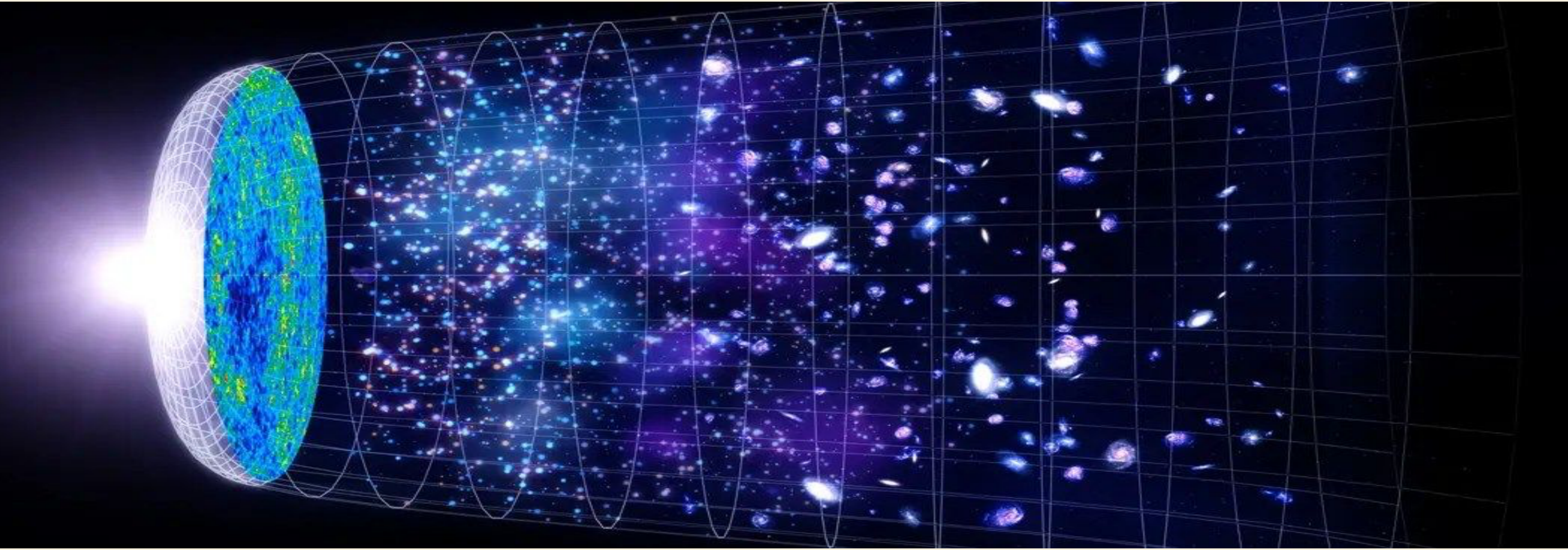
Dark Matter Freeze-Out after Inflaton Fragmentation

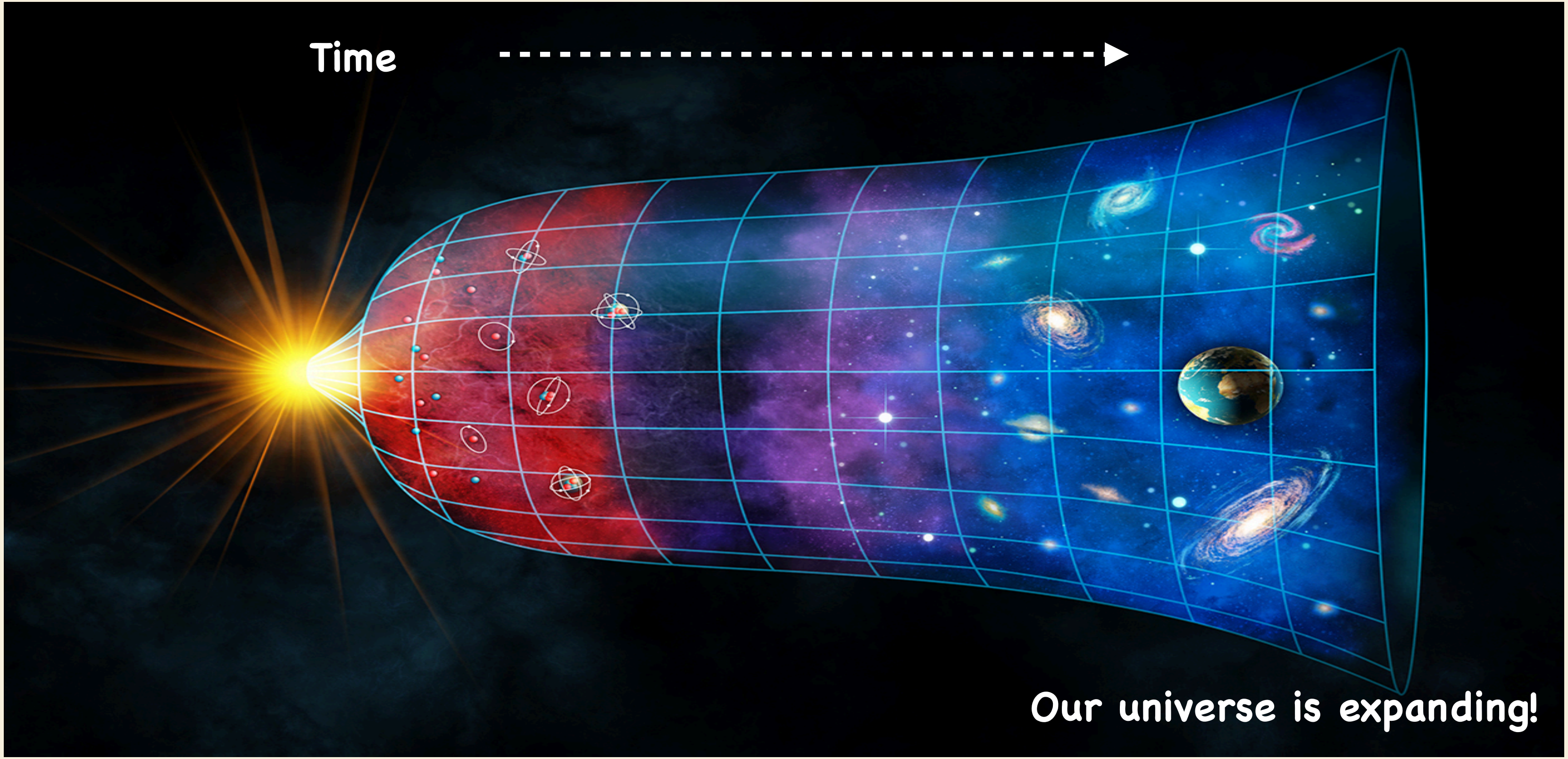
Francisco Barreto Basave



Advisor: Marcos Alejandro García García

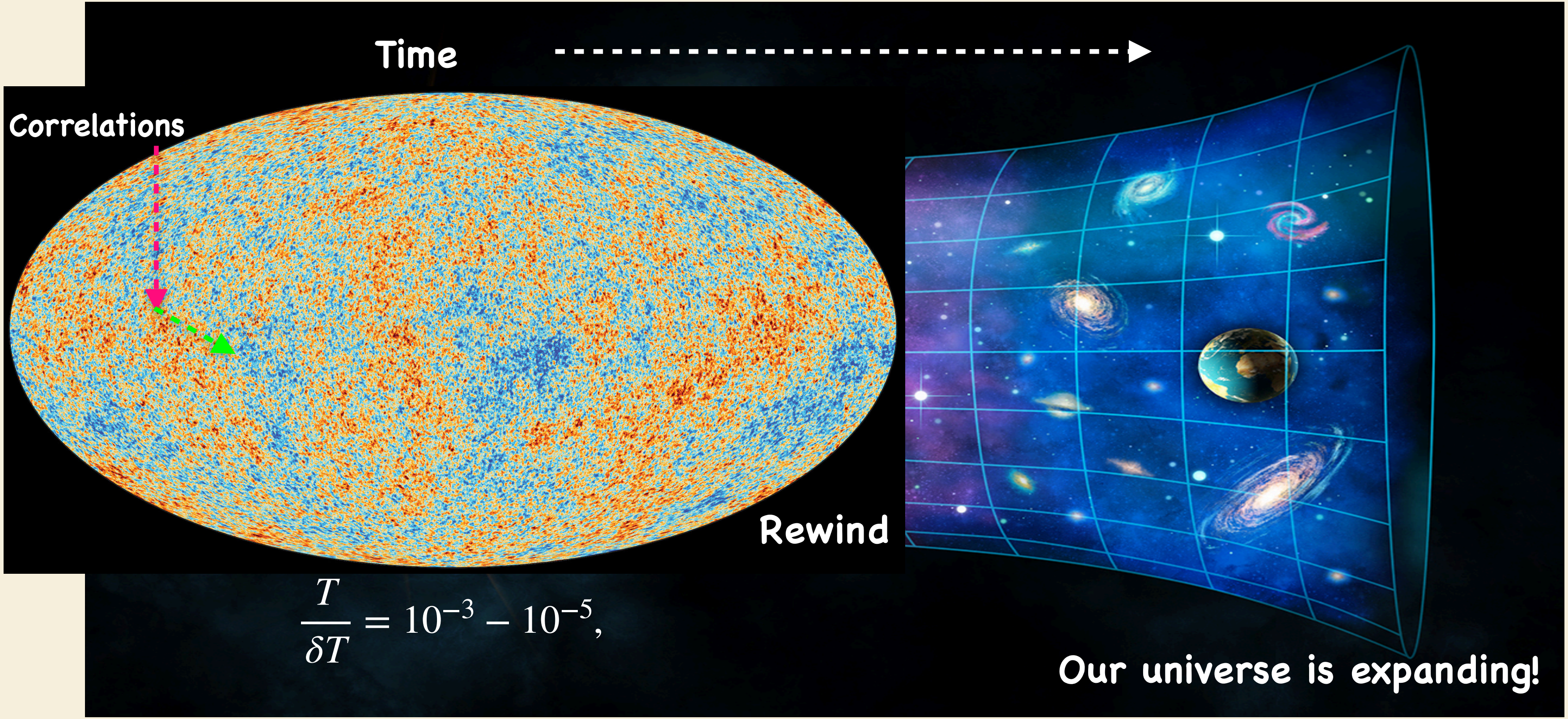
Más allá del Model Estándar y Astropartículas
Monday September 29, 2025

Thermal History of the universe



Our universe is (exponentially) expanding today.

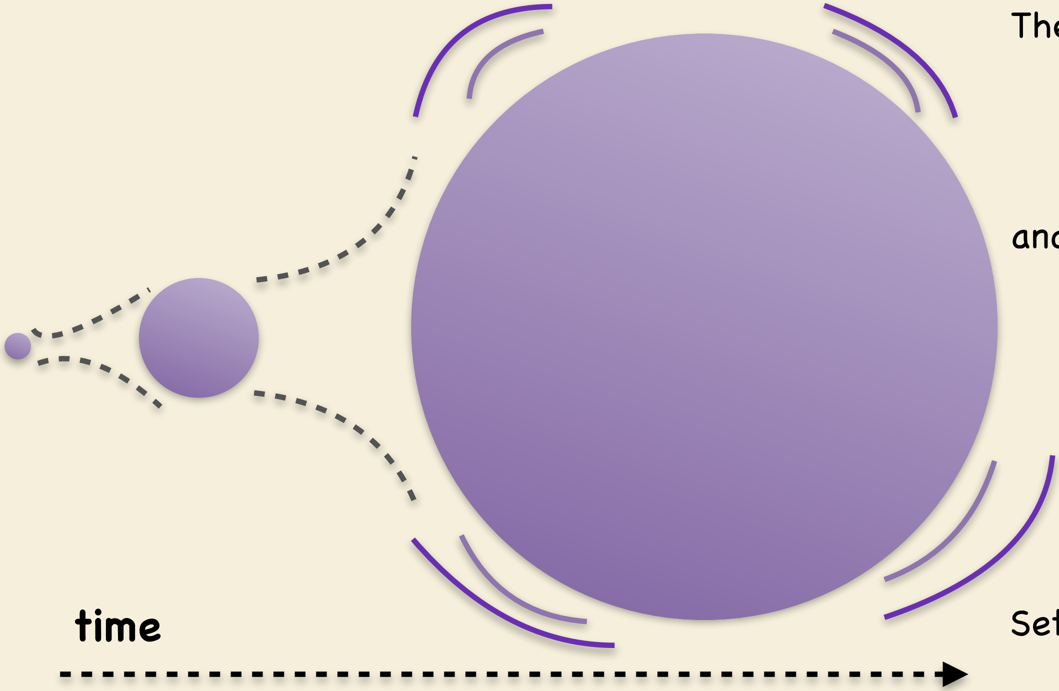
Thermal History of the universe



$$\frac{T}{\delta T} = 10^{-3} - 10^{-5},$$

Our universe is (exponentially) expanding today.

Inflation



The (single) Inflaton field, ϕ, with an action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \left(\partial_\mu \phi \right)^2 - V(\phi) \right],$$

and a T-model inflationary potential

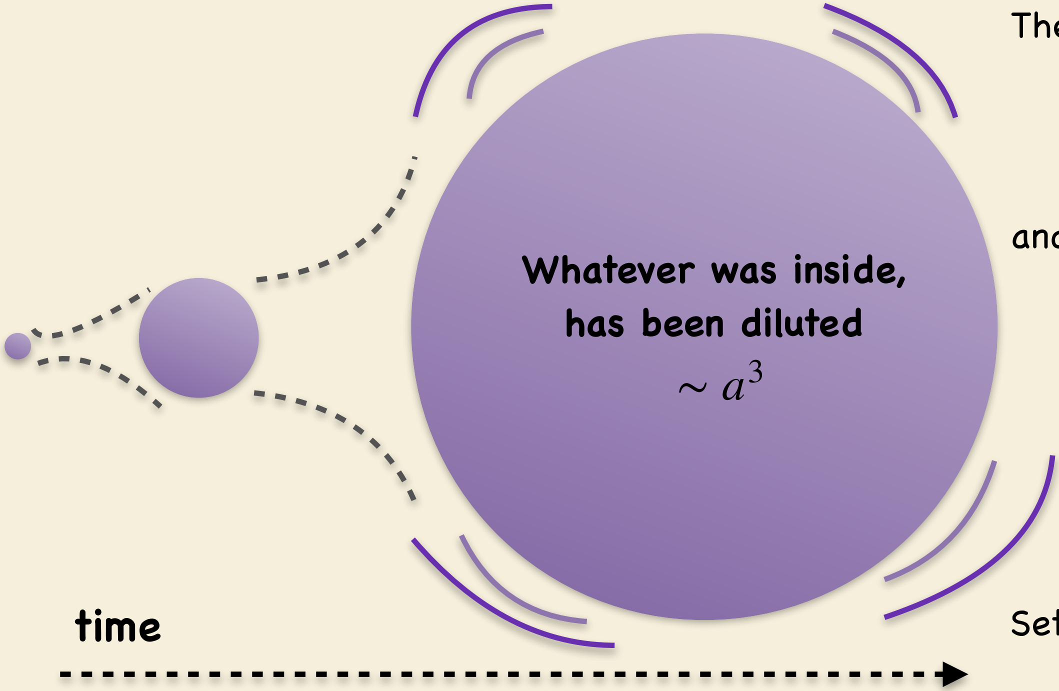
$$V(\phi) = \lambda M_p^4 \left[\sqrt{6} \tanh \frac{\phi}{\sqrt{6} M_p} \right]^k$$

$$\approx \lambda \phi^k, \qquad (\phi \ll M_p)\,.$$

Setting $$\lambda = \frac{18\pi^2 A_s}{6^{\frac{k}{2}} N_\star^2}\,.$$

The equations of motion:
$$\begin{cases} \frac{1}{2}\dot{\phi}^2 + V(\phi) = 3H^2 M_p^2, \;\; \text{Friedmann with } H = \dfrac{\dot{a}}{a}\,. \\ \ddot{\phi} + 3H\dot{\phi} - \dfrac{1}{a^2}\nabla^2\phi + V_\phi(\phi) = 0, \;\; \text{Klein} - \text{Gordon}\,. \end{cases}$$

Inflation



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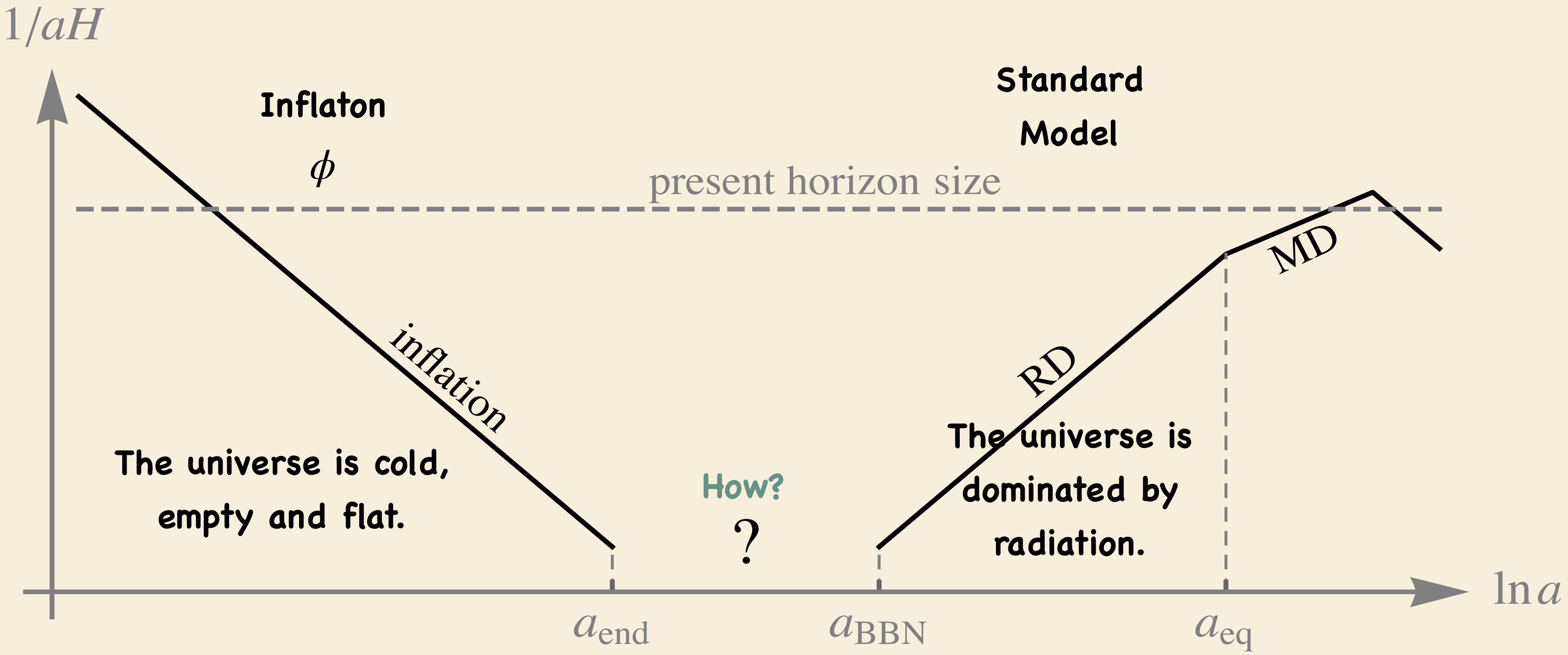
Inflation → ? → Radiation domination

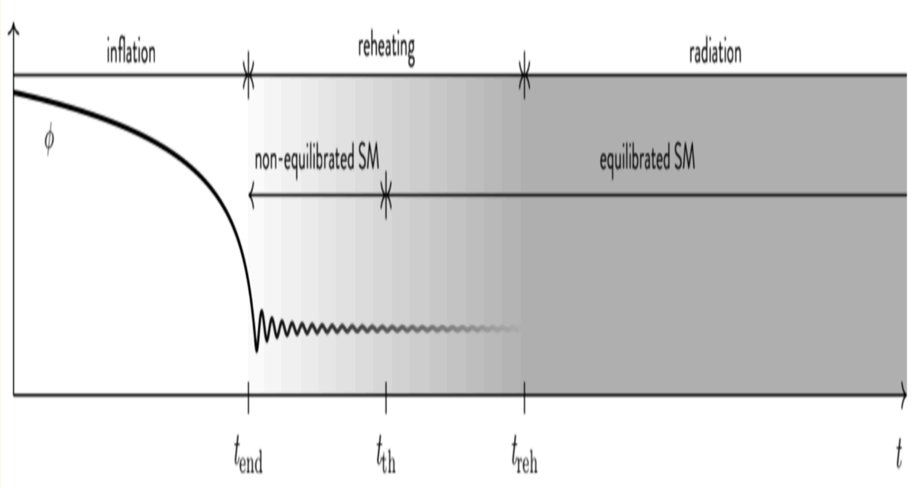


Evolution of the comoving Hubble horizon as a function of the scale factor.

arXiv:2006.16182v2

Inflation → Reheating → Radiation domination

arXiv:2306.08038v2



Solution to the Klein-Gordon equation for inflation.

Assuming a coupling of the inflaton to Standard Model (SM) fields

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V_\phi(\phi) = -\Gamma_\phi\dot{\phi}\,.$$

Averaging over oscillations arXiv:2011.14861v2

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = -\left(1+w_\phi\right)\Gamma_\phi\rho_\phi\,.$$

Using the continuity equation

$$\dot{\rho}_R + 4H\rho_R = \left(1+w_\phi\right)\Gamma_\phi\rho_\phi\,.$$

Inflation → Reheating → Radiation domination

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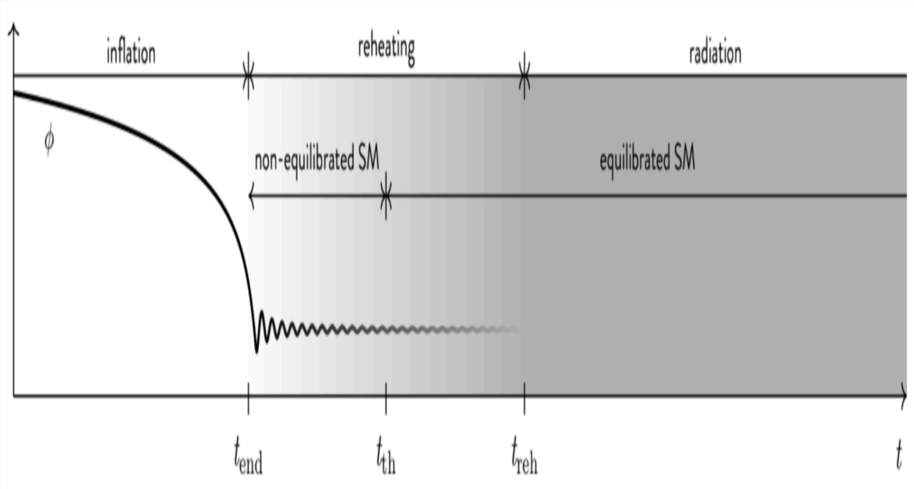

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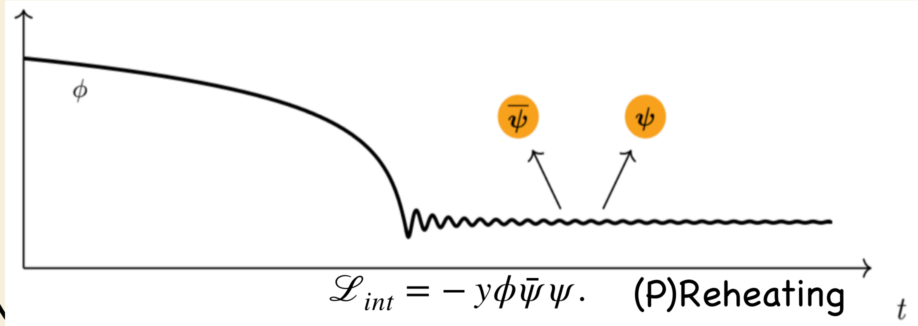

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Using the continuity equation

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Assuming a Yukawa coupling

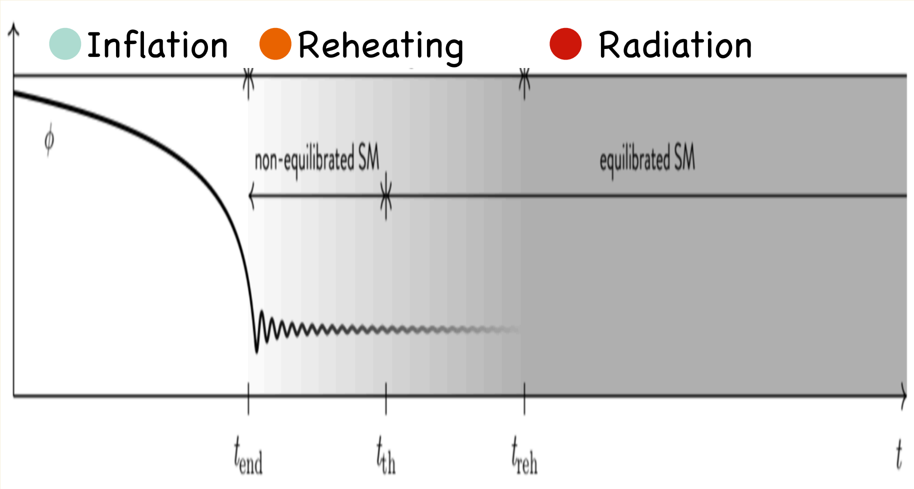
$$\mathscr{L}_{int} = -y\phi\bar{\psi}\psi,$$

We get arXiv:2012.10756v2

$$\Gamma_\phi = \alpha^2\frac{y^2}{8\pi}m_\phi, \qquad k = \{2,4\} \quad \rightarrow \quad \alpha = \{1, 0.71\}\,.$$

Coherent oscillations

arXiv:2306.08038v2



K-G equation $\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + 4\lambda\phi^3 = 0.$

Linear order $\phi(t) = \phi_{cl}(t) + \delta\phi(x,t)\,.$

Self-interactions

Classical field (Background - Bg)

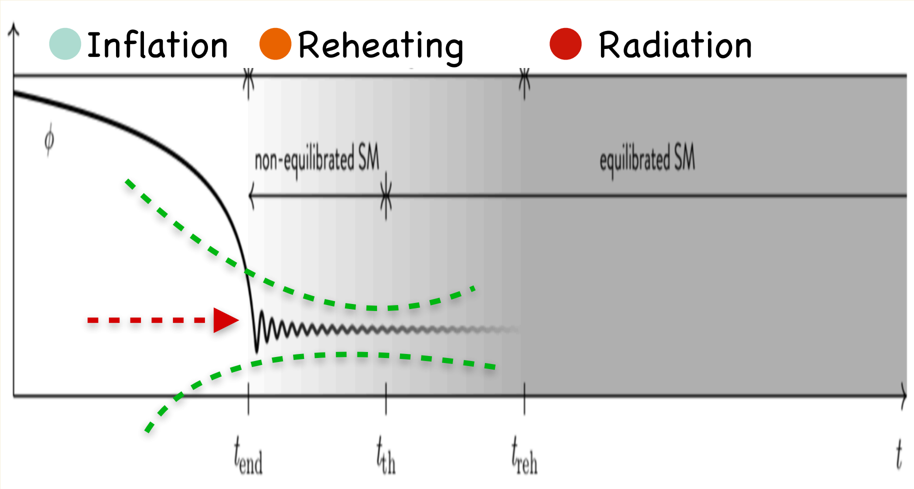
$$\ddot{\phi}_{cl} + 3H\dot{\phi}_{cl} + 4\lambda\phi_{cl}^3 = 0.$$

After the first oscillation, in conformal time τ

$$\phi_{cl}(\tau) = \,?$$

Coherent oscillations

arXiv:2306.08038v2



K-G equation

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Self-interactions

Classical field (Background - Bg)

$$\ddot{\phi}_{cl} + 3H\dot{\phi}_{cl} + 4\lambda\phi_{cl}^3 = 0.$$

After the first oscillation, in conformal time τ

$$\phi_{cl}(\tau) \approx \phi_0(\tau)\cdot\mathscr{P}(\tau)$$

Jacobi elliptic function

$$= \phi_{end}\left(\frac{a_{end}}{a(\tau)}\right)\cdot \mathrm{sn}\left(\frac{m_{end}}{\sqrt{6}}(\tau-\tau_{end}), -1\right).$$

$$\left(\langle w_\phi^{Bg}\rangle = \frac{\langle p_\phi\rangle}{\langle \rho_\phi\rangle} = \frac{k-2}{k+2}.\right)$$

Where the mass of the inflaton field is given by

$$m_\phi^2(\tau) = V_{\phi\phi}(\phi_{cl}(\tau)) = m_{end}^2\left(\frac{a_{end}}{a(\tau)}\right)^2,$$

where we define the mass of the inflaton at the end of inflation, m_{end}, as

$$m_{end}^2 \equiv 12\lambda\phi_{end}^2\,.$$

Inhomogeneities

The inhomogeneities equation of motion

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{1}{a^2}\nabla^2\delta\phi + 12\lambda\phi_{cl}^2\delta\phi = 0.$$

Introducing the canonically normalized field

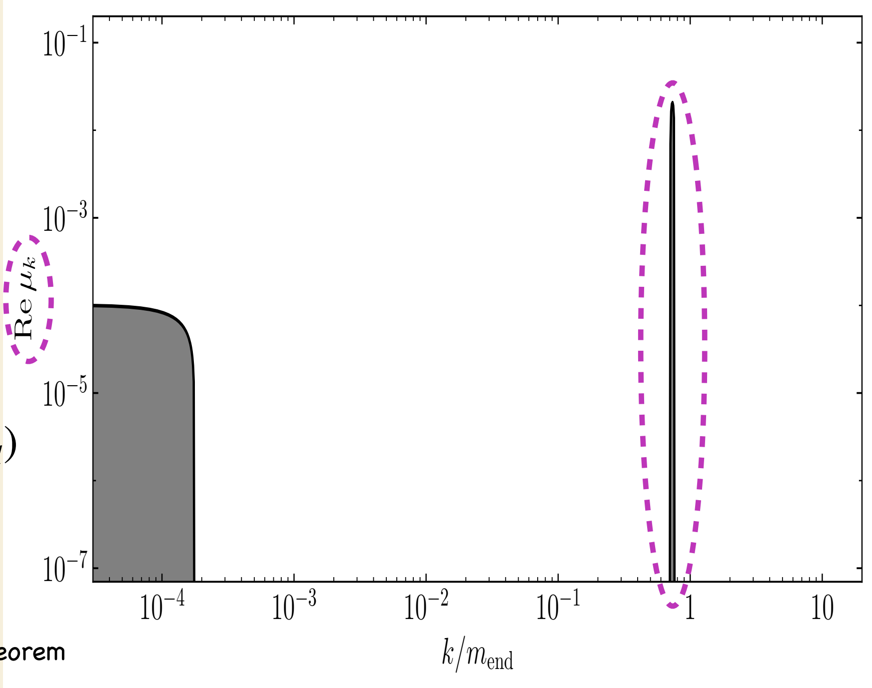
$$\hat{X} \equiv a\hat{\delta\phi} = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-ik\cdot x}\left[X_k(\tau)\hat{a}_k + X_k^*(\tau)a_{-k}^\dagger\right].$$

where X_k are the mode functions

$$X_k'' + \left(k^2 - \frac{a''}{a} + 12\lambda\phi_{cl}^2 a^2\right) X_k = 0.$$

In terms of the dimensionless variable $z = m_{end}(\tau - \tau_{end})$

$$\frac{d^2X_k}{dz^2} + \left[\left(\frac{k}{m_{end}}\right)^2 + sn\left(\frac{z}{\sqrt{6}}, -1\right)\right] X_k = 0,$$



Floquet theorem

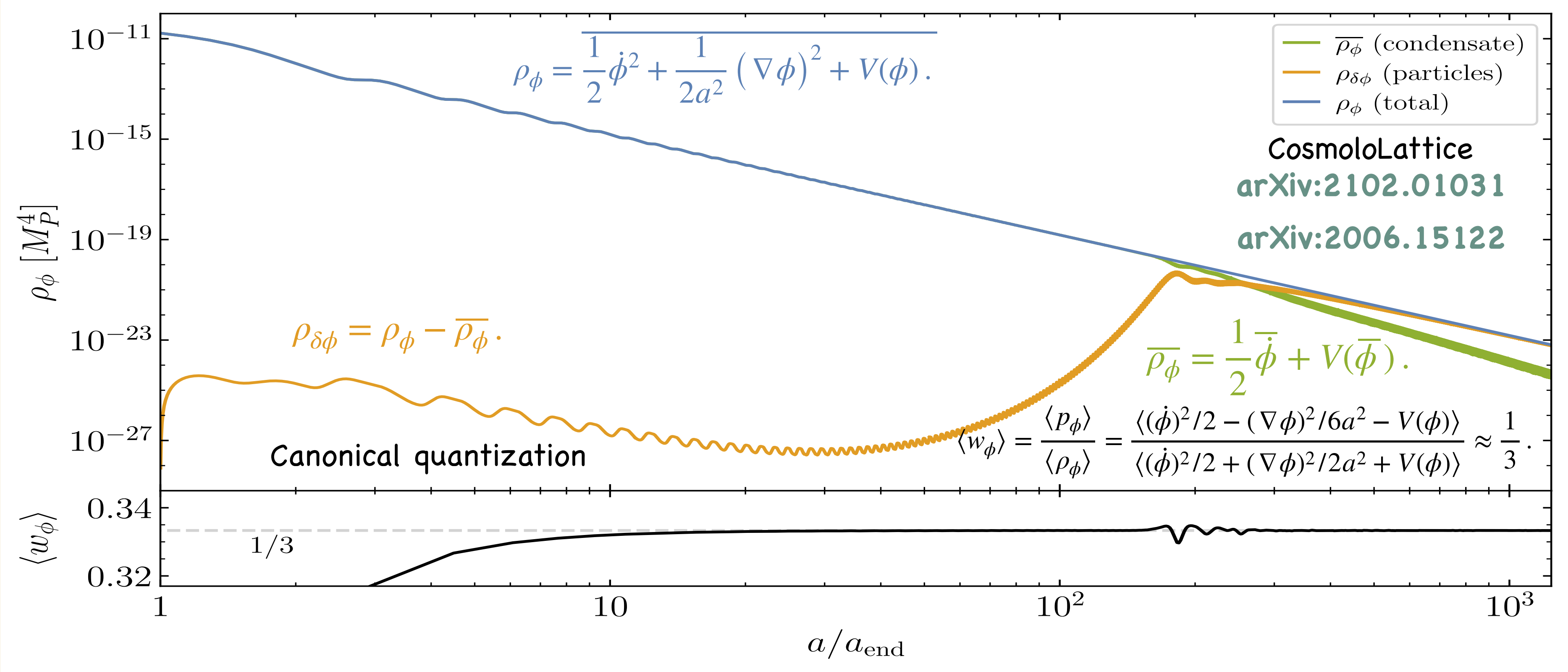
using the Floquet theorem $X_k = e^{\mu_k z} g_1(z) + e^{-\mu_k z} g_2(z)$.

$$\phi(t) = \phi_{cl}(t) + \delta\phi(x,t)\,.$$

Insuficient

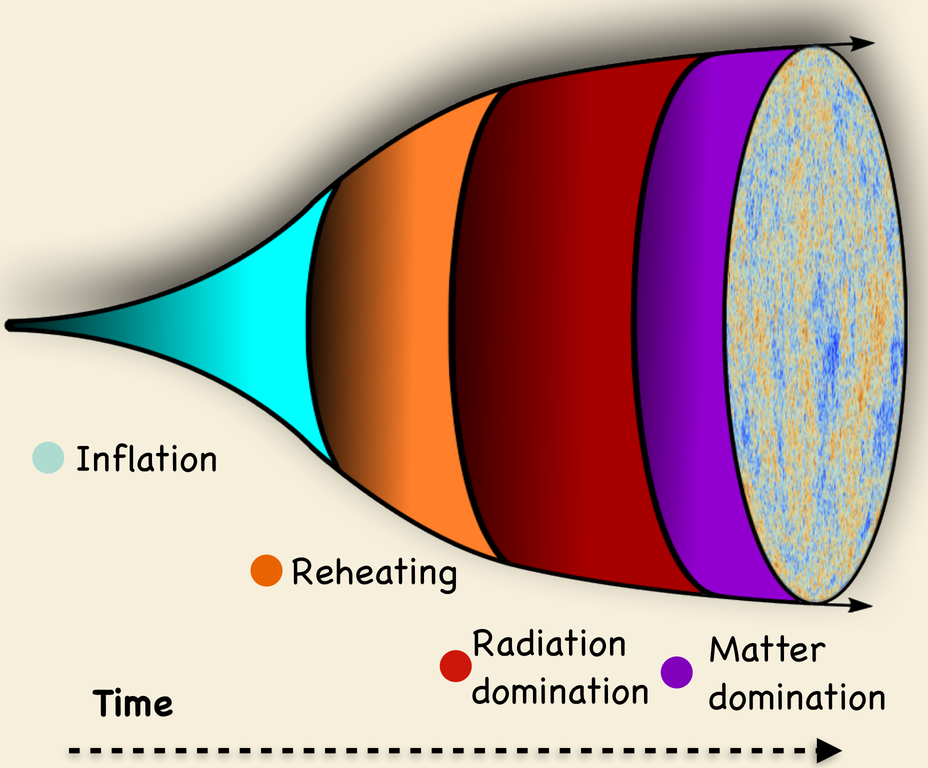
Non-linear regime

arXiv:2306.08038v2

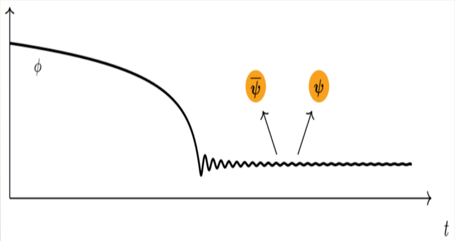


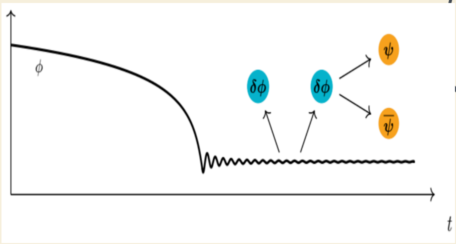
Numerical solutions to the K-G equation $\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + 4\lambda\phi^3 = 0.$

Inflation → Reheating → Radiation domination



Overview of the thermal history if the universe.





The particle production of the inhomogeneities

$$R_{\delta\phi} = \Gamma_{\delta\phi} m_\phi n_{\delta\phi} = \frac{y^2}{8\pi} m_\phi^2 n_{\delta\phi} \,.$$

Therefore, the continuity equation for radiation reads

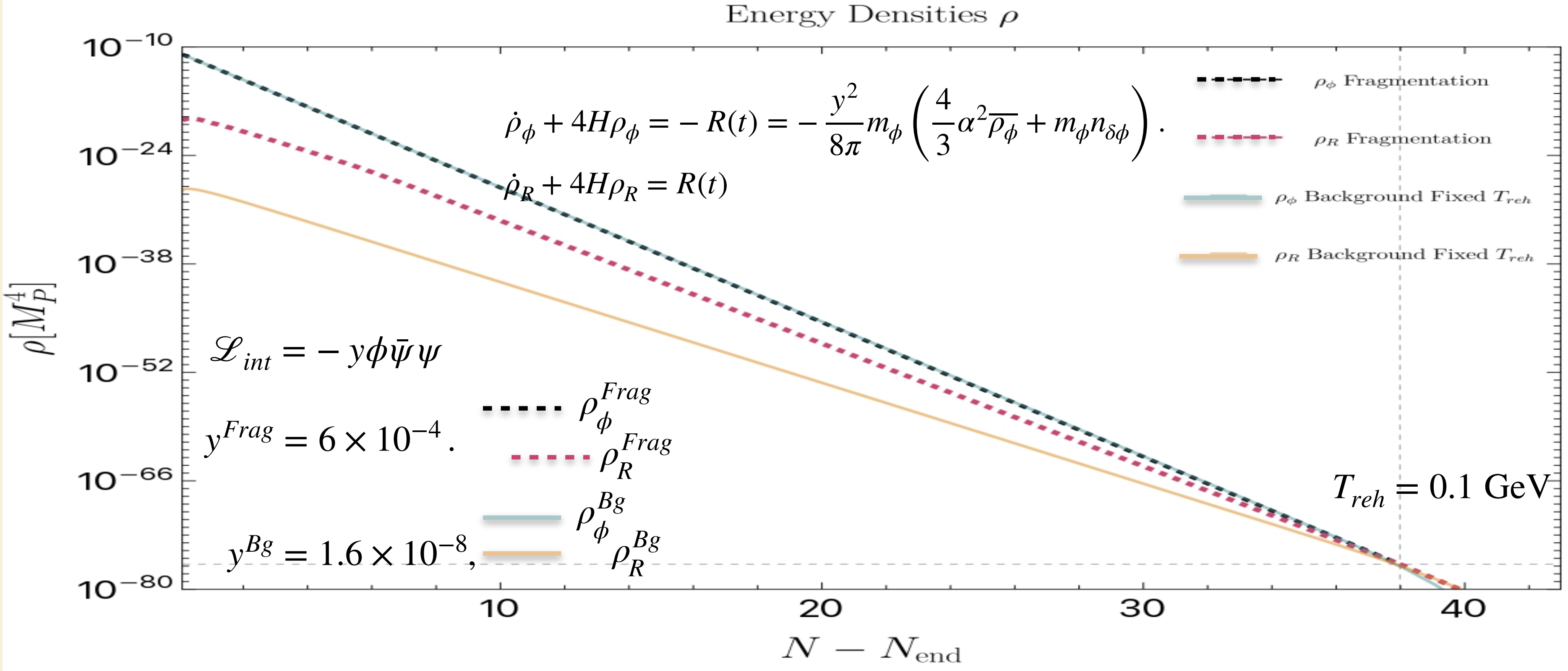
$$\dot{\rho}_\phi + 4H\rho_\phi = -\frac{y^2}{8\pi} m_\phi \left(\frac{4}{3}\alpha^2 \overline{\rho_\phi} + m_\phi n_{\delta\phi} \right) .$$

where we have used

$$\langle w_\phi \rangle = \frac{1}{3} \,.$$

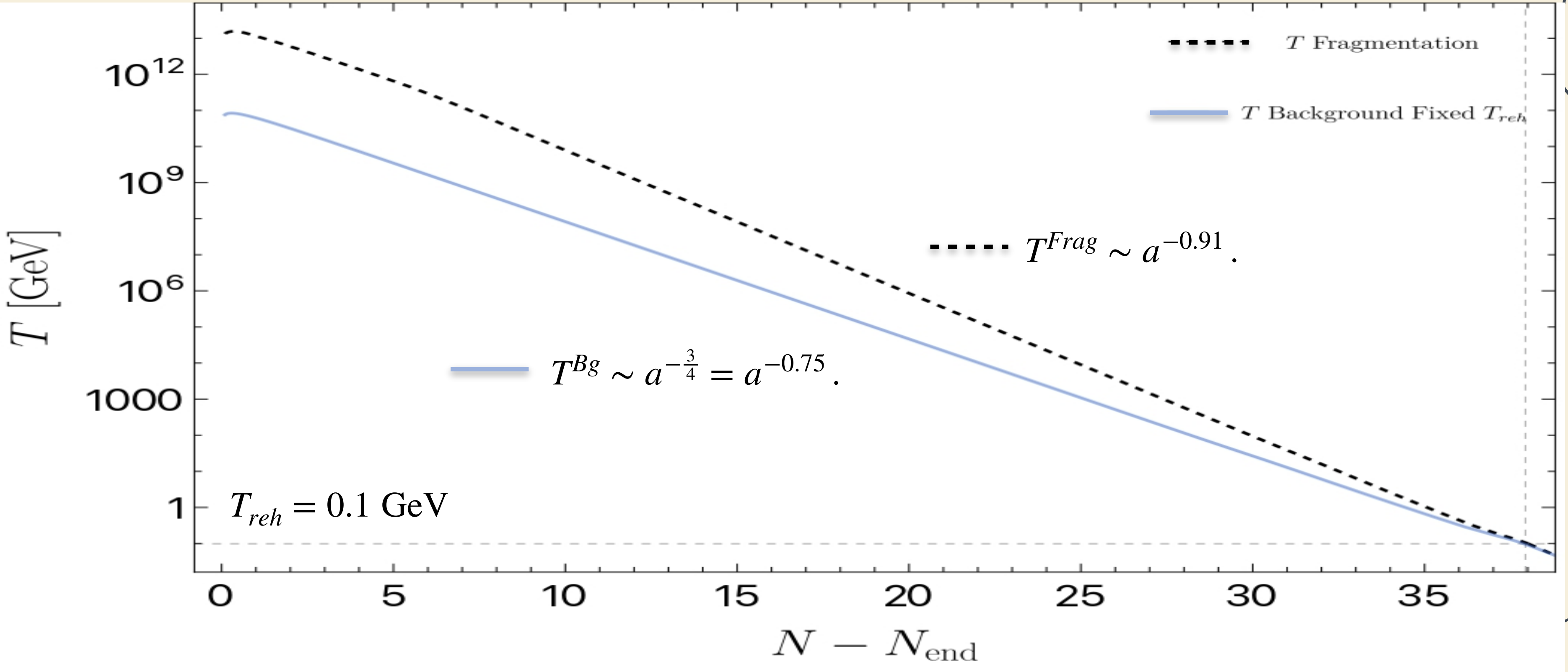
and the Friedmann equation $3H^2 M_p^2 \approx \rho_\phi \,.$

Reheating



Numerical solutions the the continuity equations system, dashed (solid) lines correspond to the fragmentation (background) scenario.

Temperature of the Thermal Bath during Reheating



Temperature of the thermal bath, T, as a function of the e-folds number, $N = \ln a/a_{end}$, the dashed (solid) line correspond to the fragmentation (background) scenario.

Dark Matter

The BE in a FLRW universe (after phase space integration): $\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\bar{\chi}\chi\to\bar{\psi}\psi} v\rangle (n_\chi^2 - n_{\chi,eq}^2)\,.$

The thermally averaged annihilation cross section $\langle \sigma_{\bar{\chi}\chi\to\bar{\psi}\psi} v\rangle = \langle \sigma v\rangle$ is obtained as

$$\langle \sigma v\rangle \approx \frac{\int\int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \left(\sigma_s + \sigma_p v^2\right) e^{-E_1/T} e^{-E_2/T}}{\int\int d^3\mathbf{p}_1 d^3\mathbf{p}_2 e^{-E_1/T} e^{-E_2/T}},$$

Wino-like Lowest Supersymmetric Particle (LSP) annihilating into W Bosons:

$$\langle \sigma v\rangle = \frac{g_2^4}{2\pi}\frac{1}{m_\chi^2}\frac{\left(1-x_W\right)^{3/2}}{\left(2-x_W\right)^2}, \quad x_W = \frac{m_W^2}{m_\chi^2}\,.$$

arXiv:astro-ph/9906527v1

Introducing the variables $Y = n/T^3$, and $N = \ln a/a_{end}$ we get

$$\frac{dY}{dN} = -3Y\left(1 + \frac{1}{T}\frac{dT}{dN}\right) - \frac{T^3\langle \sigma v\rangle}{H}\left(Y^2 - Y_{eq}^2\right).$$

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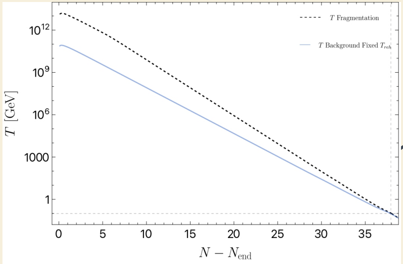

Freeze-out

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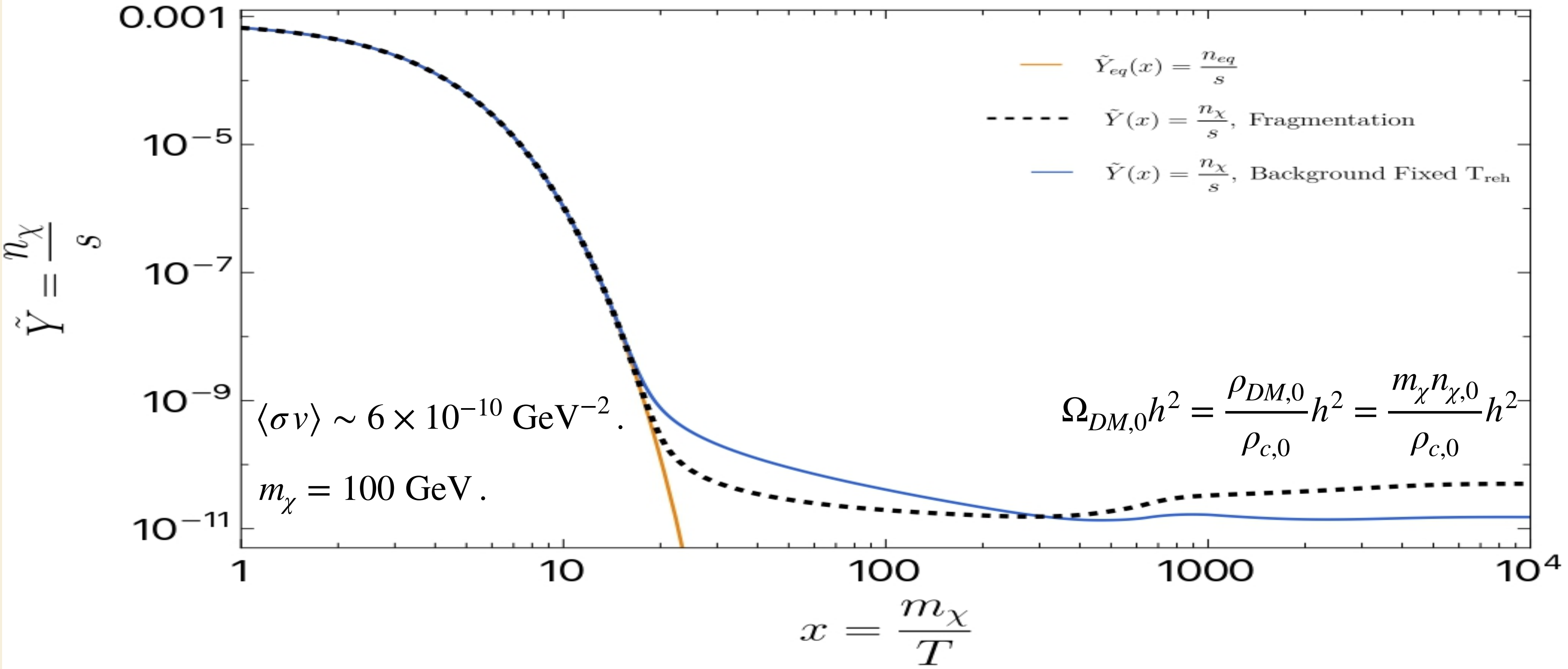

Numerical solutions to the BE for winos in the s-wave approximation, the dashed (solid) line corresponds to the fragmentation (background) scenario.

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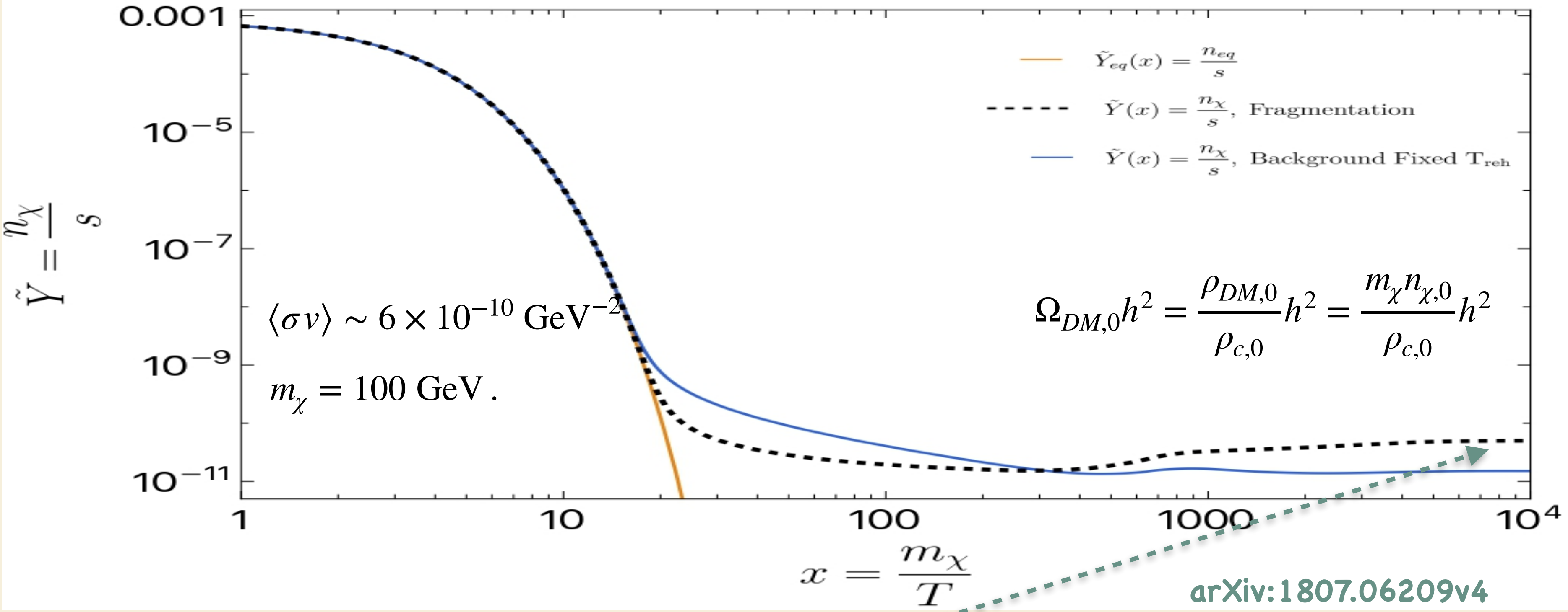
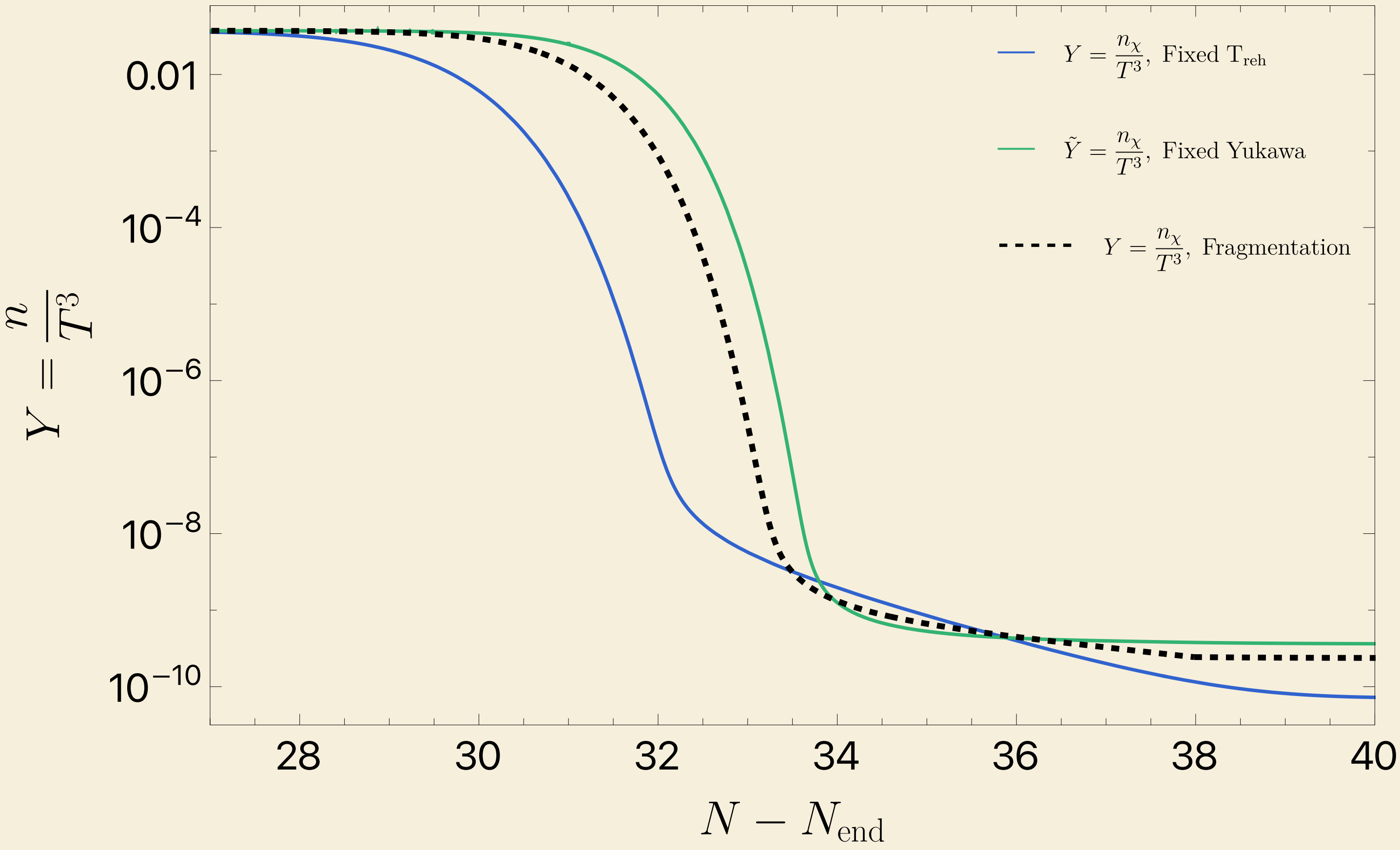


arXiv:1807.06209v4

$$\Omega_{DM,0}h^2 = \frac{\rho_{DM,0}}{\rho_{c,0}}h^2 = \frac{m_\chi n_{\chi,0}}{\rho_{c,0}}h^2 = \begin{cases} 0.114 & \text{Fragmentation,} \\ 0.037 & \text{Background}\,. \end{cases}$$

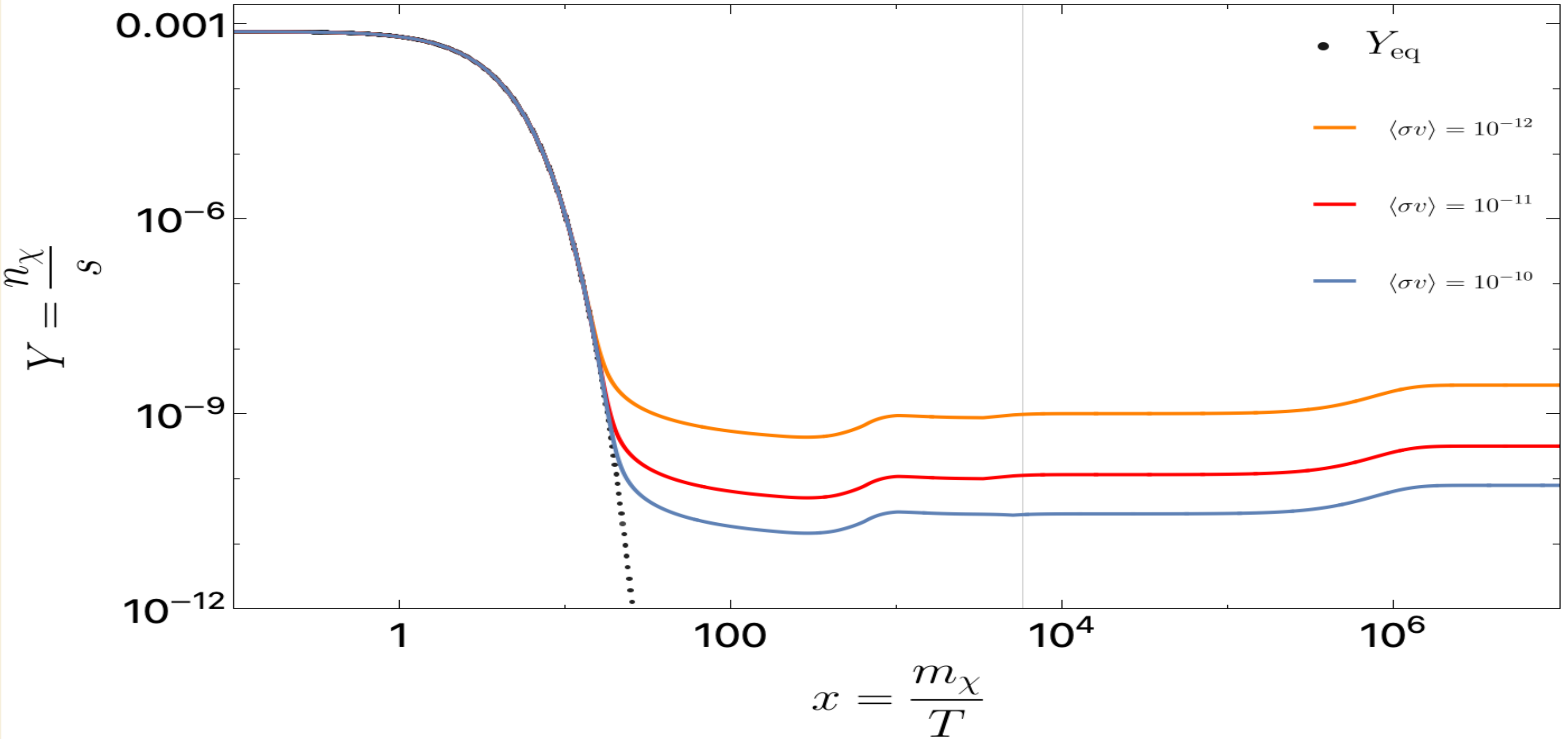
$$\Omega_{DM,0}^{\text{Planck}}h^2 = 0.11933 \pm 0.00091,$$

Freeze-out



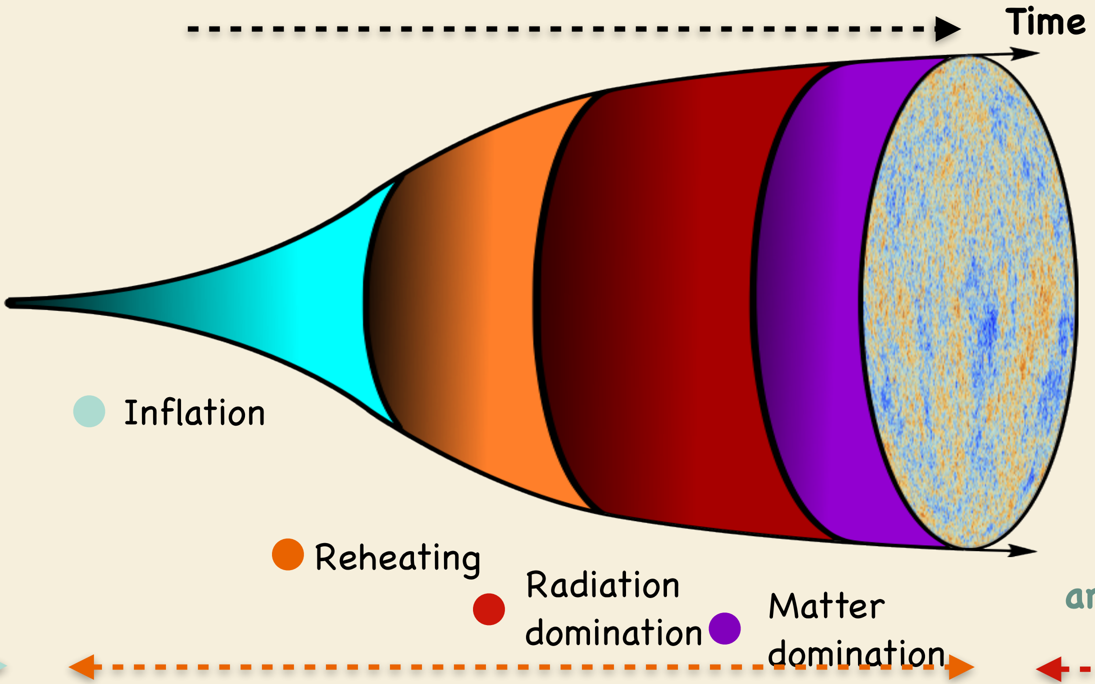
Numerical solutions to the BE for winos in the s-wave approximation, the dashed (solid) line corresponds to the fragmentation (background) scenario.

Dark Matter



Numerical solutions to the BE for winos in the s-wave approximation by fixing the Reheating temperature and varying the annihilation cross section.

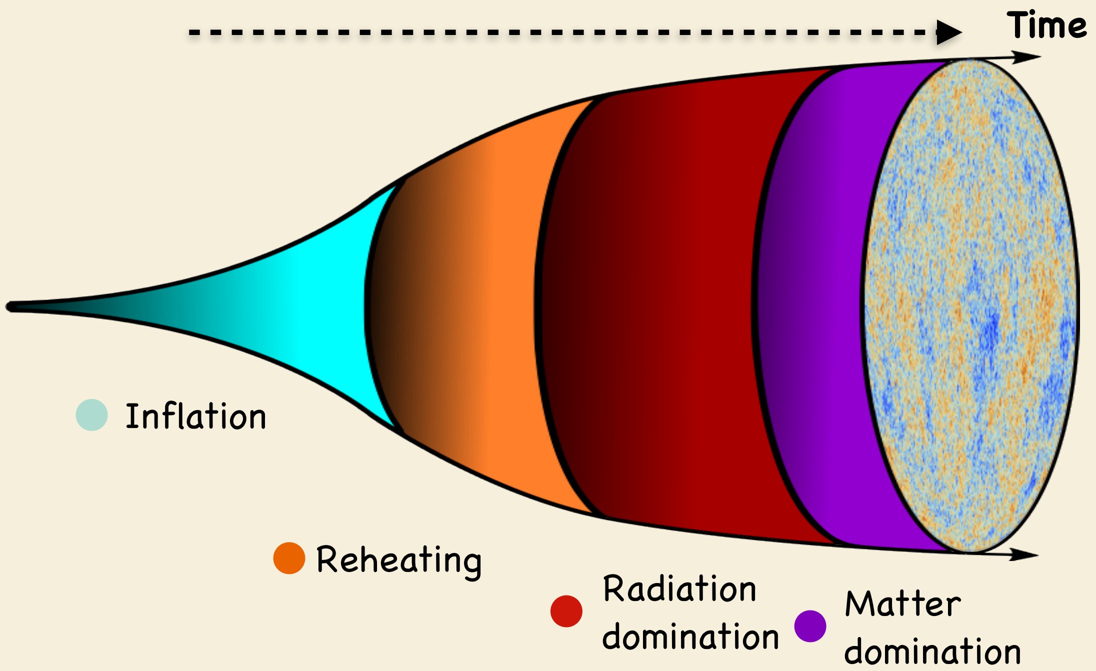
Thermal history of the universe



arXiv:1505.01076

Inflation	Fragmentation	DM freeze-out	Reheating	BBN
10^{15} GeV	10^{11} GeV	10^{3} GeV	0.1 GeV	4 MeV

Thermal history of the universe



Summary.

- Even with Fragmentation, we can complete reheating.
- WIMPs could freeze-out during reheating.
- Fragmentation has an imprint in the DM relic density.

Thanks!

Analytical approximation

The BE for DM reads

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\bar{\chi}\chi\to\bar{\psi}\psi} v\rangle (n_\chi^2 - n_{\chi,eq}^2)\,.$$

using $Y = \dfrac{n_\chi}{T^3}$,

$$\frac{dY}{dN} = -3Y\left(1 + \frac{1}{T}\frac{dT}{dN}\right) - \frac{T^3\langle\sigma v\rangle}{H}\left(Y^2 - Y_{eq}^2\right).$$

Instead, use $\tilde{Y} = \dfrac{n}{T^{\frac{3}{\alpha}}}$, and we get

$$\frac{d\tilde{Y}}{dN} = -\frac{T^{3/\alpha}\langle\sigma v\rangle}{H}\left(Y^2 - Y_{eq}^2\right).$$

The yield at reheating is given by

$$\frac{1}{Y_{reh}} = \frac{\langle\sigma v\rangle C^{\alpha/3}}{H_{end}}\frac{T_{reh}^{(3^2-\alpha^2)/3}}{-\frac{1}{3}\alpha^2 + 3}\left\{\exp\left[\left(-\frac{1}{3}\alpha^2 + 3\right)N_{reh}\right] - \exp\left[\left(-\frac{1}{3}\alpha^2 + 3\right)N_{fo}\right]\right\}.$$

where, after fragmentation, we have

$$T = Ca^{-\alpha} \qquad \alpha = 0.91$$