Abelian gauge theory with two Higgs fields on the lattice

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$U(1)_{B-L}$ exact symmetry

In the Standard Model $U(1)_{B-L}$ is an **exact** global symmetry. B baryon number, L lepton number.

This is strange, an exact symmetry is only natural when it is local.

Gauge symmetry

We promote $\mathrm{U}(1)_{B-L}$ to a local symmetry and combine it with $\mathrm{U}(1)_Y$.

We introduce new couplings α_1 and α_2 and define a new charge as

$$Y' \equiv 2\alpha_1 Y + \frac{\alpha_2}{2}(B-L)$$

$$Y'_\Phi = \alpha_1, \quad Y'_\chi = \alpha_2.$$

We take the gauge group to be $\mathrm{U}(1)_{Y'}$ and we call the gauge field $\mathcal{A}_\mu$ with h' being its gauge coupling.

Gauge anomaly

Gauge anomaly. It is cured by adding a ν_R ($L = 1$) to each generation.

Since the neutrinos have mass, it is natural to introduce a mechanism to give mass to the right-handed neutrinos.

Usually we can also give mass to the right-handed neutrinos with a Majorana mass term

$$M(\bar{\nu}_R + \nu_R^T C)(\nu_R + C\bar{\nu}_R^T).$$

However, in our scenario, this term is forbidden since it breaks the $\mathrm{U}(1)_{Y'}$ gauge symmetry.

To add a mass term solely for ν_R, independently of ν_L, we add a new Higgs field $\chi \in \mathbb{C}$

$$f_{\nu_R}(\nu_R^T \chi \nu_R + \text{c.c.}),$$

where f_{ν_R} is a Yukawa coupling.

To preserve gauge invariance, the field χ must have a charge $B - L = 2$.

Potential

$$V = \frac{m^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \frac{m'^2}{2}\chi^*\chi + \frac{\lambda'}{4}(\chi^*\chi)^2 + \frac{\kappa}{2}\Phi^\dagger\Phi\chi^*\chi$$

For the potential to be bounded from below, we need

$$\lambda > 0, \quad \lambda' > 0, \quad \kappa^2 < \lambda\lambda'.$$

We assume that $v' \gg v$ and $f_{\nu_R} \simeq O(1)$ in order to give a large mass to the right-handed neutrino.

Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(D_\mu \Phi)^\dagger D_\mu \Phi + \frac{m^2}{2}\Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2 \\
& + \frac{1}{2}(D_\mu \chi)^* D_\mu \chi + \frac{m'^2}{2}\chi^* \chi + \frac{\lambda'}{4}(\chi^* \chi)^2 \\
& + \frac{\kappa}{2}\Phi^\dagger \Phi \chi^* \chi + \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu},
\end{aligned}
$$

- $\Phi(x) = (\phi_+(x), \phi_0(x))^\mathsf{T} \in \mathbb{C}^2$, $\chi(x) \in \mathbb{C}$,
- $D_\mu \Phi = (\partial_\mu + ih' Y'_\Phi \mathcal{A}_\mu)\Phi$,
  $D_\mu \chi = (\partial_\mu + ih' Y'_\chi \mathcal{A}_\mu)\chi$
- $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$.

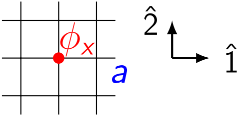
In Euclidean spacetime the probability of a configuration is

$$\frac{\exp(-S[\phi])}{Z}.$$

Lattice regularization

UV divergencies.

Lattice regularization:



- $\phi(x) \to \phi_x$
- $\partial_\mu \phi(x) \to \frac{\phi_{x+a\hat{\mu}} - \phi_x}{a}$
- $\int d^4x \to a^4 \sum_x$

Lattice units $a = 1$.

NON-PERTURBATIVE and gauge invariant approach.

Importance sampling

We generate $[\phi]_i$ with probability $\frac{1}{Z}e^{-S[\phi]_i}$, then

$$\langle O \rangle \approx \frac{1}{N}\sum_{i=1}^{N} O[\phi]_i .$$

Metropolis algorithm

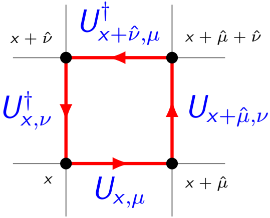
We perform a local update in a field and accept the change with probability

$$p = \min\left(1, \exp(-\Delta S)\right).$$

Abelian gauge theory on the lattice

In compact formulation we do NOT fix the gauge

$$\mathcal{A}_{\mu,x} \in \mathfrak{su}(1) \to U_{\mu,x} \in \mathrm{U}(1), \quad U_{\mu,x} = e^{i\mathcal{A}_{\mu,x}}.$$



$$U_{x,\mu\nu} = U^\dagger_{x,\nu} U^\dagger_{x+\hat{\nu},\mu} U_{x+\hat{\mu},\nu} U_{x,\mu}$$

Standard Wilson action (correct continuum limit)

$$S_{\text{gauge}}[U] = \frac{\beta}{N} \sum_x \sum_{\mu<\nu} \mathrm{Re}\left[1 - U_{x,\mu\nu}\right],$$

where $\beta = 1/h'^2$ and h' is the gauge coupling.

On the lattice the fields are

$$\Phi(x) \to \Phi_x, \quad \chi(x) \to \chi_x, \quad \mathcal{A}_\mu(x) \to \mathcal{A}_{x,\mu},$$

and the covariant derivatives take the form

$$\begin{aligned} D_\mu \Phi(x) = (\partial_\mu + ih' Y'_\Phi \mathcal{A}_\mu)\Phi(x) \;&\to\; U^1_{x,\mu}\Phi_{x+\hat{\mu}} - \Phi_x, \\ D_\mu \chi(x) = (\partial_\mu + ih' Y'_\chi \mathcal{A}_\mu)\chi(x) \;&\to\; U^2_{x,\mu}\chi_{x+\hat{\mu}} - \chi_x, \end{aligned}$$

where we have defined

$$U^1_{x,\mu} = (U_{x,\mu})^{Y'_\Phi}, \;\; U^2_{x,\mu} = (U_{x,\mu})^{Y'_\chi}, \;\; U_{x,\mu} = \exp(ih' \mathcal{A}_{x,\mu}).$$

Action on the lattice in 2d

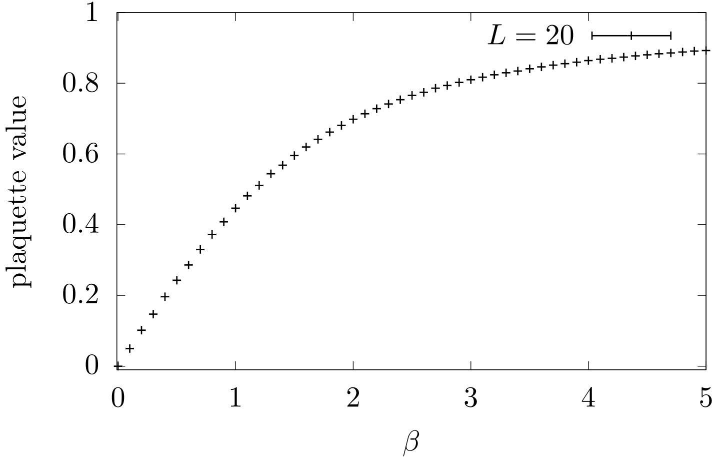
$$
\begin{aligned}
S[\Phi, \chi, U] = & \sum_x \left\{ 2\Phi_x^\dagger \Phi_x + \frac{1}{2} m^2 \Phi_x^\dagger \Phi_x + \frac{1}{4} \lambda (\Phi_x^\dagger \Phi_x)^2 \right\} \\
& + \sum_x \left\{ 2\chi_x^* \chi_x + \frac{1}{2} m'^2 \chi_x^* \chi_x + \frac{1}{4} \lambda' (\chi_x^* \chi_x)^2 \right\} \\
& - \sum_x \sum_\mu \mathrm{Re} \left[\Phi_x^\dagger \left(U^{1*}_{x-\hat{\mu},\mu} \Phi_{x-\hat{\mu}} + U^1_{x,\mu} \Phi_{x+\hat{\mu}} \right) \right] \\
& - \sum_x \sum_\mu \mathrm{Re} \left[\chi_x^* \left(U^{2*}_{x-\hat{\mu},\mu} \chi_{x-\hat{\mu}} + U^2_{x,\mu} \chi_{x+\hat{\mu}} \right) \right] \\
& + \sum_x \frac{1}{2} \kappa (\Phi_x^\dagger \Phi_x)(\chi_x^* \chi_x) + \beta \sum_x \mathrm{Re}\left[1 - U_{x,12}\right]
\end{aligned}
$$

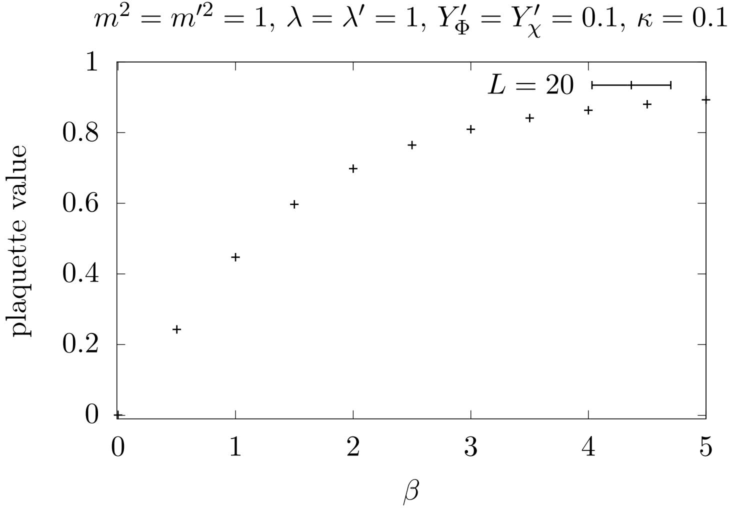
Plaquette value

The plaquette value is defined as

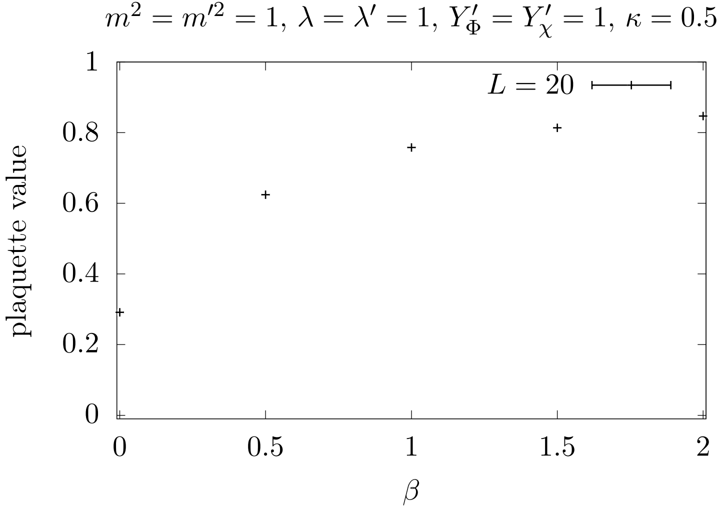
$$\text{plaquette value} \equiv \left\langle \sum_{x} \text{Re}\ U_{x,12} \right\rangle .$$

$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0,\ \kappa = 0$





$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0.1,\ \kappa = 0.1$
$L = 20$
plaquette value
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
β


$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 1,\ \kappa = 0.5$
$L = 20$
plaquette value
β
1
0.8
0.6
0.4
0.2
0
0.5
1.5
2

Topological charge

In two dimensions there exists topology of the Abelian gauge theory since

$$\pi_1(U(1)) = \mathbb{Z}.$$

On the lattice, we define

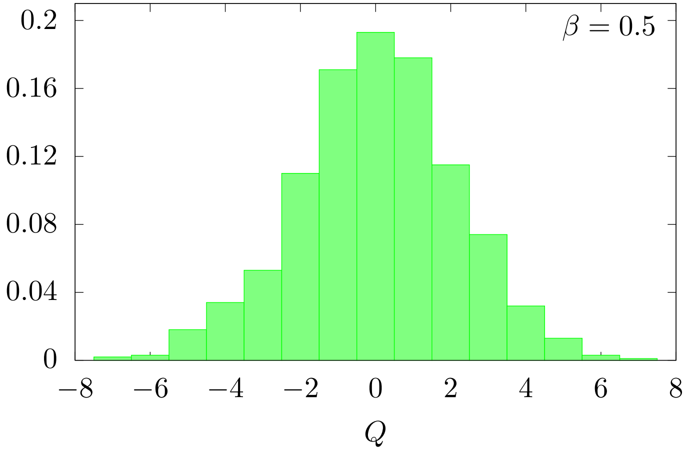
$$\theta_x = \arg U_{x,12} \in [-\pi, \pi)\,,$$

and we define the topological charge, with periodic boundary conditions, as

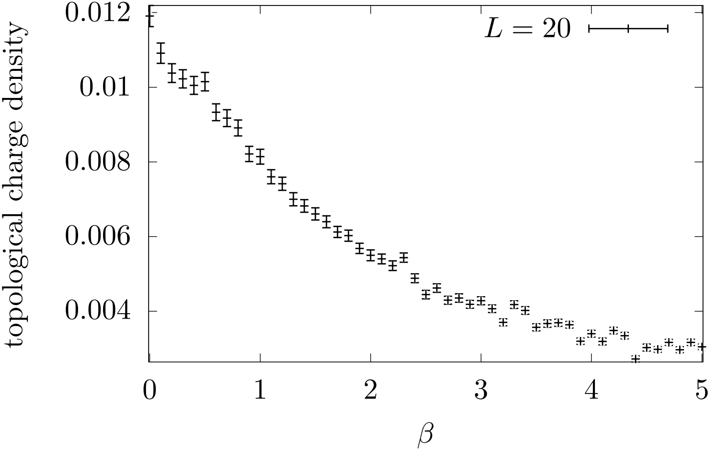
$$Q = \frac{1}{2\pi} \sum_x \theta_x \in \mathbb{Z}.$$

Topological charge histogram

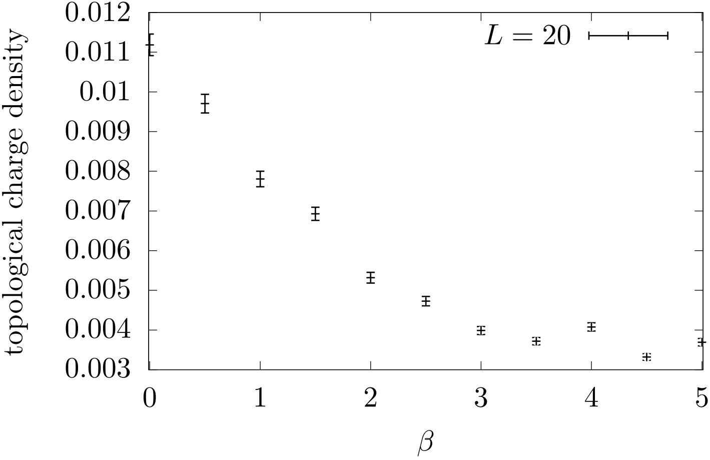
$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 1,\ \kappa = 0.5$



$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0.1,\ \kappa = 0.1$



$$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0.1,\ \kappa = 0.1$$



2-point function and renormalized mass

The 2-point function of Φ decays exponentially in Euclidean time as

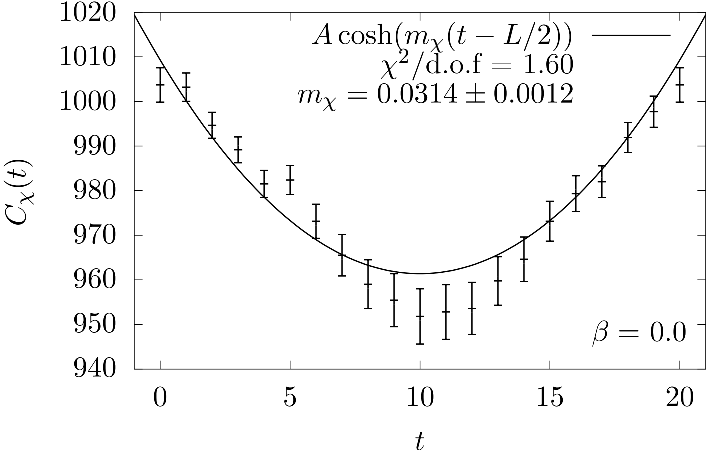
$$C_\Phi(t) = \langle \Phi(0)^\dagger \Phi(t) \rangle \propto \exp(-m_\Phi t),$$

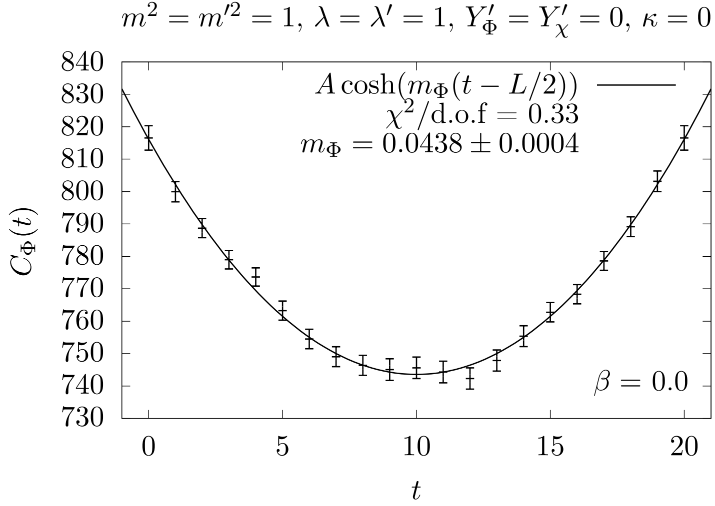
and similarly for χ

$$C_\chi(t) = \langle \chi(0)^* \chi(t) \rangle \propto \exp(-m_\chi t),$$

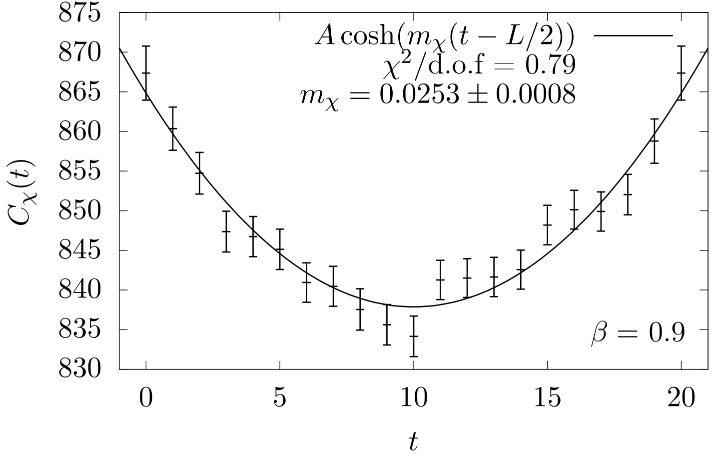
where m_Φ and m_χ are the renormalized masses of the particles associated to both Higgs fields.

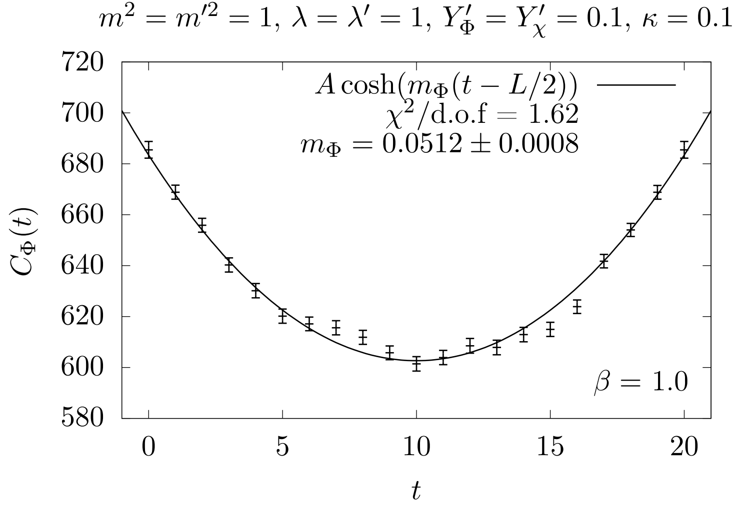
$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0,\ \kappa = 0$




$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0,\ \kappa = 0$
$A\cosh(m_\Phi(t - L/2))$
$\chi^2/\mathrm{d.o.f} = 0.33$
$m_\Phi = 0.0438 \pm 0.0004$
$\beta = 0.0$
$C_\Phi(t)$
t
840
830
820
810
800
790
780
770
760
750
740
730
0
5
10
15
20

$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0.1,\ \kappa = 0.1$




$m^2 = m'^2 = 1,\ \lambda = \lambda' = 1,\ Y'_\Phi = Y'_\chi = 0.1,\ \kappa = 0.1$
$A\cosh(m_\Phi(t - L/2))$
$\chi^2/\mathrm{d.o.f} = 1.62$
$m_\Phi = 0.0512 \pm 0.0008$
$\beta = 1.0$
$C_\Phi(t)$
t
720
700
680
660
640
620
600
580
0
5
10
15
20

Conclusions

- Turned $U(1)_{B-L}$ global symmetry to a gauge symmetry.
- Minimal model: Abelian gauge field $\mathcal{A}_\mu$, ν_R and χ.
- Simulated on the lattice in 2d. Non perturbative approach.

https://github.com/JoseAntonioLattice/scalar_gaugeU1.git