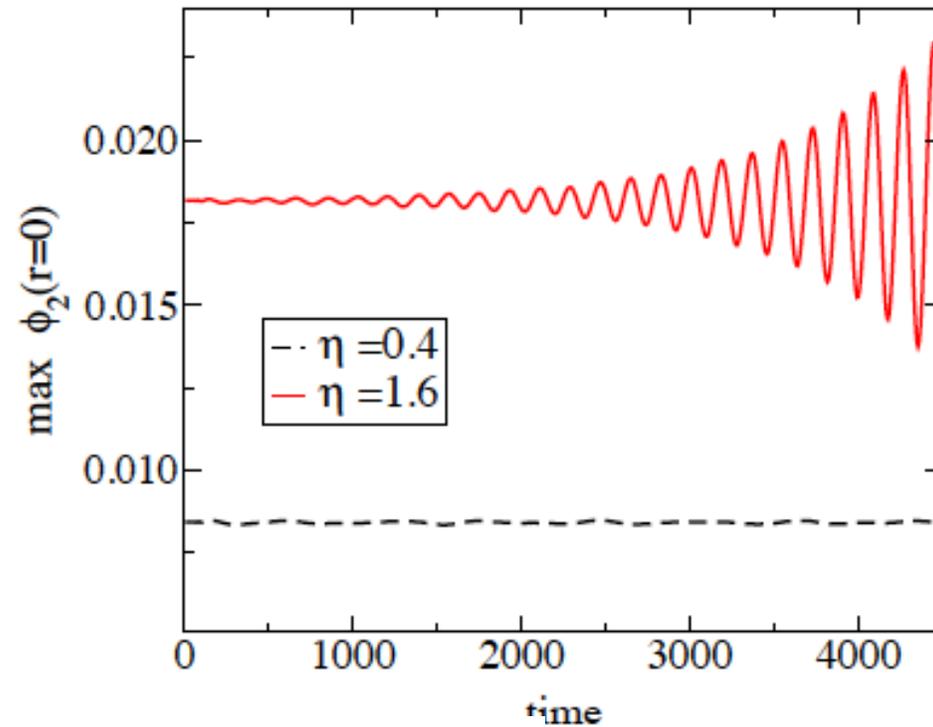
Astrophysical realization of boson stars  
relatives demands stability

# Numerical perturbation analysis: Multistate boson stars

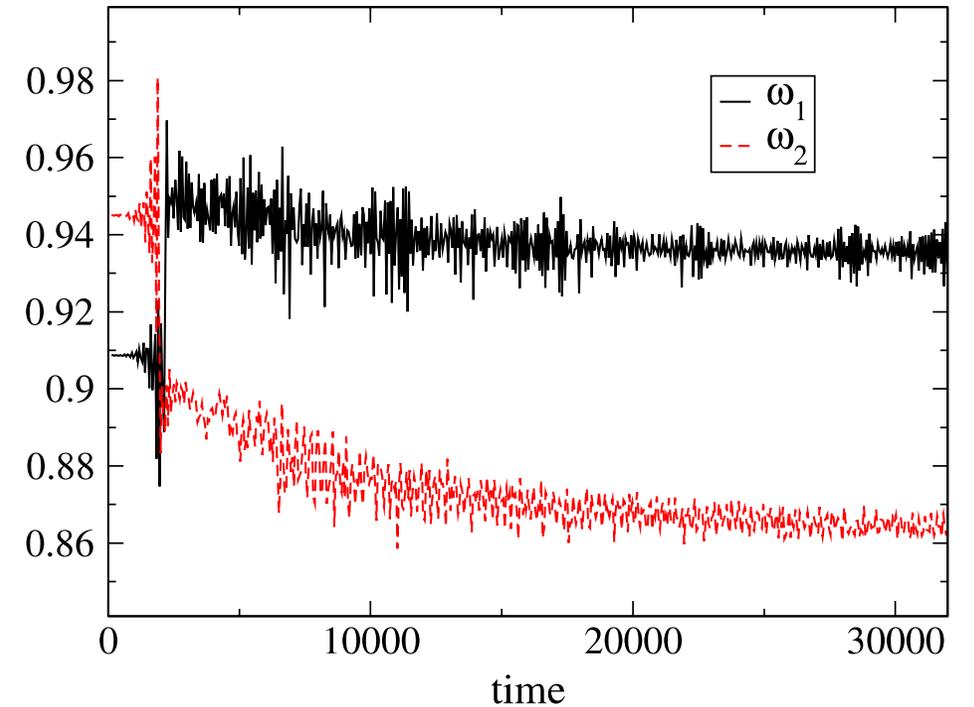
Boson stars in excited states are unstable under numerical perturbations.

Multistate boson stars, even with particles in the excited states, can be stable

$$\eta = \frac{N^{(2)}}{N^{(1)}}$$



Stable if  $\eta < 1$

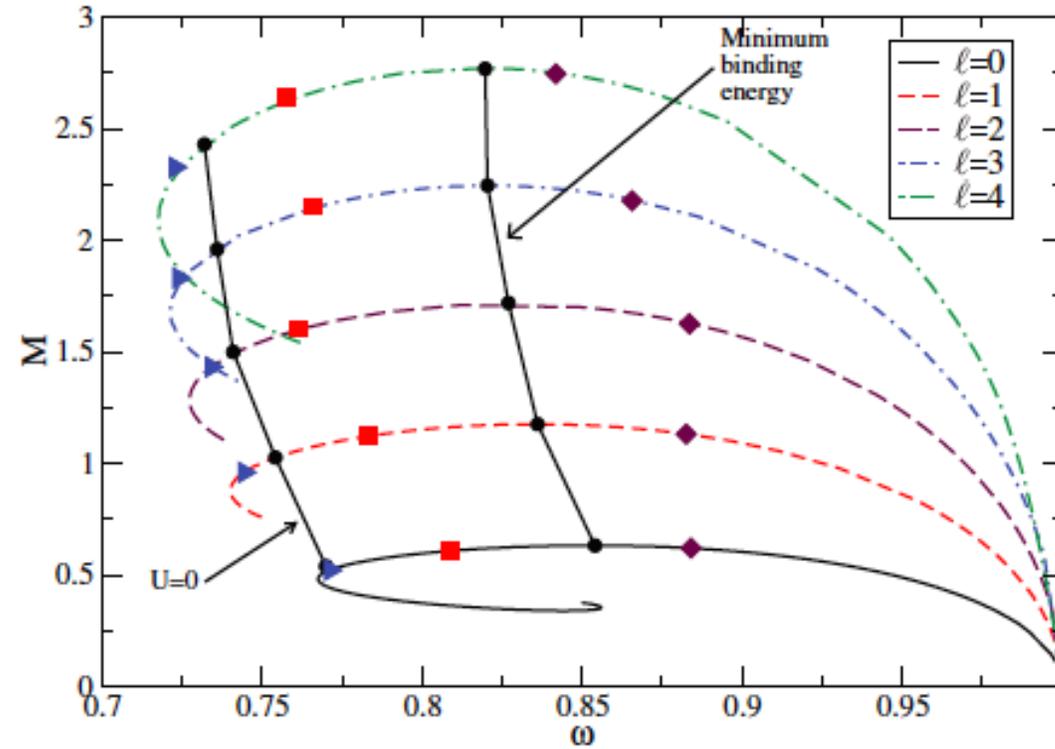


$\eta = 3$

# Numerical perturbation analysis: $\ell$ -Boson Star

$\ell$	$a_0$	$\omega$	Perturbation	$M$	$N_B$	$U$	$\epsilon/\varphi_R^{\max}$	$s$	$r_0$	End result
0	0.2	0.88401	Type 0	0.6209	0.6391	-0.0182	—	—	—	Stable
0	0.2	0.88401	Type I	0.6211	0.6394	-0.0183	+0.005	0	0.0	Stable
0	0.2	0.88401	Type I	0.6207	0.6389	-0.0182	-0.005	0	0.0	Stable
0	0.2	0.88401	Type II	0.6209	0.6391	-0.0182	+0.005	-1	0.0	Stable
0	0.2	0.88401	Type II	0.6209	0.6391	-0.0182	-0.005	-1	0.0	Stable
0	0.2	0.88401	Type III	0.6238	0.6412	-0.0174	+0.01	+1	20.0	Stable
0	0.2	0.88401	Type III	0.6237	0.6372	-0.0135	+0.01	-1	20.0	Stable
0	0.4	0.80866	Type 0	0.6088	0.6235	-0.0147	—	—	—	Black hole
0	0.4	0.80866	Type I	0.6096	0.6246	-0.0150	+0.005	0	0.0	Black hole
0	0.4	0.80866	Type I	0.6079	0.6225	-0.0146	-0.005	0	0.0	Migration to stable branch
0	0.4	0.80866	Type II	0.6087	0.6235	-0.0148	+0.005	-1	0.0	Migration to stable branch
0	0.4	0.80866	Type II	0.6088	0.6236	-0.0148	-0.005	-1	0.0	Black hole
0	0.4	0.80866	Type III	0.6193	0.6305	-0.0112	+0.01	+1	20.0	Black hole
0	0.4	0.80866	Type III	0.6193	0.6166	+0.0027	+0.01	-1	20.0	Black hole
0	0.6	0.77134	Type 0	0.5248	0.5167	+0.0081	—	—	—	Black hole
0	0.6	0.77134	Type I	0.5266	0.5190	+0.0075	+0.005	0	0.0	Black hole
0	0.6	0.77134	Type I	0.5230	0.5144	+0.0086	-0.005	0	0.0	Explosion to infinity
0	0.6	0.77134	Type II	0.5246	0.5165	+0.0081	+0.005	-1	0.0	Explosion to infinity
0	0.6	0.77134	Type II	0.5250	0.5169	+0.0081	-0.005	-1	0.0	Black hole
0	0.6	0.77134	Type III	0.5481	0.5314	+0.0167	+0.01	+1	20.0	Black hole
0	0.6	0.77134	Type III	0.5480	0.5020	+0.0460	+0.01	-1	20.0	Black hole

# Three fates of $\ell$ -Boson Star



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## Dynamical evolutions of $\ell$ -boson stars in spherical symmetry

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Juan Carlos Degollado<sup>3</sup>, Alberto Diez-Tejedor<sup>2</sup>,  
Miguel Megevand<sup>4</sup>, Darío Núñez<sup>1</sup> and Olivier Sarbach<sup>5,6</sup>

# Linear stability analysis for

# $\ell$ -Boson Star

Perturb the system  $\phi_\ell(t, r) = e^{i\omega t} [\psi_{\ell 1}(t, r) + i\psi_{\ell 2}(t, r)]$

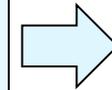
$$\delta\psi_{\ell 1}(t, r) = \psi_{\ell 0}(r)\delta\varphi_{\ell 1}(t, r),$$

$$\delta\psi_{\ell 2}(t, r) = \psi_{\ell 0}(r)\delta\varphi_{\ell 2}(t, r),$$

$$\delta\alpha(t, r) = \frac{1}{2}\alpha_0(r)\delta\nu(t, r),$$

$$\delta\gamma(t, r) = \frac{1}{2}\gamma_0(r)\delta\lambda(t, r),$$

$$\begin{aligned} & \delta\varphi''_{\ell 1} + \left[ \frac{2}{r} + \frac{\alpha'}{\alpha} - \frac{\gamma'}{\gamma} \right] \delta\varphi'_{\ell 1} + \frac{1}{\kappa_\ell r \psi_\ell^2} \delta\lambda' - \frac{\gamma^2}{\alpha^2} \delta\ddot{\varphi}_{\ell 1} \\ & + \left\{ \frac{1 - 2r\frac{\gamma'}{\gamma}}{\kappa_\ell r^2 \psi_\ell^2} + \frac{\psi'_\ell}{\psi_\ell} \left[ \frac{\alpha'}{\alpha} - \frac{\gamma'}{\gamma} + \frac{\psi'_\ell}{\psi_\ell} + \frac{1}{r} \right] - \gamma^2 \left[ \mu^2 + \frac{\ell(\ell+1)}{r^2} - \frac{\omega^2}{\alpha^2} \right] \right\} \delta\lambda \\ & - 2\gamma^2 \left\{ \mu^2 + \frac{\ell(\ell+1)}{r^2} + \frac{\omega^2}{\alpha^2} + \frac{\psi_\ell'^2}{\gamma^2 \psi_\ell^2} + \kappa_\ell r \left[ \mu^2 + \frac{\ell(\ell+1)}{r^2} \right] \psi_\ell \psi_\ell' \right\} \delta\varphi_{\ell 1} = 0, \\ \\ & \delta\lambda'' + 3 \left( \frac{\alpha'}{\alpha} - \frac{\gamma'}{\gamma} \right) \delta\lambda' + 4\kappa_\ell \left\{ 2\psi_\ell \psi_\ell' - r\gamma^2 \left[ \mu^2 + \frac{\ell(\ell+1)}{r^2} \right] \psi_\ell^2 \right\} \delta\varphi'_{\ell 1} - \frac{\gamma^2}{\alpha^2} \delta\ddot{\lambda} \\ & - 2 \left\{ 2\kappa_\ell \psi_\ell'^2 + \frac{1}{r^2} + \left( \frac{\gamma'}{\gamma} \right)' - \left( \frac{\alpha'}{\alpha} - \frac{\gamma'}{\gamma} \right)^2 - \frac{1}{r} \left( 2\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} \right) \right\} \delta\lambda \\ & + 4\kappa_\ell \left\{ 2\psi_\ell'^2 - r\gamma^2 \left[ \mu^2 + \frac{\ell(\ell+1)}{r^2} \right] \psi_\ell^2 \left[ 2\frac{\psi'_\ell}{\psi_\ell} + 2\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma} \right] + \gamma^2 \frac{\ell(\ell+1)}{r^2} \psi_\ell^2 \right\} \delta\varphi_{\ell 1} = 0, \end{aligned}$$



**Pulsation  
equations**

Pulsation equations can be rewritten:

Define  $f_1 = \delta\varphi_{\ell 1}$ ,  $f_2 = \frac{1}{\omega} \left[ \frac{\delta\lambda}{2\kappa_{\ell r}\psi_{\ell}^2} - \frac{\psi'_{\ell}}{\psi_{\ell}}\delta\varphi_{\ell 1} \right]$ .

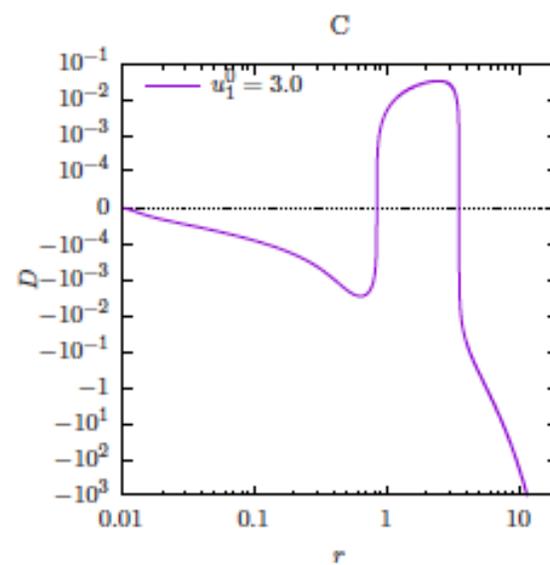
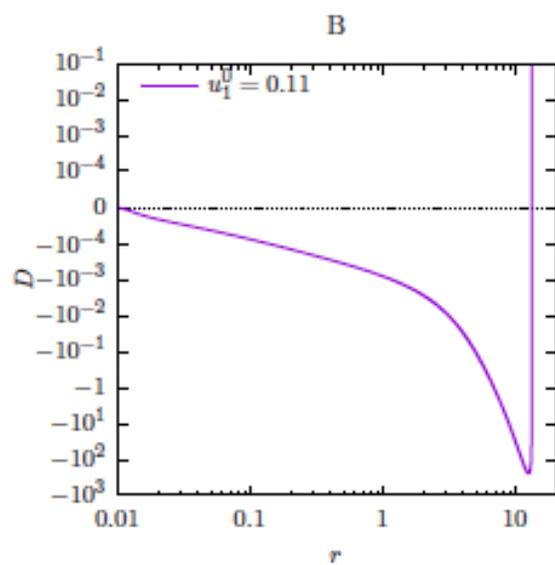
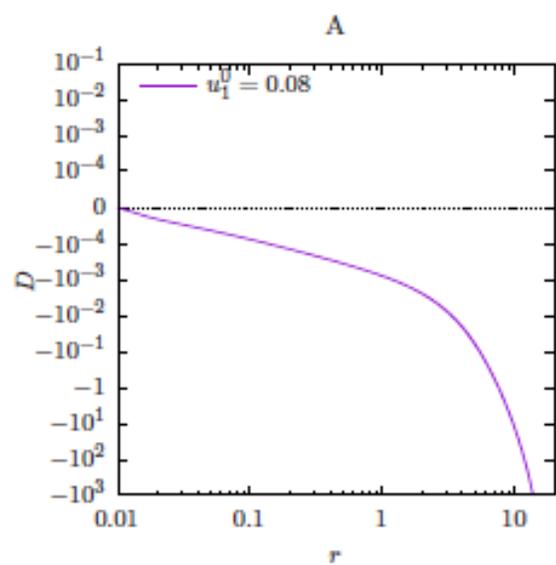
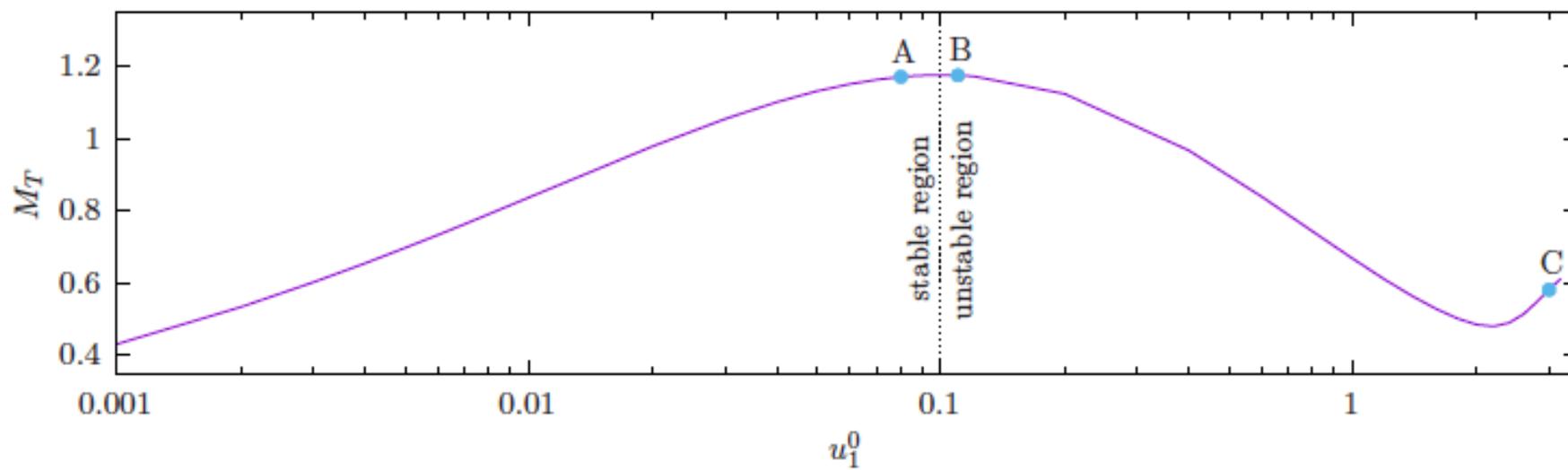
The system for  $f := \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$  can be written in the form

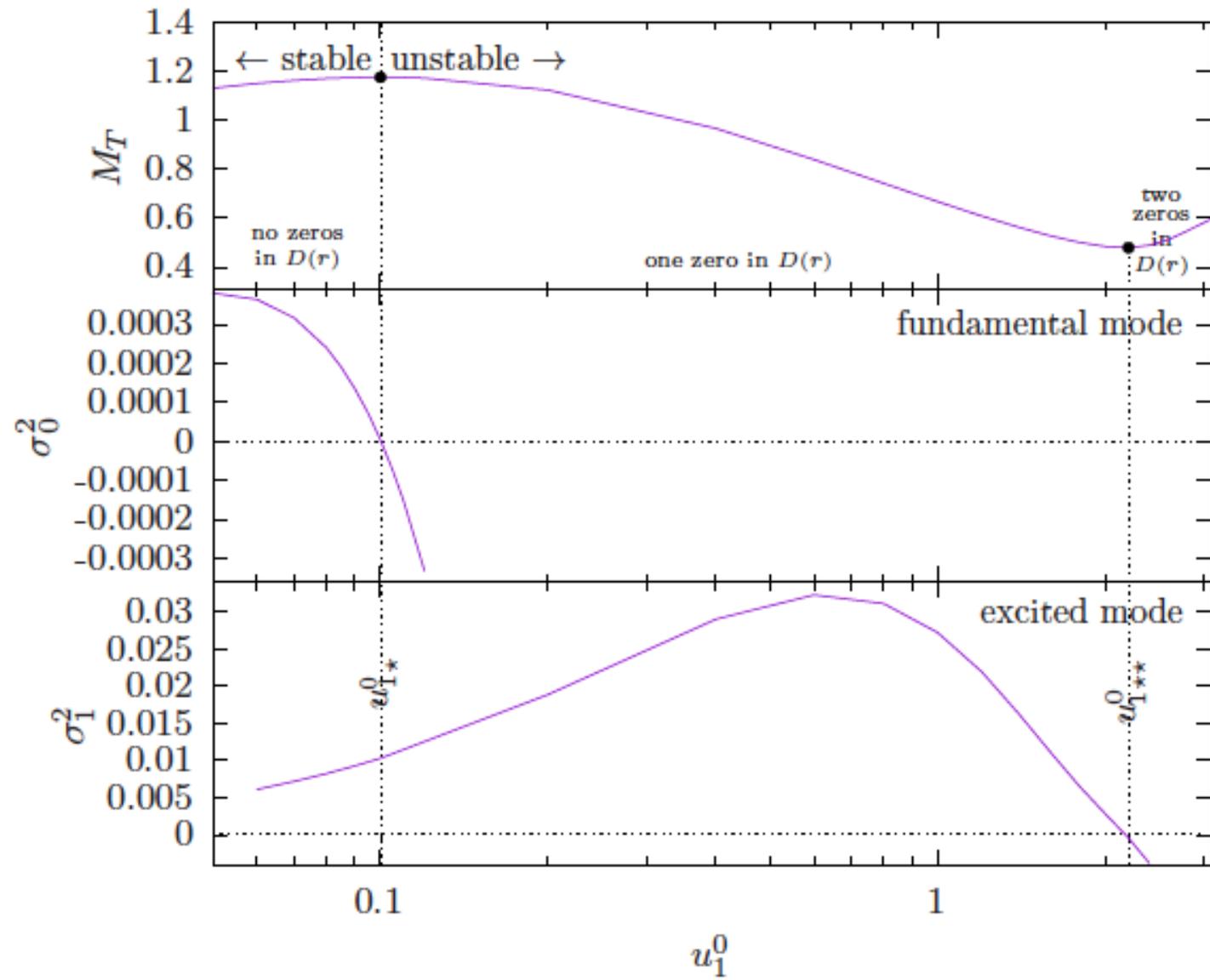
$$\frac{\gamma^2}{\alpha^2} A \ddot{f} = \frac{d}{dr} \left( A \frac{df}{dr} \right) + \frac{d}{dr} (Bf) - B^T \frac{df}{dr} + Cf,$$

the form  $\ddot{f} = -\mathcal{H}f$ , with  $\mathcal{H}$  the Schrödinger-type operator given by

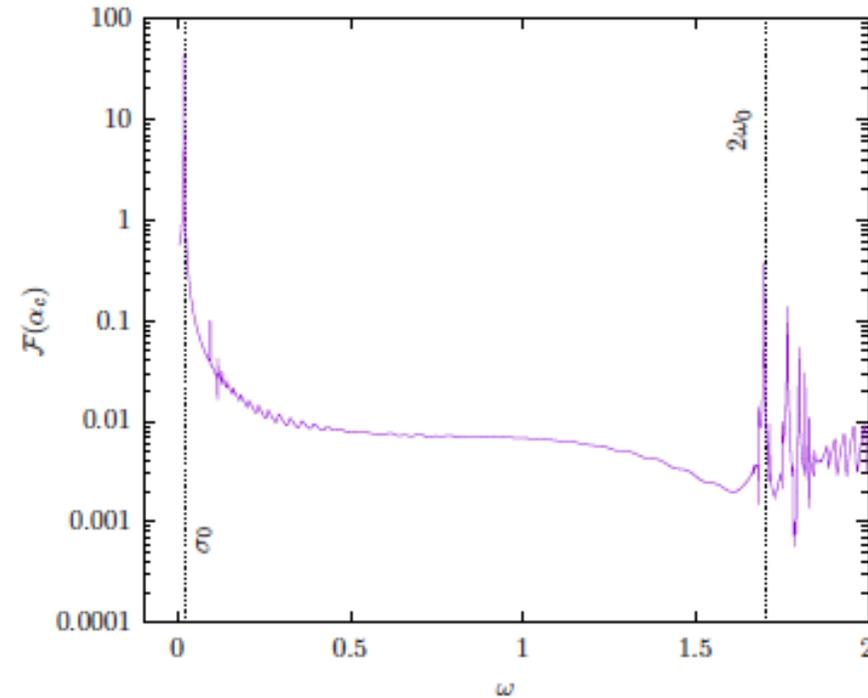
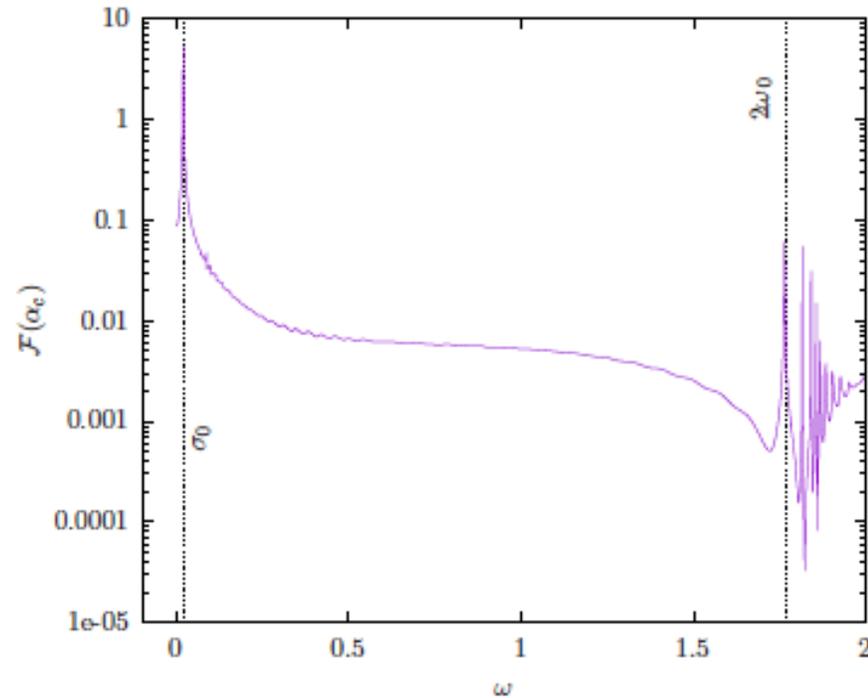
$$\mathcal{H} = \frac{\alpha^2}{\gamma^2} A^{-1} \left[ -\frac{d}{dr} A \frac{d}{dr} - \frac{d}{dr} B + B^T \frac{d}{dr} - C \right].$$

Thus it is possible to count the number of unstable modes by means of the Nodal Theorem

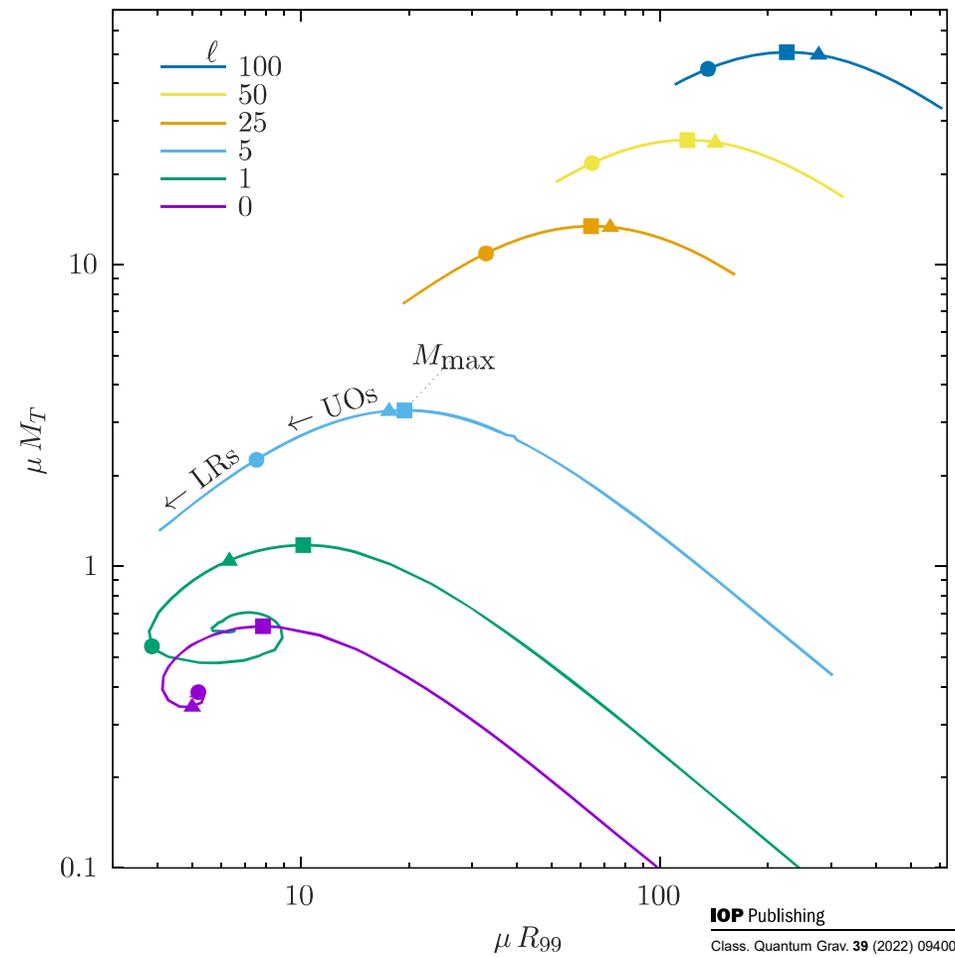
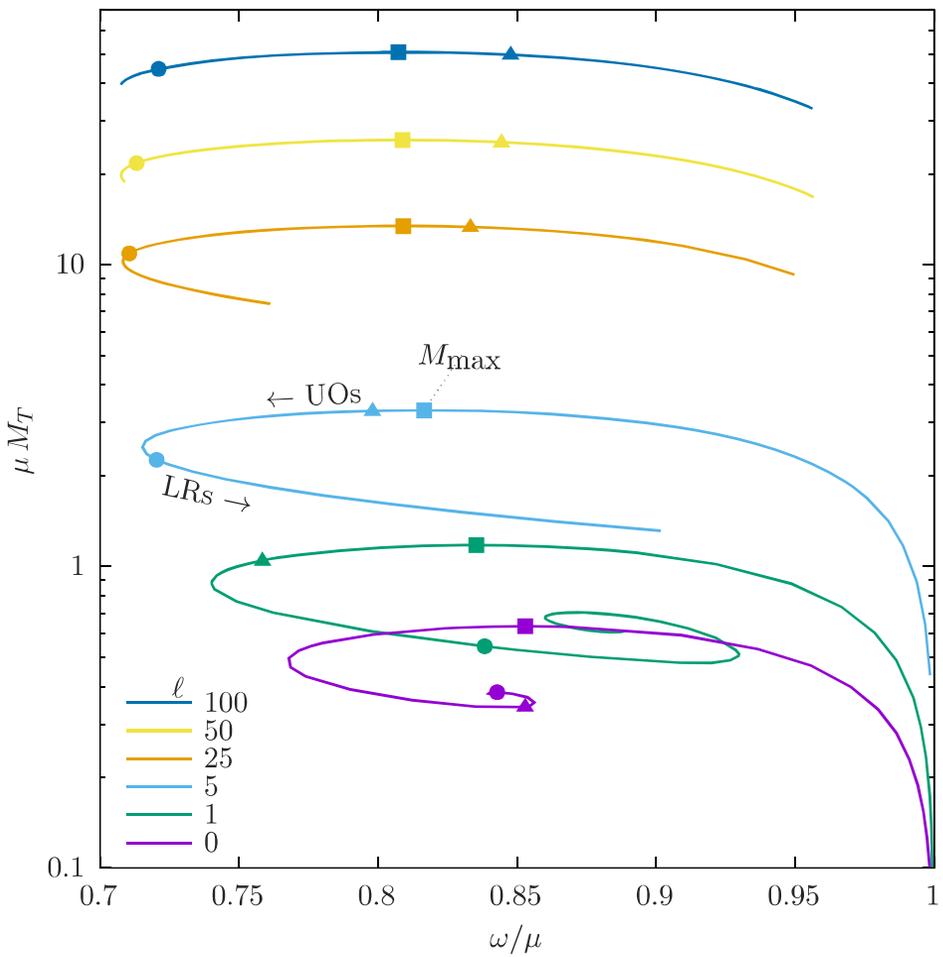




# Complementarity between numerical perturbations and linear analysis



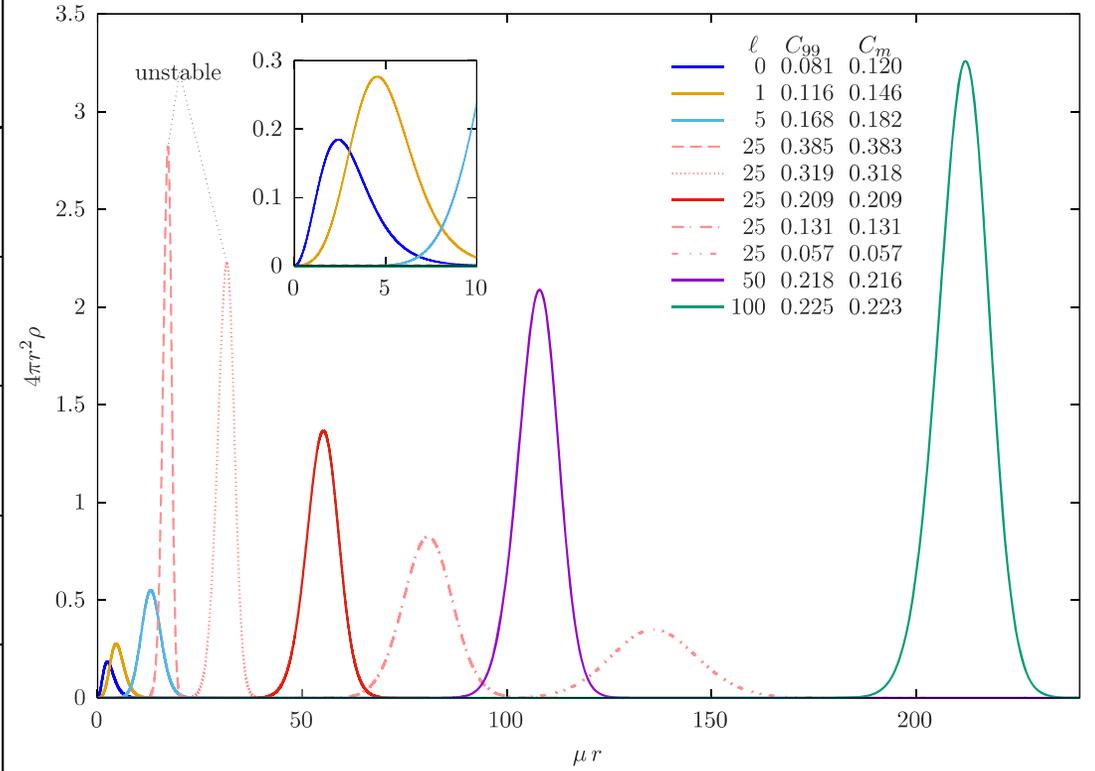
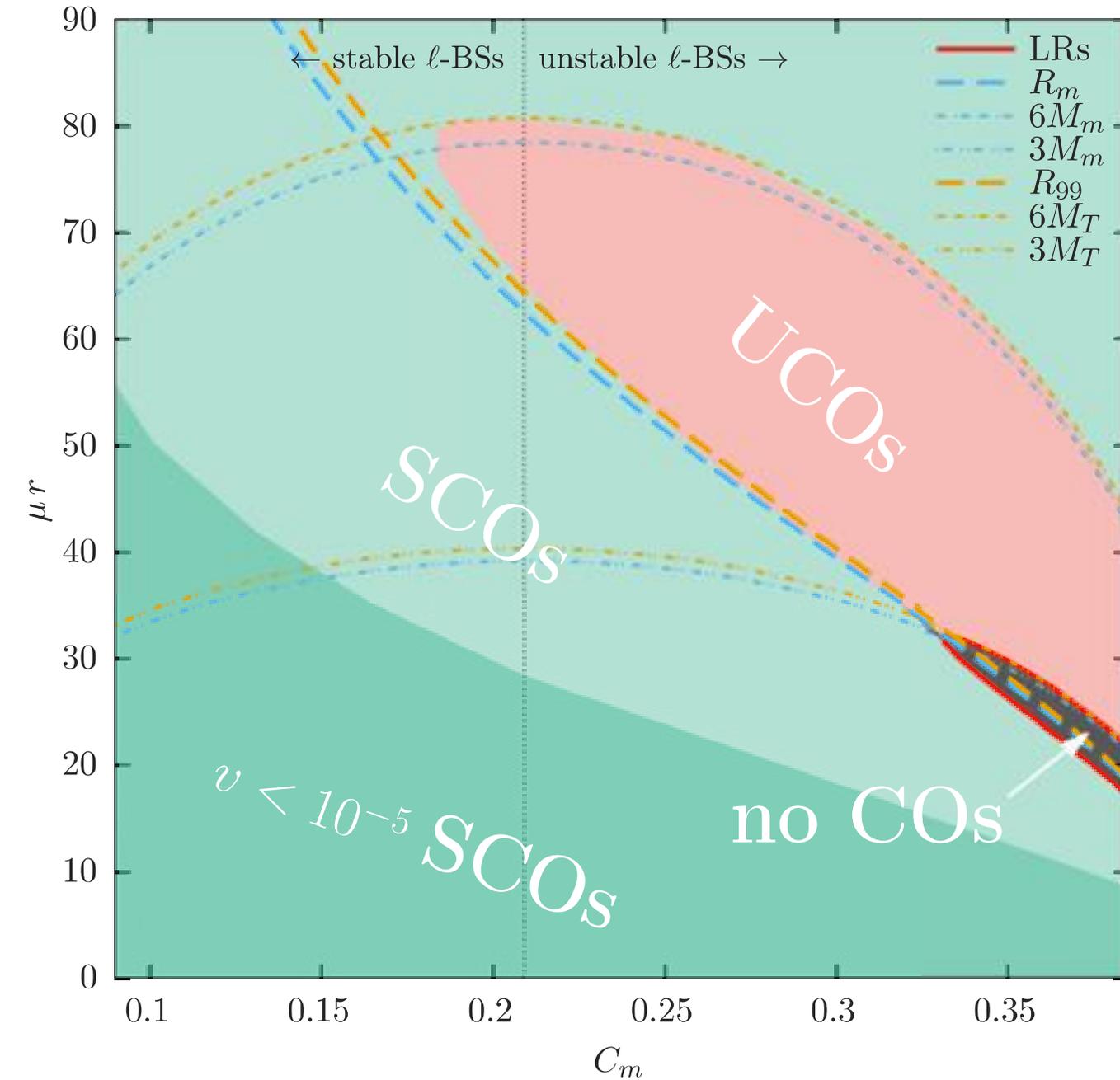
$\ell$	$(2\ell + 1)u_1^0$	$\omega$	$\sigma_0^2$	$c$
1	0.050	$8.832 \times 10^{-1}$	$3.80 \times 10^{-4}$	$-1.92 \times 10^{-2}$
1	0.080	$8.519 \times 10^{-1}$	$2.40 \times 10^{-4}$	$-2.78 \times 10^{-2}$
1	0.085	$8.475 \times 10^{-1}$	$1.91 \times 10^{-4}$	$-2.92 \times 10^{-2}$
1	0.090	$8.4341 \times 10^{-1}$	$1.35 \times 10^{-4}$	$-3.06 \times 10^{-2}$
1	0.095	$8.394 \times 10^{-1}$	$7.27 \times 10^{-5}$	$-3.20 \times 10^{-2}$
1	0.100	$8.356 \times 10^{-1}$	$3.95 \times 10^{-6}$	$-3.33 \times 10^{-2}$
1	0.105	$8.320 \times 10^{-1}$	$-7.11 \times 10^{-5}$	$-3.47 \times 10^{-2}$
1	0.110	$8.285 \times 10^{-1}$	$-1.53 \times 10^{-4}$	$-3.60 \times 10^{-2}$



### Extreme $\ell$ -boson stars

Miguel Alcubierre<sup>1</sup>, Juan Barranco<sup>2</sup>, Argelia Bernal<sup>2</sup>,  
 Juan Carlos Degollado<sup>3,\*</sup>, Alberto Diez-Tejedor<sup>2</sup>,  
 Víctor Jaramillo<sup>1</sup>, Miguel Megevand<sup>4</sup>, Darío Núñez<sup>1</sup>  
 and Olivier Sarbach<sup>5</sup>

$\ell = 25$



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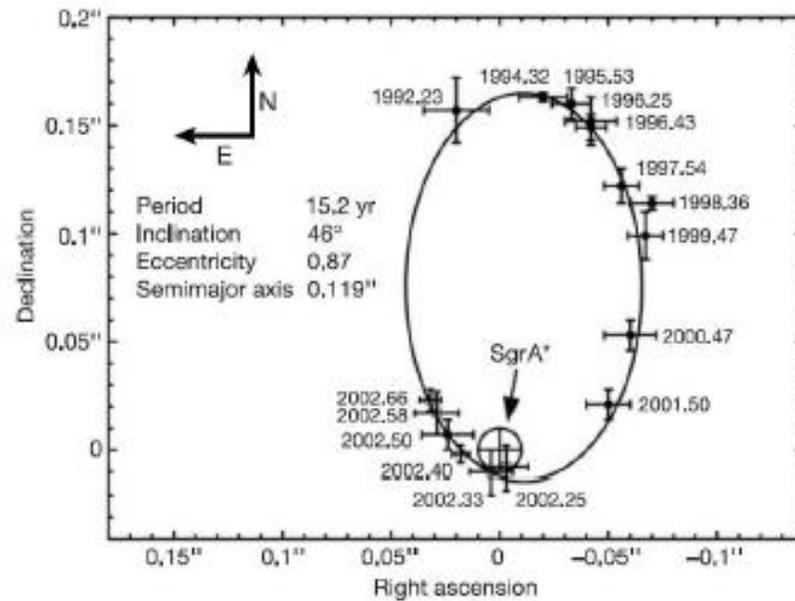
### Extreme $\ell$ -boson stars

Miguel Alcubierre<sup>1</sup>, Juan Barranco<sup>2</sup>, Argelia Bernal<sup>2</sup>,  
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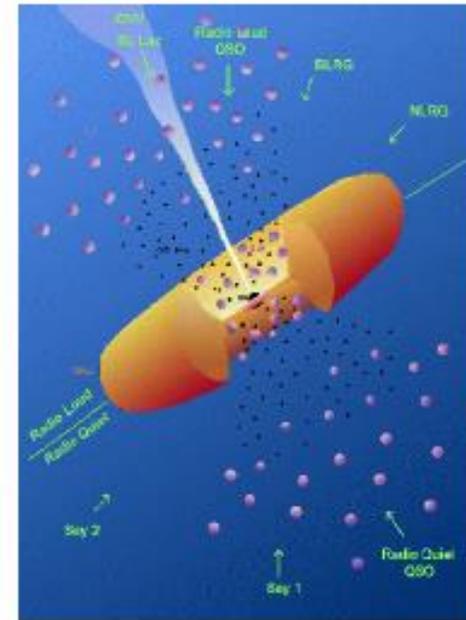
Boson stars can be treated effectively as classical configurations of multi-complex scalar field

Nature is complicated: there are supermassive black holes in the center of most of the galaxies

# Astrophysical evidence of SMBH in galaxies



$$M_{BH} = 4,3 \times 10^6 M_{\odot}$$



$$M_{BH} \sim 10^9 M_{\odot}$$

## The problem: No-hair theorems

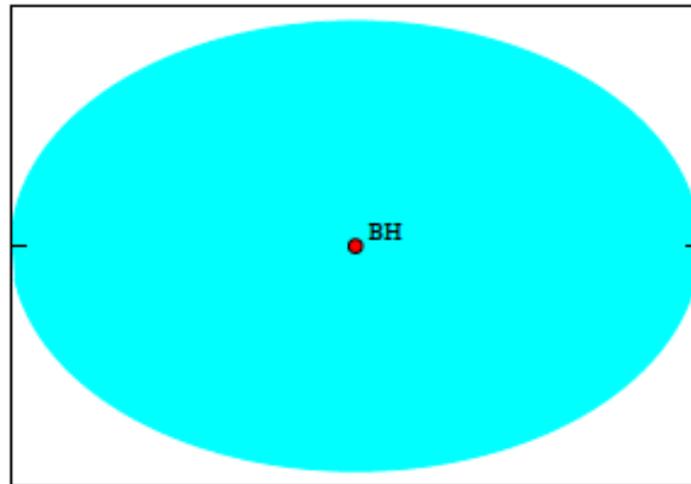


### Theorem:

A static, spherically symmetric, asymptotically flat black hole spacetime, with regular (non-extreme) horizon, which satisfies Einstein's equations with the matter fields fulfilling  $\rho := T_t^t > 0$  and  $T_\theta^\theta \leq T_r^r$ , vanishes identically and the spacetime corresponds to the Schwarzschild solution.



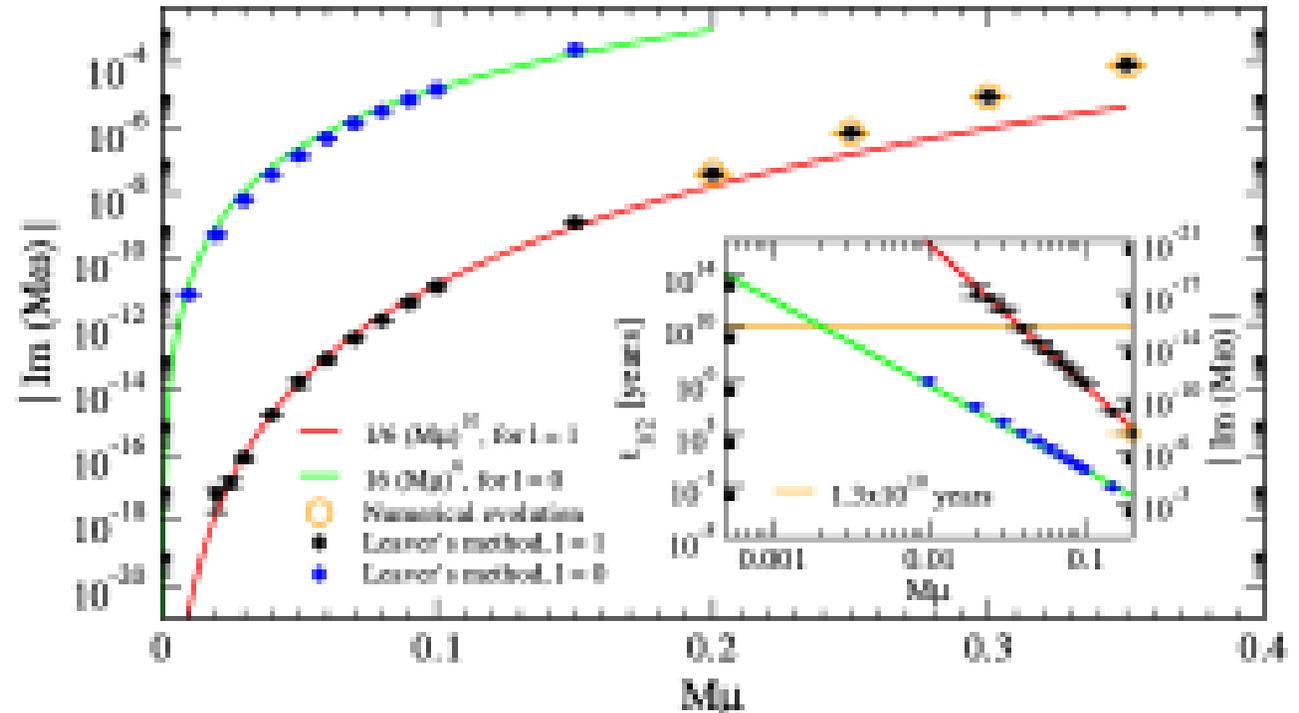
A scalar wig is a non-static configuration that can last even more time than the age of the universe



# Gravitational fine structure constant:

$$\alpha_G \equiv \frac{GM_{BH}m_\phi}{\hbar c} = 7.5 \times 10^9 \left( \frac{M_{BH}}{M_\odot} \right) \left( \frac{m_\phi c^2}{\text{eV}} \right)$$

If  $\alpha_G < 1/4$  Quasi.-bound states (in the test field limit) form



# Uncertainty relation for ultralight particles and black holes

$$\left(\frac{4GM_{BH}}{c^2}\right) \left(\frac{m_{\phi}c}{\hbar}\right) = \frac{2R_{Sch}}{\lambda_{\phi}} < 1.$$

# Conclusions

- Boson stars might arise as self-gravitating compact objects made of ultra-light spin zero DM particles
- There are many realization of Boson Stars relatives (such as Multistate boson stars, L-Boson stars, and more)
- They could be more compact, with a richer structure and because their stability, if they form, they can be astrophysical candidates.

Beyond the test field limit:

*The generalized Eddington-Finkelstein gauge*

$$ds^2 = -a(t, r)^2 dt^2 + dr^2 + \frac{2m(t, r)}{r} (a(t, r)dt + dr)^2 + r^2 d\Omega^2$$

*The  $\ell$ -boson star ansatz*

$$T_{\mu\nu} = \sum_{m=-\ell}^{\ell} \left[ \nabla_{(\mu} \Phi_m^* \cdot \nabla_{\nu)} \Phi_m - \frac{1}{2} g_{\mu\nu} (\nabla^\alpha \Phi_m^* \cdot \nabla_\alpha \Phi_m + \mu^2 |\Phi_m|^2) \right]$$

$$\Phi_m = \phi_\ell(t, r) Y^{\ell m}(\vartheta, \varphi)$$

$$\left(1 - \frac{2m}{r}\right) \psi_\ell'' = B\psi_\ell' + C\psi_\ell,$$

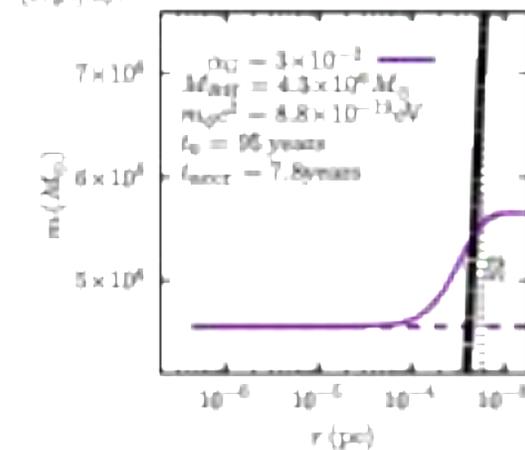
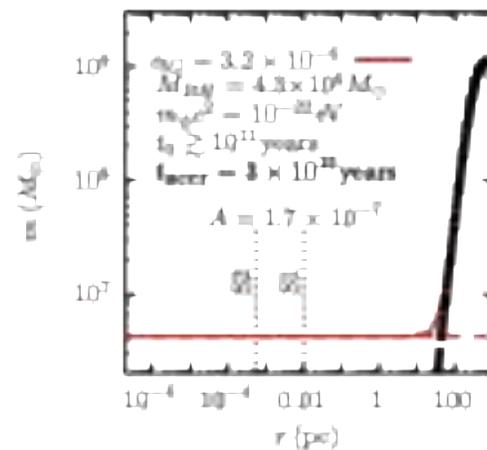
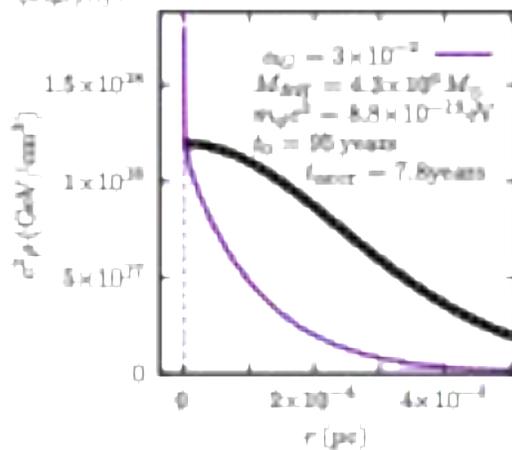
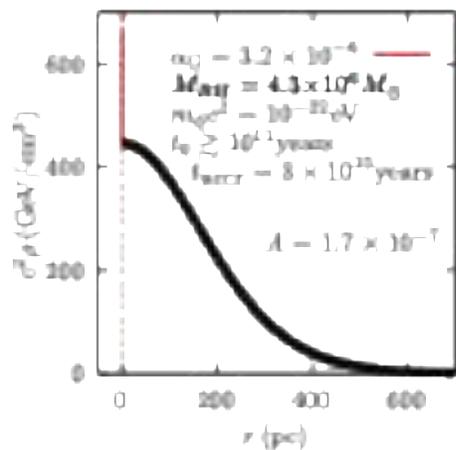
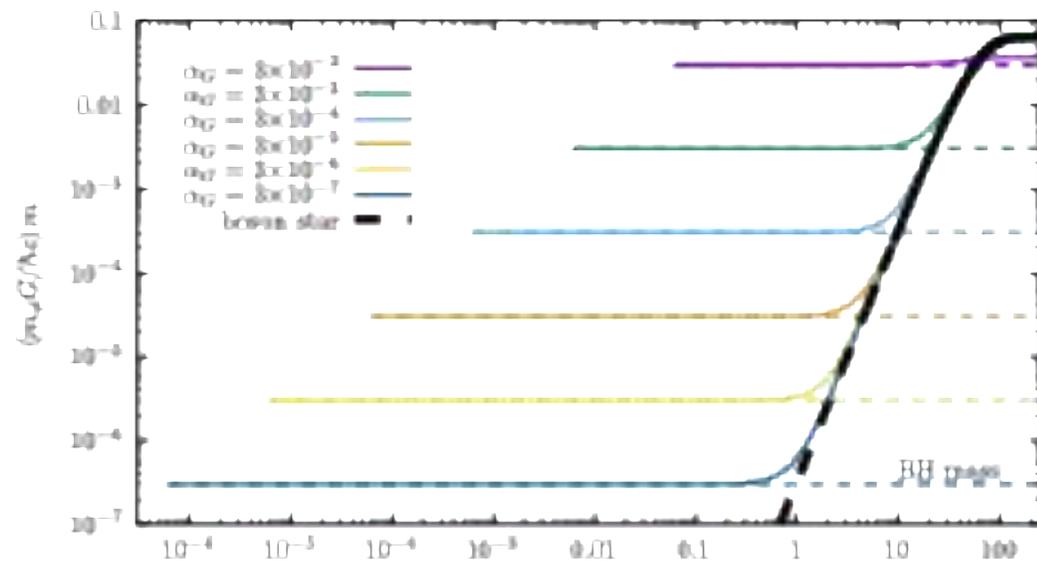
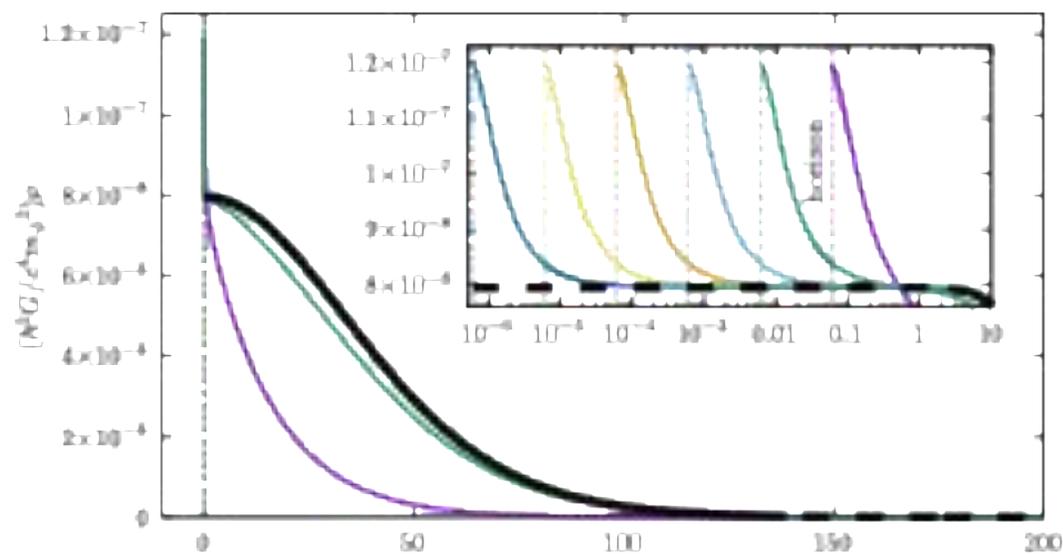
$$2m' = \kappa_\ell r^2 e^{2\sigma t} \left[ \gamma^2 \left| \frac{s}{a} \psi_\ell \right|^2 + \mu_\ell^2 |\psi_\ell|^2 + \left(1 - \frac{2m}{r}\right) |\psi_\ell'|^2 \right],$$

$$\frac{a'}{a} = \kappa_\ell r e^{2\sigma t} \left| \frac{s}{a} \psi_\ell - \psi_\ell' \right|^2,$$

$$B = -\frac{2}{r} \left(1 - \frac{m}{r}\right) - \frac{4m}{r} \frac{s}{a} + \frac{\kappa_\ell r}{\gamma^2} \left[ (\gamma^4 - 1) \left| \frac{s}{a} \psi_\ell \right|^2 + \gamma^2 \mu_\ell^2 |\psi_\ell|^2 + (1 + \gamma^2) \left(1 - \frac{2m}{r}\right) \operatorname{Re} \left( \frac{s^*}{a} \psi_\ell^* \psi_\ell' \right) \right],$$

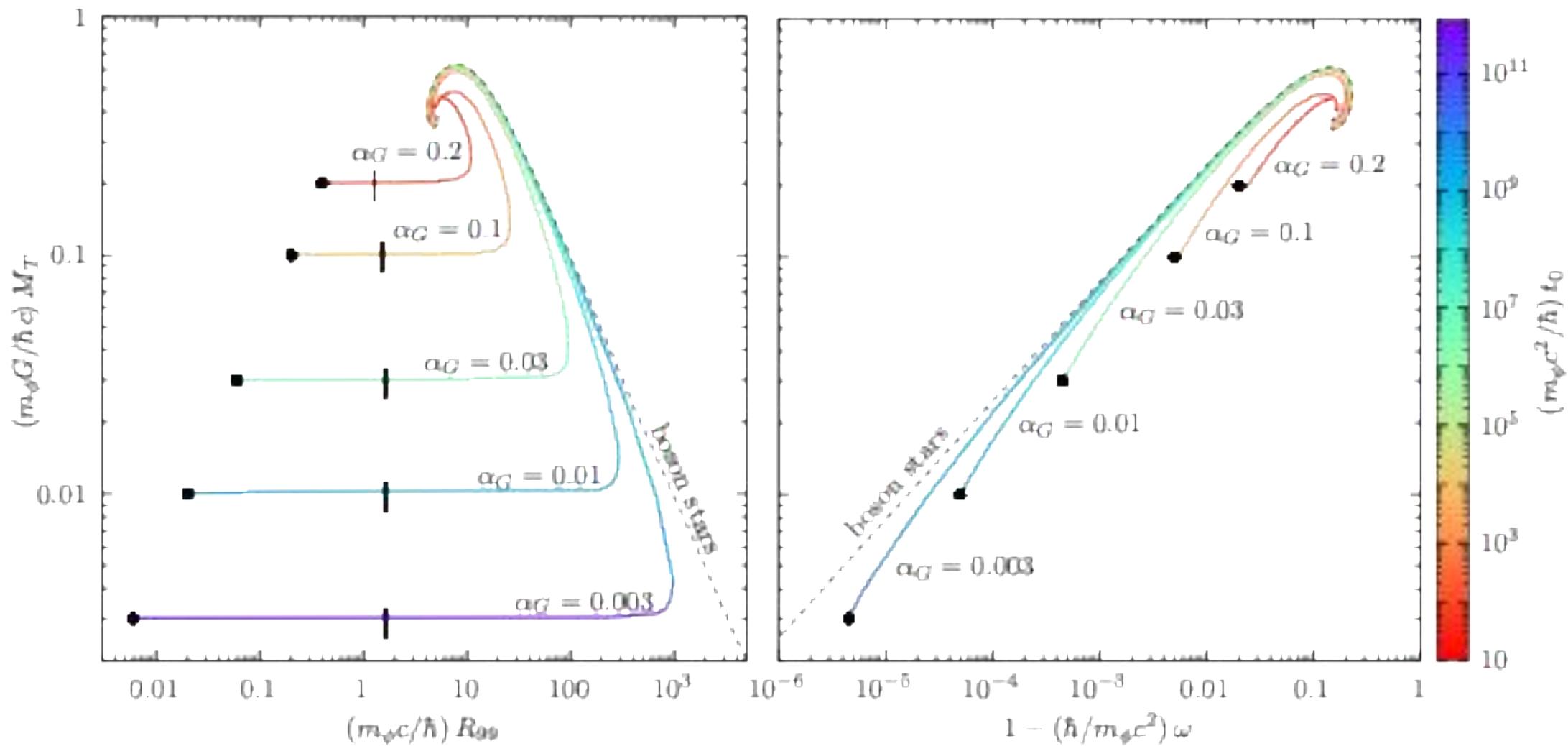
$$C = \mu_\ell^2 + \gamma^2 \left( \frac{s}{a} \right)^2 - \frac{2m}{r^2} \frac{s}{a} - \kappa_\ell r \left[ \left| \frac{s}{a} \psi_\ell \right|^2 + \mu_\ell^2 |\psi_\ell|^2 + \left(1 - \frac{2m}{r}\right) |\psi_\ell'|^2 - \left(1 - \frac{2m}{r}\right) \operatorname{Re} \left( \frac{s^*}{a} \psi_\ell^* \psi_\ell' \right) \right] \frac{s}{a}.$$

# Density profiles of solutions with $\ell = 0$

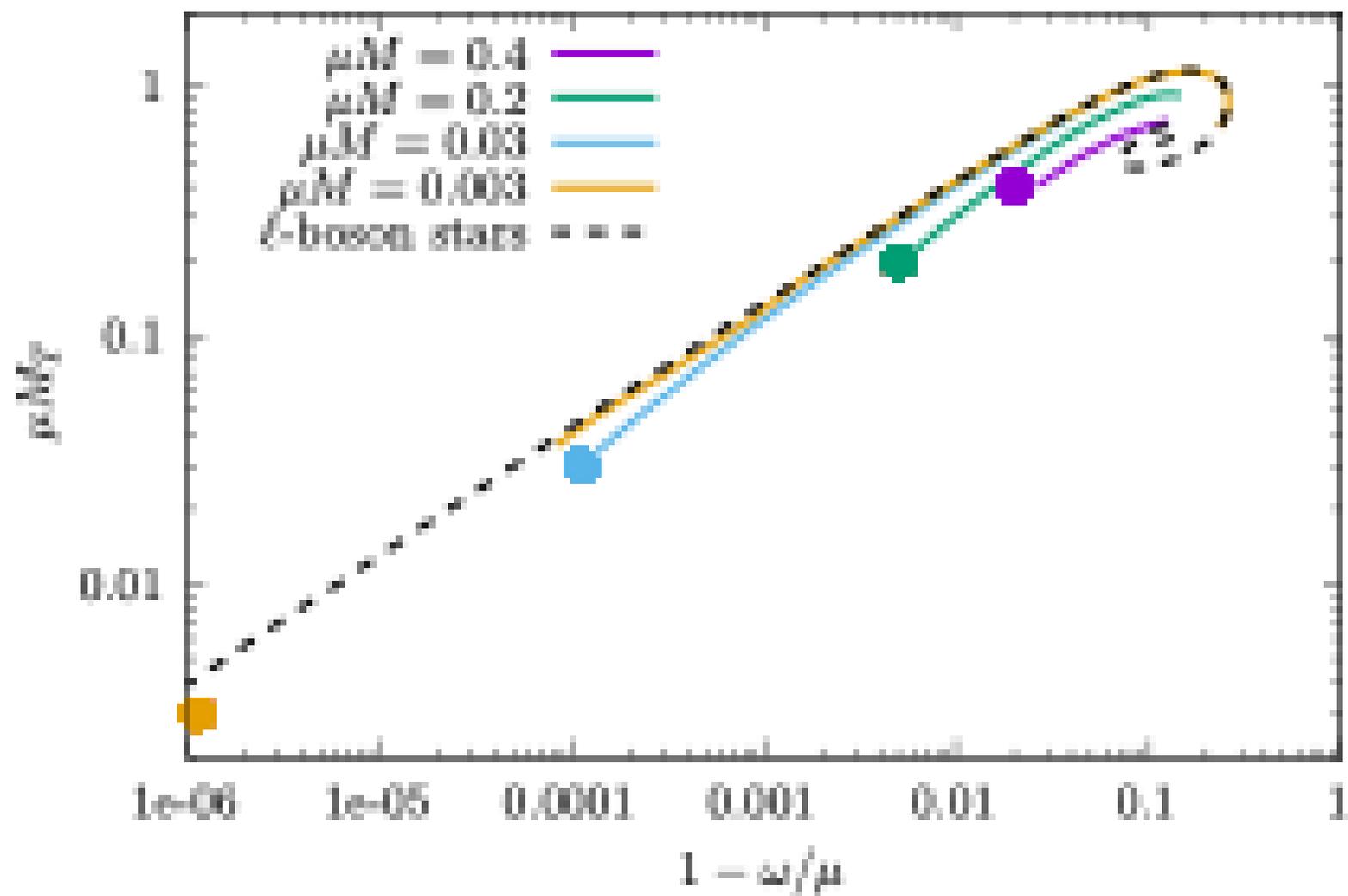


(a) Energy density

(b) Mass function



$M_T$  vs  $\omega$  of noble gravitational atoms and  $\ell$ -boson stars with  $\ell = 1$ .



- In the limit of very small fine structure gravitational constant, the accretion of scalar field by the black hole is negligible, and the scalar wig resembles the properties of a boson star or their relatives.

# Does it work? (conclusion)

