Renormalization of massless matter fields in the $(1, 0) \oplus (0, 1)$ representation.

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MWPF 2025

October 21st, 2025

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- The **Homogeneous Lorentz Group (HLG)** is the group $O(1,3)$, whose elements, $L^{\mu}{}_{\rho}$, are defined by

$$L^{\mu}{}_{\rho} g_{\mu\nu} L^{\nu}{}_{\sigma} = g_{\rho\sigma}. \tag{1}$$

- The **irreducible representations** (irreps) of the HLG are defined by two $SU(2)$ numbers:

$$\text{HLG irreps:} \qquad (a,b) \qquad a,b = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots \tag{2}$$

$$\begin{array}{c}
(0,0) \\
\left(\tfrac{1}{2},0\right) \;\; \left(0,\tfrac{1}{2}\right) \\
(1,0) \;\; \left(\tfrac{1}{2},\tfrac{1}{2}\right) \;\; (0,1) \\
\left(\tfrac{3}{2},0\right) \;\; \left(1,\tfrac{1}{2}\right) \;\; \left(\tfrac{1}{2},1\right) \;\; \left(0,\tfrac{3}{2}\right) \\
\vdots \quad\; \vdots \quad\; \vdots \quad\; \vdots \quad\; \vdots
\end{array}$$

- **Quantum fields** are defined on these irreps

Scalar:	$(0,0)$	$\rightarrow$	spin $j=0$
Spinors:	$\left(\frac{1}{2},0\right), \left(0,\frac{1}{2}\right)$	$\rightarrow$	spin $j=\frac{1}{2}$
Vector:	$\left(\frac{1}{2},\frac{1}{2}\right)$	$\rightarrow$	spin $j=1,0$
Antisymmetric tensor:	$(1,0),(0,1)$	$\rightarrow$	spin $j=1$

Fields in $(1,0) \oplus (0,1)$. Basics

- The **Standard Model** uses only a few of these irreps.

$$
\begin{array}{c}
(0,0) \\
\left(\tfrac{1}{2},0\right) \;\; \left(0,\tfrac{1}{2}\right) \\
(1,0) \;\; \left(\tfrac{1}{2},\tfrac{1}{2}\right) \;\; (0,1) \\
\left(\tfrac{3}{2},0\right) \;\; \left(1,\tfrac{1}{2}\right) \;\; \left(\tfrac{1}{2},1\right) \;\; \left(0,\tfrac{3}{2}\right) \\
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
\end{array}
$$

Scalar:	$(0,0)$	$\rightarrow$	spin $j=0$	$\rightarrow$	Higgs
Spinors:	$\left(\frac{1}{2},0\right),\left(0,\frac{1}{2}\right)$	$\rightarrow$	spin $j=\frac{1}{2}$	$\rightarrow$	leptons, quarks
Vector:	$\left(\frac{1}{2},\frac{1}{2}\right)$	$\rightarrow$	spin $j=1$	$\rightarrow$	gauge bosons
Antisymmetric tensor:	$(1,0),(0,1)$	$\rightarrow$	spin $j=1$	$\rightarrow$	?

- The $(1,0)$ and $(0,1)$ irreps are the most close-at-hand non-standard irreps. The parity invariant irrep is $(1,0) \oplus (0,1)$.

What about fields in the spin $j=1$ rep $(1,0) \oplus (0,1)$?

Fields in $(1,0)\oplus(0,1)$. Basics

- A field $\Psi \in (1,0)\oplus(0,1)$ has 6 complex components. It can be represented both as

$$\text{6-component spinor-like: } \Psi_a, \qquad a \leftrightarrow [\mu\nu] \qquad \text{2nd-rank antisymmetric tensor: } B_{\mu\nu}. \tag{3}$$

- There exists a covariant basis for operators in this rep[1]

$$\{1, \chi, S^{\mu\nu}, \chi S^{\mu\nu}, M^{\mu\nu}, C^{\mu\nu\rho\sigma}\}. \tag{4}$$

- Analogous to the Dirac field, but for spin $j=1$ instead of $j=\frac{1}{2}$

$$\begin{aligned}
\text{Dirac: } \psi \in \left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right) \quad &\sim \quad \Psi \in (1,0)\oplus(0,1) \\
\psi : \text{4-dim spinor object} \quad &\sim \quad \Psi : \text{6-dim spinor-like object} \\
\psi : \text{spin } j = \frac{1}{2} \quad &\sim \quad \Psi : \text{spin } j = 1 \\
\text{basis: } \{1, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, M^{\mu\nu}\} \quad &\sim \quad \text{basis: } \{1, \chi, S^{\mu\nu}, \chi S^{\mu\nu}, M^{\mu\nu}, C^{\mu\nu\rho\sigma}\}
\end{aligned}$$

What is the free EOM for $\Psi \in (1,0)\oplus(0,1)$?

[1] Selim Gómez-Ávila and M. Napsuciale. "Covariant basis induced by parity for the $(j,0)\bigoplus(0,j)$ representation". In: *Phys. Rev. D* 88 (9 Nov. 2013), p. 096012. DOI: 10.1103/PhysRevD.88.096012. URL: https://link.aps.org/doi/10.1103/PhysRevD.88.096012.

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The model. Massive $m \neq 0$

The most general parity-invariant free massive Lagrangian is

$$\mathcal{L} = \partial^\mu \bar{\Psi} \left(\alpha g_{\mu\nu} + \beta S_{\mu\nu} + igM_{\mu\nu}\right) \partial^\nu \Psi - m^2 \bar{\Psi}\Psi, \tag{5}$$

where $\bar{\Psi} = \Psi^\dagger S^{00}$, whose propagator has a two-pole structure

$$i\Gamma_{ab}(p) = i\,\frac{\left(\alpha p^2 - m^2\right) 1_{ab} - \beta p_\mu p_\nu (S^{\mu\nu})_{ab}}{\left[(\alpha-\beta)p^2 - m^2\right]\left[(\alpha+\beta)p^2 - m^2\right]}. \tag{6}$$

To get the physical pole at $p^2 = m^2$ the coefficients must be chosen as:

1. $\alpha = 1, \beta = 0$. **Klein-Gordon-like theory**. No restrictions $\rightarrow$ twice DOF.

$$\mathcal{L} = \partial^\mu \bar{\Psi} g_{\mu\nu} \partial^\nu \Psi - m^2 \bar{\Psi}\Psi, \qquad i\Gamma_{ab}(p) = i\,\frac{1_{ab}}{p^2 - m^2 + i\epsilon}.$$

2. $\alpha = 0, \beta = 1$. **Joos-Weinberg theory**. Unphysical pole at $p^2 = -m^2$.

$$\mathcal{L} = \partial^\mu \bar{\Psi} S_{\mu\nu} \partial^\nu \Psi - m^2 \bar{\Psi}\Psi, \qquad i\Gamma_{ab}(p) = i\,\frac{m^2 1_{ab} + p_\mu p_\nu (S^{\mu\nu})_{ab}}{\left[p^2 + m^2\right]\left[p^2 - m^2\right]}.$$

3. $\alpha = \pm\beta = 1/2$. **Shay-Good/Hammer-McDonald-Pursey (Kalb-Ramond) theory**. UV-divergent propagator (p^2/p^2-order) $\rightarrow$ non-renormalizable theory.

$$\mathcal{L} = \frac{1}{2} \partial^\mu \bar{\Psi} \left(g_{\mu\nu} \pm S_{\mu\nu}\right) \partial^\nu \Psi - m^2 \bar{\Psi}\Psi, \quad i\Gamma_{ab}(p) = i\,\frac{\left(-p^2 + 2m^2\right) 1_{ab} \pm p_\mu p_\nu (S^{\mu\nu})_{ab}}{2m^2\left[p^2 - m^2 + i\epsilon\right]}.$$

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The model. Massless $m = 0$

The massless theories do not have any of those problems. The massless Lagrangian is

$$\mathcal{L} = \partial^\mu \bar{\Psi} \left(\alpha g_{\mu\nu} + \beta S_{\mu\nu} + i g M_{\mu\nu} \right) \partial^\nu \Psi, \tag{7}$$

whose propagator is

$$i\Gamma_{ab}(p) = \frac{i}{\alpha^2 - \beta^2} \frac{\alpha p^2 1_{ab} - \beta p_\mu p_\nu (S^{\mu\nu})_{ab}}{p^4 + i\epsilon}. \tag{8}$$

Thus, the above theories become

1. $\alpha = 1, \beta = 0$. **Massless Klein-Gordon-like theory**. No restrictions $\to$ twice DOF.

$$\mathcal{L} = \partial^\mu \bar{\Psi} g_{\mu\nu} \partial^\nu \Psi, \qquad i\Gamma_{ab}(p) = i \frac{1_{ab}}{p^2 + i\epsilon}$$

2. $\alpha = 0, \beta = 1$. **Massless Joos-Weinberg theory**. No unphysical pole.

$$\mathcal{L} = \partial^\mu \bar{\Psi} S_{\mu\nu} \partial^\nu \Psi, \qquad i\Gamma_{ab}(p) = i \frac{p_\mu p_\nu (S^{\mu\nu})_{ab}}{p^4 + i\epsilon}$$

3. $\alpha = \pm\beta = 1/2$. **Massless Shay-Good/Hammer-McDonald-Pursey (Kalb-Ramond) theory**.

$$\mathcal{L} = \frac{1}{2} \partial^\mu \bar{\Psi} \left(g_{\mu\nu} \pm S_{\mu\nu} \right) \partial^\nu \Psi, \qquad i\Gamma_{ab}(p) = \text{Not defined}$$

Emergence of a gauge symmetry $\to$ add a gauge fixing. Non UV-div propagator ($1/p^2$-order).

$$\mathcal{L} = \frac{1}{2} \partial^\mu \bar{\Psi} \left[\left(1 + \frac{1}{\xi} \right) g_{\mu\nu} \pm \left(1 - \frac{1}{\xi} \right) S_{\mu\nu} \right] \partial^\nu \Psi \qquad i\Gamma_{ab} = \frac{i}{2} \frac{(1+\xi)\, p^2 1_{ab} + (1-\xi)\, p_\mu p_\nu (S^{\mu\nu})_{a}}{p^4 + i\epsilon}$$

This propagator still has the form of (8).

The model

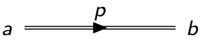
- Dimension-4 quartic self interactions

$$\mathcal{L} = \partial^{\mu}\bar{\Psi}\left(\alpha g_{\mu\nu} + \beta S_{\mu\nu}\right)\partial^{\nu}\Psi \quad \rightarrow \quad \dim\Psi = 1 \quad \rightarrow \quad \dim\left[(\bar{\Psi}A\Psi)(\bar{\Psi}B\Psi)\right] = 4. \tag{9}$$

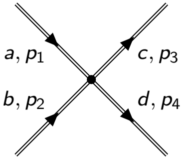
- The most general parity-invariant self-interacting massless model is

$$\mathcal{L} = \partial^{\mu}\bar{\Psi}\left(\alpha g_{\mu\nu} + \beta S_{\mu\nu}\right)\partial^{\nu}\Psi + \frac{\lambda_1}{2}(\bar{\Psi}\Psi)^2 + \frac{\lambda_2}{2}(\bar{\Psi}\chi\Psi)^2 + \frac{\lambda_3}{2}(\bar{\Psi}S^{\mu\nu}\Psi)^2 + \frac{\lambda_4}{2}(\bar{\Psi}M^{\mu\nu}\Psi)^2 \tag{10}$$

- Feynman rules



$$\equiv i\Gamma_{ab}(p) = \frac{i}{\alpha^2 - \beta^2}\,\frac{\alpha p^2 1_{ab} - \beta p_{\mu}p_{\nu}(S^{\mu\nu})_{ab}}{p^4 + i\epsilon}, \tag{11}$$



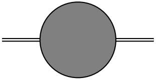
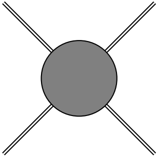
$$\equiv iV_{abcd} = \begin{array}{l} +i\lambda_1\left[1_{ab}1_{cd} + 1_{ad}1_{cb}\right] + i\lambda_2\left[\chi_{ab}\chi_{cd} + \chi_{ad}\chi_{cb}\right] \\ +i\lambda_3\left[(S^{\mu\nu})_{ab}(S_{\mu\nu})_{cd} + (S^{\mu\nu})_{ad}(S_{\mu\nu})_{cb}\right] \\ +i\lambda_4\left[(M^{\mu\nu})_{ab}(M_{\mu\nu})_{cd} + (M^{\mu\nu})_{ad}(M_{\mu\nu})_{cb}\right] \end{array} \tag{12}$$

- Observation: self-interactions explicitly break the gauge invariance of theory 3). This may bring inconsistencies in the propagating DOF of its interacting theory; to avoid this issue, we restrict to theories 1) $\alpha = 1, \beta = 0$ and 2) $\alpha = 0, \beta = 1$ from now on.

Outline

Renormalization

Superficial degree of divergence $\mathcal{D} = 4 - N$. Potentially divergent diagrams:

1) Self-energy: , 2) Vertex: . (13)

Outline

Renormalization: Counterterms

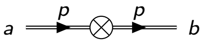
- Physical field: $\Psi = Z^{-\frac{1}{2}}\Psi_0 \rightarrow$ counterterms

$$
\begin{aligned}
\mathcal{L} =&\partial^\mu \bar{\Psi}\left(\alpha g_{\mu\nu} + \beta S_{\mu\nu}\right)\partial^\nu \Psi + \frac{\lambda_1}{2}(\bar{\Psi}\Psi)^2 + \frac{\lambda_2}{2}(\bar{\Psi}\chi\Psi)^2 + \frac{\lambda_3}{2}(\bar{\Psi}S^{\mu\nu}\Psi)^2 + \frac{\lambda_4}{2}(\bar{\Psi}M^{\mu\nu}\Psi)^2 \\
&+ \delta_Z \partial^\mu \bar{\Psi}\left(\alpha g_{\mu\nu} + \beta S_{\mu\nu}\right)\partial^\nu \Psi + \delta_{Z_1}\frac{\lambda_1}{2}(\bar{\Psi}\Psi)^2 + \delta_{Z_2}\frac{\lambda_2}{2}(\bar{\Psi}\chi\Psi)^2 + \delta_{Z_3}\frac{\lambda_3}{2}(\bar{\Psi}S^{\mu\nu}\Psi)^2 \\
&+ \delta_{Z_4}\frac{\lambda_4}{2}(\bar{\Psi}M^{\mu\nu}\Psi)^2,
\end{aligned} \tag{14}
$$

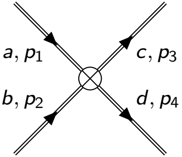
where

$$
\delta_Z = Z - 1, \qquad \delta_{Z_1} = Z_1 - 1, \qquad \delta_{Z_2} = Z_2 - 1, \qquad \delta_{Z_3} = Z_3 - 1, \qquad \delta_{Z_4} = Z_4 - 1. \tag{15}
$$

- Feynman rules



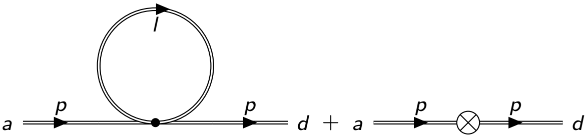
$$
\equiv i\delta\Gamma_{ab}(p) = i\delta_Z\left[\alpha p^2 1_{ab} + \beta p_\mu p_\nu (S^{\mu\nu})_{ab}\right], \tag{16}
$$



$$
\begin{aligned}
\equiv i\delta V_{abcd} = \quad & +i\lambda_1\delta_{Z_1}\left[1_{ab}1_{cd} + 1_{ad}1_{cb}\right] + i\lambda_2\delta_{Z_2}\left[\chi_{ab}\chi_{cd} + \chi_{ad}\chi_{cb}\right] \\
& +i\lambda_3\delta_{Z_3}\left[(S^{\mu\nu})_{ab}(S_{\mu\nu})_{cd} + (S^{\mu\nu})_{ad}(S_{\mu\nu})_{cb}\right] \\
& +i\lambda_4\delta_{Z_4}\left[(M^{\mu\nu})_{ab}(M_{\mu\nu})_{cd} + (M^{\mu\nu})_{ad}(M_{\mu\nu})_{cb}\right].
\end{aligned} \tag{17}
$$

Outline

Self-energy at one-loop

$i\Sigma_{ad}(p) =$  (18)

$$=\mu^{2\epsilon}\int \frac{d^D l}{(2\pi)^D} iV_{abcd}\, i\Gamma_{bc}(l) + i\delta\Gamma_{ad}(p).$$

Setting $D = 4 - 2\epsilon$, and $\epsilon \to 0$, the integral vanishes and so

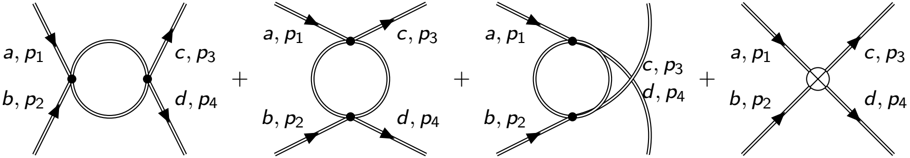
$$\text{Div}\,[i\Sigma_{ad}(p)] = i\delta_Z \left[\alpha p^2 1_{ad} + \beta p_\mu p_\nu (S^{\mu\nu})_{ad}\right] \qquad \rightarrow \qquad \delta_Z = 0. \tag{19}$$

Outline

Renormalization: Vertex

Vertex at one-loop

$$i\Lambda_{abcd}(p_1, p_2, p_3) = \tag{20}$$



$$
\begin{aligned}
=\mu^{2\epsilon} \int \frac{d^D l}{(2\pi)^D} \, [& iV_{abrn} i\Gamma_{nm}(l + p_1 + p_2) iV_{mdcs} i\Gamma_{sr}(l) \\
& + iV_{arcn} i\Gamma_{nm}(l + p_1 - p_3) iV_{mdsb} i\Gamma_{sr}(l) \\
& + iV_{adnr} i\Gamma_{nm}(l + p_1 - p_4) iV_{mcbs} i\Gamma_{sr}(l)] + i\delta V_{abcd}
\end{aligned}
$$

Renormalization: Vertex

Setting $D = 4 - 2\epsilon$, and $\epsilon \to 0$, the UV-divergent part is

$$
\begin{aligned}
\mathrm{Div}\left[i\Lambda_{abcd}(p_1,p_2,p_3)\right] &= \frac{-i}{(4\pi)^2(\alpha^2-\beta^2)^2\epsilon} \\
&\times \left\{ \left(\alpha^2 c_1^1 + \frac{\beta^2}{12} c_1^S\right)[1_{ab}1_{cd} + 1_{ad}1_{cb}] + \left(\alpha^2 c_\chi^1 + \frac{\beta^2}{12} c_\chi^S\right)[\chi_{ab}\chi_{cd} + \chi_{ad}\chi_{cb}] \right. \\
&+ \left(\alpha^2 c_S^1 + \frac{\beta^2}{12} c_S^S\right)[(S^{\mu\nu})_{ab}(S_{\mu\nu})_{cd} + (S^{\mu\nu})_{ad}(S_{\mu\nu})_{cb}] \\
&+ \left.\left(\alpha^2 c_M^1 + \frac{\beta^2}{12} c_M^S\right)[(M^{\mu\nu})_{ab}(M_{\mu\nu})_{cd} + (M^{\mu\nu})_{ad}(M_{\mu\nu})_{cb}]\right\} \\
&+ i\lambda_1\delta_{Z_1}[1_{ab}1_{cd} + 1_{ad}1_{cb}] + i\lambda_2\delta_{Z_2}[\chi_{ab}\chi_{cd} + \chi_{ad}\chi_{cb}] \\
&+ i\lambda_3\delta_{Z_3}[(S^{\mu\nu})_{ab}(S_{\mu\nu})_{cd} + (S^{\mu\nu})_{ad}(S_{\mu\nu})_{cb}] \\
&+ i\lambda_4\delta_{Z_4}[(M^{\mu\nu})_{ab}(M_{\mu\nu})_{cd} + (M^{\mu\nu})_{ad}(M_{\mu\nu})_{cb}]\,,
\end{aligned}
\tag{21}
$$

where the coefficients $\{c_1^1, c_\chi^1, c_S^1, c_M^1, c_1^S, c_\chi^S, c_S^S, c_M^S\}$ are given by

$$
\begin{aligned}
c_1^1 =& -11\lambda_1^2 - 2\lambda_1(8\lambda_4 + 12\lambda_3 + \lambda_2) - 48\left(\lambda_4^2 + 2\lambda_4\lambda_3 + 2\lambda_3^2\right) + 8\lambda_3\lambda_2 - 3\lambda_2^2, \\
c_\chi^1 =& -8\lambda_2(\lambda_1 + 2\lambda_4 - 2\lambda_3) - 48(\lambda_4 - \lambda_3)^2 - 8\lambda_2^2, \qquad c_S^1 = -8\lambda_3(\lambda_1 - 4\lambda_4 + 2\lambda_3), \\
c_M^1 =& 24\lambda_3^2 - 8\lambda_4(\lambda_1 + \lambda_4 + \lambda_2), \qquad c_1^S = -16\left(6\lambda_1^2 + \lambda_1(18\lambda_4 + 30\lambda_3 + \lambda_2) - 2(\lambda_4 + \lambda_3)(4\lambda_4 + 8\lambda_3 - 3\lambda_2)\right), \\
c_\chi^S =& 16\left(\lambda_2(\lambda_1 + 18\lambda_4 - 30\lambda_3) + 2(\lambda_4 - \lambda_3)(3\lambda_1 - 4\lambda_4 + 8\lambda_3) + 6\lambda_2^2\right), \\
c_S^S =& -3\lambda_1^2 + 32\lambda_1\lambda_4 - 2\lambda_1(4\lambda_3 + \lambda_2) - 16\left(8\lambda_4^2 + 23\lambda_3^2\right) + 8\lambda_2(4\lambda_4 + \lambda_3) - 3\lambda_2^2, \\
c_M^S =& 48\lambda_3(\lambda_1 - 8\lambda_4 + \lambda_2).
\end{aligned}
\tag{22}
$$

Renormalization: Vertex

To cancel the divergences, the counterterms $\delta_{Z_1}, \delta_{Z_2}, \delta_{Z_3}, \delta_{Z_4}$, must be chosen as (MS scheme)

$$\begin{aligned}
\delta_{Z_1} &= \frac{1}{\lambda_1}\frac{1}{(4\pi)^2(\alpha^2-\beta^2)^2\epsilon}\left(\alpha^2 c_1^1 + \frac{\beta^2}{12}c_1^S\right), \quad & \delta_{Z_2} &= \frac{1}{\lambda_2}\frac{1}{(4\pi)^2(\alpha^2-\beta^2)^2\epsilon}\left(\alpha^2 c_\chi^1 + \frac{\beta^2}{12}c_\chi^S\right), \\
\delta_{Z_3} &= \frac{1}{\lambda_3}\frac{1}{(4\pi)^2(\alpha^2-\beta^2)^2\epsilon}\left(\alpha^2 c_S^1 + \frac{\beta^2}{12}c_S^S\right), \quad & \delta_{Z_4} &= \frac{1}{\lambda_4}\frac{1}{(4\pi)^2(\alpha^2-\beta^2)^2\epsilon}\left(\alpha^2 c_M^1 + \frac{\beta^2}{12}c_M^S\right).
\end{aligned} \tag{23}$$

Outline

$$\mathcal{L} = \partial^{\mu}\bar{\Psi}\left(\alpha g_{\mu\nu} + \beta S_{\mu\nu}\right)\partial^{\nu}\Psi + \frac{\lambda_1}{2}(\bar{\Psi}\Psi)^2 + \frac{\lambda_2}{2}(\bar{\Psi}\chi\Psi)^2 + \frac{\lambda_3}{2}(\bar{\Psi}S^{\mu\nu}\Psi)^2 + \frac{\lambda_4}{2}(\bar{\Psi}M^{\mu\nu}\Psi)^2 \quad (24)$$

The inclusion of all the independent self-interactions is sufficient for renormalizing this model for any values of α, β; however, in some cases, not all of them are necessary. Renormalizable subtheories:

- $\alpha = 1, \beta = 0$. Three renormalizable subtheories (appart from $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$)

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0)\,, \quad (25)$$

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3 = 0, \lambda_4 = 0)\,, \quad (26)$$

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3 = 0, \lambda_4)\,. \quad (27)$$

- $\alpha = 0, \beta = 1$. No renormalizable subtheories; all the independent self-interactions are necessary $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$.

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Running couplings $\to$ Beta functions $\beta_{\lambda_X} = \mu d\lambda_X/d\mu$

$$\beta_{\lambda_1} = -\frac{1}{24\pi^2(\alpha^2-\beta^2)^2}\Big\{3\alpha^2\Big[11\lambda_1^2 + 2\lambda_1(\lambda_2 + 8\lambda_4 + 12\lambda_3) + 3\lambda_2^2 - 8\lambda_2\lambda_3 + 48\left(\lambda_4^2 + 2\lambda_4\lambda_3 + 2\lambda_3^2\right)\Big] + 4\beta^2\Big[6\lambda_1^2 + \lambda_1(\lambda_2 + 18\lambda_4 + 30\lambda_3) - 2(\lambda_4+\lambda_3)(-3\lambda_2 + 4\lambda_4 + 8\lambda_3)\Big]\Big\}, \tag{28}$$

$$\beta_{\lambda_2} = -\frac{1}{6\pi^2(\alpha^2-\beta^2)^2}\Big\{6\alpha^2\Big[\lambda_1\lambda_2 + \lambda_2^2 + 2\lambda_2(\lambda_4-\lambda_3) + 6(\lambda_4-\lambda_3)^2\Big] - \beta^2\Big[2\left(3\lambda_2^2 + 9\lambda_2\lambda_4 - 15\lambda_2\lambda_3 - 4\lambda_4^2 + 12\lambda_4\lambda_3 - 8\lambda_3^2\right) + \lambda_1(\lambda_2 + 6\lambda_4 - 6\lambda_3)\Big]\Big\}, \tag{29}$$

$$\beta_{\lambda_3} = -\frac{1}{96\pi^2(\alpha^2-\beta^2)^2}\Big\{96\alpha^2\left[\lambda_3(\lambda_1 - 4\lambda_4 + 2\lambda_3)\right] + \beta^2\Big[3\lambda_1^2 + 3\lambda_2^2 + 2\lambda_1(\lambda_2 + 4(\lambda_3 - 4\lambda_4)) - 8\lambda_2(4\lambda_4 + \lambda_3) + 16\left(8\lambda_4^2 + 23\lambda_3^2\right)\Big]\Big\}, \tag{30}$$

$$\beta_{\lambda_4} = -\frac{1}{2\pi^2(\alpha^2-\beta^2)^2}\Big\{2\alpha^2\Big[3\lambda_3^2 - \lambda_4(\lambda_1+\lambda_2+\lambda_4)\Big] + \beta^2\left[\lambda_3(\lambda_1+\lambda_2-8\lambda_4)\right]\Big\}. \tag{31}$$

- Theory i) ($\alpha = 1, \beta = 0$).
  - β_{λ_1}. No real fixed points.
  - β_{λ_2}. Defined by

$$\lambda_1 = -\lambda_2 + 2(\lambda_3 - \lambda_4) - \frac{6(\lambda_3 - \lambda_4)^2}{\lambda_2}. \tag{32}$$

  - β_{λ_3}. Defined by

$$\lambda_3 = 0, \qquad \text{or} \qquad \lambda_1 = 4\lambda_4 - 2\lambda_3. \tag{33}$$

  - β_{λ_4}. Defined by

$$\lambda_1 = -(\lambda_2 + \lambda_4) + \frac{3\lambda_3^2}{\lambda_4}. \tag{34}$$

Six independent non-trivial common solutions

$$\boldsymbol{\lambda}^{(1)} = (\lambda_1, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0)\,, \tag{35}$$

$$\boldsymbol{\lambda}^{(2)} = (\lambda_1, \lambda_2 = -\lambda_1, \lambda_3 = 0, \lambda_4 = 0)\,, \tag{36}$$

$$\boldsymbol{\lambda}^{(3)} = \left(\lambda_1, \lambda_2 = -\frac{6}{5}\lambda_1, \lambda_3 = 0, \lambda_4 = \frac{1}{5}\lambda_1\right)\,, \tag{37}$$

$$\boldsymbol{\lambda}^{(4)} = \left(\lambda_1, \lambda_2 = -\frac{8}{7}\lambda_1, \lambda_3 = -\frac{1}{14}\lambda_1, \lambda_4 = \frac{3}{14}\lambda_1\right)\,, \tag{38}$$

$$\boldsymbol{\lambda}^{(5)} = \left(\lambda_1, \lambda_2 = -\frac{2}{3}\lambda_1, \lambda_3 = -\frac{1}{6}\lambda_1, \lambda_4 = \frac{1}{6}\lambda_1\right)\,, \tag{39}$$

$$\boldsymbol{\lambda}^{(6)} = \left(\lambda_1, \lambda_2 = -\frac{6}{5}\lambda_1, \lambda_3 = \frac{1}{10}\lambda_1, \lambda_4 = \frac{3}{10}\lambda_1\right)\,. \tag{40}$$

- Theory ii) ($\alpha = 0, \beta = 1$).
  - β_{λ_1}. Defined by

$$\lambda_2 = \frac{-6\lambda_1^2 - 6\lambda_1(5\lambda_3 + 3\lambda_4) + 8(\lambda_3 + \lambda_4)(2\lambda_3 + \lambda_4)}{\lambda_1 + 6(\lambda_3 + \lambda_4)}. \tag{41}$$

  - β_{λ_2}. Defined by

$$\lambda_1 = \frac{-6\lambda_2^2 - 6\lambda_2(-5\lambda_3 + 3\lambda_4) + 8(-\lambda_3 + \lambda_4)(-2\lambda_3 + \lambda_4)}{\lambda_2 + 6(-\lambda_3 + \lambda_4)}. \tag{42}$$

  - β_{λ_3}. Defined by

$$\boldsymbol{\lambda} = \left(\lambda_1, \lambda_2 = \lambda_1, \lambda_3 = 0, \lambda_4 = \frac{1}{4}\lambda_1\right). \tag{43}$$

  - β_{λ_4}. Defined by

$$\lambda_3 = 0, \qquad \text{or} \qquad \lambda_1 = 8\lambda_4 - \lambda_2. \tag{44}$$

No non-trivial common solutions. Curious solution: $\boldsymbol{\lambda} = (\lambda_1, \lambda_2 = \lambda_1, \lambda_3 = 0, \lambda_4 = \lambda_1/4)$, $\beta_{\lambda_3} = \beta_{\lambda_4} = 0$ and $\beta_{\lambda_1} = -\beta_{\lambda_2} = -25\lambda_1^2/12\pi^2$.

Outline

Summary and conclusions

- There are three theories for a massive field Ψ in the $(1,0)\oplus(0,1)$ rep of the HLG:

$$\mathcal{L} = \partial^\mu\bar{\Psi}\partial_\mu\Psi - m^2\bar{\Psi}\Psi, \quad \mathcal{L} = \partial^\mu\bar{\Psi}S_{\mu\nu}\partial^\nu\Psi - m^2\bar{\Psi}\Psi, \quad \mathcal{L} = \partial^\mu\bar{\Psi}\frac{g_{\mu\nu}\pm S_{\mu\nu}}{2}\partial^\nu\Psi - m^2\bar{\Psi}\Psi.$$

  Issues: 2) unphysical pole at $p^2=-m^2$, 3) UV-divergent propagator $\rightarrow$ non-renormalizable.
- Massless theories do not have those problems

$$\mathcal{L} = \partial^\mu\bar{\Psi}\partial_\mu\Psi, \qquad \mathcal{L} = \partial^\mu\bar{\Psi}S_{\mu\nu}\partial^\nu\Psi, \qquad \mathcal{L} = \partial^\mu\bar{\Psi}\frac{g_{\mu\nu}\pm S_{\mu\nu}}{2}\partial^\nu\Psi.$$

- Dimension-4 quartic self interactions

$$\mathcal{L} = \partial^\mu\bar{\Psi}\left(\alpha g_{\mu\nu}+\beta S_{\mu\nu}\right)\partial^\nu\Psi \quad\rightarrow\quad \dim\Psi = 1 \quad\rightarrow\quad \dim\left[(\bar{\Psi}A\Psi)(\bar{\Psi}B\Psi)\right]=4.$$

- Model: general parity-invariant self-interacting massless field

$$\mathcal{L} = \partial^\mu\bar{\Psi}\left(\alpha g_{\mu\nu}+\beta S_{\mu\nu}\right)\partial^\nu\Psi + \frac{\lambda_1}{2}(\bar{\Psi}\Psi)^2 + \frac{\lambda_2}{2}(\bar{\Psi}\chi\Psi)^2 + \frac{\lambda_3}{2}(\bar{\Psi}S^{\mu\nu}\Psi)^2 + \frac{\lambda_4}{2}(\bar{\Psi}M^{\mu\nu}\Psi)^2$$

- Renormalizable at one-loop for any α,β.
- For $\alpha=1,\beta=0$ there are three renormalizable subtheories, defined by

$$\begin{aligned}\boldsymbol{\lambda} &= (\lambda_1,\lambda_2=0,\lambda_3=0,\lambda_4=0)\,,\\ \boldsymbol{\lambda} &= (\lambda_1,\lambda_2,\lambda_3=0,\lambda_4=0)\,,\\ \boldsymbol{\lambda} &= (\lambda_1,\lambda_2,\lambda_3=0,\lambda_4)\,.\end{aligned}$$

- For $\alpha=0,\beta=1$ there are no renormalizable subtheories; all the independent self-interactions are necessary.
- Large set of fixed points.

Thanks

Thanks.