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Effect of the magnetic dipole moment of the vector meson K^* on the process $e^+ e^- \to K^+ K^- \; 2\pi^0$

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Objective

To study the $e^+e^- \to K^+ K^- 2\pi^0$ process by incorporating the relevant channels to describe the experimental data, and to analyze the possibility of extracting a value for the magnetic dipole moment of the K^* meson.

Motivation

- The magnetic dipole moment (MDM) of vector mesons is a fundamental property of hadrons.
- A vector meson has 3 electromagnetic multipoles: **electric charge**, **magnetic dipole moment**, and **electric quadrupole moment**.
- There are no measurements of the MDM of the K^* meson because they have a very short lifetime ($\approx 10^{-23}$ s).
- An alternative to determine these quantities is to study processes where the electromagnetic interactions is involved, since this coupling gives rise to a multipole structure.
- We are looking for processes in which a pair of vector mesons is produced, similar to the process

$$e^+ e^- \longrightarrow W^+ W^-.$$

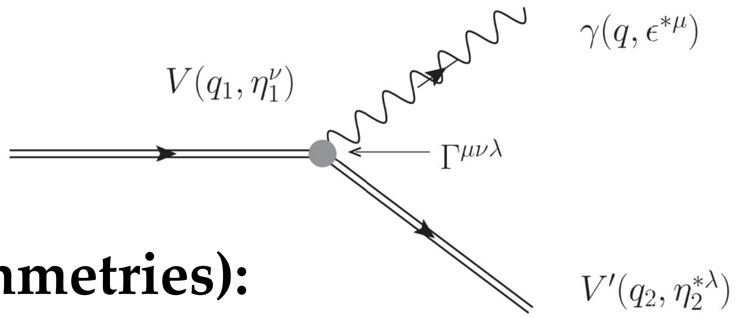
- For the case of the vector meson K^*, we propose the process

$$e^+ e^- \longrightarrow K^+ K^- \ 2\pi^0 \ .$$

The γVV vertex

- The $\Gamma^{\mu\nu\lambda}$ vertex is defined as follows:

$$V'(q_2,\ \eta_2)|J^{\mu}_{EM}(0)|V(q_1,\ \eta_1) \equiv e\Gamma^{\mu\nu\lambda}\eta_{1\nu}\eta^{*}_{2\lambda}$$



- **Parameterization (satisfying C, P and T symmetries):**

$$\begin{aligned}\Gamma^{\mu\nu\lambda} \quad = \quad & \alpha(q^2)g^{\nu\lambda}(q_1+q_2)^{\mu} + \beta(q^2)(g^{\mu\nu}q^{\lambda} - g^{\mu\lambda}q^{\nu}) - q_1^{\lambda}g^{\mu\nu} - q_2^{\nu}g^{\mu\lambda} \\ & -\frac{\zeta(q^2)}{M_V^2}\left[(q_1+q_2)^{\mu}q^{\nu}q^{\lambda} - \frac{1}{2}\left(q^{\nu}g^{\mu\lambda} + q^{\lambda}g^{\mu\nu}\right)(q_1+q_2)\cdot q\right]\end{aligned}$$

(where $\boldsymbol{\alpha(q^2)}$, $\boldsymbol{\beta(q^2)}$ and $\zeta(q^2)$ are the **electromagnetic form factors).**

José F. Nieves y Palash B. Pal: Electromagnetic properties of neutral and charged spin-1 particles. Phys Rev. D 55, 3118–3130, 1997.

- As $q^2 \to 0$, the electromagnetic form factors become the **electromagnetic multipoles**:

$$Q_V = \alpha(0), \quad \text{in units of } e; \quad \mu_V = \beta(0), \quad \text{in units of } \frac{e}{2m_V}$$

$$X_{EV} = 1 - \beta(0) + 2\zeta(0), \quad \text{in units of } \frac{e}{m_V^2}$$

Hagiwara, K., Peccei, R. D. y Zeppenfeld, D., Nucl. Phys. B282 253 (1987).

J. G. Gaemers y G. J. Gounaris: Polarization amplitudes for e+ e− → W +W − and e+ e− → ZZ. Z. Phys. C, 1:259–268, 1979.

MDM of the W and ρ meson

- **This strategy was applied for the *W* boson in good agreement with the experimental results:**

$\lvert Q_W \rvert = 1 \quad (e)$	$\lvert \mu_W \rvert = 2 \quad (e/2m_W)$	$\lvert X_W \rvert = 1 \quad (e/m^2_W)$

- The results predicted by the Standard Model are in complete agreement with those from DELPHI.

J. Abdallah et al. (The DELPHI Collaboration), Measurements of CP-conserving trilinear gauge boson couplings WWV (V ≡ γ, Z) in eþ e− collisions at LEP2, Eur. Phys. J. C 66, 35 (2010).

- **MDM of the ρ meson**. For the ρ meson, previous works involve the process:

$$e^+e^- \to \pi^+\pi^-2\pi^0.$$

(where the $\rho\rho\gamma$ vertex appears).

- This process has been studied in several experiments in the low energy regime, including BaBar.
- The $\gamma^* \to 4\pi$ vertex was described using the VMD model, including the intermediate resonances.

Phys. Rev. D 96, 092009 (2017).

- The MDM of the ρ is given by:

$$2.7 \pm 0.3, \quad \text{in units of} \quad e/2m_\rho.$$

Antonio Rojas y Genaro Toledo. Phys. Rev. D, 110:056037, Sep 2024.

The K^* meson

- **Some properties of the K^* meson:**

$I(J^P)$	Quark structure	Mass	Width of the decay	Branching ratio	Lifetime
½(1⁻).	K^{*+} $(u\bar{s})$	895.5 ± 0.8 MeV.	Γ= 46.2 ± 1.3 MeV	$Br(K^*\rightarrow K\pi)\approx 100$ %.	$\tau \approx 10^{-23}$ s.

S. Navas et al.(Particle Data Group), Phys. Rev. D 110, 030001 (2024)

- **MDM estimates for the K^* meson:**

MDM	Description, Source
$2-0.0047,\ [e/2M_{K^*}]$	Finite width modification, [1]
$2.194\ \mu_N$	Relativistic Hamiltonian, [2]
$2.4\ \mu_N$	Lattice QCD, [2], [3]
$2.08\ \mu_N$	Lattice QCD, [2], [4]
$2.04 \pm 0.4\ [e/2M_{K^*}]$	QCD sum rules, [5]
$2.23\ [e/2M_{K^*}]$	Poincaré covariant formulation, based on the Dyson-Schwinger equations of QCD, [6]
$2.37\ [e/2M_{K^*}]$	Lorentz-covariant, Dyson-Schwinger-equation-based, [7]

[1] David García Gudiño y Genaro Toledo Sánchez: Finite width induced modification to the electromagnetic form factors of spin-1 particles. Phys. Rev. D 81, 073005, 2010.

[2] A: M. Badalian y Yu. A. Simonov. Magnetic moments of mesons. ArXiv:1211.4349v2, 2013.

[3] Frank X. Lee, Scott Moerschbacher, Walter Wilcox. Magnetic moments of vector, axial and tensor mesons in lattice QCD. Phys.Rev.D 78 (2008) 094502 (Lattice QCD).

[4] J. N. Hedditch, W. Kamleh, B. G. Lasscock, D. B. Leinweber, A. G. Williams y J. M. Zanotti: Pseudoscalar and vector meson form factors from lattice QCD. Phys. Rev. D, 75:094504, May 2007.

[5] T.M. Aliev, A. ¨Ozpineci y M. Savci: Magnetic and quadrupole moments of light spin-1 mesons in light cone QCD sum rules. Physics Letters B, 678(5):470–476, 2009, ISSN 0370-2693.

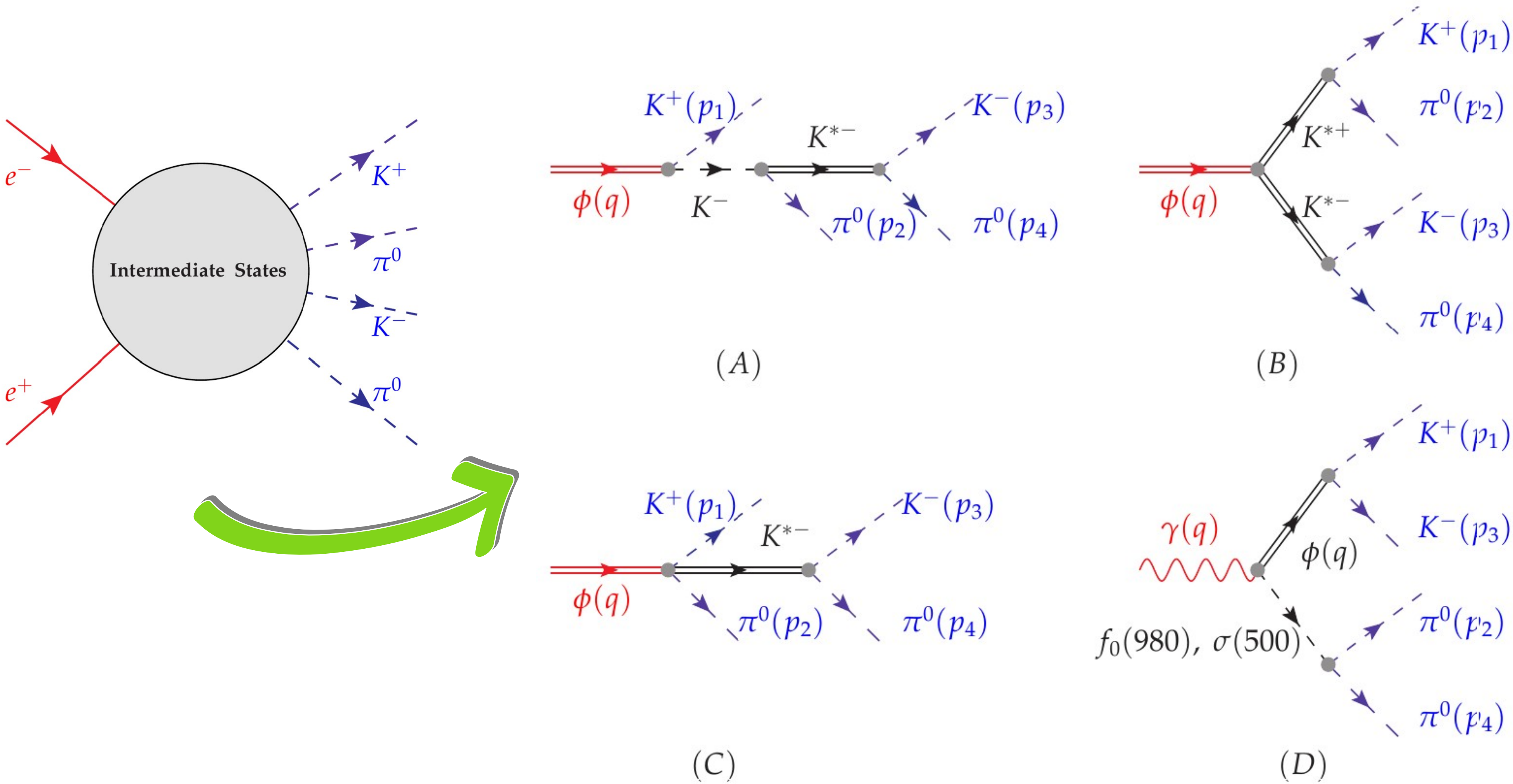
[6] M. S. Bhagwat y P. Maris: Vector meson form factors and their quark-mass dependence. Phys. Rev. C, 77:025203, Feb 2008.

[7] F. T. Hawes y M.A. Pichowsky: Electromagnetic form factors of light vector mesons. Phys. Rev. C59 1743, 1999.22

Description of the $e^+ e^- \to K^+ K^- 2\pi^0$ process

The following process is proposed to study the MDM of the K^* meson:

$$e^+ e^- \to K^+ \; K^- 2\pi^0$$



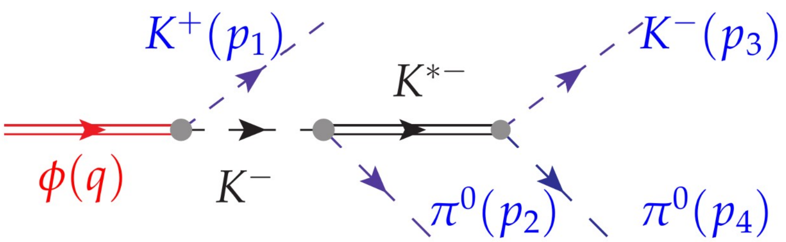
Channel A

- **Effective Lagrangian density:**

$$\mathcal{L} = \sum_V \frac{e\, m_V^2}{G_V} V_\mu A^\mu + G_{VP_1P_2} V_\mu \left(P_1 \partial^\mu P_2 - P_2 \partial^\mu P_1\right).$$

The $\gamma^* \to 2K2\pi$ vertex is modeling in the **VMD** approach.

*Phys. Rev. 157, páginas 1376–1399, 1967.
*Phys. Rev. Lett., 54:1215–1218, Mar 1985.
*Progress of Theoretical Physics, 73(4):926–941, Abril 1985.
*J. J. Sakurai: Theory of Strong Interactions. Ann. Phys. (NY) 11, páginas 1–48, 1960.



- **Amplitude:**

$$\mathcal{M}_A^\beta(p_1,p_2,p_3,p_4) = ieC_A D(K^{*-}, s_{34}) \frac{x_1^\beta}{x_1 \cdot q} r_{43} \cdot z_{12} - \bar{\Delta}(s_{34}) \left(ieC_A D(K^{*-}, s_{34}) \frac{x_1^\beta}{x_1 \cdot q} (z_{12} \cdot s_{34}) \right),$$

$$C_A \equiv \frac{G_{\phi KK} G^2_{K^* K \pi^0}}{G_\phi} D(\phi, q)$$

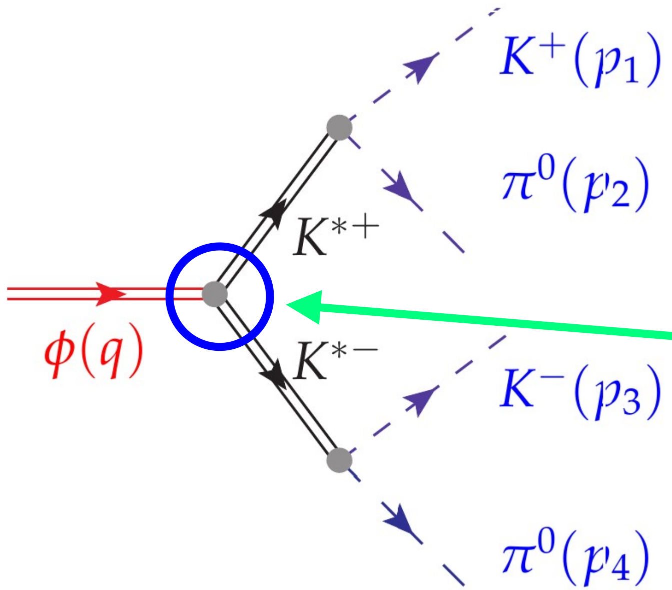
$$\bar{\Delta}(s_{34}) \equiv \frac{\Delta}{dK^*(s_{34})}, \qquad dK^*(s_{34}) \equiv m^2_{K^*} - i m_{K^*} \Gamma_{K^*}(s_{34})$$

$\Delta = m^2_\pi - m^2_K$ includes isospin breaking effects. It generalizes the $\boldsymbol{\rho}$ case.

Channel B

- **Effective Lagrangian density is the same as in the channel A:**
- **Amplitude:**

$$\mathcal{M}^{\beta}(p_1,p_2,p_3,p_4) = \frac{eG_{\phi K^*K^*}G^2_{K^*K\pi}M^2_{\phi}}{G_{\phi}}D^{\beta\gamma}(\phi,q)$$
$$\times r_{21\lambda}D^{\delta\lambda}\left(K^{*+},s_{12}\right)\Gamma^1_{\gamma\delta\epsilon}D^{\epsilon\nu}\left(K^{*-},s_{34}\right)r_{43\nu}$$



$$\Gamma^1_{\gamma\delta\epsilon} \equiv (1+i\gamma_V)\,\Gamma_{\gamma\delta\epsilon}$$

In this diagram we can see the γVV vertex in the VMD approach

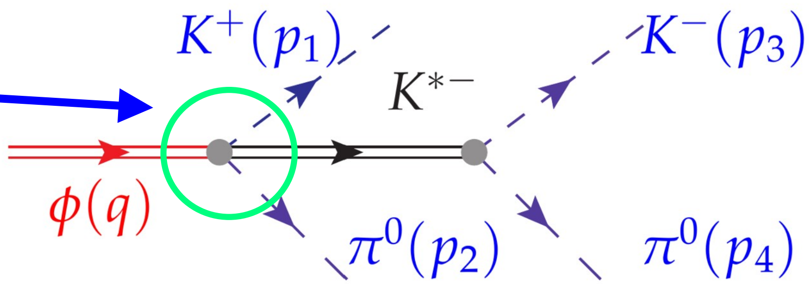
$$\gamma_V \equiv \frac{\Gamma_V}{M_V}$$

$\Gamma^1_{\gamma\delta\epsilon}$ is the vertex function at first order.

Channel C

- **Chanel C:**

Four-particle vertex, $\boldsymbol{\Gamma^{1}{}_{\gamma\delta}}$ (the corresponding coupling constant is $\boldsymbol{G_{\Phi K^{*}K\pi}}$).



- **Amplitude:**

$$\mathcal{M}_{C}^{\beta}(p_1,p_2,p_3,p_4) = ieC_{C}^{\beta\gamma}\Gamma^{1}_{\gamma\delta}D^{\delta\eta}(K^{*-},s_{34})r_{43\eta},$$

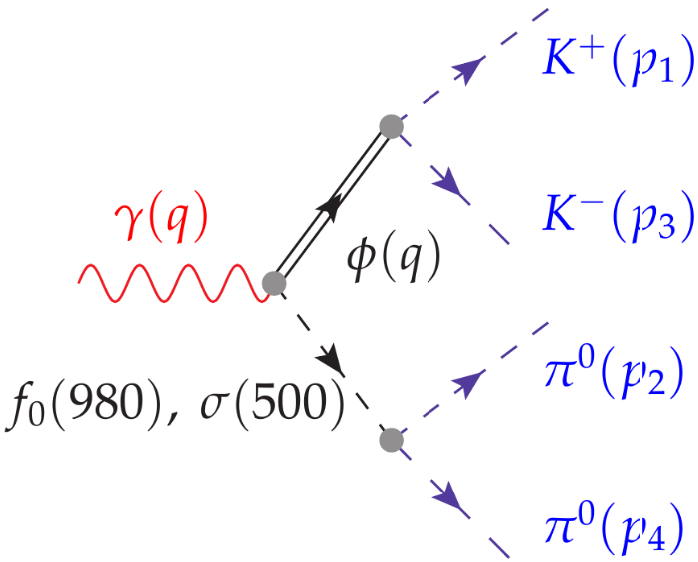
$$C_{C}^{\beta\gamma} = \frac{M_{\phi}^{2}G_{\phi K^{*}K\pi}G_{K^{*}K\pi}}{G_{\phi}}D^{\beta\gamma}(\phi,q).$$

- The effective coupling, $\boldsymbol{G_{\Phi K^{*}K\pi}}$, and the general vertex, $\boldsymbol{\Gamma^{1}{}_{\gamma\delta}}$, are not predicted by the assumed hadronic model.
- However, they are fixed by requiring the gauge invariance to the sum of the A, B and C amplitudes.

Channels D and E

- **Effective Lagrangian density:**

$$\mathcal{L}_S = iG_{VVS}\left(g_{\alpha\beta}g_{\mu\nu}\,\partial^\alpha V_1^\mu\,\partial^\beta V_2^\nu - \partial_\nu V_1^\mu\,\partial_\mu V_2^\nu\right)S + iG_{SPP}\,S\,P_1\,P_2,$$



Amplitude:

$$\mathcal{M}_D^\mu(p_1,p_2,p_3,p_4) = ieC_D D(\phi,s_{13})D(f,s_{24})\frac{\left(q\cdot r_{31}\,s_{13}^\mu - q\cdot s_{13}\,r_{31}^\mu\right)}{q\cdot s_{13}}$$

$$C_D \equiv G_{\phi\gamma f_0}G_{\phi KK}G_{f_0\pi^0\pi^0}$$

- **For these two channels we define the amplitude:**

$$\mathcal{M}_{DE}^\mu = \mathcal{M}_D^\mu + e^{i\theta_1}\mathcal{M}_E^\mu$$

This amplitude is gauge invariance by itself.

Gauge invariance condition

- Symmetries of **BE** and **C** must be taken into account, which are obtained by exchanging the corresponding momenta, which is visible in the dependence of each amplitude. .

- They combine appropriately to produce a **gauge-invariant amplitude**:

$$\mathcal{M}^{\beta}_{ABC_1} = \mathcal{M}^{\beta}_{A}(p_1,p_2,p_3,p_4) + \mathcal{M}^{\beta}_{A}(p_3,p_4,p_1,p_2) + \mathcal{M}^{\beta}_{B}(p_1,p_2,p_3,p_4) + \mathcal{M}^{\beta}_{C}(p_1,p_2,p_3,p_4) + \mathcal{M}^{\beta}_{C}(p_3,p_4,p_1,p_2).$$

$\mathcal{M}_{ABC_2}$ is obtained by $p_2 \leftrightarrow p_4$

- Channels ***A, B, C*** are related by the **gauge invariance condition**:

$$q_\beta \mathcal{M}^{\beta}_{ABC_1} = 0 \quad \text{and} \quad q_\beta \mathcal{M}^{\beta}_{ABC_2} = 0.$$

- This implies that

$$G_{\phi K^* K^*} = G_{\phi KK}.$$

$$\Gamma^1_{\gamma\delta} = \frac{x_{3\gamma}}{x_3 \cdot q}\left(z_{12\delta} - r_{21\delta}\right)$$

$$G_{\phi K^* K\pi} = G_{\phi KK}\, G_{K^* K\pi}$$

- We can define: $\mathcal{M}^{\beta}_{ABC} = \mathcal{M}^{\beta}_{ABC_1} + \mathcal{M}^{\beta}_{ABC_2}$

Total amplitude and cross section

- Total Amplitude:

$$\mathcal{M} = \frac{e}{q^2} l^{\mu} J_{\mu}$$

The leptonic current , l^{μ}, is common to each channel.

- J_{μ} is the electromagnetic hadron current:

$$J_{\mu} = \sum_{\text{channel}} \mathcal{M}^{\mu}_{\text{channel}} = \mathcal{M}^{\mu}_{ABC} + e^{i\theta_2} \mathcal{M}^{\mu}_{DE}$$

J_{μ} fulfills **BE** symmetry and **C** invariance.

- Total amplitude squared was calculated using FeynCalc in Mathematica:

$$\overline{|\mathcal{M}|^2} = \frac{e^2}{q^4} l_{\mu\nu} h^{\mu\nu}$$

- A Fortran subroutine, **Vegas**, was used to perform the integration of the cross section:

$$d\sigma = \frac{(2\pi)^4 \, \overline{|\mathcal{M}|^2}}{4\sqrt{(k_+ \cdot k_-)^2}} \, \delta^4\Big(q - \sum_{i=1}^{n} p_i\Big) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 \, 2E_i}.$$

S. Navas et al.(Particle Data Group), Phys. Rev. D 110, 030001 (2024)

- For the kinematics, the work of R. Kumar is being taken into account.

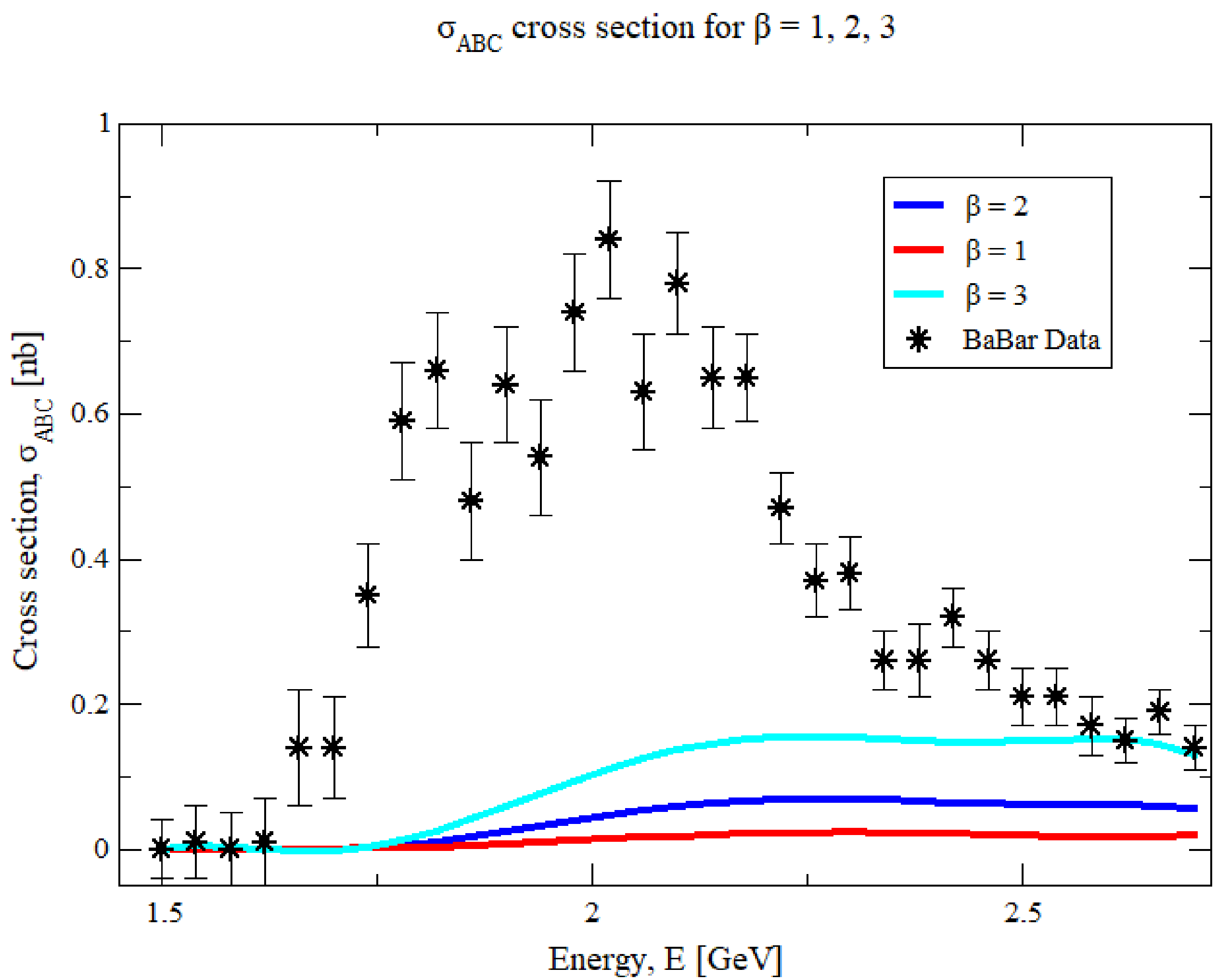
Rajendra Kumar: Covariant Phase-Space Calculations of n-Body Decay and ProductionProcesses. Phys. Rev. 185, páginas 1865–1875, 1969.

Fit to Data

Free parameters fitted with Minuit:

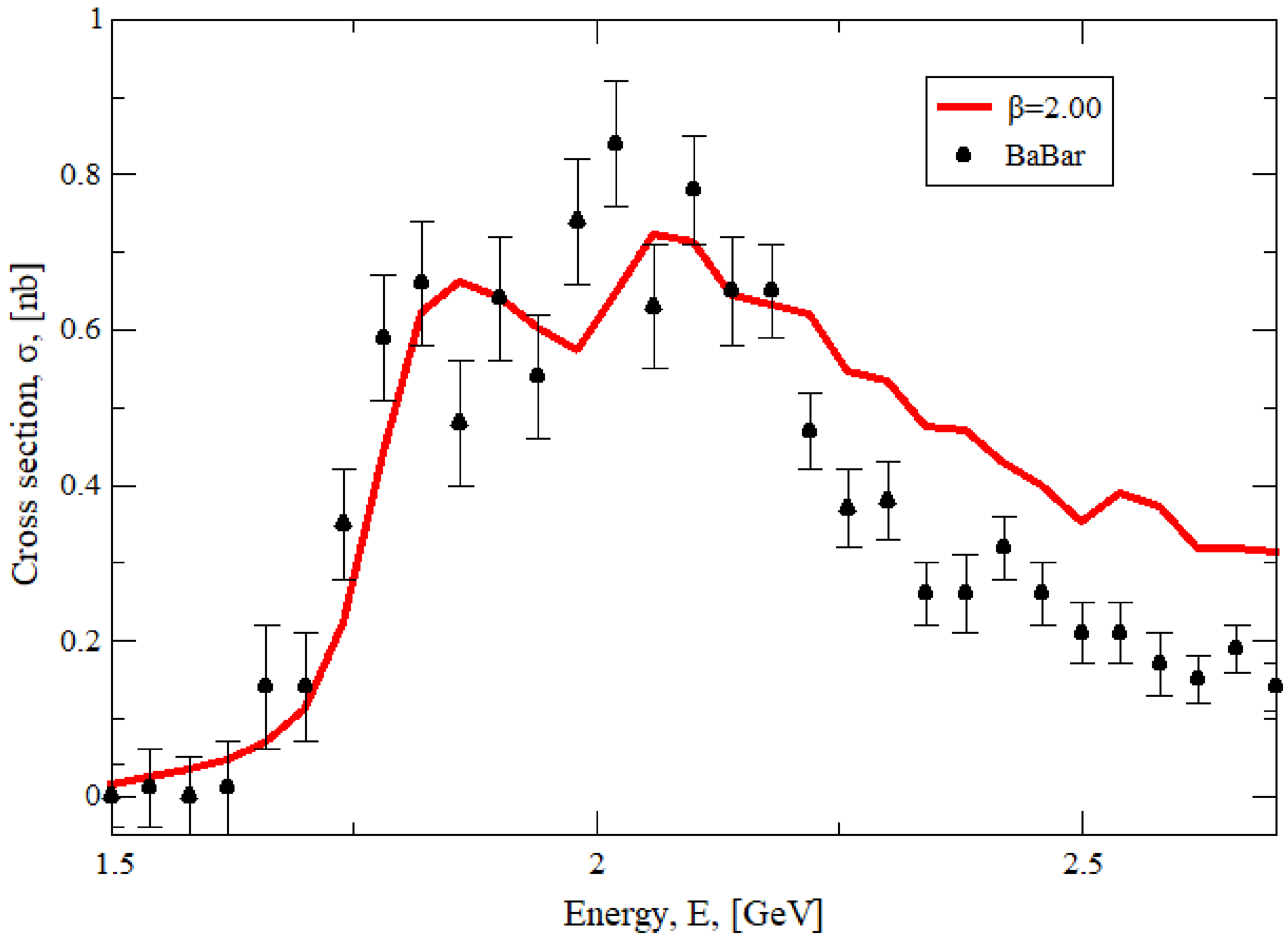
Parameter	Fitted value	PDG or computed
θ_1	2.00 ± 0.29	$-$
m_{f_0}	0.97 ± 0.35 GeV	0.990 ± 0.02 GeV
Γ_{f_0}	$(0.76 \pm 0.34) \times 10^{-2}$ GeV	$0.010 - 0.010$ GeV
m_σ	0.71 ± 0.43 GeV	$0.400 - 0.800$ GeV
Γ_σ	0.60×10^{-1} GeV	$0.100 - 0.800$ GeV
$G_{f_0\pi^0\pi^0}$	2.42 ± 0.79	1.23 ± 0.65 GeV
$G_\sigma = G_{\phi\gamma\sigma}G_{\sigma\pi^0\pi^0}$	1.2567 ± 0.64503	$0.45^{+0.78}_{-0.31}$ GeV^2
β	2.00 ± 0.73	$-$
θ_2	2.54 ± 0.48	$-$

Preliminary results

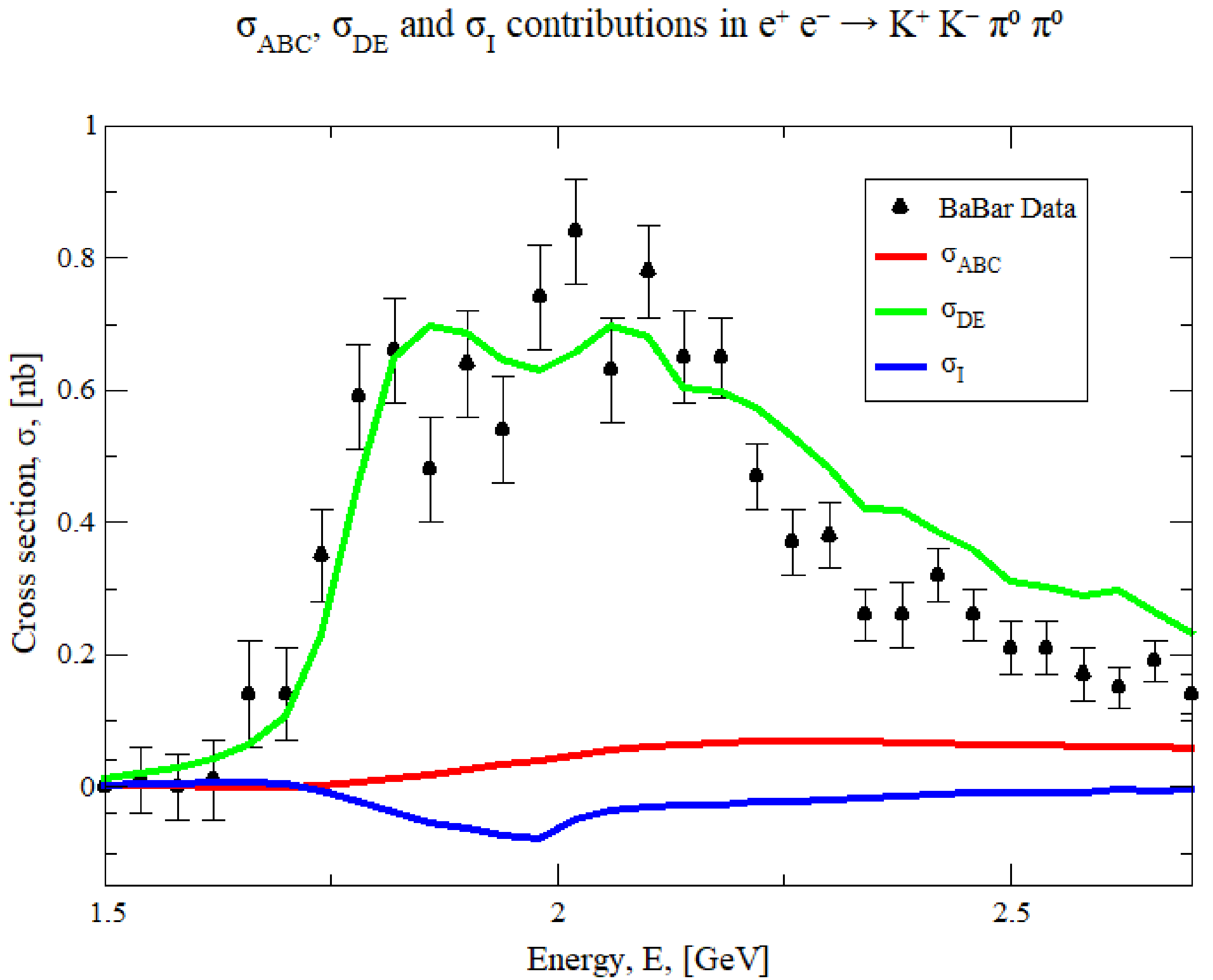


Preliminary results

Experimental and Theoretical Cross section for the $e^+ e^- \to K^+ K^- \pi^0 \pi^0$ process

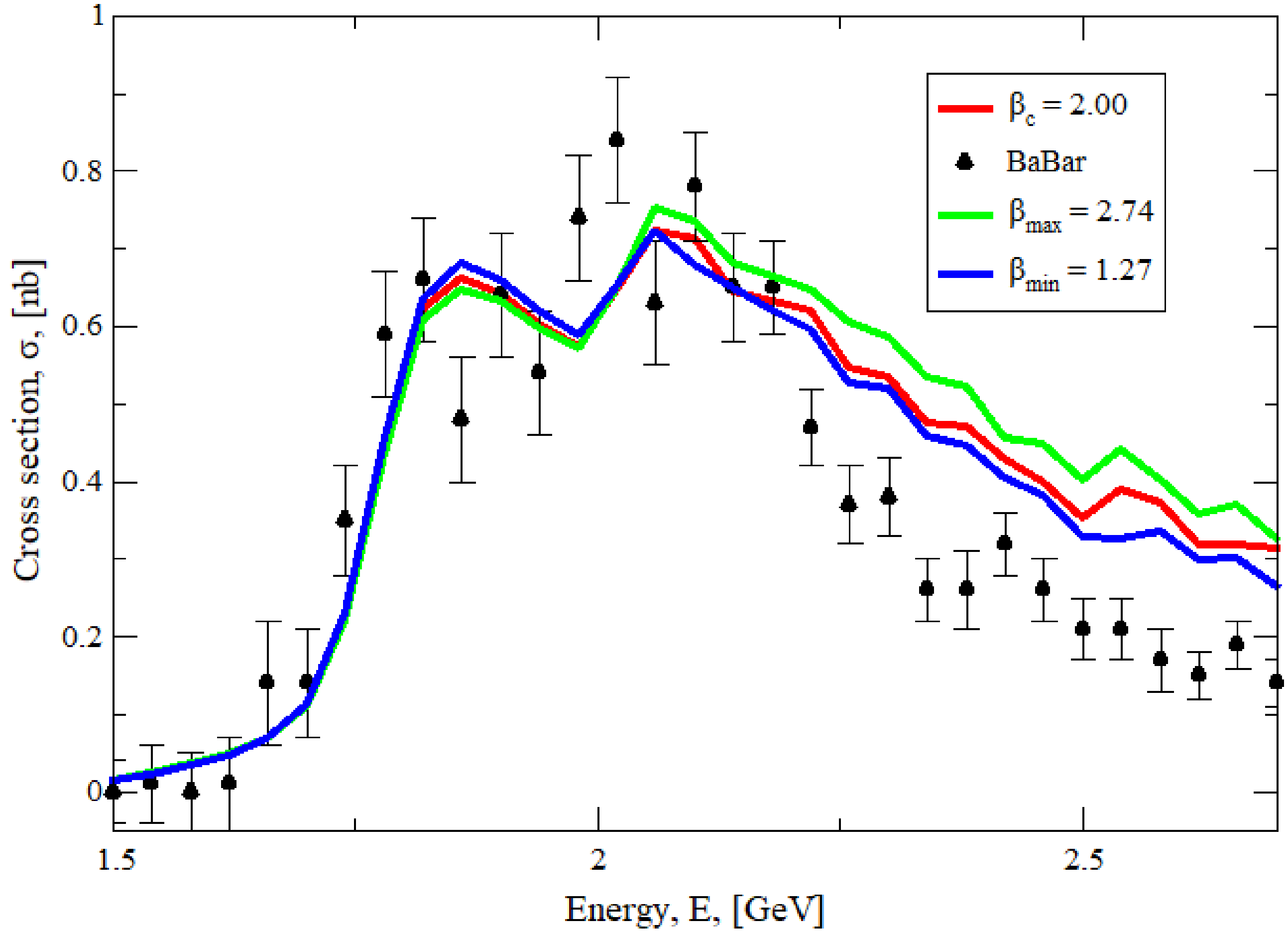


Preliminary results



Preliminary results

Cross sections of $e^+ e^- \to K^+ K^- \pi^0 \pi^0$ for β_{min}, β_c, β_{max}



Electric Form Factor

- Electric form factor as $q^2 \to 0$,

$$\left|F_\phi(0)\right| = \lim_{q^2\to 0}\left|\frac{G_{\phi KK}\,M_\phi^2}{G_\phi} D\left(\phi, q^2\right)\right| = 1.$$

- In this work,

$$\left|F_\phi^{exp}(0)\right| = 0.33.$$

- We need to normalize so that the electric charge has the correct value in the correct units,

$$F(0) \equiv \frac{F_\phi(0)}{F_\phi^{exp}(0)} = 1$$

- The correct expression to compute the mdm of the K^*:

$$\mu_{K^*} = \frac{\beta_{final}}{F_\phi^{exp}(0)} = 6.1 \pm 2.21.$$

The systematic error is still being evaluated.

where

$$\beta_{final} = \beta \pm \Delta\beta_{stat} \pm \Delta\beta_{sys}.$$

Conclusions

- A theoretical description of the process $e^+ e^- \rightarrow K^+ K^- 2\pi^0$ was developed within the Vector Meson Dominance (VMD) framework.

- The channel *B* involves the γVV vertex. This vertex contains the multipole structure of the meson.

- Channel *C* involves contact vertices and coupling constants. These quantities were fixed by requiring gauge invariance among the amplitudes of channels *A*, *B*, and *C*.

- The free parameters of the model were fitted to BaBar data using MINUIT, achieving a good agreement in the 1.5 – 2.1 GeV energy range.

- A sensitivity of the total cross section to variations of β is observed.

Thank you for your attention.

BACKUP

MDM of the W and ρ meson

- **This strategy was applied for the *W* boson in good agreement with the experimental results:**

$\lvert Q_W \rvert$	$\lvert \mu_W \rvert$	$\lvert X_W \rvert = \kappa_\gamma - \lambda_\gamma$
1 (e)	2 $(e/2m_W)$	1 (e/m^2_W)

The SM values imply that $\kappa_\gamma = 1, \lambda_\gamma = 0$.

- **DELPHI** found:

$$\lambda_\gamma = 0.002^{+0.035}_{-0.035}$$

$$\Delta\kappa_\gamma = 0.024^{+0.077}_{-0.081}$$

J. Abdallah et al. (The DELPHI Collaboration), Measurements of CP-conserving trilinear gauge boson couplings WWV (V ≡ γ, Z) in eþ e− collisions at LEP2, Eur. Phys. J. C 66, 35 (2010).

- **MDM of the ρ meson**. For the ρ meson, previous works involve the process:

$$e^+e^- \to \pi^+\pi^- 2\pi^0.$$

(where the $\rho\rho\gamma$ vertex appears).

- This process has been studied in several experiments in the low energy regime, including BaBar. Phys. Rev. D 96, 092009 (2017).
- The $\gamma^* \to 4\pi$ vertex was described using the VMD model, including the intermediate resonances.
- The MDM of the ρ is given by:

$$2.7 \pm 0.3, \quad \text{in units of} \quad e/2m_\rho.$$

Antonio Rojas y Genaro Toledo. Phys. Rev. D, 110:056037, Sep 2024.

Coupling constants and Fit to Data

Coupling Constant	Value	Obtained from:
$G_{\phi KK}$	4.47 ± 0.03	$\phi \to K^+K^-$
$G_{\phi K^*K^*}$	4.47 ± 0.03	$= G_{\phi KK}$ (from G I)
$G_{K^*K\pi}$	3.05 ± 0.05	$K^* \to K^+\pi^0$
$G_{\phi K^*K\pi}$	13.66 ± 0.33	$= G_{\phi KK}G^2_{K^*K\pi}$ (from G I)
G_ϕ	13.40 ± 0.09	$\phi \to e^+e^-$
$G_{\phi\gamma f_0}$	$0.14^{+0.12}_{-0.035}$ GeV	$\phi \to \gamma f_0$
$G_{\phi\gamma\sigma}$	$0.14^{+0.12}_{-0.035}$ GeV	$= G_{\phi\gamma f_0}$
$G_{f_0\pi\pi}$	$1.38^{+0.50}_{-0.80}$ GeV	$f_0 \to \pi^0\pi^0$
$G_{\sigma\pi\pi}$	$3.18^{+1.59}_{-1.84}$ GeV	$\sigma \to \pi^0\pi^0$

S. Navas et al.(Particle Data Group), Phys. Rev. D 110, 030001 (2024)

The full decay width of f_0 is used when calculating $f_0 \to \pi^0\,\pi^0$, because there are no branching ratio available for this process. The same for $\sigma \to \pi^0\,\pi^0$.

In addition, there is no branching ratio for $\phi \to \gamma\sigma$, but an approximation is to take:

$$G_{\phi\gamma\sigma} = G_{\phi\gamma f}.$$

Results of the fit with MINUIT

Parameter	**Fitted value**	**PDG or computed**
θ_{D-E}	2.00 ± 0.29	–
m_{f_0}	0.97 ± 0.35 GeV	0.990 ± 0.02 GeV
Γ_{f_0}	$(0.76 \pm 0.34) \times 10^{-2}$ GeV	$0.010 - 0.010$ GeV
m_σ	0.71 ± 0.43 GeV	$0.400 - 0.800$ GeV
Γ_σ	0.60×10^{-1} GeV	$0.100 - 0.800$ GeV
$G_{f_0\pi^0\pi^0}$	2.42 ± 0.79	1.23 ± 0.65 GeV
$G_\sigma = G_{\phi\gamma\sigma}G_{\sigma\pi^0\pi^0}$	1.2567 ± 0.64503	$0.45^{+0.78}_{-0.31}$ GeV^2
β	2.00 ± 0.73	–
θ_{ABC-DE}	2.54 ± 0.48	–

However, **some coupling constants were left as free parameters**:

$G_{f\pi\pi}$, $G_{\sigma\pi\pi}$ and $G_{\phi\gamma\sigma}$.

Other parameters are masses, widths, phases and the parameter β.

Conclusions

- A theoretical description of the process $e^+ e^- \rightarrow K^+ K^- 2\pi^0$ was developed within the Vector Meson Dominance (VMD) framework, including the relevant intermediate channels and their interferences.
- The effective amplitudes for each channel were constructed from effective hadronic Lagrangians.
- The channel B involves the γVV vertex, which is parameterized in a general form that satisfies C, P, and T symmetries and their combinations. This vertex contains the multipole structure of the meson.
- Channel *C* involves contact vertices and coupling constants that cannot be predicted by the assumed hadronic model. These quantities were fixed by requiring gauge invariance among the amplitudes of channels *A*, *B*, and *C*.
- Some coupling constants associated with the decays $\boldsymbol{f_0 \rightarrow \pi^0 \pi^0}$, $\boldsymbol{\phi \rightarrow \gamma\sigma}$ and $\boldsymbol{\sigma \rightarrow \pi^0 \pi^0}$ cannot be calculated, since no branching ratios are available. Therefore, they were treated as free parameters and determined from a fit to the experimental data.
- The free parameters of the model were fitted to BaBar data using MINUIT, achieving a good agreement in the 1.5 – 2.1 GeV energy range.
- A sensitivity of the total cross section to variations of β is observed.
- A preliminary value for the magnetic dipole moment (MDM) of the *K** meson has been estimated. Although this result does not agree with previous estimations, further fits using MINUIT are being carried out to obtain the final value.