Sizing the double pole resonant enhancement in $e^+e^- \to \pi^0\pi^0\gamma$ cross section and $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$ decay

XIX Mexican Workshop on Particles and Fields
October 20 - 24, 2025
León, Guanajuato
Presenter: Leonardo Esparza Arellano
Collaboration with: Dr. Genaro Toledo Sánchez, Dr. Antonio Rojas Ramos
Published Dec 1, 2023 [*Phys.Rev.D* 108 (2023) 11, 116020]

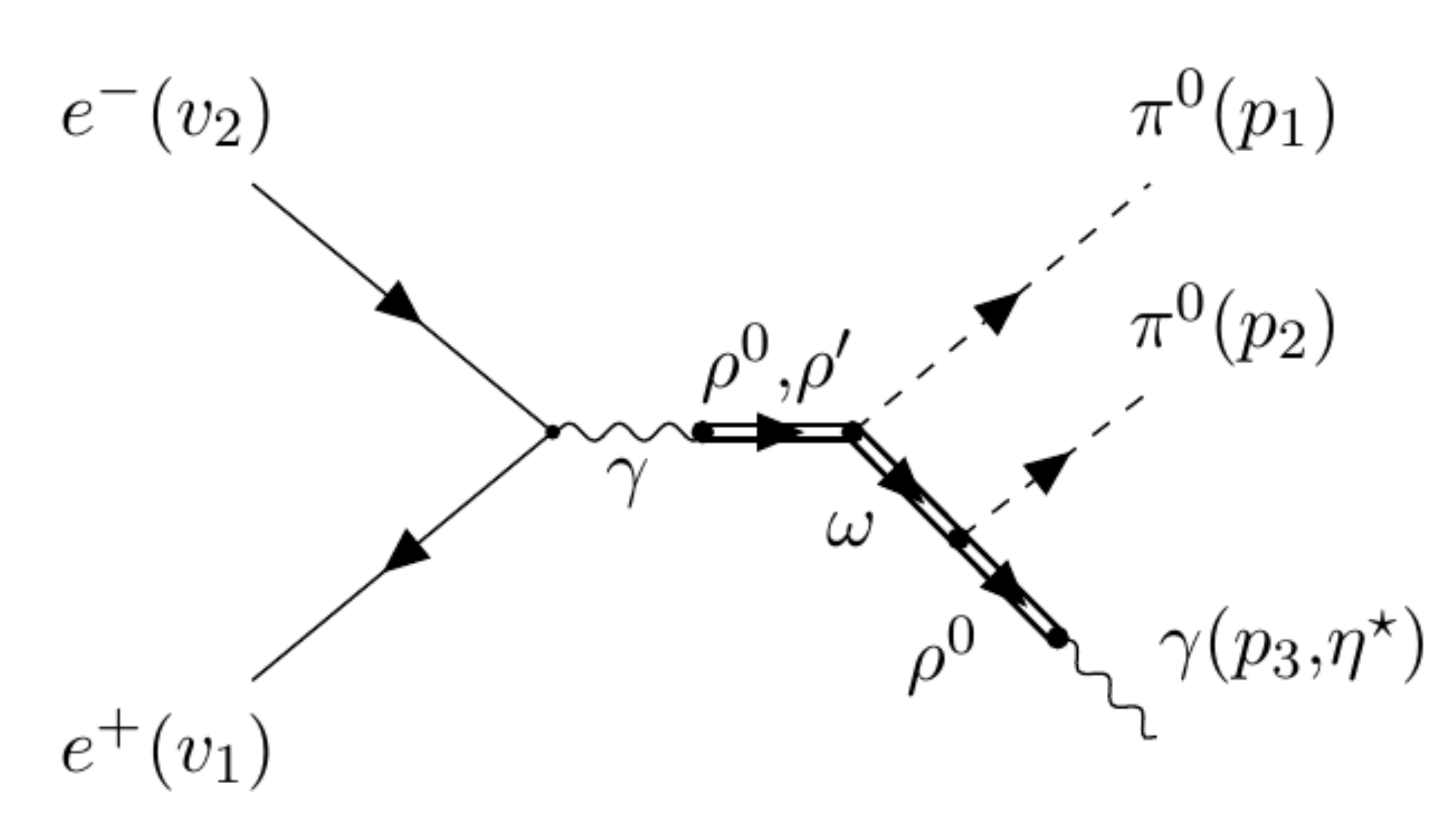
Content

- Motivation
- $e^+e^- \to \pi^0\pi^0\gamma$ scattering
- $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$ Decay
- Conclusions

Motivation $e^+e^- \to \pi^0\pi^0\gamma$

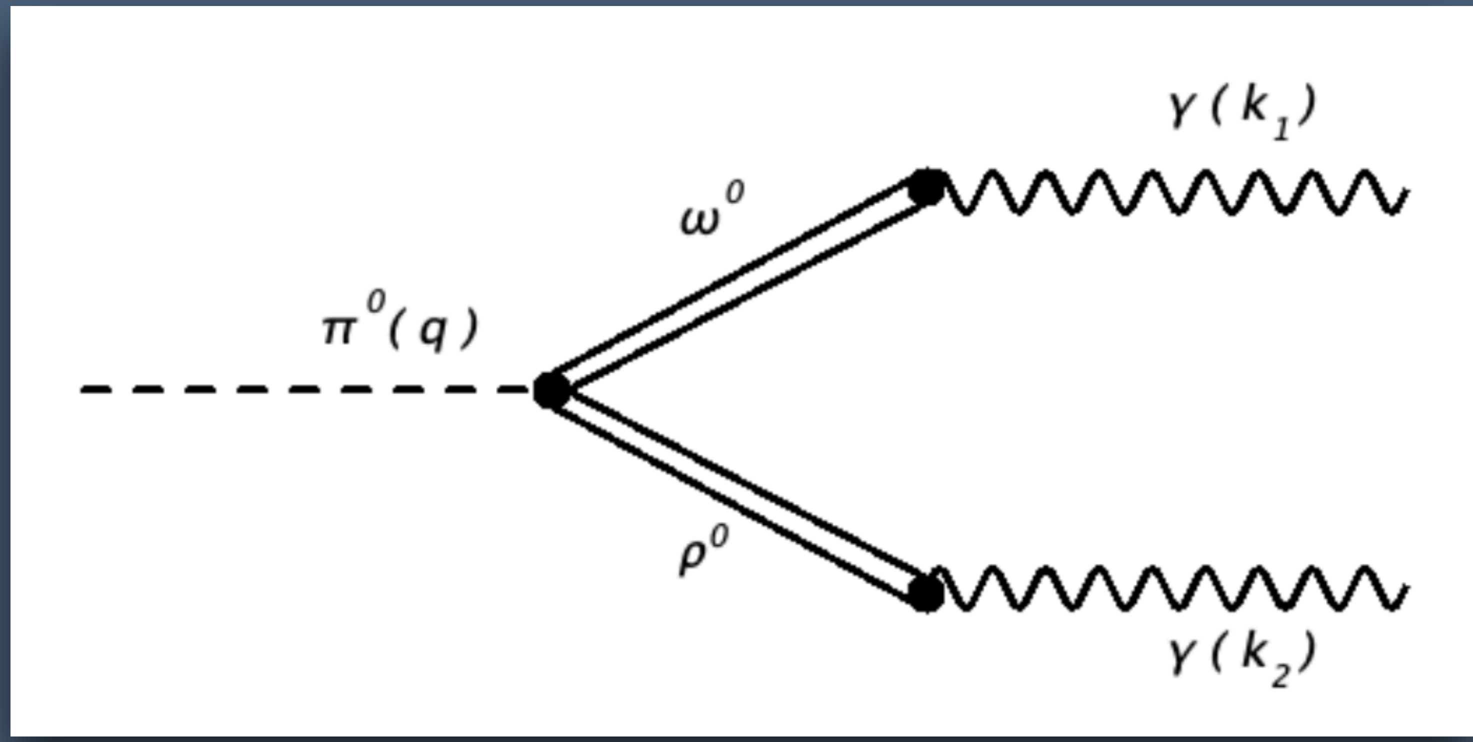
- Scattering $e^+e^- \to \pi^+\pi^-$ is the lowest-order main hadronic contribution to muon g-2, where resonances ρ, ω can be found carrying the same amount of momenta. Only difference between them relies on their masses and decay widths.

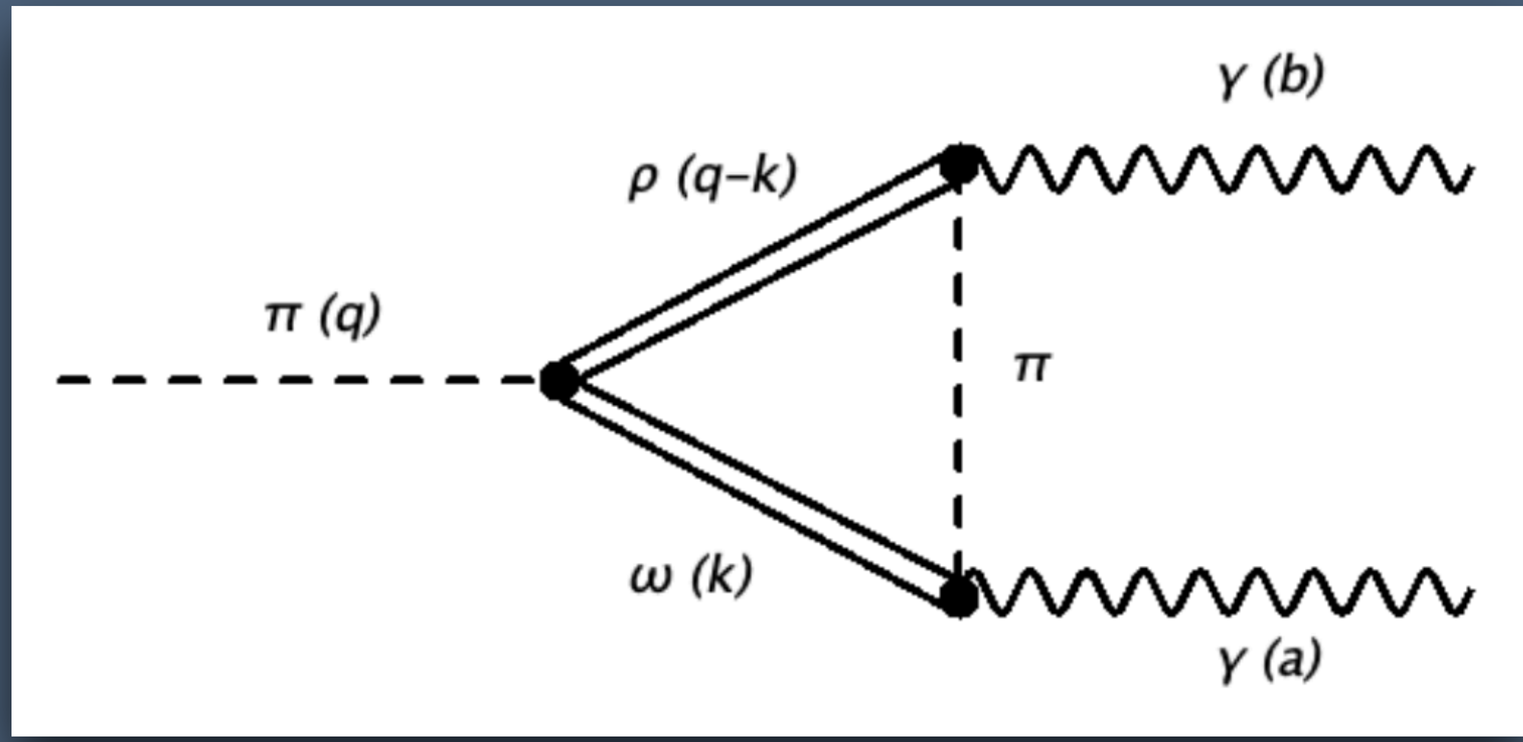
- Scattering $e^+e^- \to \pi^0\pi^0\gamma$ (our work) is the radiative (EM) hadronic correction. It contains the resonances ρ, ω as well, however, unlike in $e^+e^- \to \pi^+\pi^-$, they carry different momenta. We introduce the $g_{\rho\omega\pi}$ coupling which is linked to the chiral anomaly.



$g_{\rho\omega\pi}$ coupling

- Correction to coupling constant $g_{\rho\omega\pi}$ using tree level $\pi^0 \to \rho^0\omega^0 \to \gamma\gamma$ and loop correction $\pi^0 \to \rho^0\omega^0\pi^0 \to \gamma\gamma$

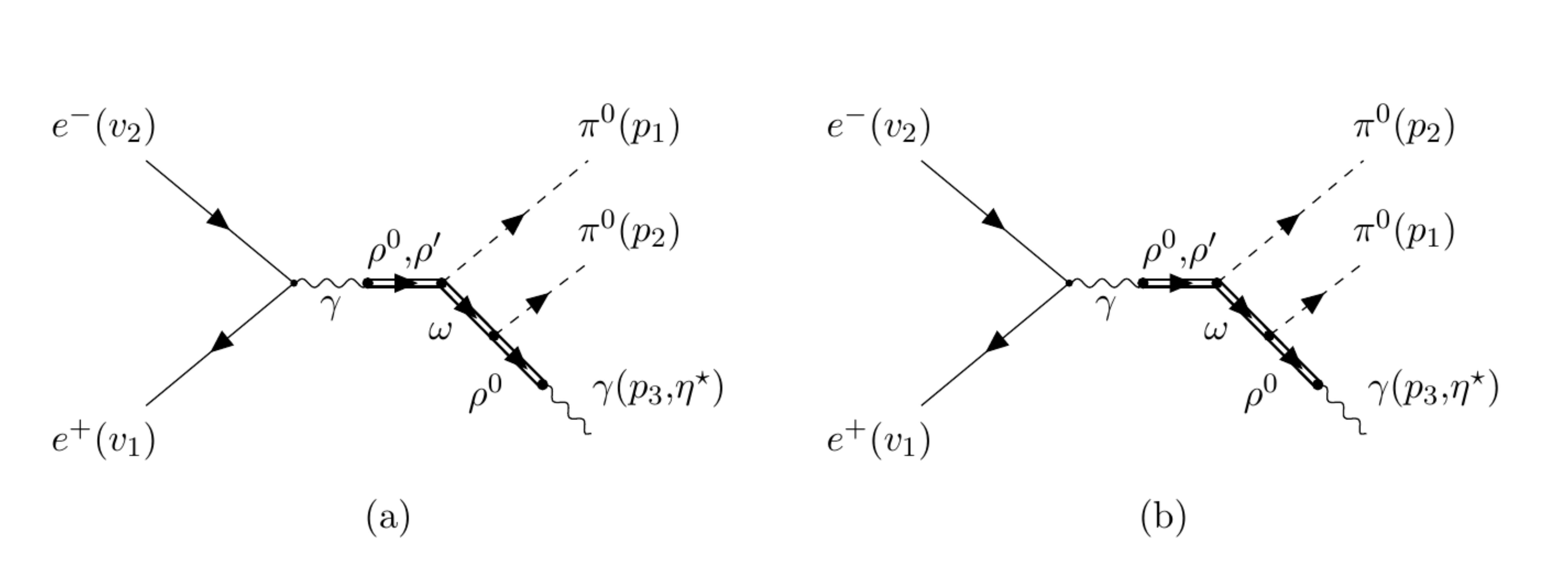




$$\hat{g}_{\rho\omega\pi} = 11.712\, GeV^{-1}$$

$$g_{\rho\omega\pi}(q^2) \equiv \hat{g}_{\rho\omega\pi}(1 + \hat{g}^2_{\rho\omega\pi}G_{loop})$$

Scattering $e^+e^- \to \pi^0\pi^0\gamma$ with intermediate states ρ, ρ', ω. Difference in momenta due to on-shell π emission.



$$\mathcal{M}_{(a)} = \frac{e^2}{q^2}\left(C_\rho + e^{i\theta}C_{\rho'}\right)\epsilon_{\mu\sigma\epsilon\lambda}\, q^\sigma\,(q-p_1)^\epsilon\,\epsilon_{\alpha\beta\nu}{}^{\lambda}\,(q-p_1)^\alpha\, p_3{}^\beta\,\eta^{*\nu}\, l^\mu,$$

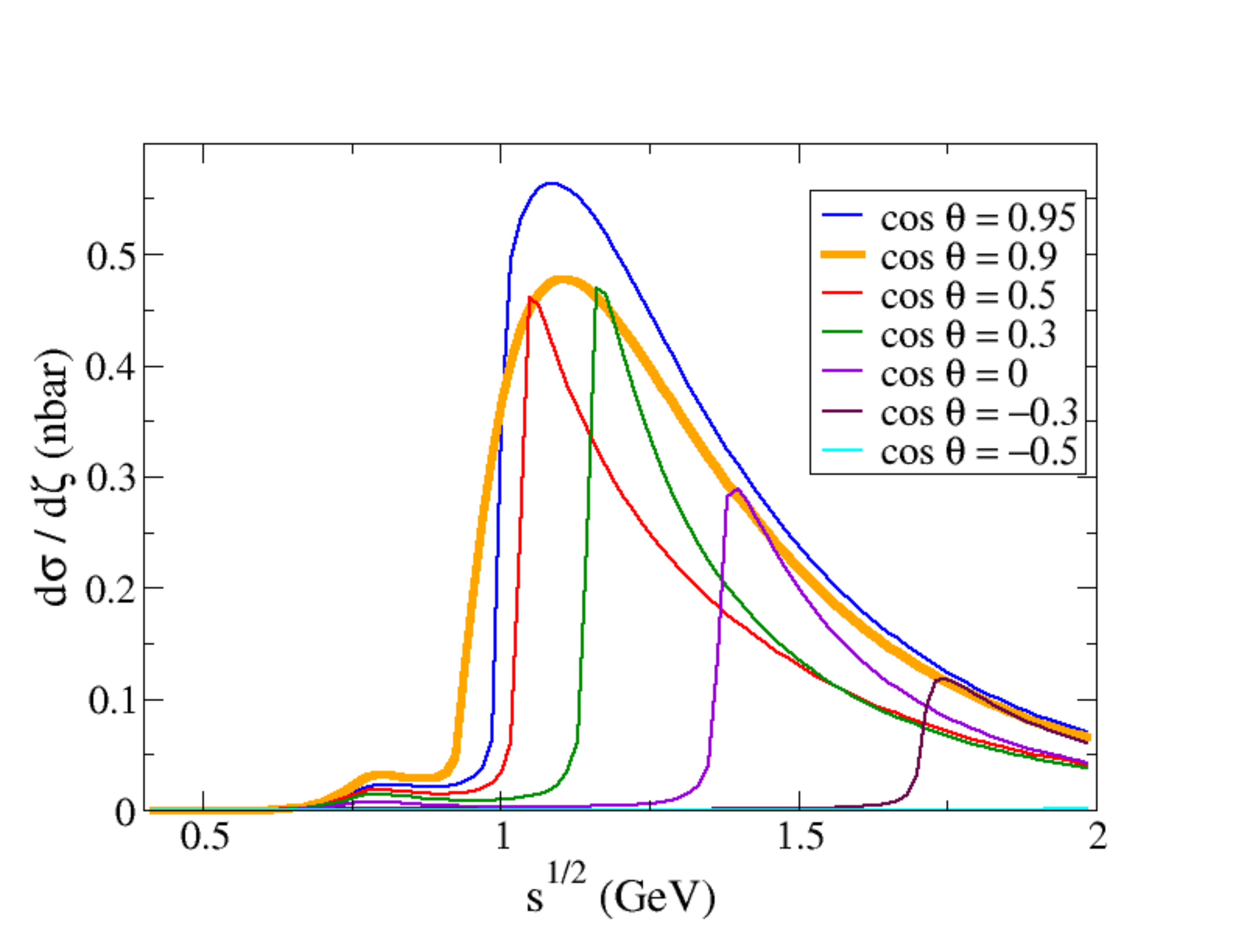
$$C_\rho = \left(\frac{g_{\omega\rho\pi}}{g_\rho m_\omega}\right)^2 f_\rho[s] f_\omega[s_1] \quad C_{\rho'} = \frac{g_{\omega\rho'\pi}\, g_{\omega\rho\pi}}{g_\rho\, g_{\rho'} m_\omega^2}\, f_{\rho'}[s] f_\omega[s_1]$$

Parameter	Value
g_ρ	4.962 ± 0.093
g_ω	16.652 ± 0.473
$g_{\rho'}$	12.918 ± 1.191
$g_{\omega\rho\pi}$ (GeV^{-1})	11.314 ± 0.383
$g_{\omega\rho'\pi}$ (GeV^{-1})	3.477 ± 0.963
θ/π	0.872 ± 0.051

- G. Ávalos, A. Rojas, M. Sánchez and G. Toledo, Phys. Rev. D 107, no.5, 056006 (2023)
- M. N. Achasov et al. Phys. Rev. D 94, no.11, 112001 (2016)
- R. R. Akhmetshin et al. [CMD-2], Phys. Lett. B 562, 173-181 (2003)

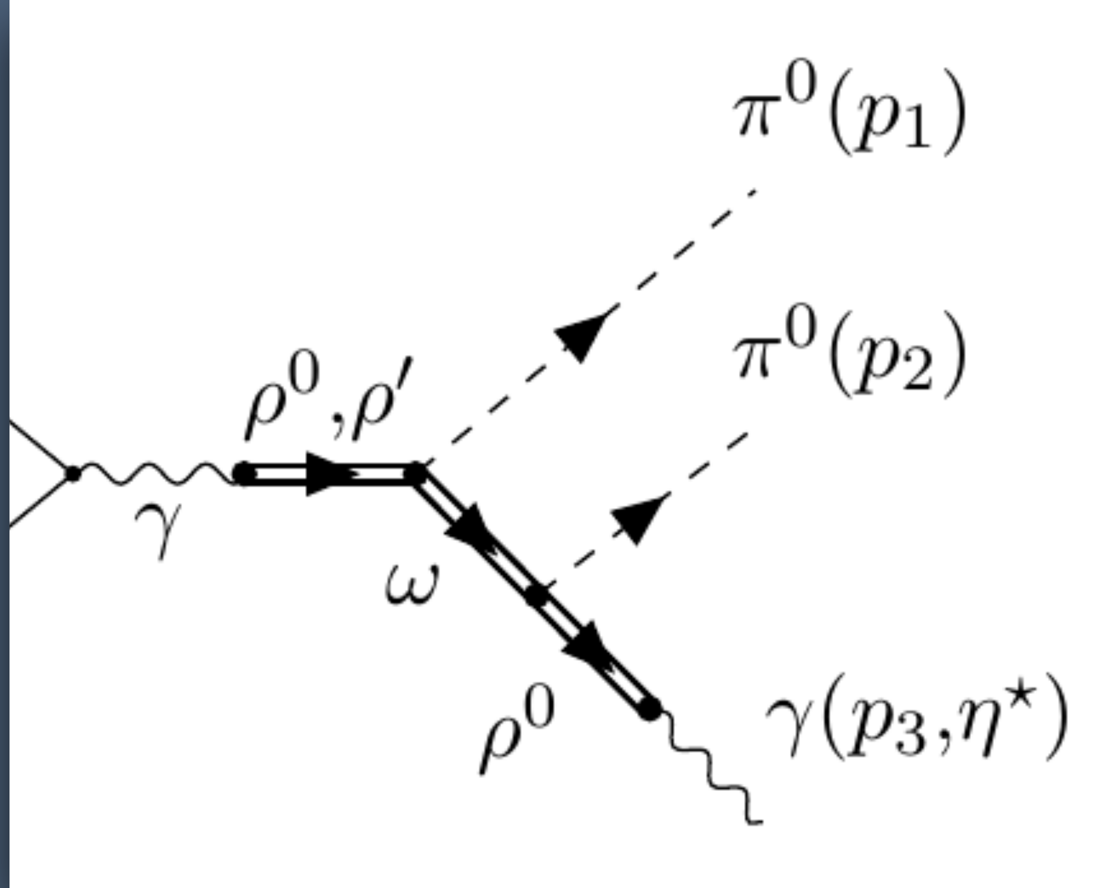
- Total cross section measured by SND [Phys. Lett. B 486, 29-34 (2000)] and CMD-2 [Phys. Lett. B 562, 173-181 (2003)]

Differential cross section with respect to scattered angle $\zeta = \cos\theta$.



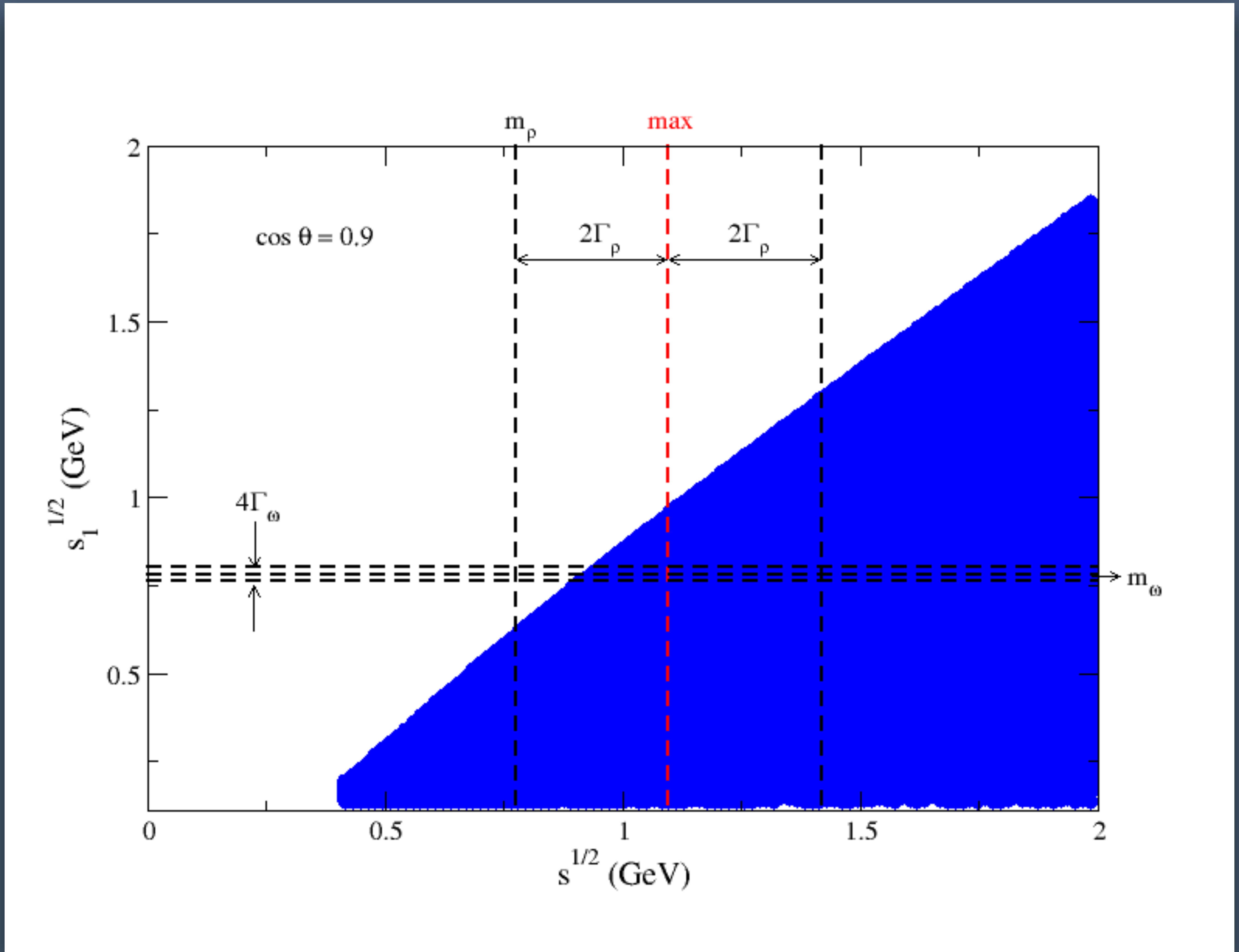
- Cross section amplifies when both off-shell propagators resonate. Negative angles are suppressed (recoil).

Dalitz plot $\zeta = 0.9$ of kinematical variables s and s_1.

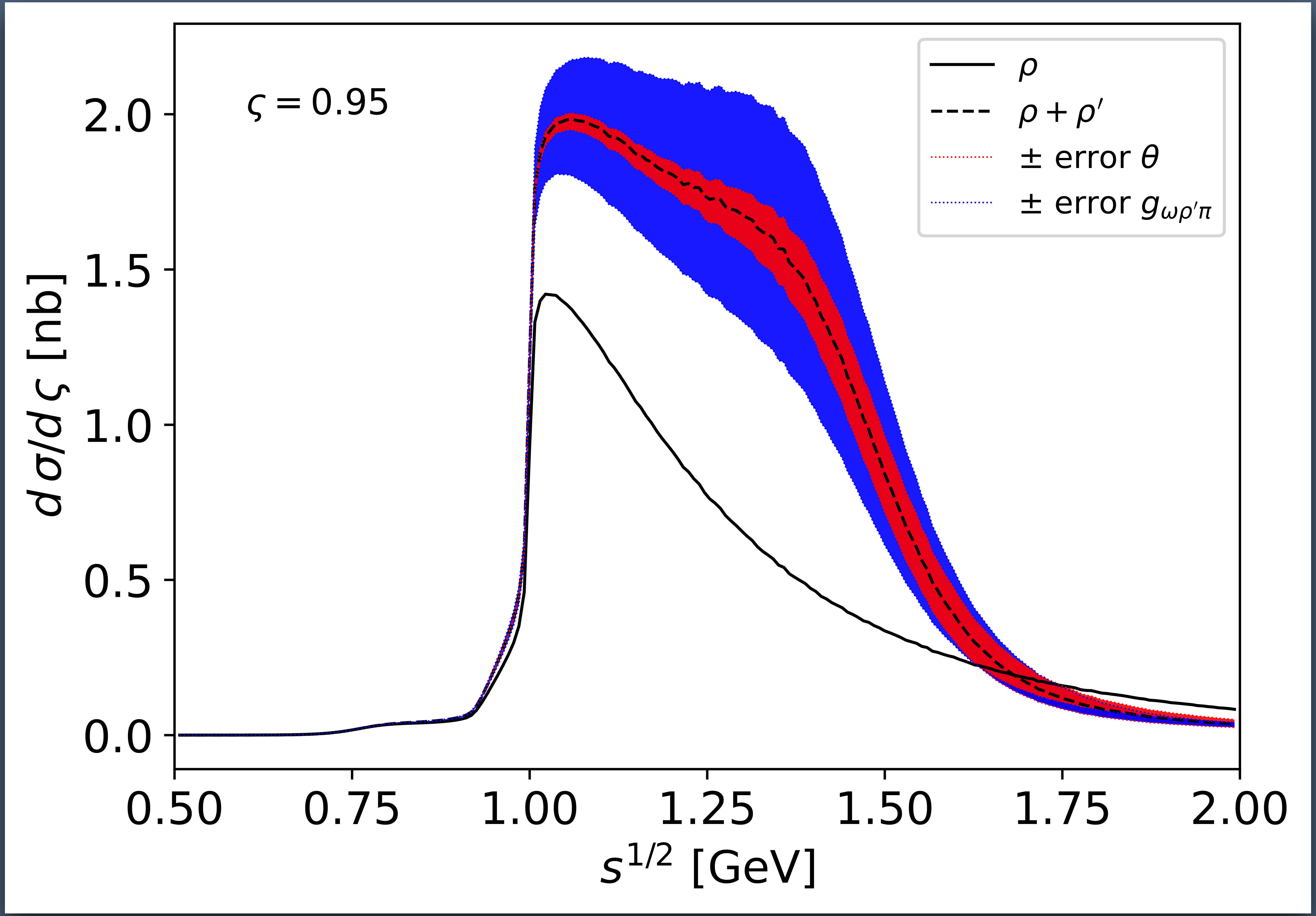


$$\rho(s) = \frac{m_\rho^2}{m_\rho^2 - s + i m_\rho \Gamma_\rho}$$

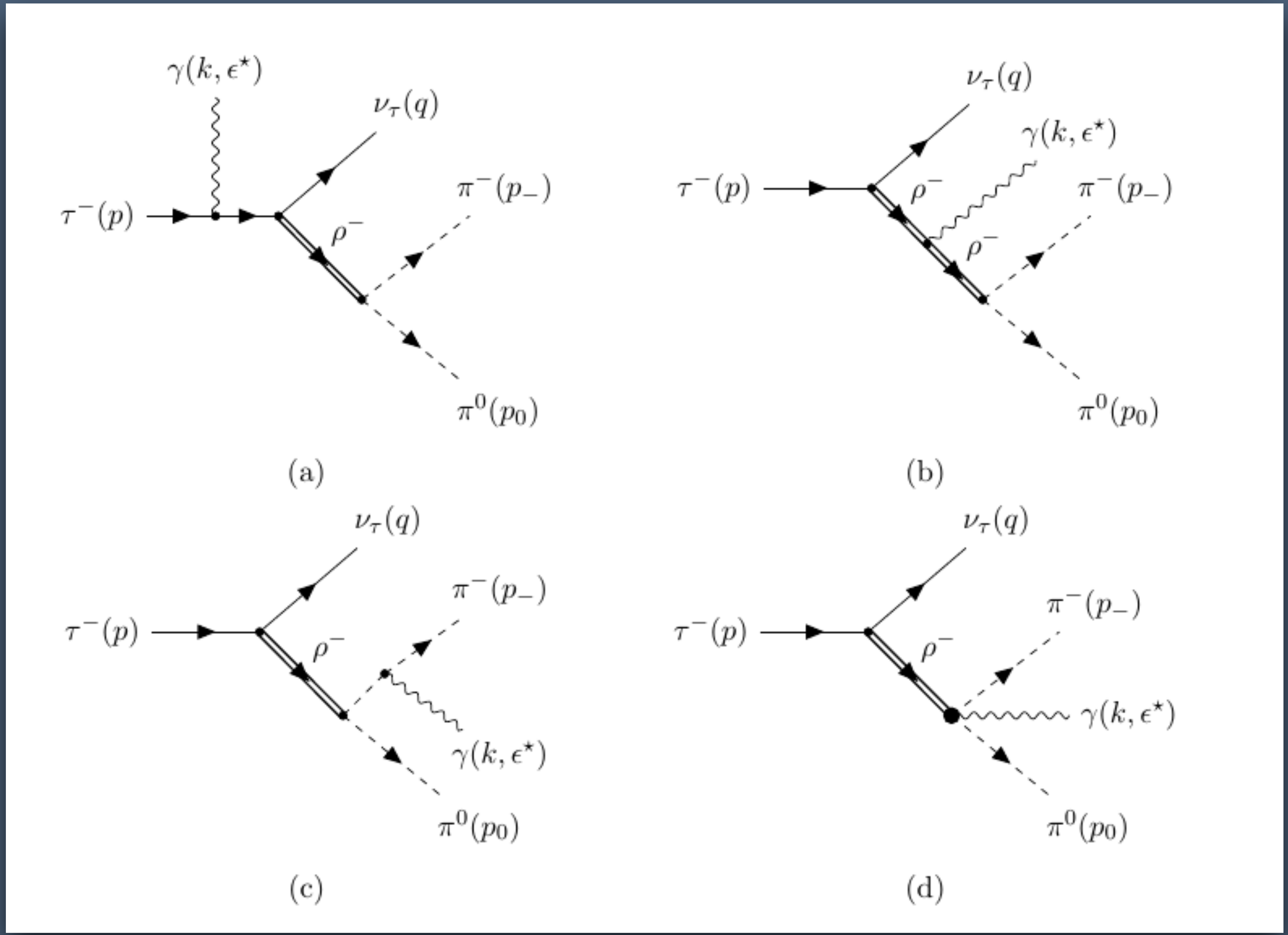
$$\omega(s_1) = \frac{m_\omega^2}{m_\omega^2 - s_1 + i m_\omega \Gamma_\omega}$$



- Cross section $\zeta = 0.95$ with ρ resonance (solid line) and adding the ρ′ contribution (dotted line). Error bands due to uncertainty in $g_{\omega\rho'\pi}$ and θ



Decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$



- We have various radiative decay channels (Structure Independent). We compute the interference of these SI channels with the MD channel.

We identify the MI part, in accordance to the Low theorem, as those contributions of order $O(k^{-1})$ and $O(k^0)$:

$$\mathscr{M}_{Low} = eG_F V^*_{ud} \epsilon^{*\mu} \left\{ f_+[t]\, L_\mu(p, p_-) Q_\nu + 2\, p_0 \cdot k\, L_\mu(p_0, p_-) \frac{df_+[t]}{dt}\, Q_\nu - \right.$$

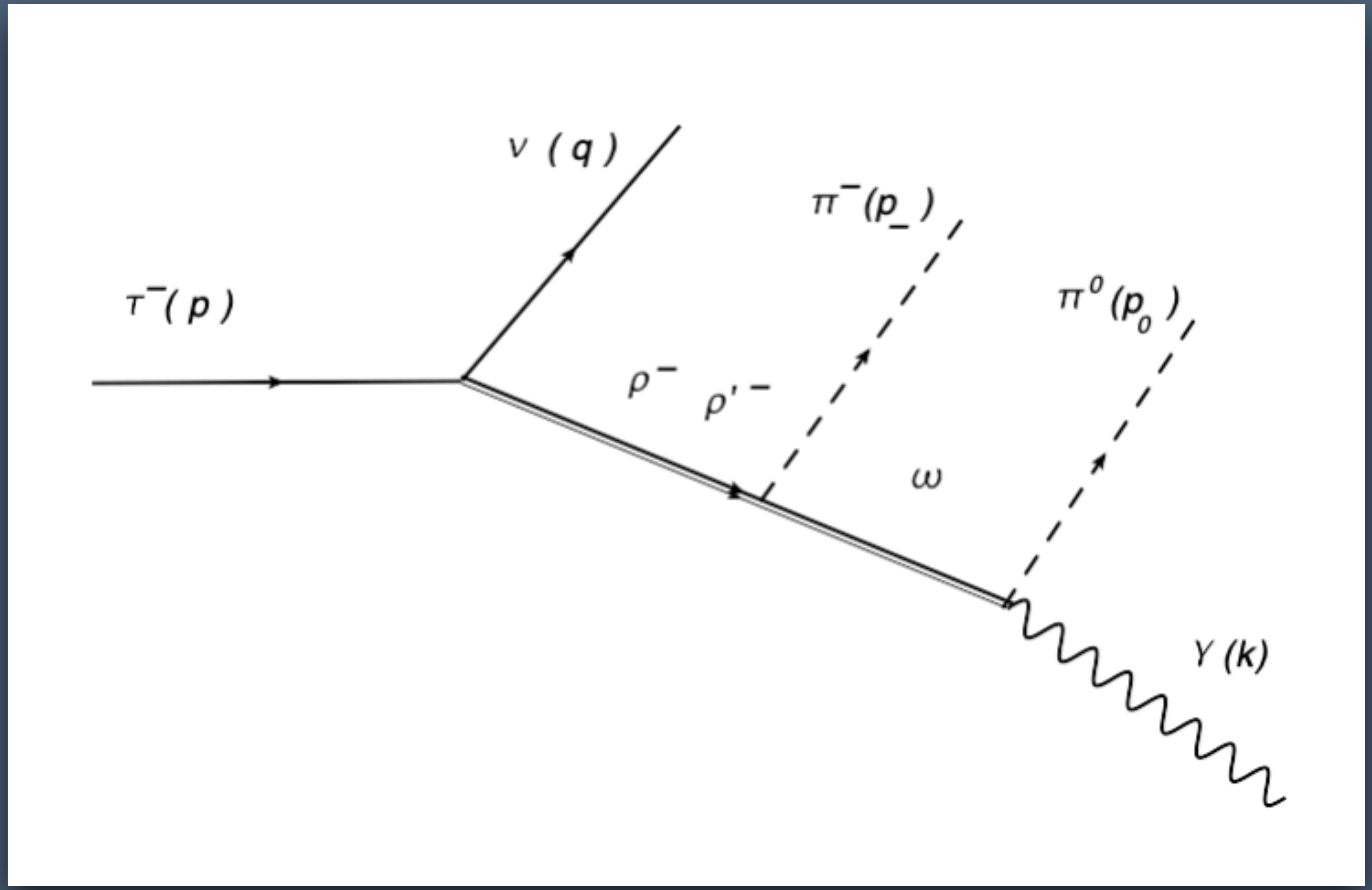
$$\left. - \frac{f_+[t]}{2\, p \cdot k} \left[F_{\mu\nu}(Q) Q \cdot k + i Q^\alpha k^\beta \epsilon_{\nu\alpha\beta\mu} \right] - f_+[t]\, F_{\mu\nu}(p_-) \right\} l^\nu$$

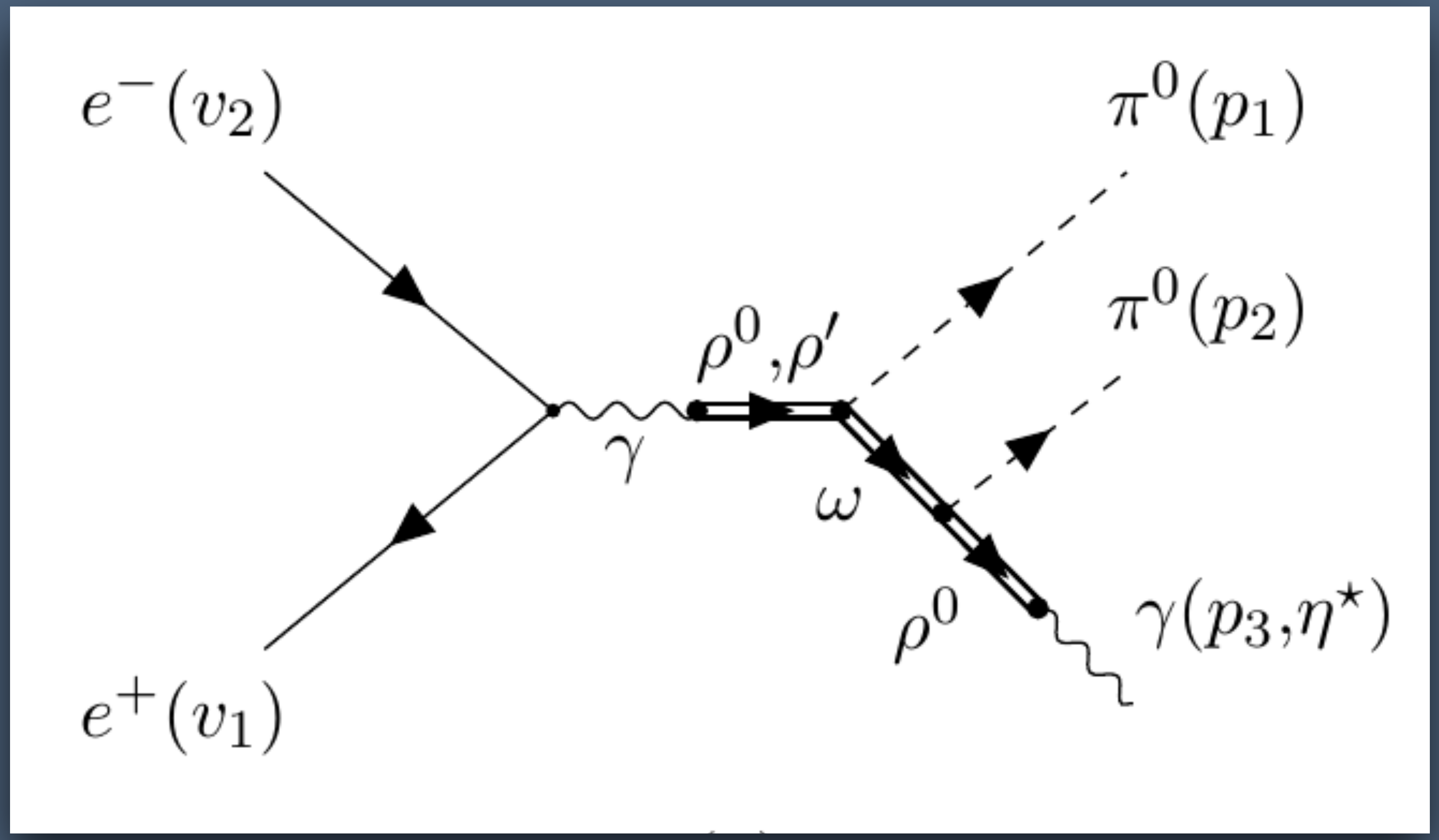
The form factor $f_+[t]$ is obtained from a fit to the two pion invariant mass distribution of the non radiative decay, measured by Belle (Phys. Rev. D 78, 072006 (2008)):

$$f_+[t] = \frac{1}{1 + \beta + \gamma} \left\{ f_\rho[t] + \beta f_{\rho'}[t] + \gamma f_{\rho''}[t] \right\}$$

Where β, γ are numerical values and θ is the relative phase between the ρ and ρ' contribution to the ω channel.

Decay $\tau^- \rightarrow \pi^-\pi^0\nu_\tau\gamma$ through the so-called "ω-channel", which is the dominant channel. Same structure as the scattering process

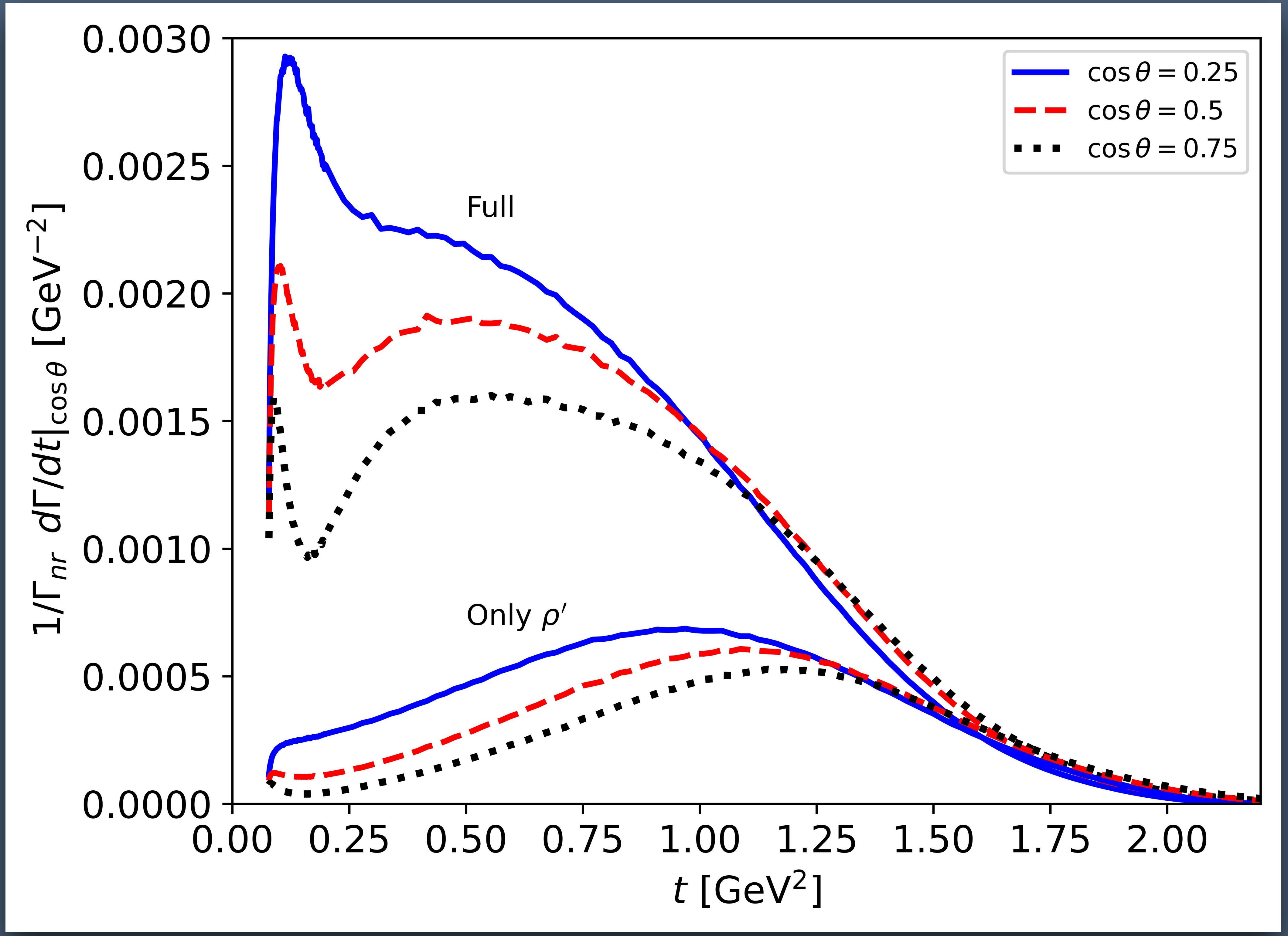




$$\mathscr{M}_\omega = eG_F V^*_{ud} \frac{G_\rho}{\sqrt{2}} \frac{g^2_{\omega\rho\pi} e^{i\theta_w}}{g_\rho m^2_\rho m^2_\omega} f_\omega[r] f_o[t'] \epsilon_{\alpha\sigma\mu}{}^{\lambda}\epsilon_{\phi\lambda\chi}{}^{\nu} k^\sigma p_0^\alpha (p_0+k)^\phi p_-^\chi \epsilon^{*\mu}\ell^\nu$$

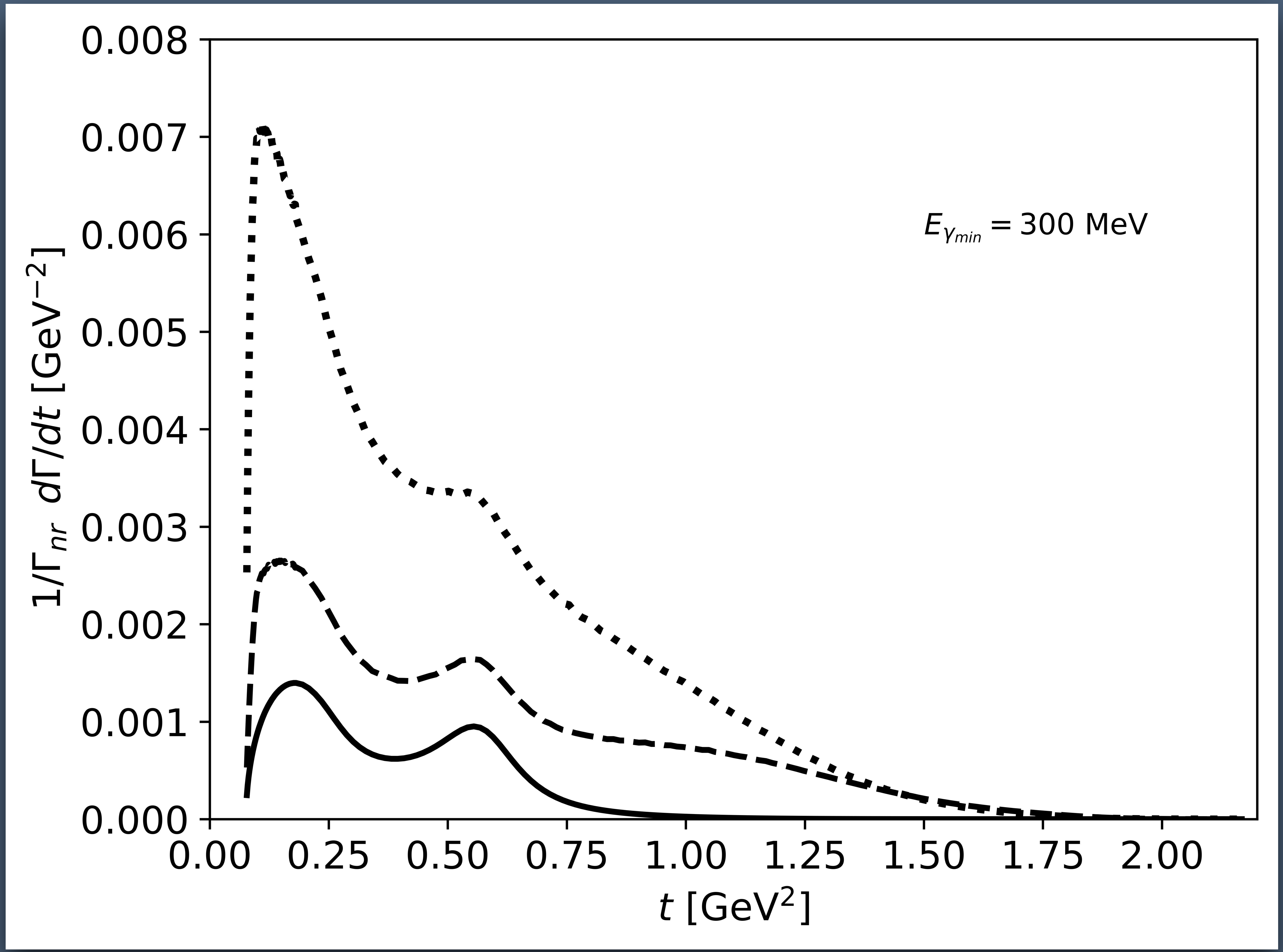
- In this decay, the kinematical variable denoted as t is known as the "dipion invariant mass"

Decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$ through ω-channel (Model Dependent). We study the differential decay rate with respect to the emission angle of π^- and the Mandelstam variable $t = (m_{\pi^-} + m_{\pi^0})^2$.



- Same resonant behavior as in $e^+e^- \to \pi^0\pi^0\gamma$ scattering. Small angles are favored. No experimental info yet.

Differential decay rate (integrated for all angles)

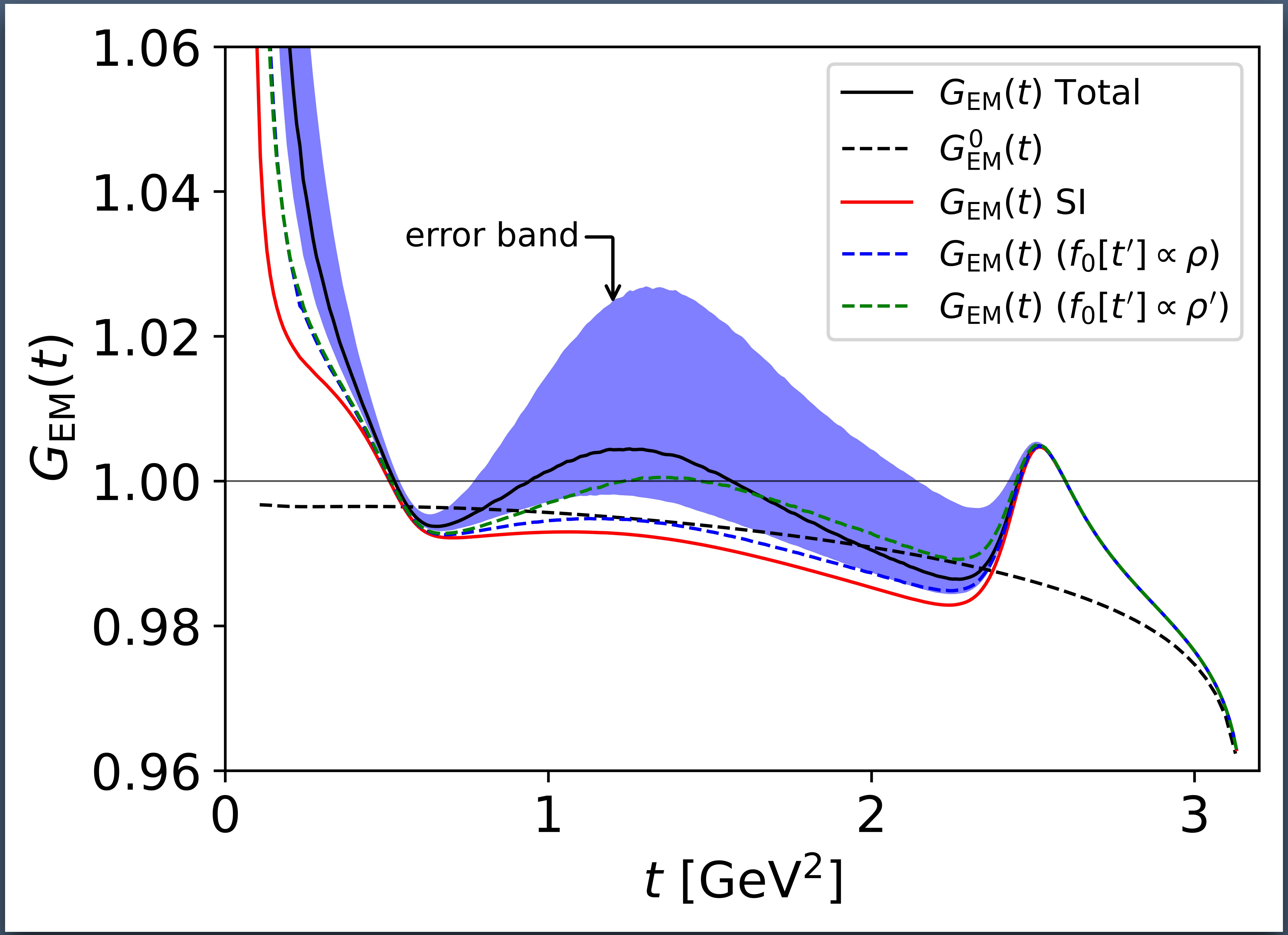


- Dotted line: both ρ, ρ' (SI + interference)
- Dashed line: ρ only (ω channel)
- Solid line: SI only

- Cut off of the photon of 300 MeV such that the photon energy can not go lower than that.

Factor G_{EM}

$$\frac{d\Gamma_{2\pi(\gamma)}}{dt} = \frac{d\Gamma^0_{2\pi}}{dt} G_{EM}(t)$$



- Relation between the radiative and non-radiative decay widths: G_{EM} factor.

Conclusions

- Addition of ρ' is as relevant as the ρ, contrary to previous beliefs.
- Models are sensitive enough to different angles of pion emission.
- Difference in momenta of ρ and ω mesons plays an important role in $e^+e^- \to \pi^0\pi^0\gamma$ scattering.