Semileptonic τ decays to $I \neq 0$ mesons

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Objectives

- We want to improve a previous analysis [1] in $\tau \to KK\pi\nu_\tau$.
- We want first to reproduce that analysis.
- Then, we want to compute all the $\tau \to 3h\,\nu_\tau$, where $h = \pi^\pm, \pi^0, K^\pm, K^0, \overline{K}^0$.
- All computed within $R\chi$T[2], an extention of χPT.

[1] D. Gómez-Dumm *et al.*, PRD **81** (2010) 034031.

[2] Ecker *et al.*, Nucl. Phys. **B321** (1989) 311; Ecker *et al.*, PLB **223** (1989) 425.

Motivation

- τ decays provide the cleanest scenario to test low-energy hadronic interactions.
- Such τ decays give a large background in measuring observables for other processes.
- Expressing the decay amplitudes in terms of some parameters, MC methods can be used to generate such events.
- Therefore, one can include these amplitudes in MC generators as TAUOLA [3].

[3] Jadach *et al.* Comput.Phys.Commun. **64** (1990) 275

χPT

- Chiral Perturbation Theory relies on the symmetry of the quark term in the QCD Lagrangian *in the chiral limit*.

$$\mathcal{L}_q = \sum_f \bar{q}_f \left(i\gamma_\mu D^\mu - m_f \right) q_f,$$

where $f = u, d, s, \dots$

- When projecting the quark fields into left/right parts $q_{L/R} = \frac{1}{2}(1 \mp \gamma_5)q$,

$$\bar{q}\gamma_\mu D^\mu q = \bar{q}_L \gamma_\mu D^\mu q_L + \bar{q}_R \gamma_\mu D^\mu q_R,$$

- while

$$-m\bar{q}q = -m\bar{q}_R q_L - m\bar{q}_L q_R.$$

χPT

- So, if the mass terms is disregarded one gets an $SU(3)_R \otimes SU(3)_L$ symmetry.
- This means, $u_\chi \leftrightarrow d_\chi \leftrightarrow s_\chi \leftrightarrow u_\chi$, for $\chi = R, L$.
- The terms in the Lagrangian are constructed imposing such symmetry.
- The lowest order operators are

$$\mathcal{L}_2 = \frac{f^2}{4}\langle (D^\mu U)^\dagger D_\mu U \rangle + \frac{f^2 B_0}{2}\langle M U^\dagger + U^\dagger M \rangle.$$

χPT

- Here f and B_0 are low energy constants, $U = \exp\left[\sqrt{2}\phi/f\right]$,

$$\phi = \sum_a \frac{1}{\sqrt{2}}\phi_a\lambda_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\phi_+ & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}\phi_- & K^0 \\ K^- & \overline{K^0} & \phi_s \end{pmatrix},$$

- where

$$\phi_\pm = \pm\pi^0 + C_q\eta + C_q'\eta', \qquad \text{and} \qquad \phi_s = -C_s\eta + C_s'\eta'$$

RχT

- The vector resonances are included following the flavor structure and chiral symmetry

$$\mathcal{L}_2^V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i}{2\sqrt{2}} G_V \langle V_{\mu\nu}[u^\mu, u^\nu]\rangle,$$

- and similarly for the axial resonances

$$\mathcal{L}_2^A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle,$$

- where $u^\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger]$, $f^\pm = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$ and $U = u^2$.

Form factors

- The decay amplitude is

$$\mathcal{M} = -4G_F\overline{V}_{uq}\overline{u}_{\nu_\tau}\gamma^\mu(1-\gamma_5)u_\tau T_\mu,$$

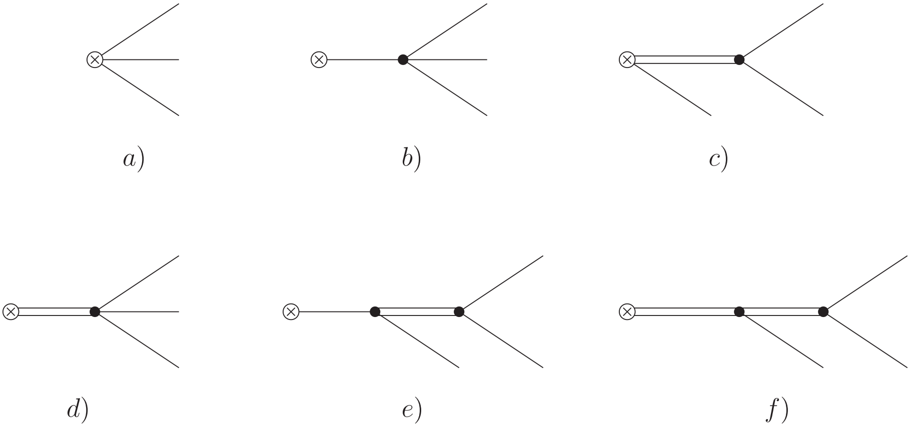
- where the hadronic matrix element is given by

$$T^\mu = V_1^\mu F_1 + V_2^\mu F_2 + V_3^\mu F_3 + Q^\mu F_4,$$

where V_i are the Form Factors, $V_{1/2}^\mu = (g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2})(p_{2/3} - p_1)_\nu$ and $V_{3\mu} = i\varepsilon_{\mu\nu\rho\sigma}p_1^\nu p_1^\rho p_3^\sigma$.

τ decays

- We find ten decay channels, where for each channel the diagrams that contribute are



Reproducing results

- The ten decay channels are for $\tau^- \to \nu_\tau h_1 h_2 h_3$

Channel	h_1	h_2	h_3
1	π^-	π^+	π^-
2	π^-	π^0	π^0
3	π^-	K^+	K^-
4	K^-	π^+	π^-
5	K^-	K^+	K^-
6	K^-	π^0	π^0
7	π^-	$\overline{K}^0$	K^0
8	K^-	$\overline{K}^0$	K^0
9	K^-	π^0	K^0
10	π^-	$\overline{K}^0$	π^0

Reproducing results

- We found that for channels 3 and 7, the form factors have the same functional form.

- Dividing F_3 into contributions from zero, one and two resonance exchange

$$F_3^{NR} = -\frac{1}{12f},$$

$$F_3^{1R} = -\frac{1}{24}\frac{F_V G_V}{f^3}\left[\frac{g_3^{1R}}{M_\rho^2 - s} + \frac{h_3^{1R}}{M_{K^\star}^2 - t}\right],$$

$$F_3^{2R} = \frac{1}{12}\frac{F_A G_V}{f^3}\frac{Q^2}{M_{a_1}^2 - Q^2}\left[\frac{g_3^{2R}}{M_\rho^2 - s} + \frac{h_3^{2R}}{M_{K^\star}^2 - t}\right].$$

Reproducing previous results

- For these decays, we have the same Feynman diagrams and Form Factors.
- Thus, we can use these decays as test.
- But... we haven't reproduced such observables. (Serious lack of students!)
- All the form factors for the other channels have been computed.

Conclusions and pending work

- We have been able to reproduce previous results for $\tau \to KK\pi\nu_\tau$ decays.
- Such results include Feynman diagrams and all the form factors.
- Such results show we are on the right path!
- We are making the numerical programs to compute the Q^2 invariant mass spectra.
- Stay tuned for the final chapter!

Thank you!

Back up!

Isospin structure of mass operator

- $SU(3)_R \otimes SU(3)_L$ breaks spontaneously to $SU(3)_V$, which means q_R transforms exactly as q_L.
- Let's analize the isospin structure of the $\overline{q}\lambda_i q$ operators within $SU(3)_V$.

$$\overline{q}\lambda_i q \xrightarrow{SU(3)_V} \overline{q}U^\dagger\lambda_i U q.$$

- Taking the trace

$$\text{Tr}[U^\dagger\lambda_i U] = \text{Tr}[\lambda_i] = 0,$$

one finds that

$$U^\dagger\lambda_i U = R_{ij}\lambda_j,$$

since any traceless matrix is a linear combination of Gell-Mann matrices.

Isospin structure of mass operator

- On the other hand, isospin transformations form a subgroup $SU(2)_I \subset SU(3)_V$.
- In flavor space this means for $U \in SU(3)_V$

$$U = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}, \qquad V \in SU(2)_I$$

- Since only λ_1, λ_2 and λ_3 transform thusly under $SU(2)_I$,

$$U^\dagger \lambda_i U = R_{ij} \lambda_j \qquad \text{for } i,j = 1,2,3.$$

- Also, since

$$2\delta_{ij} = \langle \lambda_i \lambda_j \rangle = \langle (U^\dagger \lambda_i U)(U^\dagger \lambda_j U) \rangle = R_{ik} R_{jl} \langle \lambda_k \lambda_l \rangle = 2(RR^T)_{ij},$$

R is an orthogonal 3×3 matrix.

Isospin structure of mass operator

- $\Rightarrow \bar{q}\lambda_i q$ for $i = 1, 2, 3$, transform under $SU(3)_V$ exactly as pion fields.
- Thus, $\bar{q}\lambda_3 q$ generates $\Delta I = 1, \Delta I_3 = 0$ transitions.
- Therefore, the $\bar{q}\lambda_3 q$ operator has exactly the same structure as a π^0 field operator.
- We can now use this to compute the $\eta \to \pi^0\pi^+\pi^-$ decay amplitude.

$\eta \to 3\pi$ decay amplitude

- The decay amplitude is defined as

$$i(2\pi)^4\delta^{(4)}(p_1 - p_2 - p_3 - p_4)\mathcal{A}^{ijk}_{\eta\to3\pi} = \langle\pi^i(p_2)\pi^j(p_3)\pi^k(p_4)|iT|\eta(p_1)\rangle,$$

- where

$$T = -\frac{m_u - m_d}{2}\int d^4x\bar{q}(x)\lambda^3q(x),$$

- which gives

$$\mathcal{A}^{ijk}_{\eta\to3\pi} = -\frac{m_u - m_d}{2}\langle\pi^i\pi^j\pi^k|\bar{q}(0)\lambda^3q(0)|\eta\rangle.$$

- On the other hand, one can define an amplitude for a general isospin index s.t.

$$\mathcal{A}^{ijk,l}_{\eta\to3\pi} = -\frac{m_u - m_d}{2}\langle\pi^i\pi^j\pi^k|\bar{q}(0)\lambda^lq(0)|\eta\rangle.$$

$\eta \to 3\pi$ decay amplitude

- This means

$$\mathcal{A}^{ijk,3}_{\eta\to 3\pi} = \mathcal{A}^{ijk}_{\eta\to 3\pi}.$$

- $\overline{q}\lambda^l q$ transforms exactly as a pion under isospin transformations.

- Therefore $\mathcal{A}^{ijk,l}_{\eta\to 3\pi}$ must have the exact same isospin structure as $\pi\pi \to \pi\pi$.

- This means that it can be written as $\mathcal{A}^{ijkl}_{\pi\pi\to\pi\pi}$,

$$\mathcal{A}^{ijk}_{\eta\to 3\pi} = \mathcal{A}^{ijk,3}_{\eta\to 3\pi} = A_1(s,t,u)\delta^{ij}\delta^{k3} + A_2(s,t,u)\delta^{ik}\delta^{j3} + A_3(s,t,u)\delta^{i3}\delta^{jk}.$$

$\eta \to 3\pi$ decay amplitude

- Crossing symmetry gives

$$A_1(s,t,u) = A_1(s,u,t), \qquad A_2(s,t,u) = A_1(t,s,u), \qquad A_3(s,t,u) = A_1(u,t,s)$$

- Therefore, the decay amplitude is given by a single function

$$\mathcal{A}^{ijk}_{\eta\to 3\pi}(s,t,u) = A(s,t,u)\delta^{ij}\delta^{k3} + A(t,u,s)\delta^{ik}\delta^{j3} + A(u,s,t)\delta^{i3}\delta^{jk}.$$

- There are only two decay channels for physical pions

$$\mathcal{A}^{+-0}_{\eta\to 3\pi} = \mathcal{A}^{113}_{\eta\to 3\pi} = A(s,t,u),$$

$$\mathcal{A}^{000}_{\eta\to 3\pi} = \mathcal{A}^{333}_{\eta\to 3\pi} = A(s,t,u) + A(t,u,s) + A(u,s,t).$$

- Thus, the neutral channel can be obtained directly from the charged one.

From η decays to η' decays

- The great advantage of this development is the straightforward use in η' decays.

- Following the previous procedure, we define the decay amplitude

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = \langle \pi^i(p_2)\pi^j(p_3)\eta(p_4)|iT|\eta'(p_1)\rangle,$$

- where

$$T = -\frac{m_u - m_d}{2}\int d^4x\bar{q}\lambda^3 q.$$

- So, we construct the amplitude

$$\mathcal{A}^{ij,k}_{\eta'\to\eta\pi\pi} = -\frac{m_u - m_d}{2}\langle \pi^i\pi^j\eta|\bar{q}\lambda^k q|\eta'\rangle,$$

which fulfills $\mathcal{A}^{ij,3}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{ij}_{\eta'\to\eta\pi\pi}$, for which we'll use the same arguments.

$\eta' \to \eta\pi\pi$ decays

- The $\mathcal{A}^{ij,k}_{\eta'\to\eta\pi\pi}$ has the same isospin structure as $\mathcal{A}^{ijk}_{\eta\to 3\pi\pi}$, which means

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{ij,3}_{\eta'\to\eta\pi\pi} = A_1(s,t,u)\delta^{ij} + A_2(s,t,u)\delta^{i3}\delta^{j3} + A_3(s,t,u)\delta^{i3}\delta^{j3}.$$

- Crossing symmetry relates all the previous functions, such that

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = A(s,t,u)\delta^{ij} + \left[A(t,u,s) + A(u,s,t)\right]\delta^{i3}\delta^{j3}.$$

- Finally, for the physical pions we have

$$\mathcal{A}^{+-}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{11}_{\eta'\to\eta\pi\pi} = A(s,t,u),$$

$$\mathcal{A}^{00}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{33}_{\eta'\to\eta\pi\pi} = A(s,t,u) + A(t,u,s) + A(u,s,t).$$