Photon propagation in a charged Bose–Einstein condensate model

Sarira Sahu

In Collaboration with José F. Nieves

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Introduction

FIVE STATES OF MATTER

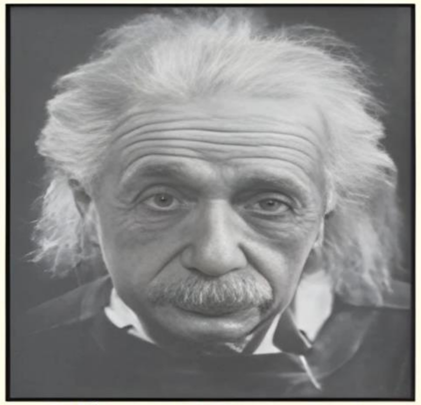
VHTC

Now scientists are talking of five states of matter: Solid, Liquid, Gas, Plasma and Bose-Einstein Condensate.

Plasma: The state consists of super energetic and super excited particles. These particles are in the form of ionised gases. The **fluorescent tube and neon sign bulbs consist of plasma.** Inside a neon sign bulb there is neon gas and inside a fluorescent tube there is helium gas or some other gas. The gas gets ionised, that is, gets charged when electrical energy flows through it. This charging up creates a plasma glowing inside the tube or bulb. The plasma glows with a special colour depending on the nature of gas. The Sun and the stars glow because of the presence of plasma in them. The plasma is created in stars because of very high temperature.

Bose-Einstein Condensate: In 1920, Indian physicist Satyendra Nath Bose had done some calculations for a fifth state of matter. Building on his calculations, Albert Einstein predicted a new state of matter – the Bose-Einstein Condensate (BEC). In 2001, Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman of USA received the Nobel prize in physics for achieving "Bose-Einstein condensation".

S.N. Bose



Albert Einstein

The BEC is formed by cooling a gas of extremely low density, about one-hundred thousandth the density of normal air, to super low temperatures.

- Particle propagation has interesting consequences in thermal backgrounds
- Extension of the Standard Model
- Additional scalar particle (Neutral and Charged) interacting particularly with Neutrino and Photons have implications in Nuclear Physics, Astrophysics, Cosmological Plasma, additional contribution to the effective potential.
- Propagation of fermion in a thermal background that contains a scalar BEC.
- Problem of Dark Matter can also be addressed through the self-interactiong scalar fields forming DM.
- In Condensed Matter Physics, study of photon propagation in atomic BEC
- Reduction of Light velocity (c) to 17 m/s
- Useful for Quantum Technology

Scalar BEC Model & SSB

The Lagrangian for this model is

$$L_\phi = (d^\mu \phi)^*(d_\mu \phi) - V_\phi \,,$$

$$V_\phi = m^2 \phi^* \phi + \lambda(\phi^* \phi)^2 \,.$$

$$d_\mu \equiv \partial_\mu - i\mu u_\mu \,,$$

$\boldsymbol{\mu}$ → Chemical Potential
Φ→ Charged Scalar
u_μ=(1,0), in the medium rest-frame
$v_\mu = \boldsymbol{\mu}\, u_\mu$

$$L_\phi = (\partial^\mu \phi)^*(\partial_\mu \phi) + i[\phi^*(v \cdot \partial\phi) - (v \cdot \partial\phi^*)\phi] - U(\phi) \,,$$

$$U \equiv V_\phi - \mu^2 \phi^* \phi = -(\mu^2 - m^2)\phi^* \phi + \lambda(\phi^* \phi)^2 \,.$$

Φ=0 is the minimum

If $m^2 > \mu^2$, U corresponds to standard massive complex scalar field with mass $m^2 - \mu^2$.

Let us consider the case when a $\mu^2 > m^2$, Φ=0 is not the minimum, it develops a non-zero vacuum expectation value and U(1) symmetry is broken.

We can write

$$\phi = \frac{1}{\sqrt{2}}(\phi_0 + \phi_1 + i\phi_2) \ ,$$

with

$$\langle\phi\rangle \equiv \frac{1}{\sqrt{2}}\phi_0 \ , \qquad \phi_0^2 = \frac{\mu^2 - m^2}{\lambda} \ .$$

Then the Lagrangian density is given by

$$L_\phi = \frac{1}{2}\left[(\partial^\mu\phi_1)^2 + (\partial^\mu\phi_2)^2\right] \underline{+ \phi_2 v \cdot \partial\phi_1 - \phi_1 v \cdot \partial\phi_2} - U(\phi)$$

$$U(\phi) = -\frac{1}{2}(\mu^2 - m^2)[(\phi_0 + \phi_1)^2 + \phi_2^2] + \frac{1}{4}\lambda[(\phi_0 + \phi_1)^2 + \phi_2^2]^2 \ .$$

Mass of ϕ_1

$$m_1^2 = 2(\mu^2 - m^2) \ ,$$

And ϕ_2 is massless

Mixed through v

To find the modes which have definite dispersion relations

Expressing as Matrix

Lagrangian density in momentum space will be

$$\hat{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

$$L_\phi^{(2)}(k) = \frac{1}{2}\hat{\phi}^*(k)\Delta_\phi^{-1}(k)\hat{\phi}(k),$$

where

$$\Delta_\phi^{-1}(k) = \begin{pmatrix} k^2 - m_1^2 & 2i\,v\cdot k \\ -2i\,v\cdot k & k^2 \end{pmatrix}.$$

Determinant (D) of Δ_ϕ^{-1},

D=0, gives the dispersion relation

$$\omega_\pm^2(\kappa) = \kappa^2 + \frac{1}{2}m_\rho^2 \pm \sqrt{\frac{1}{4}m_\rho^4 + 4\mu^2\kappa^2},$$

$$m_\rho^2 = m_1^2 + 4\mu^2,$$

$$\omega = u\cdot k,$$
$$\kappa = \sqrt{\omega^2 - k^2}.$$

$$\omega \neq \kappa,$$

In the rest frame of the medium

$$k^\mu = (\omega, \vec{\kappa}),$$

$$\omega_+(0) = m_\rho,$$
$$\omega_-(0) = 0.$$

The combinations of $\phi_{1,2}$ that have the definite dispersion relations $\omega_\pm$ (

Goldstone mode

EM field interacting with the charged scalar BEC

$$L = -\frac{1}{4}F^2 + L_{\phi A},$$ where $F_{\mu\nu}$ is the electromagnetic field tensor and

$$L_{\phi A} = (\hat{D}^\mu \phi)^* (\hat{D}_\mu \phi) - V_\phi, \qquad \hat{D}_\mu = d_\mu + iqA_\mu = \partial_\mu - i v_\mu + iqA_\mu,$$

Spectrum of the Model

To determine the spectrum of the model we adopt the unitary gauge, which is the convenient one to use for this purpose.

Thus we parametrize ϕ in the form

$$\phi = \frac{1}{\sqrt{2}}(\phi_0 + \rho)e^{i\theta/\phi_0}.$$

The field θ does not appear in V_ϕ, and by a gauge transformation it disappears also from the kinetic term. To be clear that we are employing the unitary gauge we will denote by V_μ the transformed vector potential

$$V_\mu = A_\mu + \frac{1}{q\phi_0}\partial_\mu\theta.$$

$$L_{\phi A} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(v - qV)^\mu(v - qV)_\mu(\phi_0 + \rho)^2 - V_\phi,$$

By expanding it

$$L_{\phi A} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(q\phi_0)^2 V^2$$
$$+\frac{1}{2}q^2V^2\rho^2 + q^2\phi_0\rho V^2 - qv\cdot V(\phi_0 + \rho)^2 - U_\phi,$$

$$U_\phi = -\frac{1}{2}(\mu^2 - m^2)(\phi_0 + \rho)^2 + \frac{1}{4}(\phi_0 + \rho)^4.$$

Again we consider $\mu^2 > m^2,$

and choose ϕ_0 such that U_ϕ has the minimum at $\rho = 0$, thus,

$$\phi_0^2 = \frac{\mu^2 - m^2}{\lambda}.$$

Bilinear part of L (including the EM Tensor)

$$L^{(2)} = -\frac{1}{4}F^2 + \frac{1}{2}m_V^2 V^2 + \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}m_1^2\rho^2 - 2m_V(v \cdot V)\rho\,,$$

where

$$\boxed{\begin{aligned} m_V &= q\phi_0, \\ m_1^2 &= 2(\mu^2 - m^2). \end{aligned}}$$

The physical picture that emerges is this: the field θ becomes the longitudinal component of V, and we end up with two fields, V and ρ. The longitudinal component of V is mixed with the scalar field ρ (the term $\rho v \cdot V$). As we show, the consequence is that the propagating modes with definite dispersion relations involve a superposition of the longitudinal component of V and the ρ.

Eigenmodes and the Dispersion relations

In momentum space different terms of $L^{(2)}$ have the form

$$-\frac{1}{4}F^2 \to -V^{*\mu}k^2\tilde{g}_{\mu\nu}V^\nu, \quad \frac{1}{2}(\partial\rho)^2 \to k^2\rho^*\rho, \quad \frac{1}{2}m_V^2V^2 \to m_V^2V^{*\mu}V_\mu,$$

$$\frac{1}{2}m_1^2\rho^2 \to m_1^2\rho^*\rho, \qquad (u\cdot V)\rho \to (u\cdot V)^*\rho + c.c,$$

and therefore

$$L^{(2)}(k) = -V^{*\mu}[k^2\tilde{g}_{\mu\nu} - m_V^2 g_{\mu\nu}]V^\nu + \rho^*(k^2 - m_1^2)\rho$$
$$-2\underline{\mu m_V[(u\cdot V)^*\rho + c.c]},$$

where

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}.$$

Mixes V with ρ

We decompose V^μ in the form

$$V^\mu = V_T^\mu + V_L e_3^\mu + \frac{k^\mu}{\sqrt{k^2}} V_k,$$

where $V_T^\mu = \sum_{i=1,2} V_i e_i^\mu \qquad e_{1,2}^\mu = (0, \vec{e}_{1,2}), \quad e_3^\mu = \frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}},$

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu = u_\mu - \frac{(k \cdot u) k_\mu}{k^2}, \qquad \sqrt{-\tilde{u}^2} = \frac{\kappa}{\sqrt{k^2}}.$$

Thus the quadratic part of the Lagrangian density is

Decoupled

No K.E., not a dynamical variable

$$L^{(2)}(k) = \boxed{-(k^2 - m_V^2) V_T^* \cdot V_T} + (k^2 - m_V^2) V_L^* V_L + m_V^2 V_k^* V_k + (k^2 - m_1^2) \rho^* \rho$$

$$+ 2\mu m_V \sqrt{-\tilde{u}^2} \left[V_L^* \rho + c.c. \right] - \frac{2\mu m_V (k \cdot u)}{\sqrt{k^2}} \left[V_k^* \rho + c.c \right].$$

Eliminate this term using Lagrange equation

$$m_V^2 V_k - \frac{2\mu m_V (k \cdot u)}{\sqrt{k^2}} \rho = 0, \qquad \longrightarrow \qquad V_k = \frac{2\mu (k \cdot u)}{m_V \sqrt{k^2}} \rho.$$

Finally we get

$$L^{(2)}(k) = -(k^2 - \cancel{m_V^2})V_T^* \cdot V_T + (k^2 - m_V^2)V_L^* V_L + \left(k^2 - m_1^2 - \frac{4\mu^2(k\cdot u)^2}{k^2}\right)\rho^*\rho$$
$$+2\mu m_V\sqrt{-\tilde{u}^2}\,[V_L^*\rho + c.c.]\,.$$

Equations of motion

$$\boxed{\begin{aligned}(k^2 - m_V^2)V_L + 2\mu m_V\sqrt{-\tilde{u}^2}\rho &= 0,\\ 2\mu m_V\sqrt{-\tilde{u}^2}V_L + \left[k^2 - m_1^2 - \frac{4\mu^2(k\cdot u)^2}{k^2}\right]\rho &= 0,\end{aligned}}$$

For $\kappa = 0$ V_L and ρ decouple

$$\boxed{\begin{aligned}(\omega^2 - m_V^2)V_L &= 0,\\ (\omega^2 - m_\rho^2)\rho &= 0,\end{aligned}}$$

$$m_\rho^2 = m_1^2 + 4\mu^2.$$

For $\kappa \neq 0$ the dispersion relations are

$$\boxed{\omega^2 = \kappa^2 + \frac{1}{2}(m_\rho^2 + m_V^2) \pm \left[\frac{1}{4}(m_\rho^2 - m_V^2)^2 + 4\mu^2\kappa^2\right]^{\frac{1}{2}}.}$$

Interpreting the Dispersion Relations under different conditions

Final Dispersion relations

$$\omega^2 = \kappa^2 + \frac{1}{2}(m_\rho^2 + m_V^2) \pm \left[\frac{1}{4}(m_\rho^2 - m_V^2)^2 + 4\mu^2\kappa^2\right]^{\frac{1}{2}} .$$

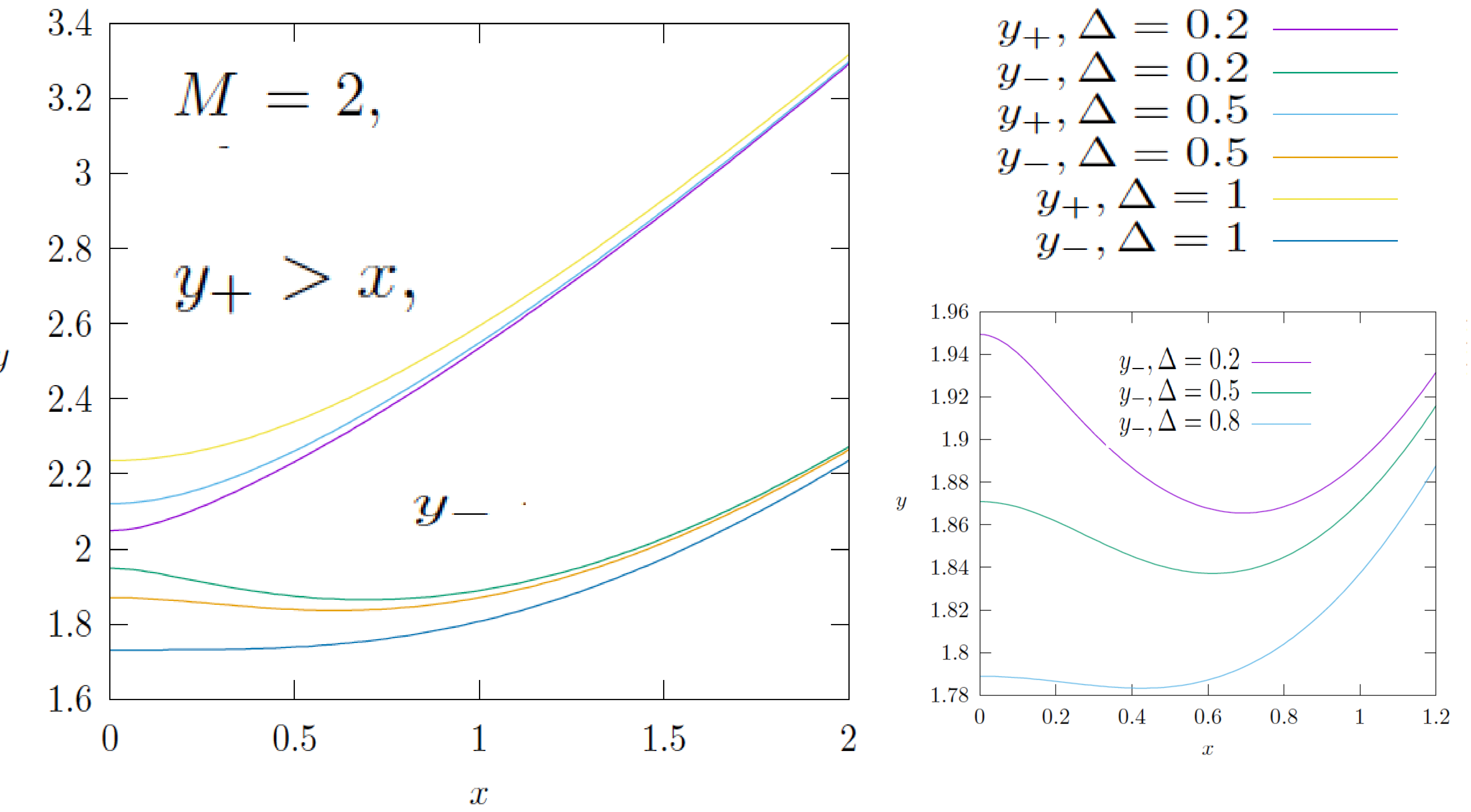
This can be written in a normalized form and study under different conditions

$$x^2 = \frac{\kappa^2}{2\mu^2}, \qquad M^2 = \frac{m_+^2 + m_V^2}{4\mu^2},$$

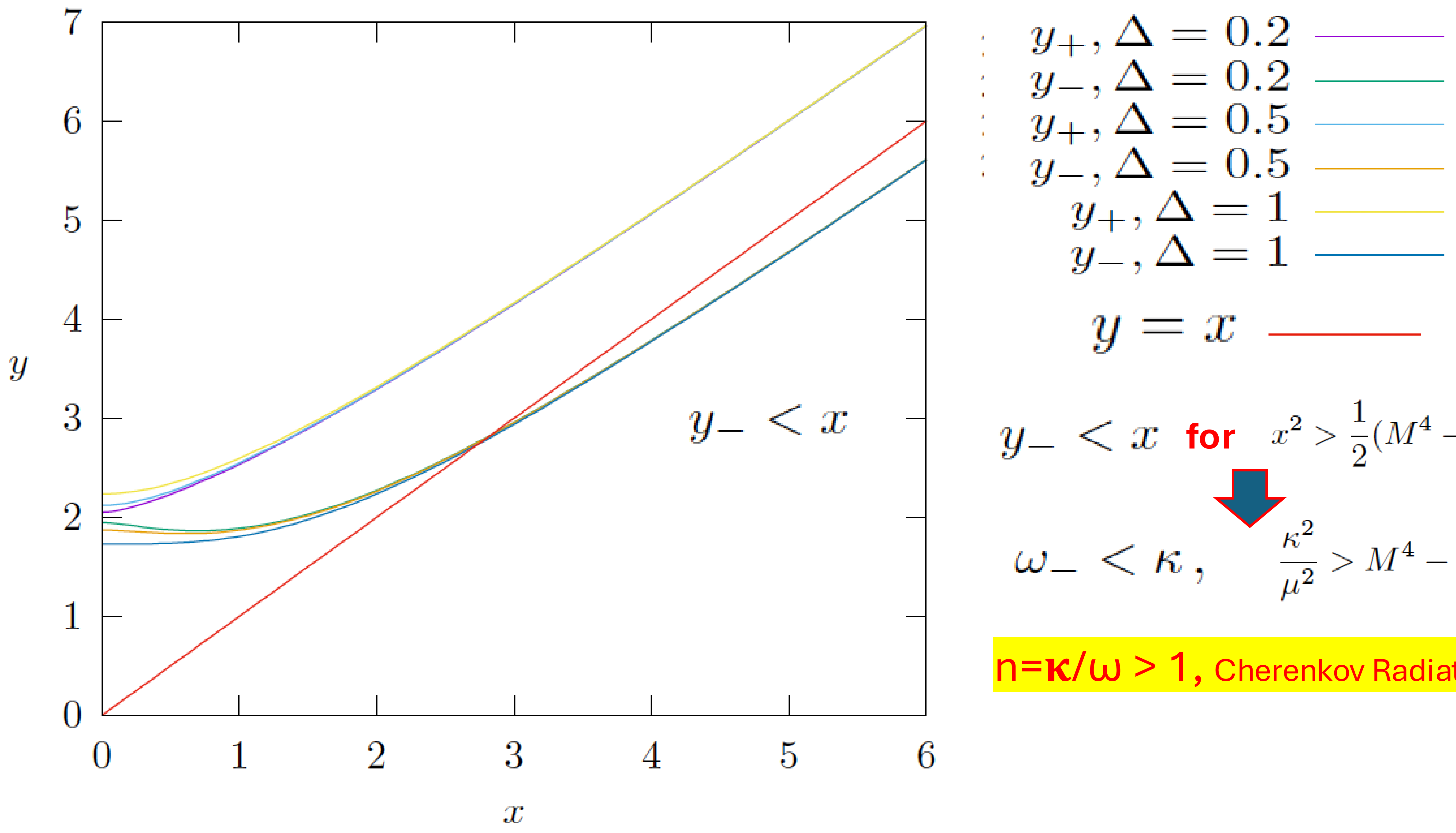
$$y^2 = \frac{\omega^2}{2\mu^2}, \qquad \Delta = \frac{|m_+^2 - m_V^2|}{4\mu^2}.$$

$$\Delta < M^2, \qquad M^2 > 1 .$$

$$\boxed{y_\pm^2 = x^2 + M^2 \pm \sqrt{\Delta^2 + 2x^2} .}$$



$M = 2,$
$y_+ > x,$
y_-
$y_+, \Delta = 0.2$
$y_-, \Delta = 0.2$
$y_+, \Delta = 0.5$
$y_-, \Delta = 0.5$
$y_+, \Delta = 1$
$y_-, \Delta = 1$
y
x
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
0
0.5
1
1.5
2
$y_-, \Delta = 0.2$
$y_-, \Delta = 0.5$
$y_-, \Delta = 0.8$
1.96
1.94
1.92
1.9
1.88
1.86
1.84
1.82
1.8
1.78
0
0.2
0.4
0.6
0.8
1
1.2



$y_- < x$ **for** $x^2 > \frac{1}{2}(M^4 - \Delta^2)\,.$

$$\omega_- < \kappa\,, \qquad \frac{\kappa^2}{\mu^2} > M^4 - \Delta^2\,,$$

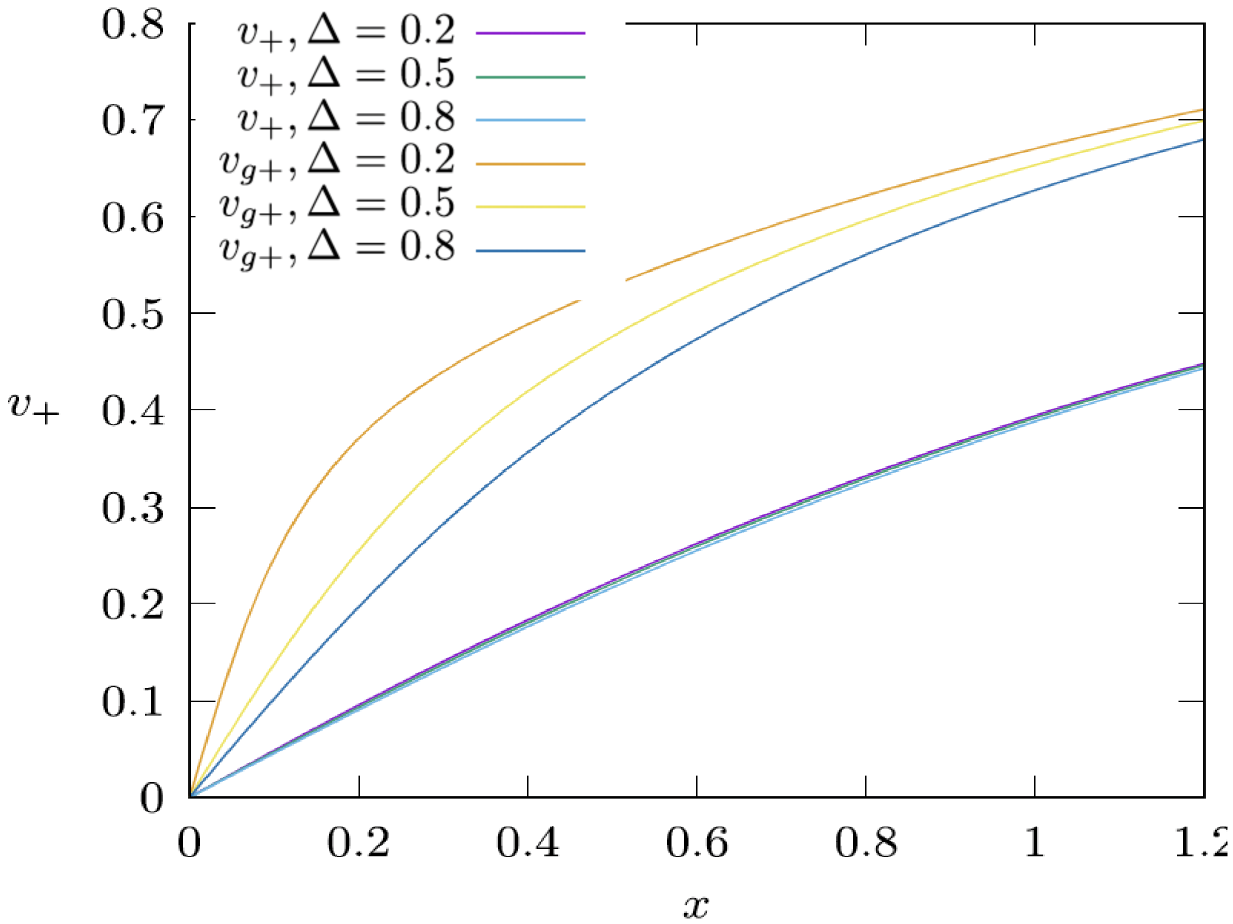
n=κ/ω > 1, Cherenkov Radiation

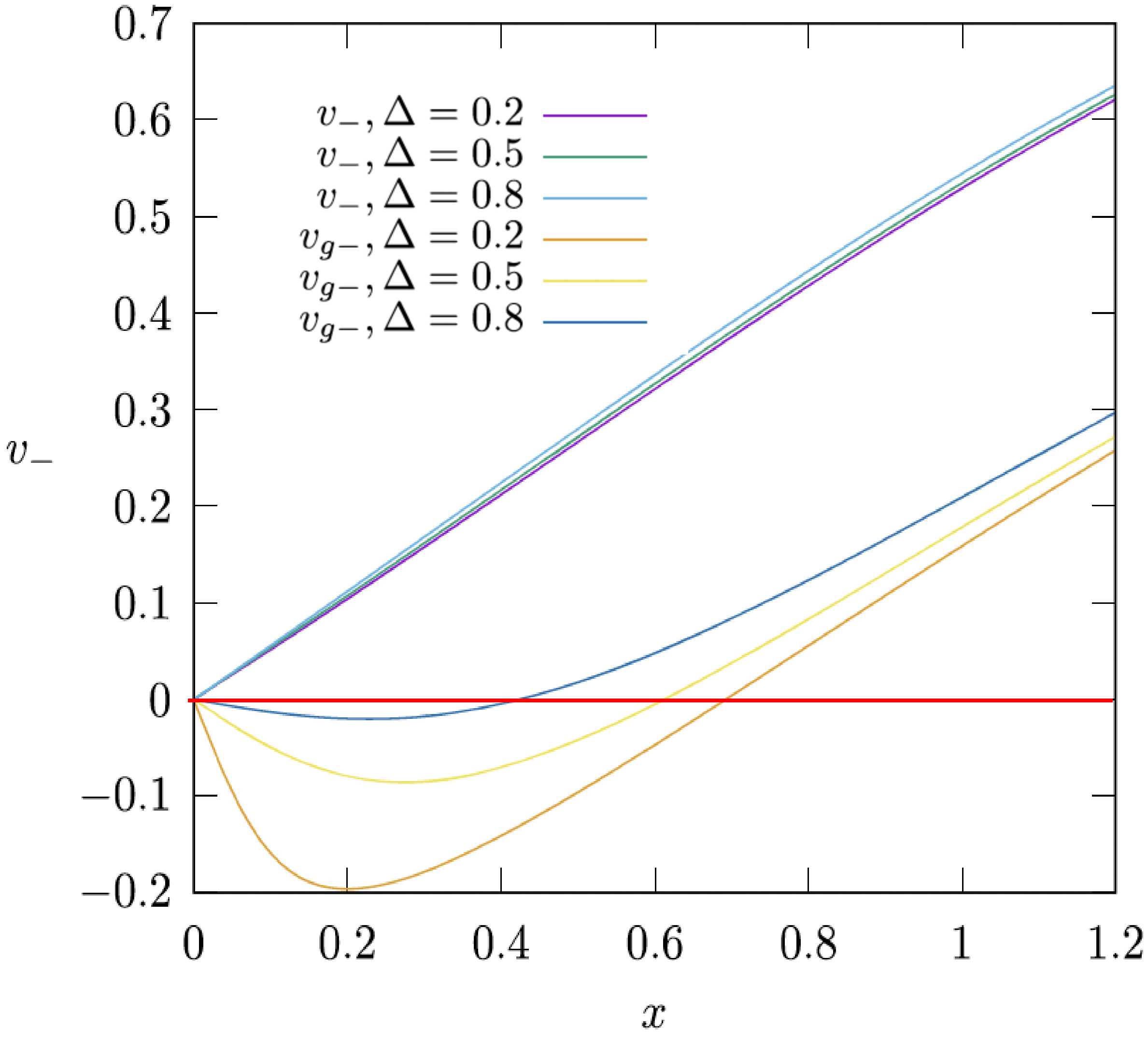
The group velocity for each branch is

Phase velocity

$$v_{\pm} \equiv \kappa/\omega_{\pm}$$

$$v_g = \frac{\partial \omega}{\partial \kappa} = \frac{x}{y}\left[1 \pm \frac{1}{\sqrt{\Delta^2 + 2x^2}}\right].$$





Summary & Outlook

- Studied Photon propagation in a BEC of charged scalar.
- Derived the dispersion relations for the Collective Modes in BEC
- For Photons: Two modes are usual → Transverse Photons
- Additional Two modes (+) and (--) are combination of Longitudianl Photon and massive Scalar field.
- Their dispersion relations are very different.
- The (+) increases steadly as momentum increases and always $\omega > \boldsymbol{\kappa}$
- The(--) has very interesting feature: Slowing down of Light in BEC, Cherenkov Radiation
- Nontrivial Optical properties

Thank You