

G.G. Barnaföldi, L. Gyulai, G. Bíró, R, Vértesi

Support: Hungarian NKFIH grants 2021-4.1.2-NEMZ KI-2024-00031, 2024-1.2.5-TÉT-2024-00022, Wigner Scientific Computing Laboratory

Refs: J.Phys.G 51 (2024) 8, 085103, IJMPA (arXiv 2409.01085)

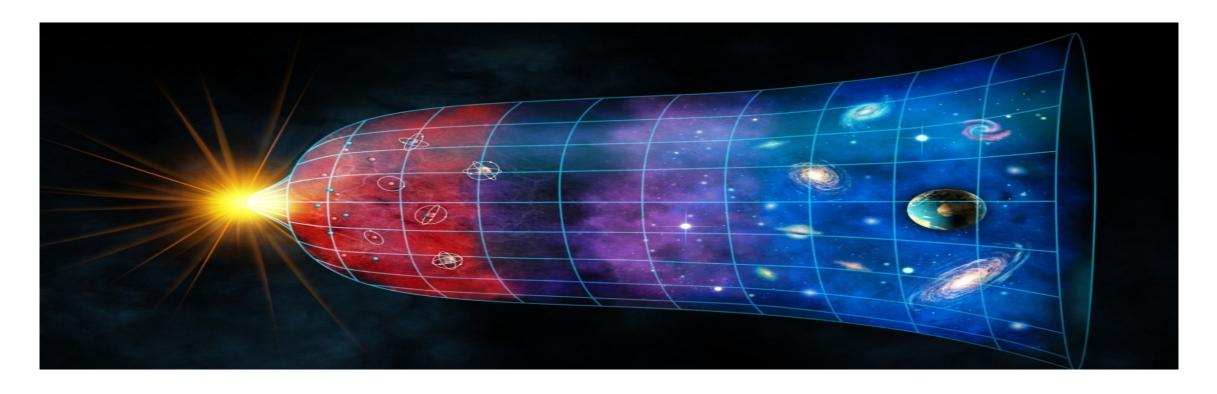








### How far can we see back in time?

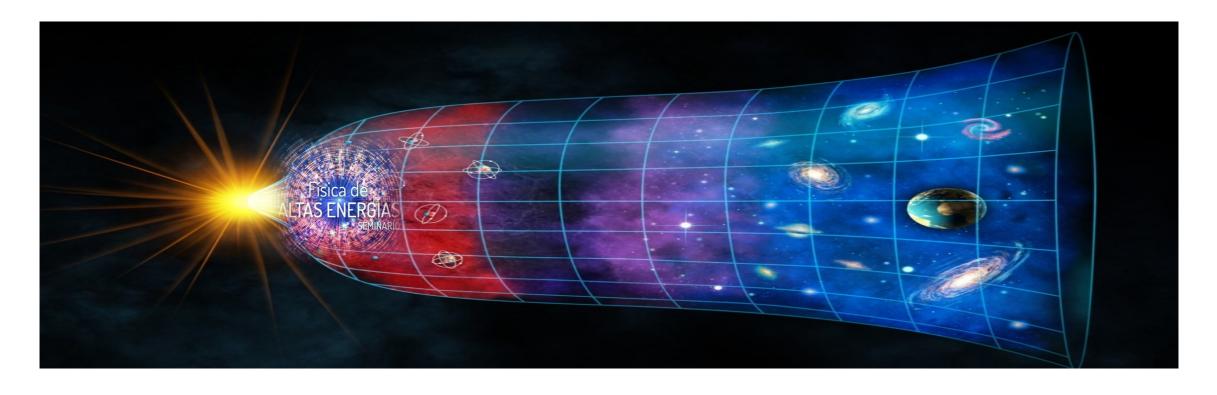


Recent: Astrophysical observation, >280 ky

Early times: particle accelerators, ~1 ms

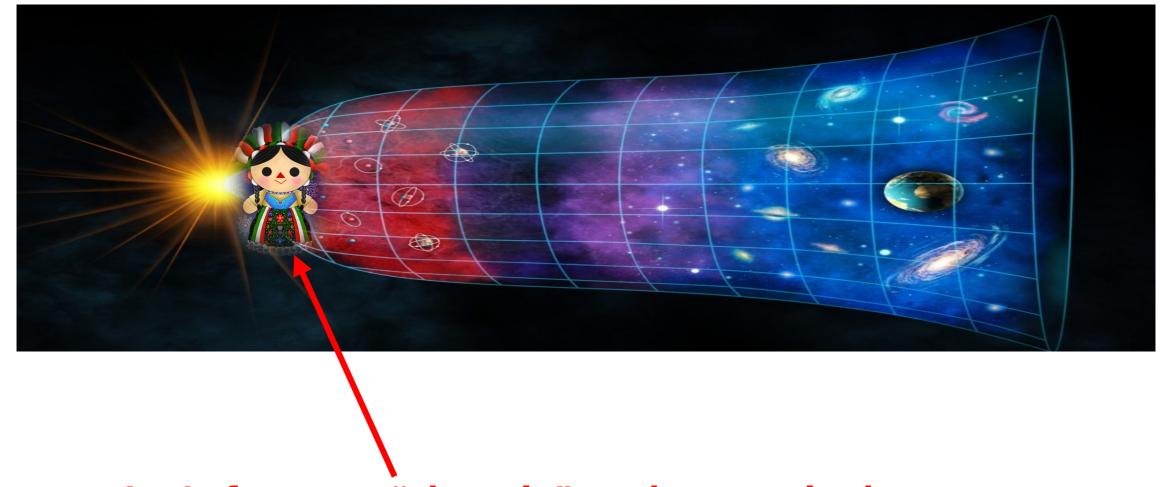
Even earlier: computer simulations, <1ms

### How far can we see back in time in a HIC?



The hottest topic ever: Quark-Gluon Plasma as our origin at the birth of the Universe at 1/million seconds after the Big Bang.

### How far can we see back in time with charm?

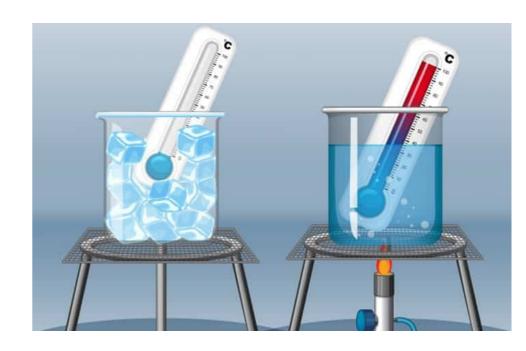


Let's focus on "charmly" to these early times....

# Motivation for the talk...

#### Our aims here are:

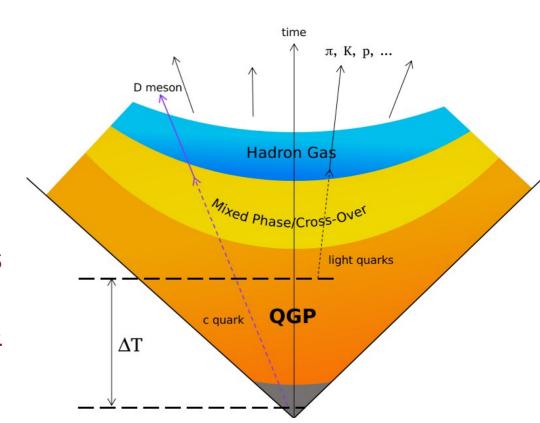
- define a thermometer
- check the feasibility to define a scale



### Motivation for the talk...

#### Our aims here are:

- define a thermometer
- check the feasibility to define a scale
- find similarities between light and heavy flavours
- find traces of different production mechanisms & timelines



All within the non-extensive statistical framework

### Related works

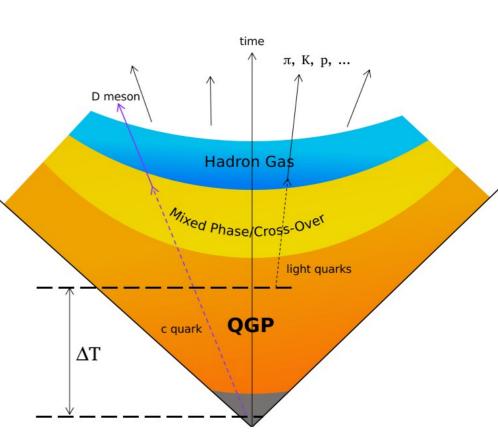
Previous studies (K Shen, G Bíró, TS Biró, AN Mishra, GGB)

Light-flavoured hadrons (K,  $\pi$ , p,  $\Lambda$ ,  $\Phi$ ,  $\Sigma$ ,  $\Xi$ ,  $\Omega$ ) have already been studied in the non-extensive statistical framework in the broad range of collision systems and multiplicities

[JPG 47 (2020) 10, 105002, JPG 50 (2023) 9, 095004]

Recent works (L Gyulai, R. Vértesi, G. Bíró, G. Paic, GGB)

In our study we expand the list of investigated particles with D mesons (containing c quark), which are mostly produced in hard interactions early in the collisions [JPG 51 (2024) 8, 085103, IJMPA (arXiv:2409.01085)]



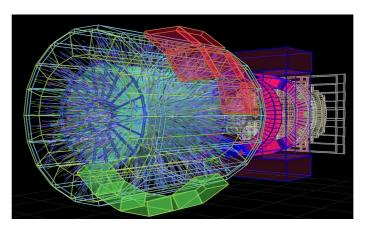
# Non-extensive statistics and the Tsallis thermometer

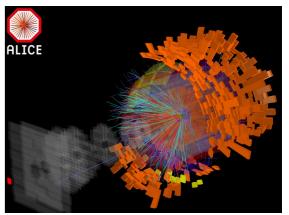
### Extensive vs. non-extensive statistics

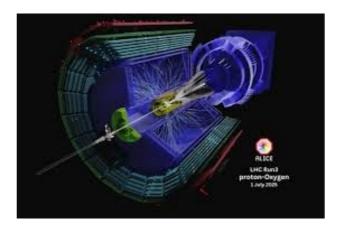
If it is hot, need to define a temperature, but...

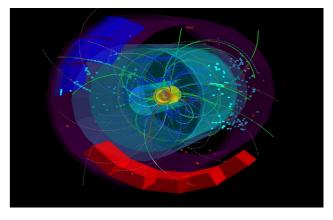


VS.









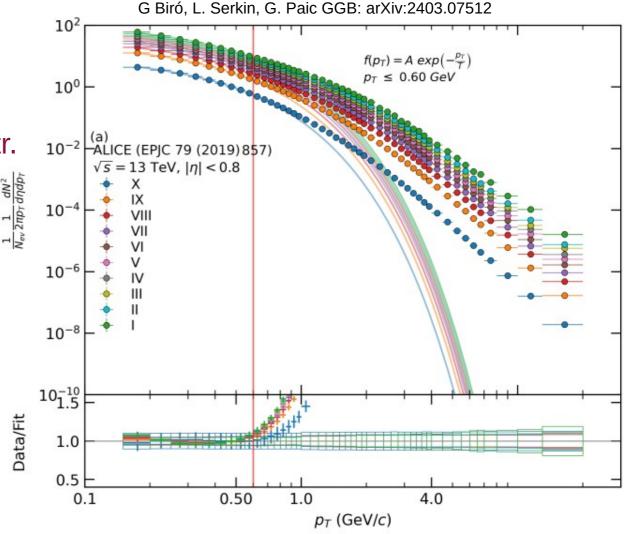
Ideal fluid with  $N_A \sim 10^{23}$  d.o.f. vs.

Strongly interacting fluid with N~10-10<sup>6</sup> d.o.f.

### Hadron spectra vs. extensive statistics

#### Identified particle spectrum:

- Low-p<sub>T</sub> part:
  - soft particle production
  - exponential-like (Boltzmann-Gibbs) distr.
  - stemming from a thermal equilibrium



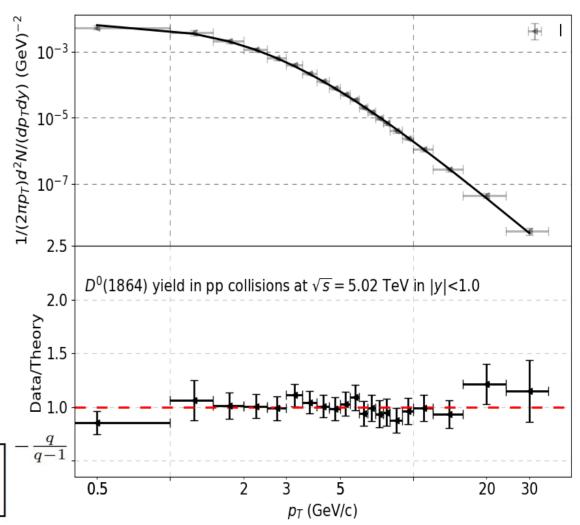
### Hadron spectra vs. non-extensive statistics

#### Identified particle spectrum:

- Low-p<sub>T</sub> part:
  - soft particle production
  - exponential-like (Boltzmann-Gibbs) distr.
  - stemming from a thermal equilibrium
- High-p<sub>⊤</sub> part:
  - jet-like origin
  - power-law tail distribution (non-thermal)
  - described by the perturbative QCD

#### **Tsallis-Pareto distribution smoothly connects both:**

$$\frac{\mathrm{d}^2 N}{2\pi p_T \mathrm{d} p_T \mathrm{d} y} \bigg|_{u \approx 0} = A m_T \left[ 1 + \frac{q-1}{T} (m_T - m) \right]$$



# Quantify and compare hadron spectra data

#### Precise spectra description

from low- to high-p<sub>+</sub>

$$f(m_T) = A \cdot \left[ 1 + \frac{q-1}{T_s} (m_T - m) \right]^{-\frac{1}{q-1}}$$

- in multiplicity classes (pp, pA, AA)

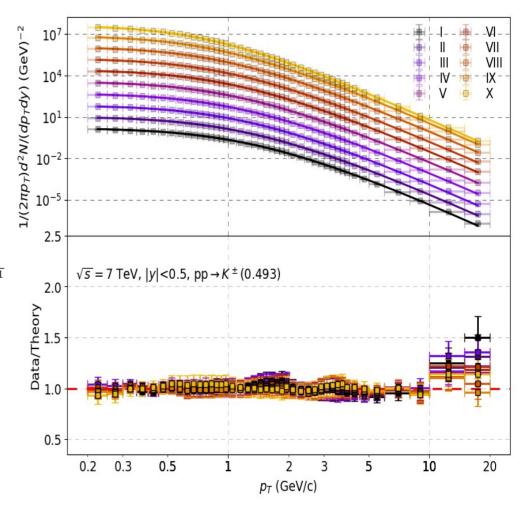
$$\frac{dN_{ch}}{dy}\Big|_{y=0} = 2\pi A T_s \left[ \frac{(2-q)m^2 + 2mT_s + 2T_s^2}{(2-q)(3-2q)} \right] \times \left[ 1 + \frac{q-1}{T_s} m \right]^{-\frac{1}{q-1}}$$

- With PID:

$$\pi^{\pm}, K^{\pm}, K_s^0, K^{*0}, p(\bar{p}), \Phi, \Lambda, \Xi^{\pm}, \Sigma^{\pm}, \Xi^0, \Omega$$

- Wide range:

	pp	рА	AA
CM energy (GeV)	7000, 13000	5020	130-5020
Multiplicity range	2.2-25.7	4.3-45	13.4-2047



# Identifying scalings in hadron spectra

#### QCD-inherited scaling properties

$$f(m_T) = A \cdot \left[ 1 + \frac{q-1}{T_s} (m_T - m) \right]^{-\frac{1}{q-1}}$$

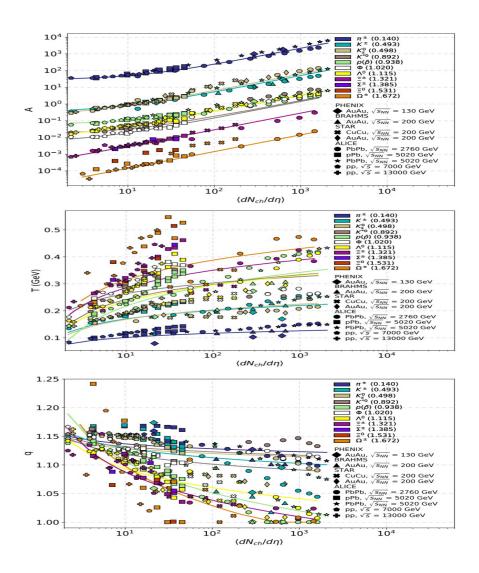
Parameter scaling with √s & multiplicity

$$A(\sqrt{s_{NN}}, \langle N_{ch}/\eta \rangle, m) = A_0 + A_1 \ln \frac{\sqrt{s_{NN}}}{m} + A_2 \langle N_{ch}/\eta \rangle$$
$$T(\sqrt{s_{NN}}, \langle N_{ch}/\eta \rangle, m) = T_0 + T_1 \ln \frac{\sqrt{s_{NN}}}{m} + T_2 \ln \ln \langle N_{ch}/\eta \rangle,$$
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#### Details:

G. Biró et al: J.Phys.G 47 (2020) 10, 105002

K. Shen et al Eur. Phys. J. A 55 (2019) 8, 126



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$$f(p_T) = A \exp\left(-\frac{p_T}{T}\right)$$

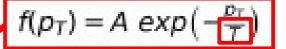
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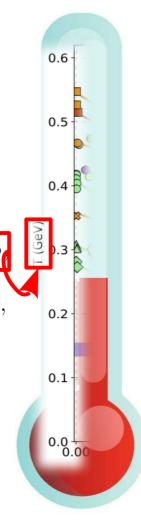
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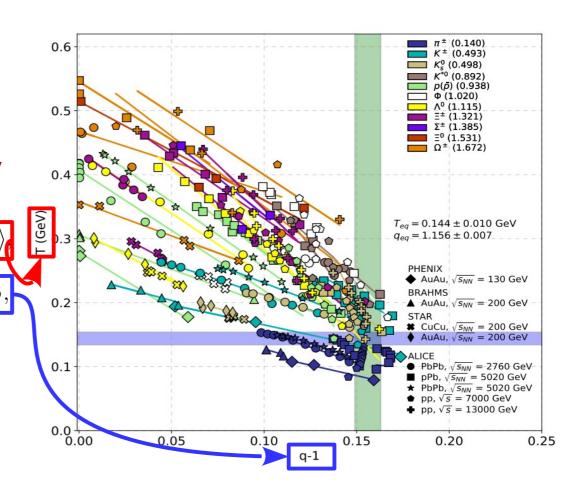
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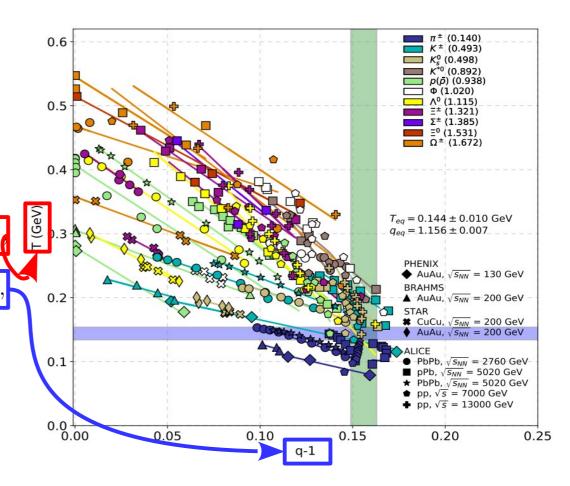
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- Light Flavour (LF)
  - Strong dependence on event multiplicity
  - Mass hierarchy presents for light flavour
  - LF grouping:  $T_{eq} \approx 0.14$  GeV and  $q_{eq} \approx 1.15$



QCD-inherited scaling properties

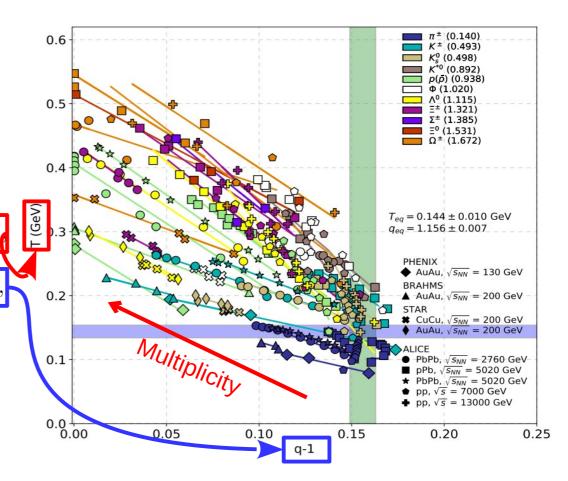
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Non-extensive entropy does not need thermal equilibrium:  $S(E_1 + E_2) \neq S(E_1) + S(E_2)$ 

$$\frac{1}{T} = \langle S'(E) \rangle = \langle \beta \rangle$$
$$q = 1 - \frac{1}{C} + \frac{\Delta \beta^2}{\langle \beta \rangle^2}.$$

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IF charged hadron multiplicity is NBD

$$T = \frac{E}{\langle n \rangle},$$

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 Biro, Van, BGG

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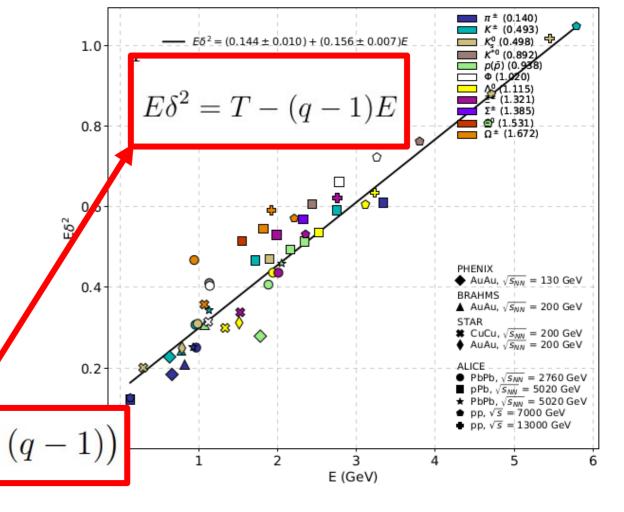
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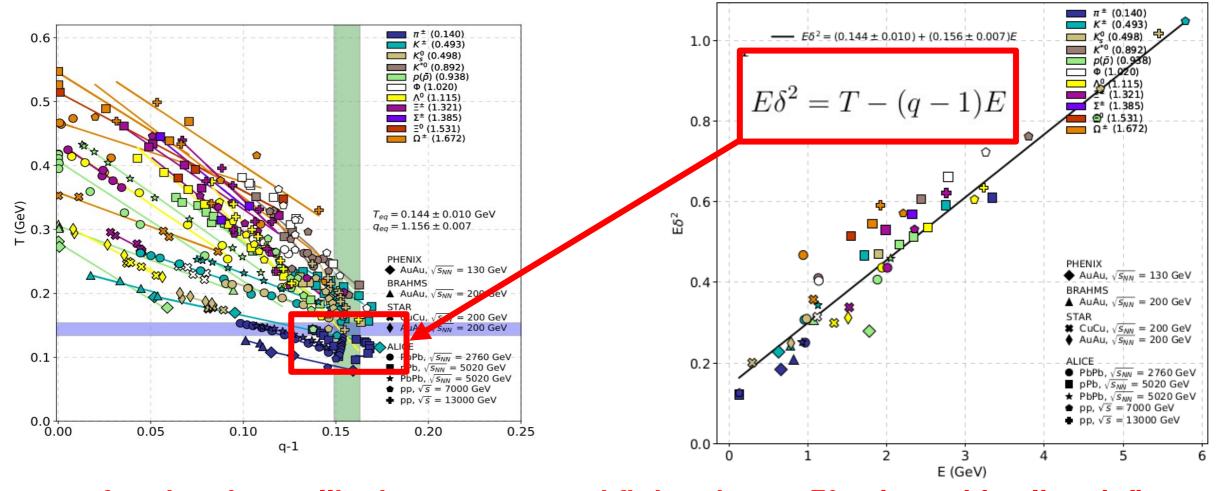
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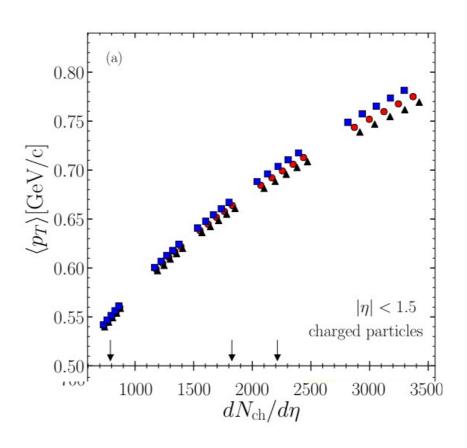
Transforming the Tsallis-thermometer and fitting the E- $E\delta^2$  points with a line defines the (linearized) equilibrium values for the: T (offset) and q (slope) parameters.

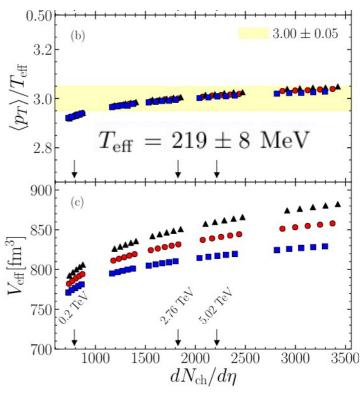
### Connection to mean transverse momenta

For given particle species one can see the Tsallis temperature values and connect them to the mean transverse momenta.

Similarly as for Boltzmann, here the well-fitted Tsallis-Pareto function can have relation to the  $p_T$ ,

$$f(p_T) = A \ exp\left(-rac{p_T}{T}
ight)$$
  $\langle p_T 
angle \simeq 3 \ T_{
m eff}$ 



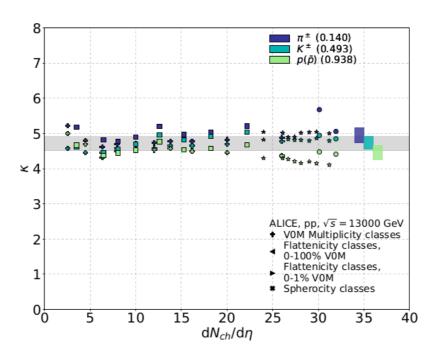


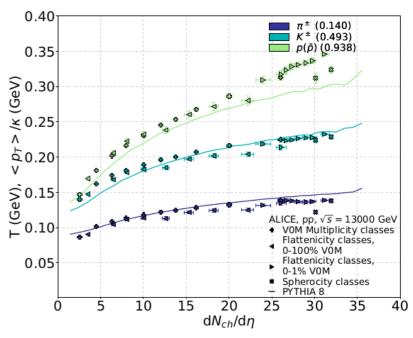
FG Gardim, AV Giannini, J-Y Ollitrault: PLB 856 (2024) 138937

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Gyulai, Bíró, Vértesi BGG: in preparetion

# Testing the thermodynamical consistency

#### QCD-inherited scaling properties

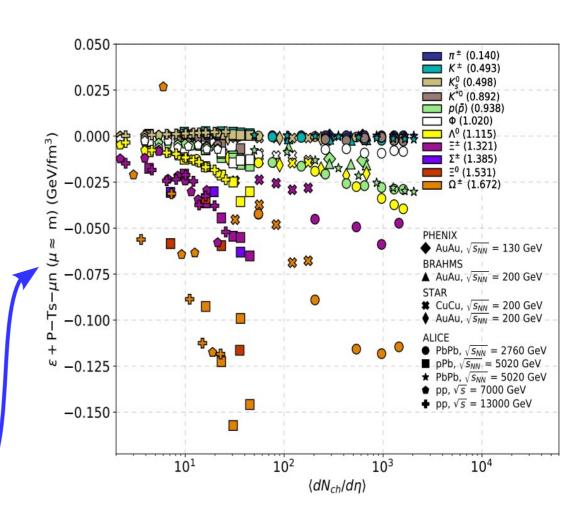
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Parameter scaling with √s & multiplicity

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#### Thermodynamical consistency

P = g 
$$\int \frac{d^3p}{(2\pi)^3} Tf$$
, N = nV = gV  $\int \frac{d^3p}{(2\pi)^3} f^q$ ,  
s = g  $\int \frac{d^3p}{(2\pi)^3} \left[ \frac{E-\mu}{T} f^q + f \right]$ ,  $\varepsilon = g \int \frac{d^3p}{(2\pi)^3} Ef$ 

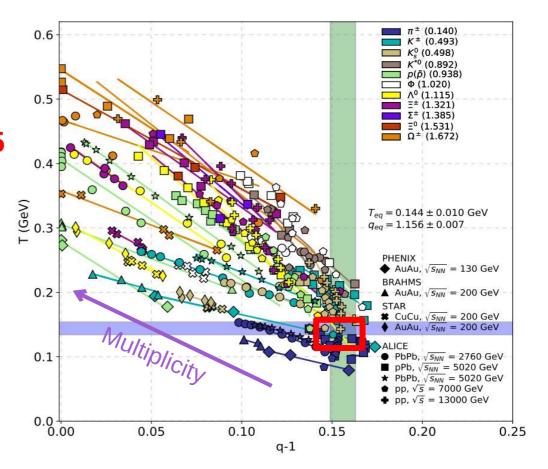


# Applying the Tsallis thermometer

# Tsallis-thermometer of light flavours

#### **Light Flavour (LF)**

- Strong dependence on event multiplicity
- Mass hierarchy presents for LF
- LF grouping:  $T_{eq} \approx 0.14$  GeV and  $q_{eq} \approx 1.15$

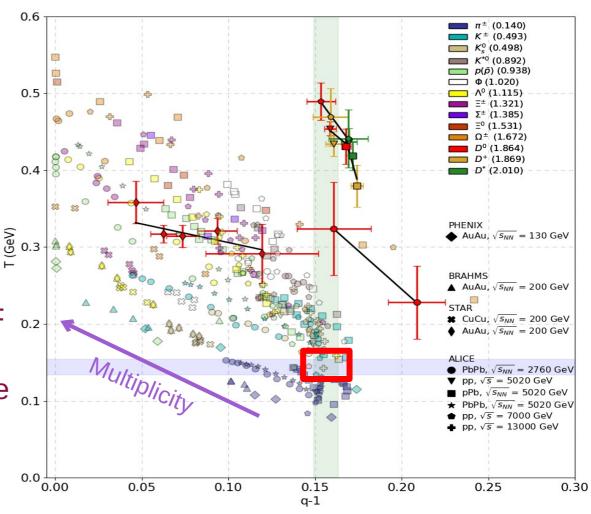


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### D mesons (HF)

- Dependence on the collision energy for HF is more prominent, than for LF
- A HF grouping is also present, however the  $_{\scriptscriptstyle 0.1}$  "center" is shifted compared to the LF
- HF grouping:  $T_{eq} \approx 0.25$  GeV and  $q_{eq} \approx 1.21$

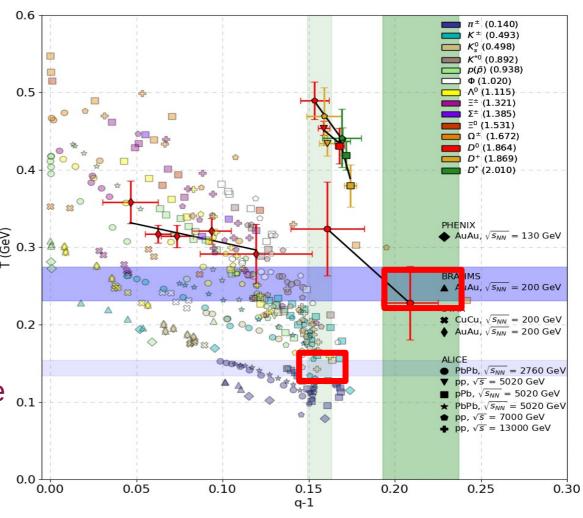


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### D mesons (HF)

- Dependence on the collision energy for HF 0.2 is more prominent, than for LF
- A HF grouping is also present, however the one center" is shifted compared to the LF
- HF grouping:  $T_{eq} \approx 0.25$  GeV and  $q_{eq} \approx 1.21$

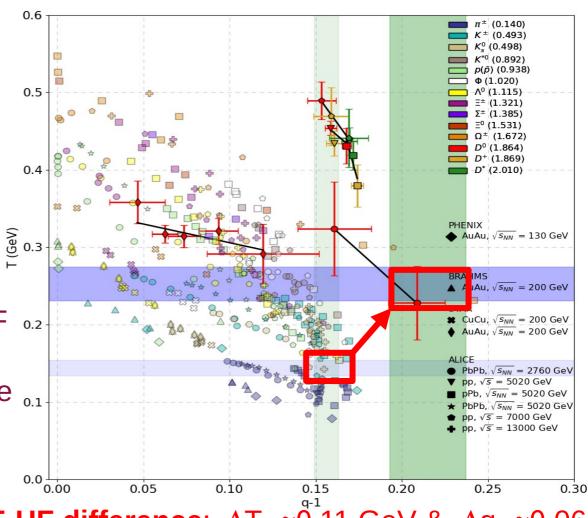


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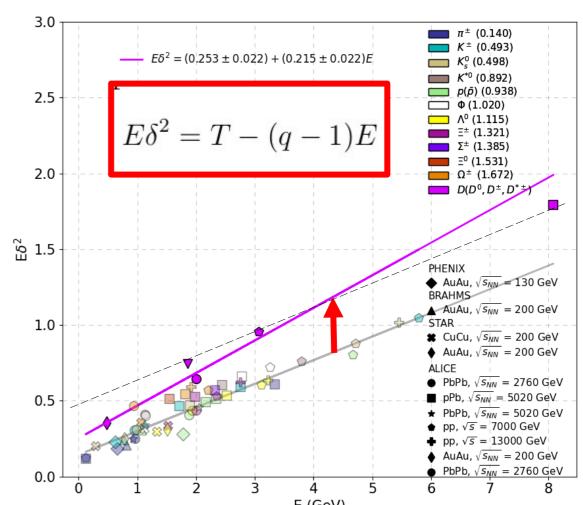
**LF-HF difference**:  $\Delta T_{eq} \approx 0.11 \text{ GeV } \& \Delta q_{eq} \approx 0.06$ 

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- Dependence on the collision energy for HF is more prominent, than for LF
- A HF grouping is also present, however the "center" is shifted compared to the LF
- HF grouping:  $T_{eq} \approx 0.25$  GeV and  $q_{eq} \approx 1.21$

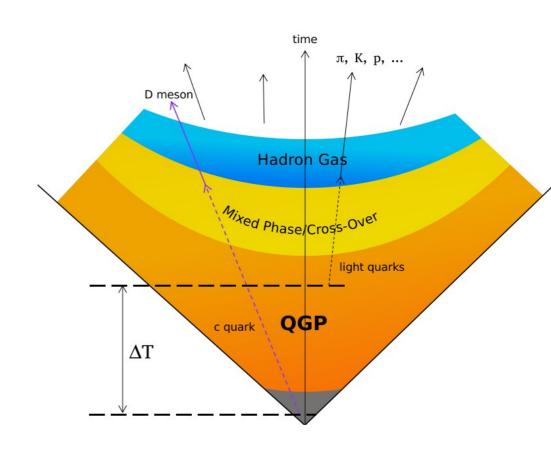


**LF-HF difference**:  $\Delta T_{eq} \approx 0.11$  GeV &  $\Delta q_{eq} \approx 0.06$ 

So, how far we can see back in time?

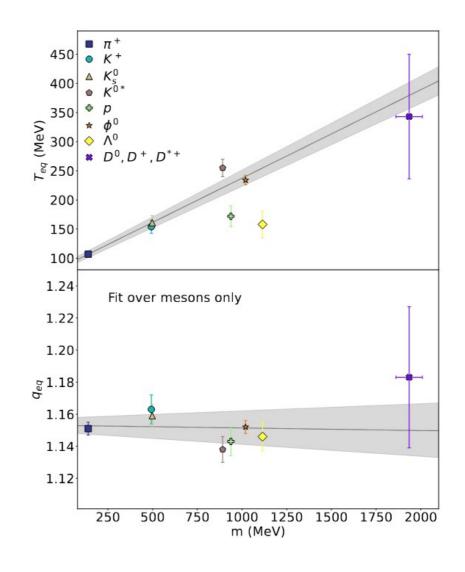
#### What we have learned so far...

- Effective temperature of heavy flavour (HF) hadron spectra is **higher**, thank the one obtaine from light flavour (LF) hadron spectra.
- We also know, that heavy flavour is formed earlier, therefore carries information of eariler stages of the reactions
- We know about **differences of** the effective temperatures of **mesons and baryons.** 
  - → Let's estimate, how earlier are HF vs. LF?



#### **Further properties of the fix point**

- Temperature ( $T_{eq}$ ) of the common fix points for mesons are linearly increase with the hadron masses.
- Temperature,  $T_{eq}$  is smaller for baryons than the same mass mesons.
- Non-extensivity parameter,  $q_{eq}$  does not present significant mass dependence



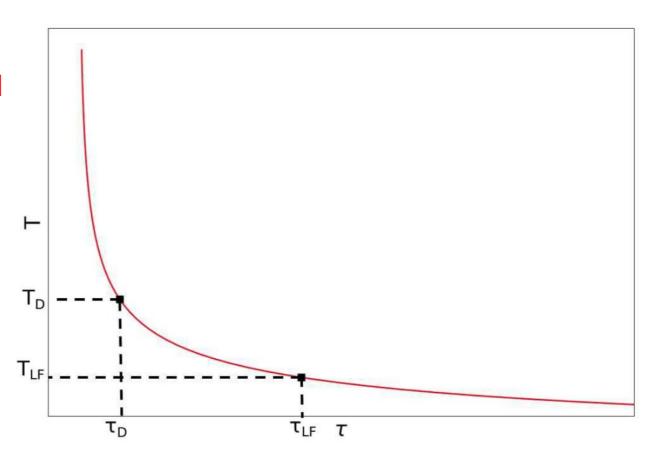
- Bjorken-model DOES NOT say anything on the thermodynamical description
  - → temperature scales can be connected

$$\tau = \tau_0 \left(\frac{T_0}{T}\right)^3$$

• Once we know the temperature values, we could turn this to measure the time scales, using the approximated fix point value:  $T_{\rm eq}$ 

$$au_{
m D} = au_{
m LF} \left(rac{T_{
m LF}}{T_{
m D}}
ight)^3$$

- Taking all light flavours as reference,
  - → D-meson formation relative to all LF



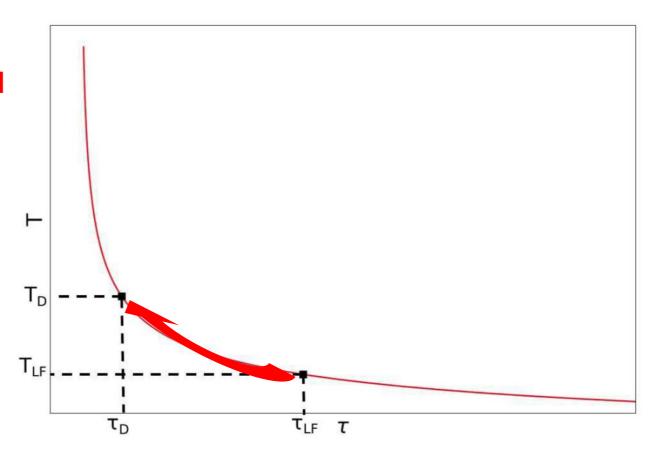
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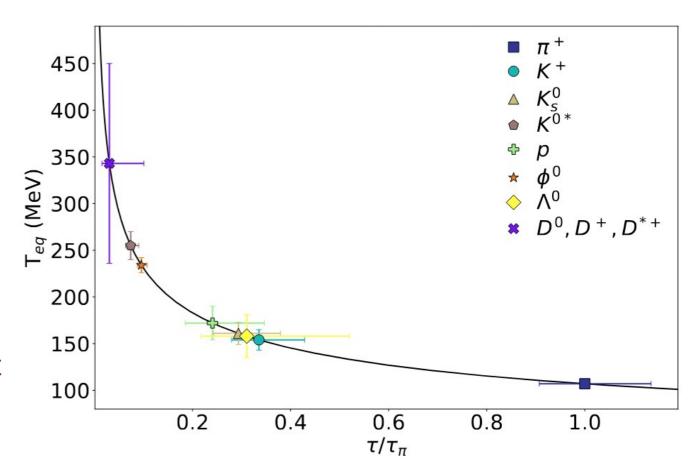
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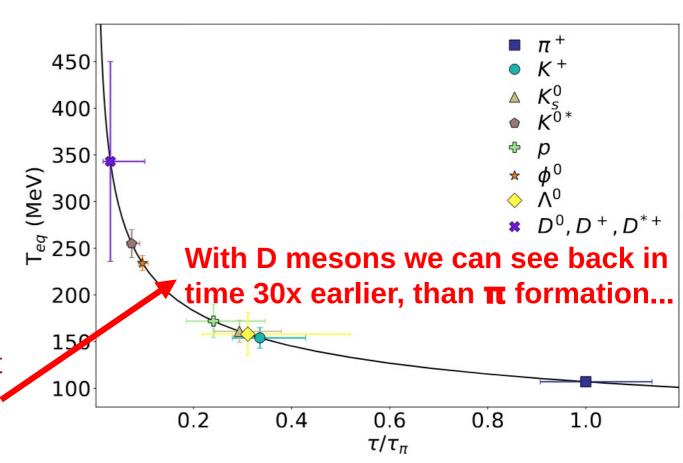
#### Adding more identified hadrons

- Pion formation is the latest one
- Formation time has mass order: the lighter the hadron is, it forms later.
- Heavier baryons forms later than other mesons with the same mass
- Taking all PID & D-mesons (here only at LHC energies) → D-meson formation relative to π is 30x earlier...



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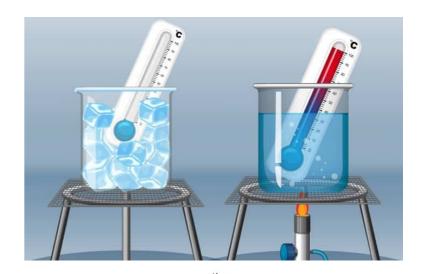
### Conclusions

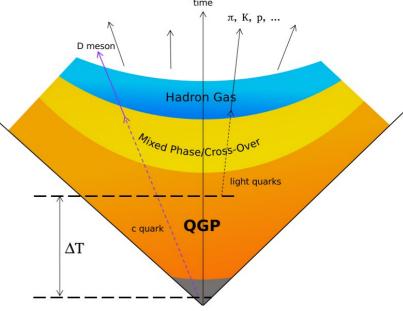
#### Non-extensive statistical framework

- Based on the data, our model is working for both LF and D-meson production
- Works from RHIC to LHC energies at the highest p<sub>T</sub>
- Tsallis-Pareto fits well in all multiplicities

#### Comparing LF & HF via Tsallis-thermometer

- Tsallis-thermometer present similar trends, but scales are different between LF and HF.
- Mass hierarchy is present and stronger for HF
- Overall grouping is different between mesons & baryons, and between LF & HF
- → To take away... Bjorken model is compatible with the Tsallis-thermometer, and relative formation time can be estimated.







# Backups

## Thermodynamical consistency?

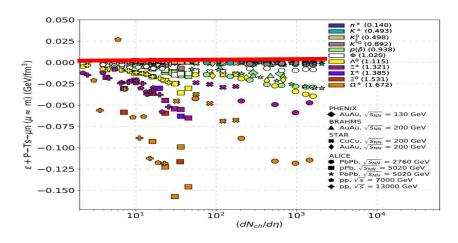
Thermodynamical consistency: fulfilled up to a high degree

$$P = g \int \frac{d^3p}{(2\pi)^3} Tf,$$

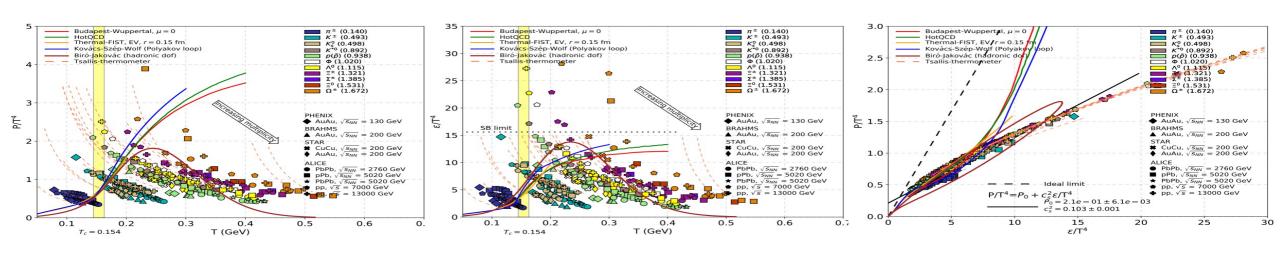
$$N = nV = gV \int \frac{d^3p}{(2\pi)^3} f^q,$$

$$s = g \int \frac{d^3p}{(2\pi)^3} \left[ \frac{E-\mu}{T} f^q + f \right],$$

$$\varepsilon = g \int \frac{d^3p}{(2\pi)^3} Ef$$



Compare EoS to data: Lattice QCD (parton) & Biró-Jakovác parton-hadron



## Thermodynamical consistency?

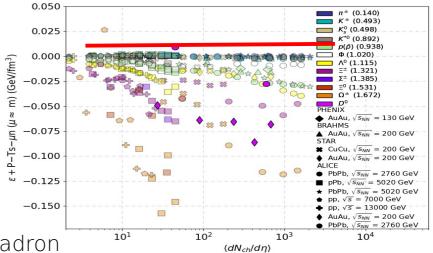
Thermodynamical consistency: fulfilled up to a high degree

$$P = g \int \frac{d^3p}{(2\pi)^3} Tf,$$

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