Gravitational physics from quantum scattering amplitudes

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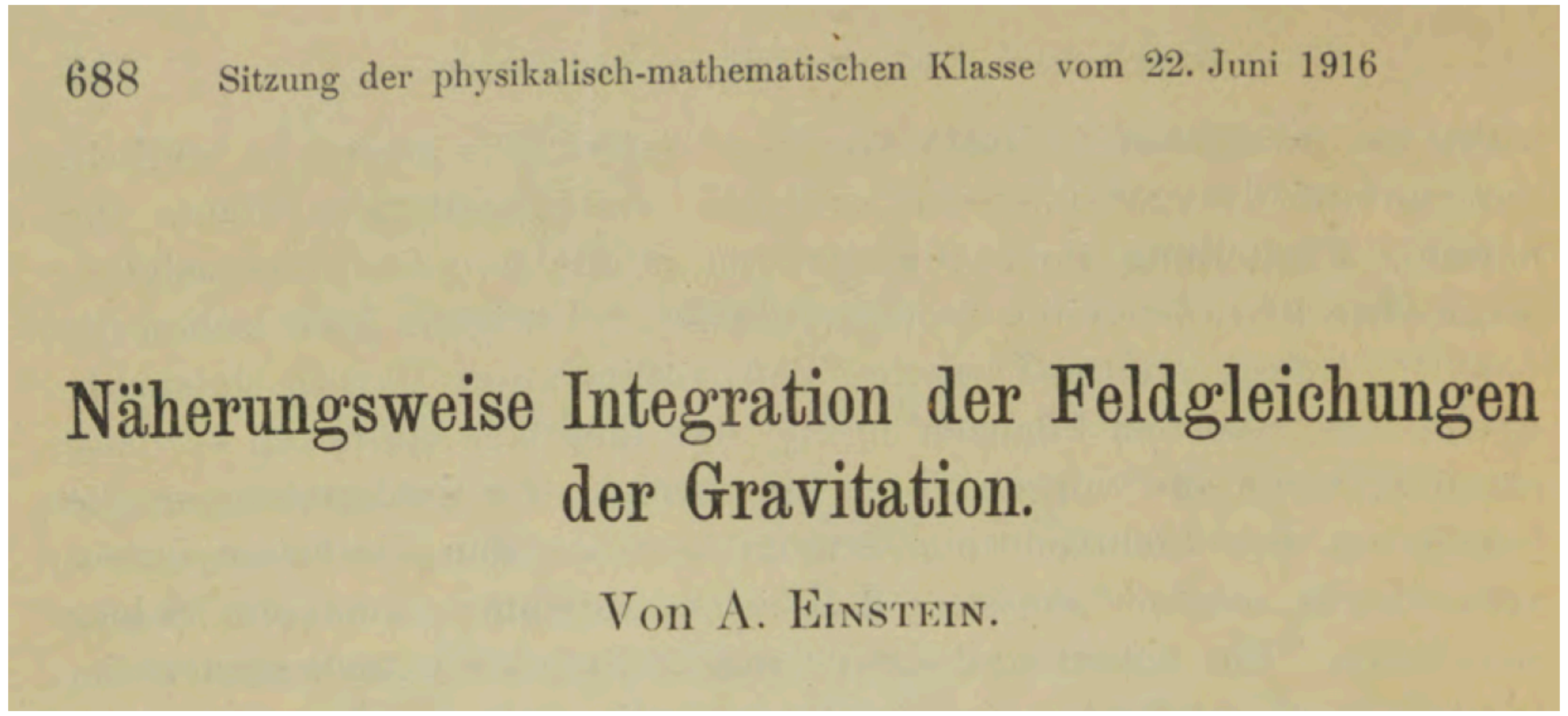
In collaboration with Manfred Kraus

XIX Mexican Workshop on Particles and Fields
October 21, 2025

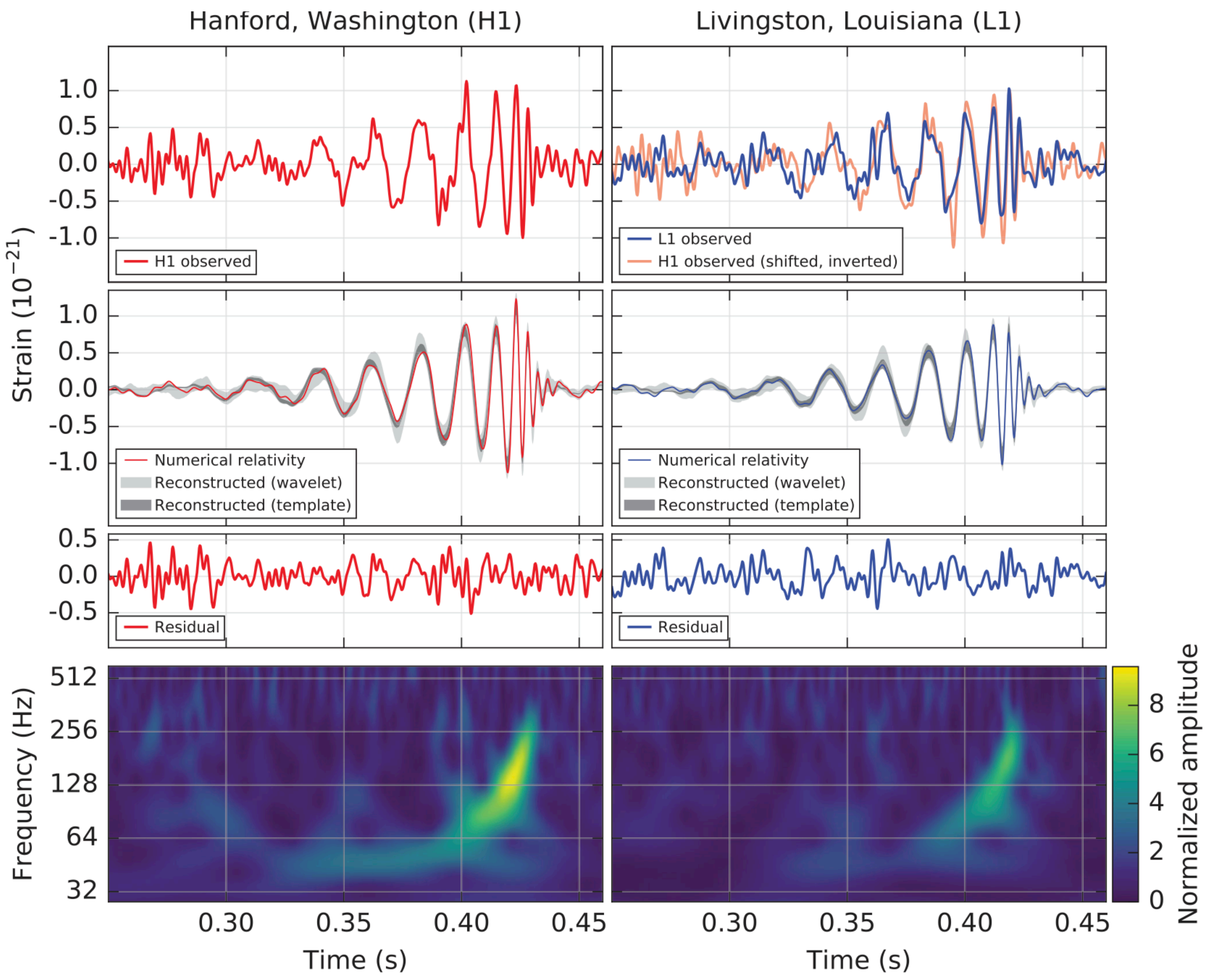
New Era of Gravitational-Wave Science


688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

[Abbott *et al.* ’16]



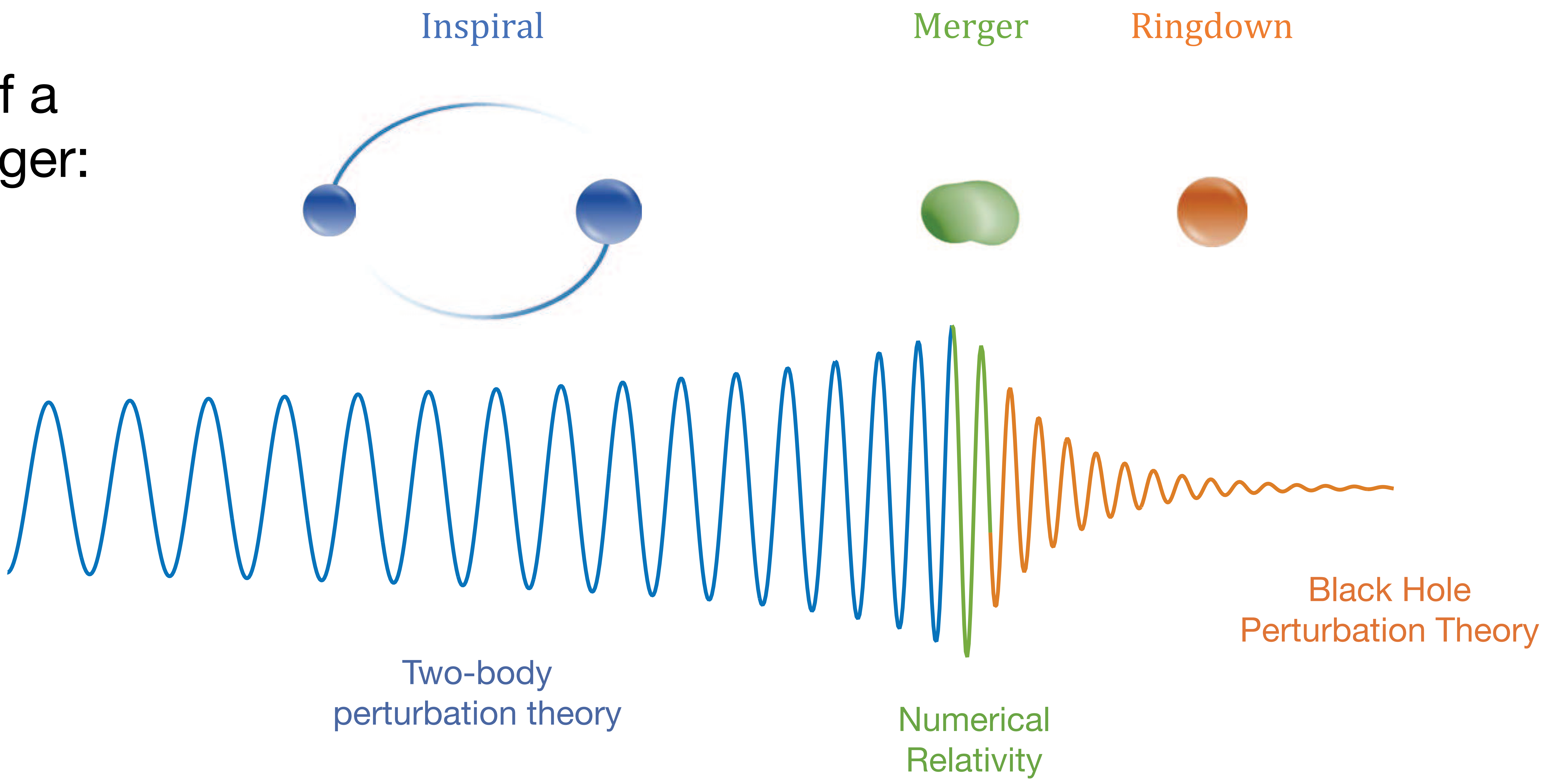
- First detection of a GW, nearly 100 years after Einstein’s prediction

- Source of GW signal: binary black hole merger

Gravitational Waveform Production

- Need to solve $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$ Hard problem
- In principle, the solution is simple: Solve the equations in a computer!
  - Computationally expensive: ~250k templates for GW150914
  - Challenging for EMRI $m_1 \ll m_2$, large eccentricities
  - New simulation required for each point in parameter space
- Strategy: approximate analytical methods + numerical methods

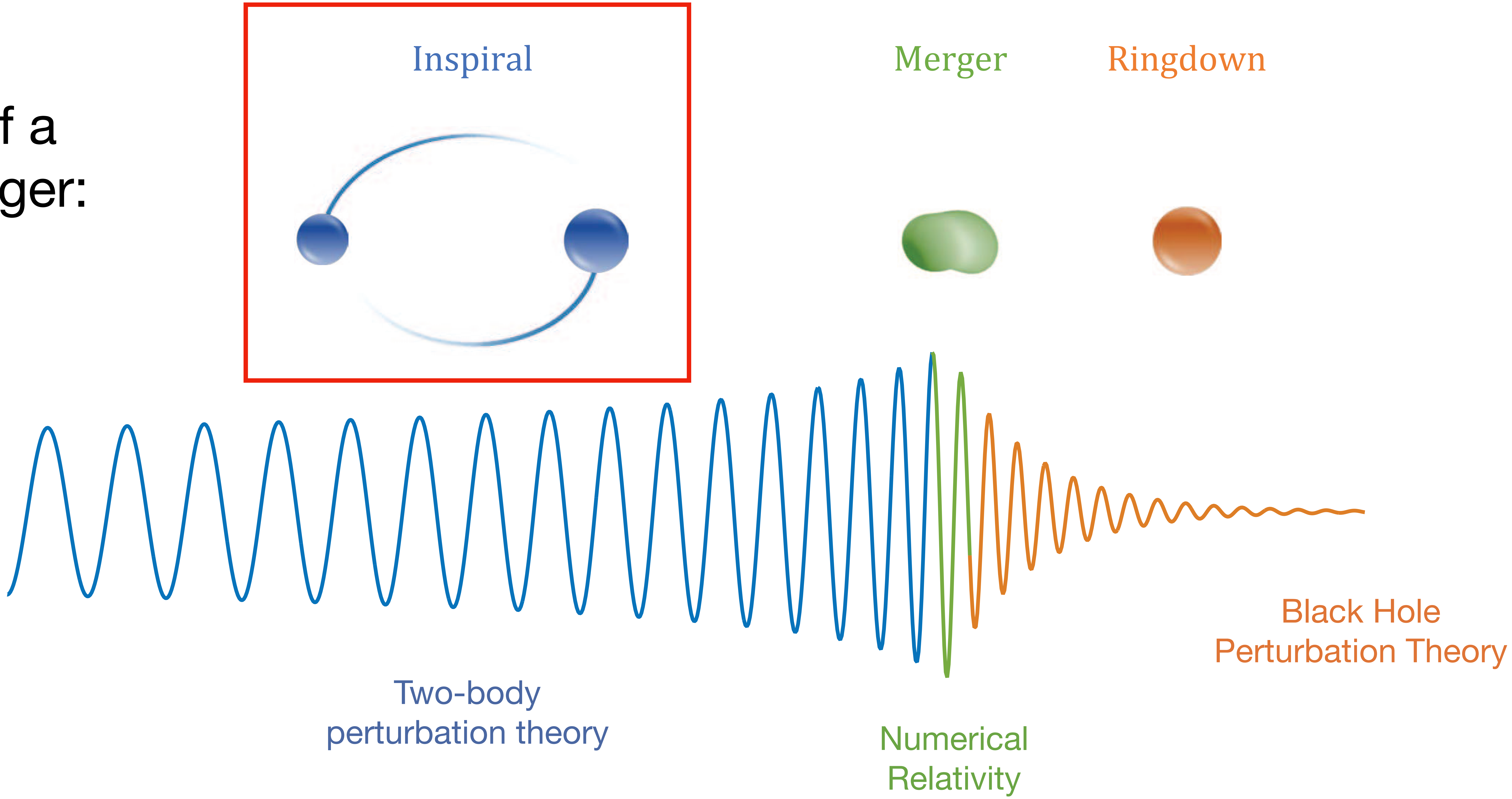
Phases of a binary merger:



[From Antelis & Moreno, arXiv:1610.03567]

We focus on

Phases of a binary merger:



[From Antelis & Moreno, arXiv:1610.03567]

Analytical Methods For Inspiral

- Post-Newtonian (weak field, nonrelativistic)
  - Double expansion in $v^2 \sim \frac{GM}{|\boldsymbol{r}|} \ll 1$

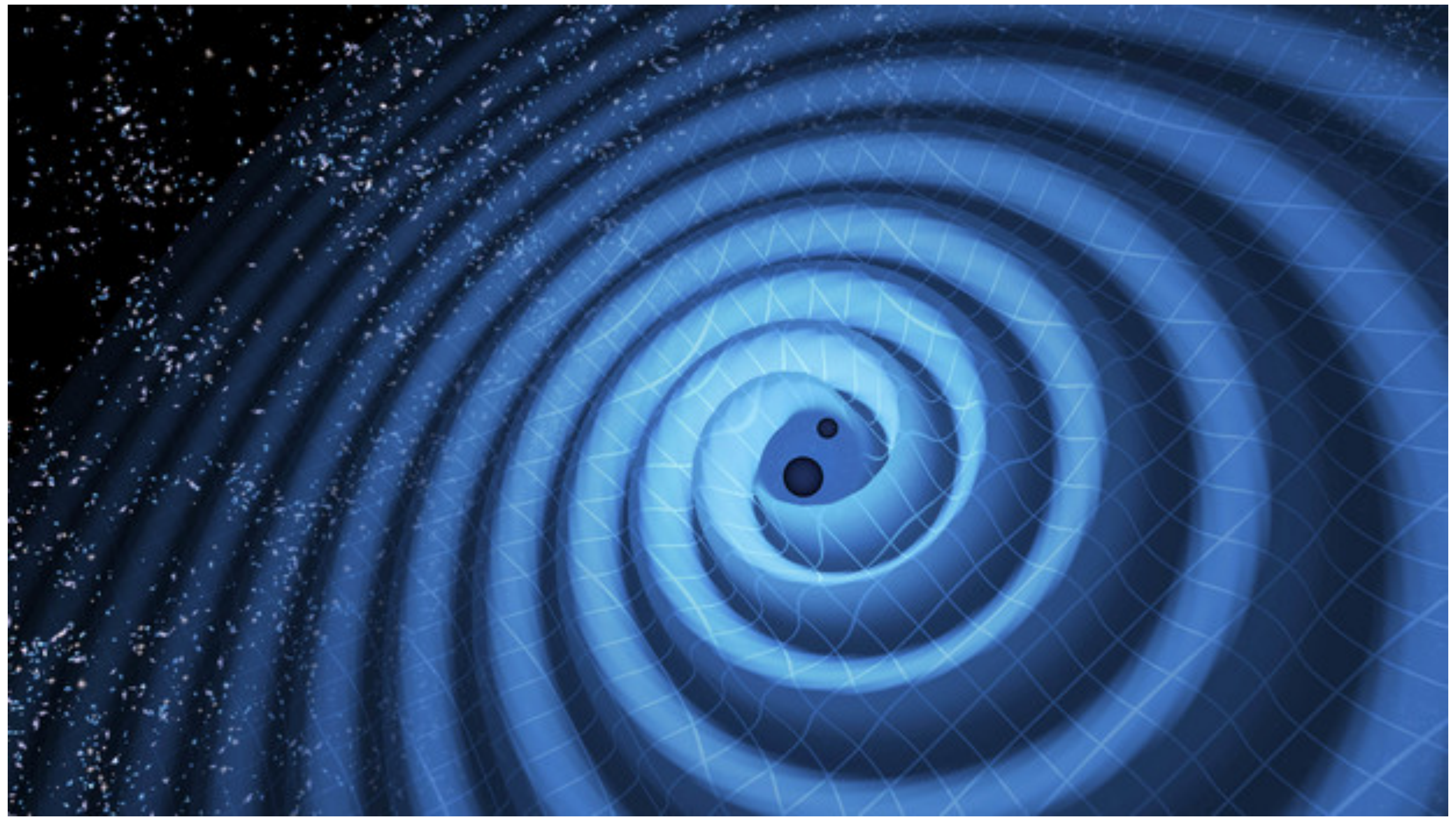
❖ Post-Minkowskian (weak field, relativistic)
  - Expansion in G $\frac{GM}{|\boldsymbol{r}|} \ll v^2 \sim 1$

- Gravitational self-force (strong field)
  - Expansion in small mass ratio $v^2 \sim \frac{GM}{|\boldsymbol{r}|} \sim 1$

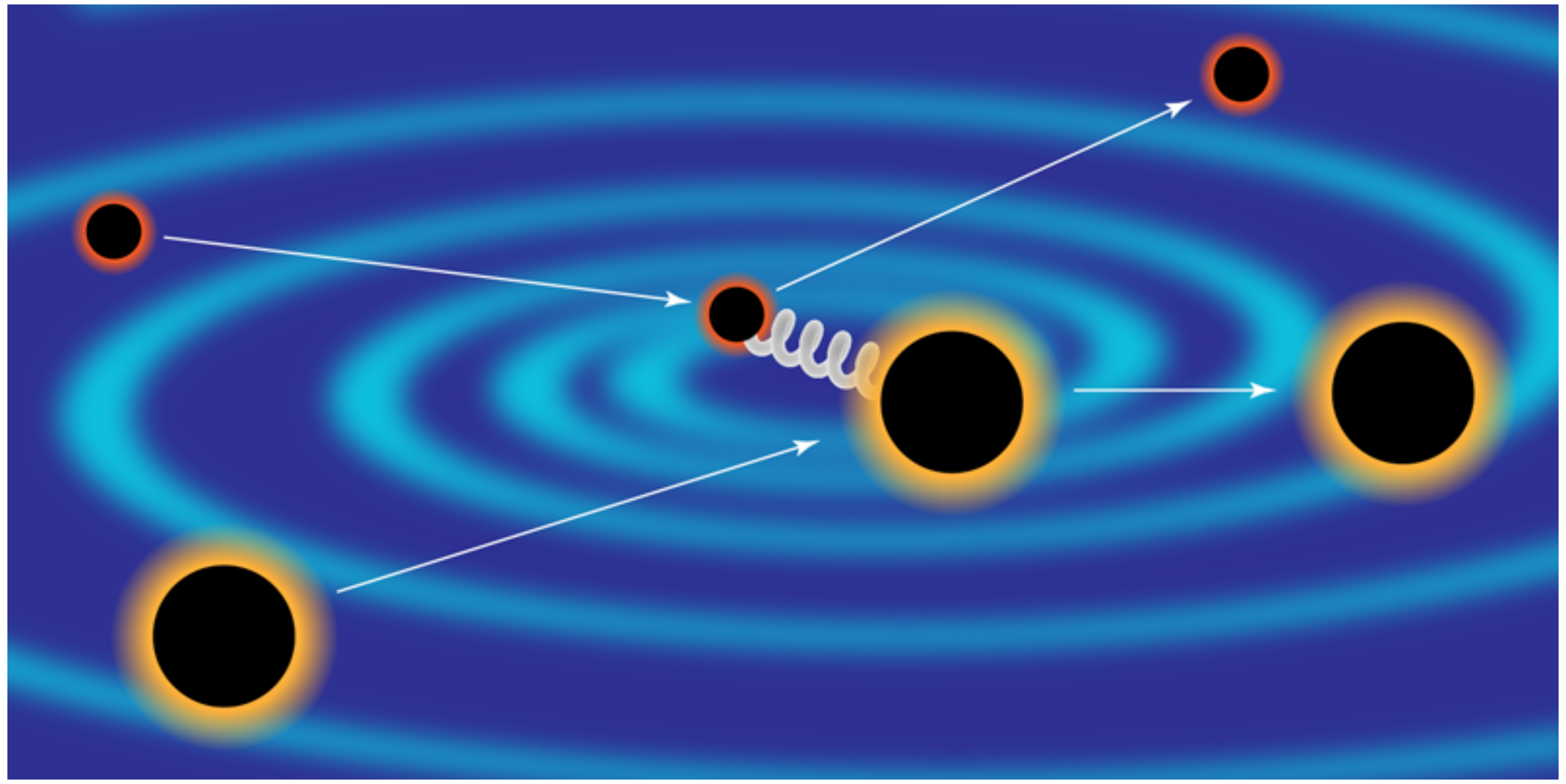
    $m_2/m_1 \ll 1$

PM approach naturally suited for scattering problem

Gravitational Scattering



Credit: LIGO/T. Pyle

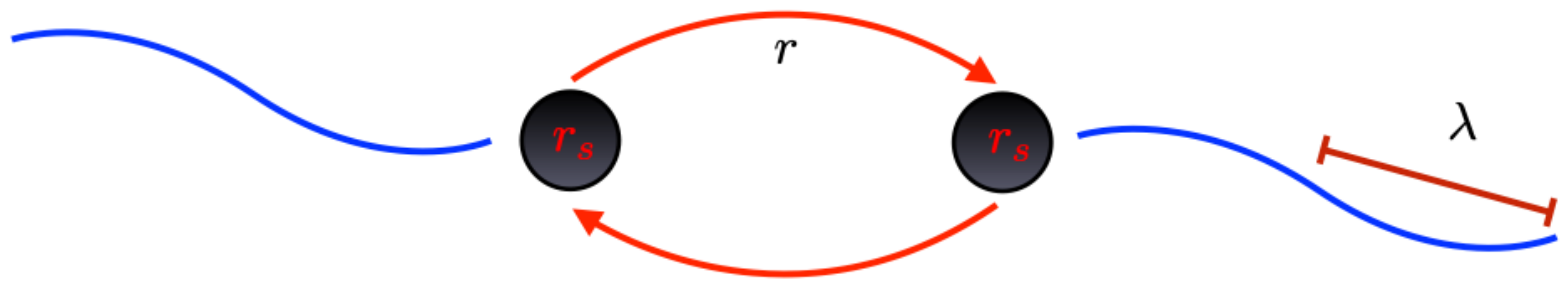


Credit: APS/Alan Stonebraker

- Scattering and bound dynamics governed by same Hamiltonian*
- Our strategy is to extract Hamiltonian from scattering problem using scattering amplitudes and then translate to bound problem

How do scattering amplitudes come into play?

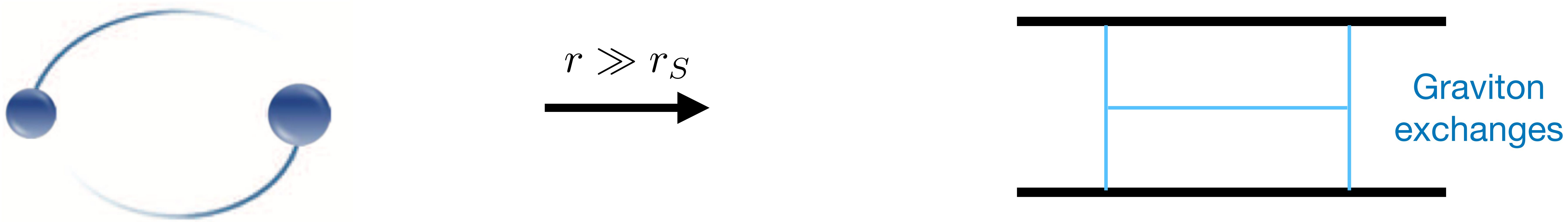
Separation of Scales



[From Walter Golberger, Yale & KITP 03]

- Inspiral phase: $r_S \ll r \ll \lambda$
- Can treat compact objects as point particles using effective field theory!
  [Goldberger, Rothstein; Cheung, Rothstein, Solon; Damgaard, Haddad, Helset;…]

Point Particle EFT



- Spinless compact objects interacting gravitationally replaced by massive scalar fields minimally coupled to gravity

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_i (\nabla^\mu \phi_i \nabla_\mu \phi_i - m_i^2 \phi_i^2) \right]$$

System of Interest

- We study the dynamics of electrically charged, compact binary objects
- Not much attention received in the literature, but there has been recent progress:
  - 3-body Hamiltonian at 2PM [Jones, Solon]
  - 2-body Hamiltonian at 2PN [Placidi, Grilli, Orselli, Pegorin, Bartolo, Mastrolia]
  - Scattering angle at 3PM [Wilson-Gerow]
- Puts constraints on black hole charges from GW signals

Charged Binary Black Holes

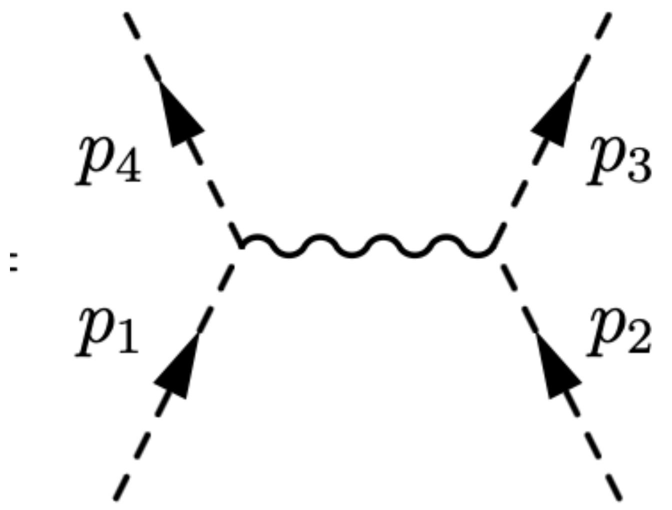
Graviton & photon exchanges

- Spinless compact objects interacting gravitationally and electromagnetically replaced by massive complex scalar fields minimally coupled to gravity

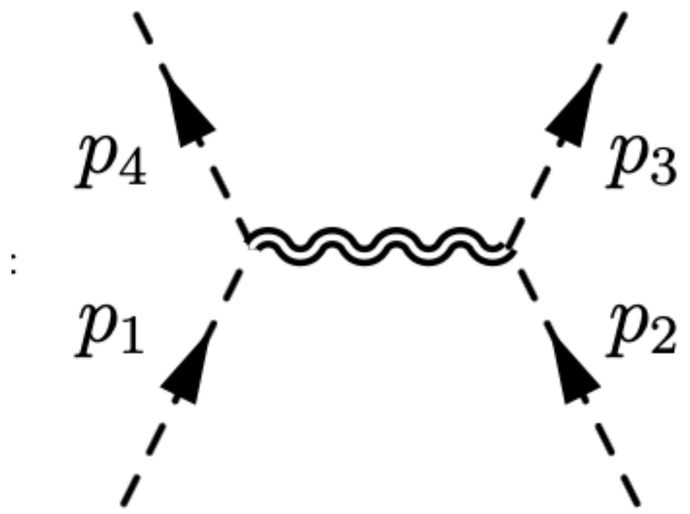
$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{16\pi G}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_i (|\nabla_\mu \phi_i|^2 - m_i^2 |\phi_i|^2) \right]$$

Two-body Potential and Amplitudes

- Fourier transform of non-analytic pieces of tree-level amplitude gives classical potential



$$\sim \frac{e^2 Q_1 Q_2 (4 m_1 m_2 \sigma)}{q^2} \xrightarrow{\text{F.T.}} \sim \frac{e^2 Q_1 Q_2}{4\pi r}(1 + \mathcal{O}(v^2))$$



$$\sim \frac{G m_1^2 m_2^2 (1 - 2\sigma^2)}{q^2} \xrightarrow{\text{F.T.}} \sim \frac{G m_1 m_2}{r}(1 + \mathcal{O}(v^2))$$

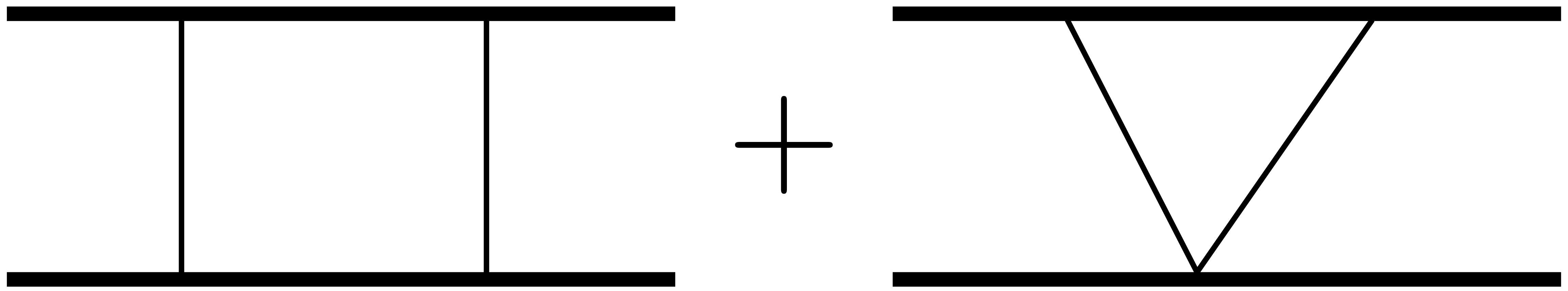
Loop Diagrams

- Classical physics applies when de Broglie wavelength of particles is much smaller than their separation

$$\lambda_{\text{dB}} = \frac{1}{|\boldsymbol{p}|} \ll |\boldsymbol{b}| \sim \frac{1}{|\boldsymbol{q}|} \implies J = |\boldsymbol{p} \times \boldsymbol{b}| \gg 1\,, |\boldsymbol{p}| \gg |\boldsymbol{q}|$$

- Classical limit expansion parameter: $\mathcal{O}(1/J) \sim \mathcal{O}(|\boldsymbol{q}|)$

- Loops have classical pieces too!



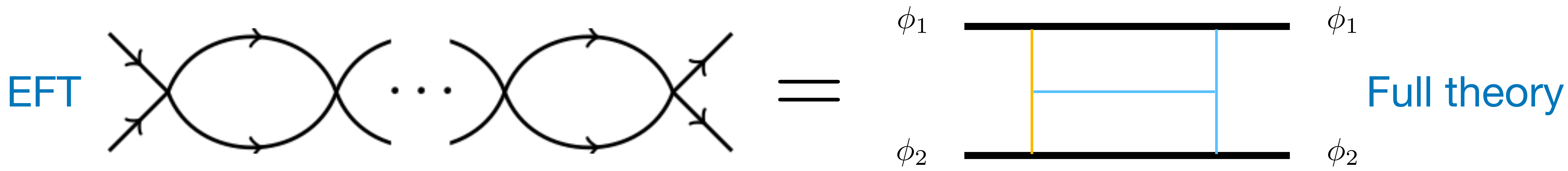
$$+ \cdots \longleftarrow \quad G^2\,, Ge^2\,, e^4$$

Nonrelativistic EFT

$$\mathcal{L} = \sum_{i=1}^{2} \int_{\boldsymbol{k}} \phi_i^\dagger(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \phi_i(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k'}} V(\boldsymbol{k},\boldsymbol{k'}) \phi_1^\dagger(\boldsymbol{k'}) \phi_1(\boldsymbol{k}) \phi_2^\dagger(-\boldsymbol{k'}) \phi_2(-\boldsymbol{k})$$

- Integrated out massless force carriers mediating near-instantaneous interactions. Scalars interact through long-distance potential

- No antiparticles, no particle creation



EFT Matching determines the potential

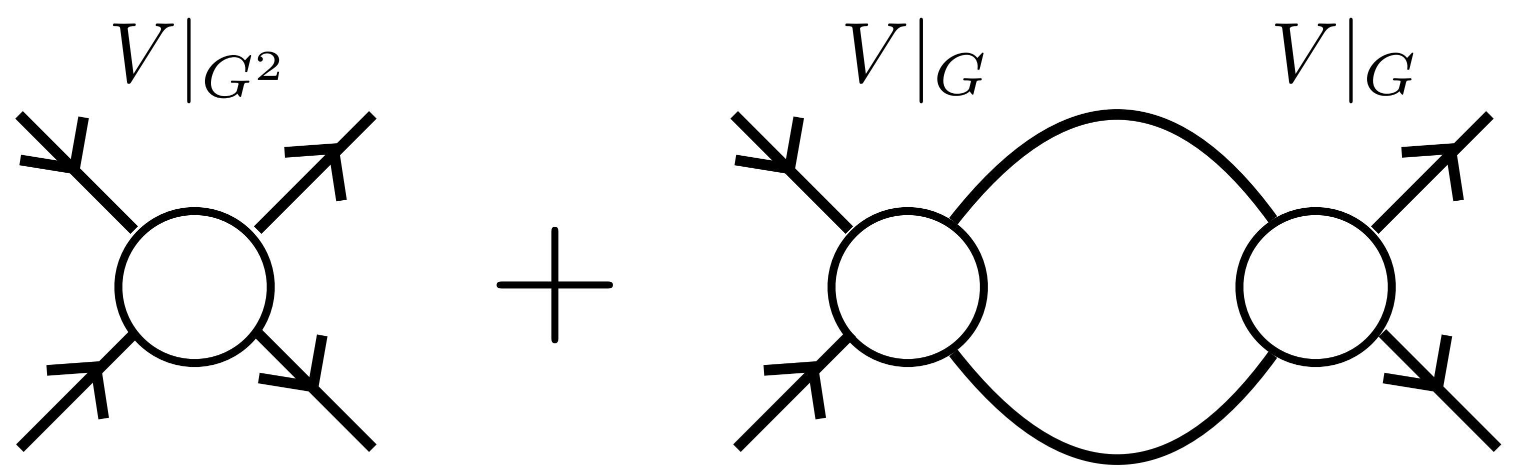
Cancellation of Superclassical Pieces

- In the small $|\boldsymbol{q}|$ expansion, diagrams appear which have larger classical scaling. These are called superclassical terms

- In the EFT, superclassical terms are iterations of lower order potentials

- At 1-loop:

$$\mathcal{M}^{\mathrm{EFT}}_{1-\mathrm{loop}} =$$



- Superclassical pieces cancel in EFT matching between full theory and effective theory amplitudes

Two-body Effective Hamiltonian

$$H^{\rm 2PM}(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + V^{\rm 2PM}(\boldsymbol{p},\boldsymbol{r})\,,$$

$$V^{\rm 2PM}(\boldsymbol{p},\boldsymbol{r}) = \frac{c_1(\boldsymbol{p}^2)}{|\boldsymbol{r}|} + \frac{c_2(\boldsymbol{p}^2)}{|\boldsymbol{r}|^2}$$

Coefficient functions:

$$c_1 = \frac{G m_1^2 m_2^2}{E_1 E_2}(1-2\sigma^2) + \frac{e^2 Q_1 Q_2}{4\pi E_1 E_2}(m_1 m_2 \sigma)$$

Bern, Gatica, Herrmann, Luna, Zeng (2021)

Lorentz factor

$$\sigma = \frac{p_1\cdot p_2}{m_1 m_2} = \frac{1}{\sqrt{1-v^2}}$$

$$c_2 = G^2\left[\frac{3m_1^2m_2^2(m_1+m_2)(1-5\sigma^2)}{4E_1E_2} - \frac{4m_1^3m_2^3\sigma(1-2\sigma^2)(E_1+E_2)}{E_1^2E_2^2} - \frac{m_1^4m_2^4(1-2\sigma^2)^2(E_1^2+E_1E_2+E_2^2)}{2E_1^3E_2^3(E_1+E_2)}\right]$$

Westpfahl (1985)

$$+\frac{e^2Q_1^2Q_2^2}{16\pi^2}\left[-\frac{m_1+m_2}{2E_1E_2} + \frac{m_1m_2\sigma(E_1+E_2)}{E_1^2E_2^2} - \frac{m_1^2m_2^2\sigma^2(E_1^2+E_1E_2+E_2^2)}{2E_1^3E_2^3(E_1+E_2)}\right]$$

Bern, Gatica, Herrmann, Luna, Zeng (2021)

$$+\frac{Ge^2}{4\pi}\left[\frac{3Q_1Q_2m_1m_2(m_1+m_2)\sigma}{E_1E_2} - \frac{m_1m_2(Q_1^2m_2+Q_2^2m_1)(1-3\sigma^2)}{4E_1E_2} + \frac{Q_1Q_2m_1^2m_2^2(1-6\sigma^2)(E_1+E_2)}{E_1^2E_2^2} - \frac{Q_1Q_2m_1^3m_2^3\sigma(1-2\sigma^2)(E_1^2+E_1E_2+E_2^2)}{E_1^3E_2^3(E_1+E_2)}\right]$$

Our result

Original checks:

- Hamiltonian reproduces pure gravity result in the neutral limit $(Q_1, Q_2) \to (0, 0)$
  [Bern, Cheung, Roiban, Shen, Solon, Zeng]

- Hamiltonian reproduces scalar QED result in the limit $G \to 0$
  [Bern, Gatica, Herrmann, Luna, Zeng]

- Scattering angle agreement
  [Wilson-Gerow]

Ongoing check:

- Compare to 2PN Hamiltonian in the overlap region published recently
  [Placidi *et.al.*, arXiv:2509.20432v2]

Conclusions

- Quantum scattering amplitudes have pushed post-Minkowskian expansion to higher orders
- Can incorporate effects of spin, finite size, and radiation
- Complements post-Newtonian, self-force, and numerical relativity and offers additional insights
- Methods are not exhausted and can be improved

Thank you!