Propagators for Negative-energy and Tachyonic Solutions in Relativistic Equations*

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Abstract

It is well known that the relativistic equations have acausal solutions, which have generally been ignored. This is particularly true for higher spins. We consider spin 1/2 and spin 1 in this talk. We analyze corresponding propagators which may indicate if a theory is local or non-local. Negative-energy and tachyonic solutions are also considered. The conclusions are paradoxical in both spins.

The algebraic characteristic equations of the Dirac equation are $Det(\hat{p}-m)=0$ and $Det(\hat{p}+m)=0$, $\hat{p}=p^\mu\gamma_\mu$ for $u-$ and $v-$ 4-spinors of the spin-1/2. They have solutions with $p_0=\pm E_p=\pm\sqrt{\mathbf{p}^2+m^2}$. The recent problems of superluminal neutrinos, negative-mass squared neutrinos, various schemes of oscillations including sterile neutrinos, require attention. Recently, the concept of the *bi-orthonormality* has been proposed; the (anti) commutation relations and statistics are assumed to be different for *neutral* particles. Next, Sakharov in 1967, Ref. [1], introduced the idea of two universes with opposite arrows of time, born from the same initial singularity (i.e. Big Bang). Next, Debergh et al. constructed (within the framework of the present-day quantum field theory) negative-energy fields for spin-1/2 fermions, Ref. [2]. Currently, the predominating consensus is the existence of dark matter (DM) and dark energy (DE) paradigms. Possible particle candidates have been proposed for the DM, but to date, the search for these candidates has not been successful. This suggests that something was missing in the foundations of relativistic quantum theories. Modifications appear to be necessary in the Dirac sea concept, and in the even more sophisticated Stueckelberg concept of backward propagation in time. The

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Dirac sea concept is intrinsically related to the Pauli principle. However, the Pauli principle is intrinsically related to the Fermi statistics and the anticommutation relations of fermions. We propose relevant modifications in the basics of relativistic quantum theory below.

The general scheme for construction of the field operator has been presented in [3]. In the case of the $(1/2,0)\oplus(0,1/2)$ representation we have:

$$\begin{aligned}
&\Psi(x) = \frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2-m^2)e^{-ip\cdot x}\Psi(p) = \\
= &\frac{1}{(2\pi)^3}\sum_h \int d^4p\,\delta(p_0^2-E_p^2)e^{-ip\cdot x}u_h(p_0,\mathbf{p})a_h(p_0,\mathbf{p}) = \qquad (1)\\
= &\frac{1}{(2\pi)^3}\sum_h \int \frac{d^3\mathbf{p}}{2E_p}\theta(p_0) \\
&\left[u_h(p)a_h(p)|_{p_0=E_p}e^{-i(E_pt-\mathbf{p}\cdot\mathbf{x})} + u_h(-p)a_h(-p)|_{p_0=E_p}e^{+i(E_pt-\mathbf{p}\cdot\mathbf{x})}\right],
\end{aligned}$$

where $a_h, b_h^\dagger$ are the annihilation/creation operators, and in textbook cases

$$u_h(\mathbf{p}) = \begin{pmatrix}\exp(+\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}/2)\phi_R^h(\mathbf{0})\\ \exp(-\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}/2)\phi_L^h(\mathbf{0})\end{pmatrix} \qquad (2)$$

$cosh(\varphi) = E_p/m$, $sinh(\varphi) = |\mathbf{p}|/m$. During the calculations above we had to represent $1 = \theta(p_0) + \theta(-p_0)$ in order to get positive- and negative-frequency parts [4]. In the Dirac case we should assume the following relation in the field operator:

$$\sum_h v_h(p)b_h^\dagger(p) = \sum_h u_h(-p)a_h(-p)\,. \qquad (3)$$

We need $\Lambda_{\mu\lambda}(p) = \bar{v}_\mu(p)u_\lambda(-p)$. By direct calculations, we find

$$-mb_\mu^\dagger(p) = \sum_\lambda \Lambda_{\mu\lambda}(p)a_\lambda(-p)\,. \qquad (4)$$

Hence, $\Lambda_{\mu\lambda} = -im(\boldsymbol{\sigma}\cdot\mathbf{n})_{\mu\lambda}$, $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$. In the $(1,0)\oplus(0,1)$ representation a similar procedure leads to a different situation:

$$a_\mu(p) = [1-2(\mathbf{S}\cdot\mathbf{n})^2]_{\mu\lambda}a_\lambda(-p)\,. \qquad (5)$$

This signifies that in order to construct the Sankaranarayanan-Good field operator [5] to satisfy $[\gamma_{\mu\nu}\partial_\mu\partial_\nu - \frac{(i\partial/\partial t)}{E}m^2]\Psi(x) = 0$, we need additional postulates.

We have, in fact, $u_h(E_p,\mathbf{p})$ and $u_h(-E_p,\mathbf{p})$ originally, which satisfy the equations: $\left[E_p(\pm\gamma^0) - \boldsymbol{\gamma}\cdot\mathbf{p} - m\right]u_h(\pm E_p,\mathbf{p}) = 0\,.$ Due to the properties

$U^\dagger\gamma^0 U = -\gamma^0$, $U^\dagger\gamma^i U = +\gamma^i$ with the unitary matrix $U = \gamma^0\gamma^5$ we have in the negative-energy case: $\left[E_p\gamma^0 - \boldsymbol{\gamma}\cdot\mathbf{p} - m\right] U^\dagger u_h(-E_p,\mathbf{p}) = 0$. The explicit forms $\gamma^5\gamma^0 u(-E_p,\mathbf{p})$ are different from the textbook "positive-energy" Dirac spinors. After the space inversion operation, we have ($R = (\mathbf{x}\to-\mathbf{x},\mathbf{p}\to-\mathbf{p})$)

$$PR\tilde{u}(p) = PR\gamma^5\gamma^0 u_\uparrow(-E_p,\mathbf{p}) = -\tilde{u}(p)\,, \qquad (6)$$

$$PR\tilde{\tilde{u}}(p) = PR\gamma^5\gamma^0 u_\downarrow(-E_p,\mathbf{p}) = -\tilde{\tilde{u}}(p)\,. \qquad (7)$$

Similar formulations have been presented in Refs. [6] and [7]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [8], who first presented the theory in the 2-dimensional representation of the inversion group in 1956 (later called "the Bargmann-Wightman-Wigner-type quantum field theory" in 1993). Barut and Ziino [7] proposed yet another model. They considered the γ^5 operator to be the operator of charge conjugation. Thus, the charge-conjugated Dirac equation has a different sign compared to the ordinary formulation:

$$[i\gamma^\mu\partial_\mu + m]\Psi^c_{BZ} = 0\,, \qquad (8)$$

and the charge conjugation so defined applies to the whole system, fermion+electromagnetic field, $e\to -e$ in the covariant derivative. The superpositions of the Ψ_{BZ} and Ψ^c_{BZ} also give us the "doubled Dirac equation", the equations for $\lambda-$ and $\rho-$ self/anti-self charge conjugate spinors. The concept of doubling the Fock space has been developed in the Ziino program (cf. Refs. [8, 9]) within the framework of the quantum field theory. In the BZ case the charge conjugate states are simultaneously the eigenstates of the chirality. Here, the relevant paper is Ref. [10]. It is straightforward to merge $u(\mathbf{p})$ and $v(\mathbf{p})$ spinors in one doublet of "positive energy" and $v(\mathbf{p})$ and $u(\mathbf{p})$ spinors, in another doublet of "negative energy" , as Markov and Fabbri did. However, the point of my paper is that both $u(p_0,\mathbf{p})$ and $v(p_0,\mathbf{p})$ contains contributions to both positive- and negative- energies, cf. Ref. [11].

We study the problem of construction of causal propagators in spin $S = 1/2$ and higher-spin theories. The hypothesis is: in order to construct analogues of the Feynman-Dyson propagator we actually need four field operators connected by the dual and parity transformation. We use the standard methods of quantum field theory. Thus, the number of components in the *causal* propagators is enlarged accordingly. The conclusion under discussion is that if we did not expand the number of components in the fields (in the propagator) we would not be able to obtain the causal propagator.

According to the Feynman-Dyson-Stueckelberg conception, the $S = 1/2$ causal propagator S_F has to be constructed on using the formula (*e.g.*, Ref. [12, p.91])

$$S_F(x_2, x_1) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \left[\theta(t_2 - t_1)\, a\; u^\sigma(p) \overline{u}^\sigma(p) e^{-ip\cdot x} + \right.$$
$$\left. + \theta(t_1 - t_2)\, b\; v^\sigma(p) \overline{v}^\sigma(p) e^{ip\cdot x} \right], \tag{9}$$

where $x = x_2 - x_1$. In the spin $S = 1/2$ Dirac theory, it results in

$$S_F(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{\hat{p} + m}{p^2 - m^2 + i\epsilon}, \tag{10}$$

where $a = -b = 1/i$, ϵ defines the rules of work near the poles.

However, attempts to construct the causal covariant propagator in this way failed in the framework of the Weinberg theory, Ref. [13], which is a generalization of Dirac's ideas to higher spins. The propagator proposed in Ref. [14] is the causal propagator. However, the old problem remains: the Feynman-Dyson propagator is not the Green function of the Weinberg equation. As mentioned, the covariant propagator proposed by Weinberg propagates kinematically spurious solutions [14]. We construct the propagator in the framework of the model given in Ref. [9]. The concept of the Weinberg field *doubles* has been proposed there. For the functions $\psi_1^{(1)}$ and $\psi_2^{(1)}$, connected with the former by the dual (chiral, $\gamma_5 = diag(1_{3\times 3}), -1_{3\times 3})$) transformation, the equations are[1]

$$(\gamma_{\mu\nu} p_\mu p_\nu + m^2)\psi_1^{(1)} = 0, \tag{11}$$
$$(\gamma_{\mu\nu} p_\mu p_\nu - m^2)\psi_2^{(1)} = 0, \tag{12}$$

with $\mu, \nu = 1, 2, 3, 4$. For the field functions connected with $\psi_1^{(1)}$ and $\psi_2^{(1)}$ by the $\gamma_5\gamma_{44}$ transformations the set of equations is written:

$$\left[\widetilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2\right] \psi_1^{(2)} = 0, \tag{13}$$
$$\left[\widetilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2\right] \psi_2^{(2)} = 0, \tag{14}$$

where $\widetilde{\gamma}_{\mu\nu} = \gamma_{44}\gamma_{\mu\nu}\gamma_{44}$ is connected with the $S = 1$ Barut-Muzinich-Williams $\gamma_{\mu\nu}$ matrices [16]. In the cited paper I have used the plane-wave expansion. Thus, $u_1^{(2)}(\mathbf{p}) = \gamma_5\gamma_{44} u_1^{(1)}(\mathbf{p})$, $\overline{u}_1^{(2)} = \overline{u}_1^{(1)}\gamma_5\gamma_{44}$, $u_2^{(2)}(\mathbf{p}) =$

[1] I have to use the Euclidean metrics here in order a reader to be able to compare the formalism with the classical cited works.

$\gamma_5\gamma_{44}\gamma_5 u_1^{(1)}(\mathbf{p})$ and $\overline{u}_2^{(2)}(\mathbf{p}) = -\overline{u}_1^{(1)}\gamma_{44}$. Now we check whether the sum of the four equations

$$
\begin{aligned}
&[\gamma_{\mu\nu}\partial_\mu\partial_\nu - m^2] *\\
&* \int \frac{d^3p}{(2\pi)^3 2E_p}\left[\theta(t_2-t_1)\, a\;\; u_1^{\sigma\,(1)}(p)\overline{u}_1^{\sigma\,(1)}(p)e^{ip\cdot x} + \theta(t_1-t_2)\, b\;\; v_1^{\sigma\,(1)}(p)\overline{v}_1^{\sigma\,(1)}(p)e^{-ip\cdot x}\right] +\\
&[\gamma_{\mu\nu}\partial_\mu\partial_\nu + m^2] *\\
&* \int \frac{d^3p}{(2\pi)^3 2E_p}\left[\theta(t_2-t_1)\, a\;\; u_2^{\sigma\,(1)}(p)\overline{u}_2^{\sigma\,(1)}(p)e^{ip\cdot x} + \theta(t_1-t_2)\, b\;\; v_2^{\sigma\,(1)}(p)\overline{v}_2^{\sigma\,(1)}(p)e^{-ip\cdot x}\right] +\\
&[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu + m^2] *\\
&* \int \frac{d^3p}{(2\pi)^3 2E_p}\left[\theta(t_2-t_1)\, a\;\; u_1^{\sigma\,(2)}(p)\overline{u}_1^{\sigma\,(2)}(p)e^{ip\cdot x} + \theta(t_1-t_2)\, b\;\; v_1^{\sigma\,(2)}(p)\overline{v}_1^{\sigma\,(2)}(p)e^{-ip\cdot x}\right] +\\
&[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu - m^2] *\\
&* \int \frac{d^3p}{(2\pi)^3 2E_p}\left[\theta(t_2-t_1)\, a\;\; u_2^{\sigma\,(2)}(p)\overline{u}_2^{\sigma\,(2)}(p)e^{ip\cdot x} + \theta(t_1-t_2)\, b v_2^{\sigma\,(2)}(p)\overline{v}_2^{\sigma\,(2)}(p)e^{-i\cdot px}\right]\\
&= \quad \delta^{(4)}(x_2-x_1) \qquad (15)
\end{aligned}
$$

can be satisfied by a definite choice of a and b. Simple calculations give

$$
\begin{aligned}
&\partial_\mu\partial_\nu\left[a\,\theta(t_2-t_1)\, e^{ip(x_2-x_1)} + b\,\theta(t_1-t_2)\, e^{-ip(x_2-x_1)}\right] =\\
&-\left[a\, p_\mu p_\nu \theta(t_2-t_1)\exp\left[ip(x_2-x_1)\right] + b\, p_\mu p_\nu \theta(t_1-t_2)\exp\left[-ip(x_2-x_1)\right]\right]\\
&+a\left[-\delta_{\mu 4}\delta_{\nu 4}\delta\,'(t_2-t_1) + i(p_\mu\delta_{\nu 4} + p_\nu\delta_{\mu 4})\delta(t_2-t_1)\right]\\
&\exp\left[i\mathbf{p}\cdot(\mathbf{x}_2-\mathbf{x}_1)\right] + b\;\left[\delta_{\mu 4}\delta_{\nu 4}\delta\,'(t_2-t_1) +\right.\\
&\left. i(p_\mu\delta_{\nu 4} + p_\nu\delta_{\mu 4})\delta(t_2-t_1)\right]\exp\left[-i\mathbf{p}(\mathbf{x}_2-\mathbf{x}_1)\right]\;; \qquad (16)
\end{aligned}
$$

We conclude as follows: the generalization of the notion of causal propagators is admitted by the use of the Wick-like formula for the time-ordered particle operators provided that $a = b = 1/4im^2$. It is necessary to consider all four equations, Eqs. (11)-(14). Obviously, this is related to the 12-component formalism, which I presented in Ref. [9].

Meanwhile, I propose to use the 8-component (or 16-component) spin-1/2 formalism in similarity with the 12-component formalism of this discussion. If we calculate

$$
\begin{aligned}
S_F^{(+,-)}(x_2,x_1) = &\int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\left[\theta(t_2-t_1)\, a\;\; \Psi^\sigma_\pm(p)\overline{\Psi}^\sigma_\pm(p)e^{-ip\cdot x}\right.\\
&\left. + \theta(t_1-t_2)\, b\;\; \Psi^\sigma_\mp(p)\overline{\Psi}^\sigma_\mp(p)e^{ip\cdot x}\right] = \int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\frac{(\hat{p}\pm m)}{p^2-m^2+i\epsilon}, \qquad (17)
\end{aligned}
$$

(with Ψ doublets in the field operator) we readily come to the result that the corresponding Feynman-Dyson propagator gives the local theory in the sense:

$$\sum_{\pm}[i\Gamma_\mu\partial_2^\mu \mp m]S_F^{(+,-)}(x_2-x_1) = \delta^{(4)}(x_2-x_1), \qquad (18)$$

even in the case of self/anti-self charge conjugate states. [2]

We should use the set of Weinberg propagators obtained in the perturbation calculus of scattering amplitudes. In Ref. [17] the amplitude for the interaction of two $2(2S+1)$ bosons has been obtained on the basis of the use of one field only, and it is obviously incomplete, see also Ref. [16]. But, it is interesting to note that the spin structure was proved there to be the same, whether we consider the two-Dirac-fermion interaction or the two-Weinberg $S=1$-boson interaction. However, the denominator differs slightly ($1/\vec{\Delta}^2 \to 1/2m(\Delta_0 - m)$) from the fermion-fermion case in the cited papers [17], where $\Delta_0, \vec{\Delta}$ is the momentum-transfer 4-vector in Lobachevsky space. More accurate considerations of the fermion-boson and boson-boson interactions in the framework of the Weinberg theory have been reported elsewhere, Ref. [18]. So, the conclusion is: one can construct analogs of the Feynman-Dyson propagators for the $2(2S+1)$ model and, hence also local theories, provided that the Weinberg states are quadrupled ($S=1$ case), and the neutral particle states are doubled.

What is the physical sense of the mathematical formalism presented here? In the $S=1$ Weinberg equation [13] we have 12 solutions.[3] Apart from $p_0 = \pm E_p$ we have tachyonic solutions $p_0 = \pm E_p' = \pm\sqrt{\mathbf{p}^2 - m^2}$, *i. e.* $m \to im$. This is easily checked by using the algebraic equations and solving them with respect to p_0:

$$Det[\gamma^{\mu\nu} p_\mu p_\nu \pm m^2] = 0\,. \tag{19}$$

In constructing the field operator, Ref. [19] we generally need $u(-p) = u(-p_0, -\mathbf{p}, m)$ which should be transformed to $v(p) = \gamma^5 u(p) = \gamma^5 u(+p_0, +\mathbf{p}, m)$. On the other hand, when we calculate the parity properties we need $\mathbf{p} \to -\mathbf{p}$. The $u(p_0, -\mathbf{p}, m)$ satisfies

$$[\tilde{\gamma}^{\mu\nu} p_\mu p_\nu + m^2] u(p_0, -\mathbf{p}, m) = 0\,. \tag{20}$$

The $u(-p_0, \mathbf{p}, m)$ "spinor" satisfies:

$$[\tilde{\gamma}^{\mu\nu} p_\mu p_\nu + m^2] u(-p_0, +\mathbf{p}, m) = 0\,, \tag{21}$$

that is the same as above. The tilde signifies $\tilde{\gamma}^{\mu\nu} = \gamma_{00}\gamma^{\mu\nu}\gamma_{00}$ that is analogoues to the $S=1/2$ case $\tilde{\gamma}^\mu = \gamma_0 \gamma^\mu \gamma_0$. The $u(-p_0, -\mathbf{p}, m)$ satisfies:

$$[\gamma^{\mu\nu} p_\mu p_\nu + m^2] u(-p_0, -\mathbf{p}, m) = 0\,. \tag{22}$$

[2] The dilemma of the (non)local propagators for the spin $S=1$ has also been analyzed in Ref. [15] within the Duffin-Kemmer-Petiau (DKP) formalism or the Dirac-Kähler formalism [15].

[3] In Ref. [16] we have causal solutions only for the S=1 Tucker-Hammer equation.

This case is opposite to the spin-1/2 case where the spinor $u(-p_0, \mathbf{p}, m)$ satisfies

$$[\tilde{\gamma}^\mu p_\mu + m] u(-p_0, +\mathbf{p}, m) = 0\,, \tag{23}$$

and $u(p_0, -\mathbf{p}, m)$,

$$[\tilde{\gamma}^\mu p_\mu - m] u(p_0, -\mathbf{p}, m) = 0\,. \tag{24}$$

In general we can use $u(-p_0, +\mathbf{p}, m)$ or $u(p_0, -\mathbf{p}, m)$ to construct the causal propagator in the spin-1/2 case. However, we do not need to use both because a) $u(-p_0, +\mathbf{p}, m)$ satisfies a similar equation to $u(+p_0, -\mathbf{p}, m)$ and b) we have an integration over $\mathbf{p}$. This integration is invariant with respect to $\mathbf{p} \rightarrow -\mathbf{p}$. The situation is different for spin 1. The tachyonic solutions of the original Weinberg equation

$$[\gamma^{\mu\nu} p_\mu p_\nu + m^2] u(p_0, +\mathbf{p}, m) = 0 \tag{25}$$

are just some solutions of the equation with the opposite square of $m \rightarrow im$). We cannot transform the propagator of the original equation (25) to that solely by a change of variables, as in the spin-1/2 case. The mass squared changes the sign, just as in the case of $v-$ "spinors". When we construct the propagator we have to take this solution into account, as well as the superposition $u(p, m)$ and $u(p, im)$, and corresponding equations. The conclusion is paradoxical: in order to construct the causal propagator for spin 1 we have to take acausal (tachyonic) solutions of homogeneous equations into account. It is not surprising that the propagator is not causal for the Tucker-Hammer equation because it does not contain the tachyonic solutions.

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