Flavoring the Dark Energy: Modular Quintessence and the Swampland

Hansel Gordillo-Ruiz

XIX Mexican Workshop on Particles and Fields

October 21, 2025

In collaboration with M. Hernandez-Segura, I. Portillo-Castillo, S. Ramos-Sanchez and I. Zavala: 2509.22781

1. Motivation
2. Modular-invariant effective action
3. De-Sitter landscape
4. Modular quintessence
5. Conclusions

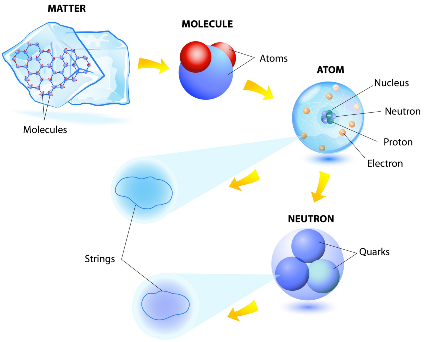
What is dark energy?

- The ΛCDM model assumes a cosmological constant within the FLRW framework to account for the present cosmic acceleration.
- However, recent observations from DESI[1] suggest that dark energy may not be a true cosmological constant.
- This motivates the search for a theoretical setting where dark energy could have a dynamical nature.
- String theory provides a natural context in which dark energy can be dynamical.

[1] DESI: 2503.14739, 2503.14738

String theory

- A candidate to unify quantum mechanics and general relativity.
- Each particle represents a vibrational state of a string.
- Heterotic strings are a combination of a bosonic string and a superstring.
- It is a 10D theory but no evidence of additional dimensions has been found; they must be compactified.
- The geometry of the compactification of the extra dimensions defines the spectrum of fields and their interactions.



Modular fields

- The 4D spectrum contains many additional fields to the Standard Model.
- Some of these fields could account for DE, DM, or even drive inflation.
- In particular, modular fields parameterize the size and shape of the extra six dimensions.
- They are neutral under the gauge group.
- Hence, modular fields are good candidates for *quintessence*.[2]

[2] Olguin-Trejo, Parameswaran, Zavala: 1810.08634

1 Motivation

2 Modular-invariant effective action

3 De-Sitter landscape

4 Modular quintessence

5 Conclusions

Modular group

Modular symmetries of the torus are inherited by the effective field theory.[3]

The modular group $\Gamma := \mathrm{SL}(2,\mathbb{Z})$ is defined as

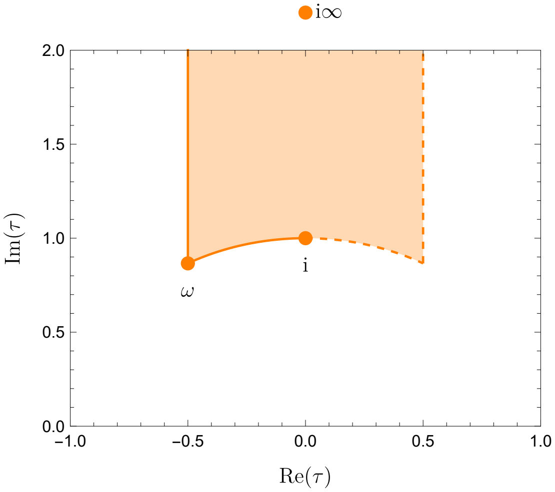
$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \; a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

Γ acts over the modulus τ as

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}\,, \qquad \gamma \in \Gamma$$

[3] Parameswaran, Ramos-Sanchez, Zavala: 1009.3931

Fundamental domain of SL$(2, \mathbb{Z})$



Our model

We take an orbifold $\mathbb{T}^6/\mathbb{Z}_6 - \mathrm{II}$. This model has an eclectic flavor symmetry: a traditional flavor symmetry $\Delta(54)$ and a finite modular symmetry $\Gamma_3' \cong T'$.[4]

Yukawa couplings are given by modular forms, which are holomorphic functions that transform as

$$Y(\tau) \;\to Y(\gamma\tau) \;:=\; (c\tau+d)^{n_Y}\,\rho_Y(\gamma)\,Y(\tau)\,, \qquad \gamma\in\Gamma$$

We know the modular forms for T'

$$\hat{Y}^{(1)}(\tau) \;=\; \begin{pmatrix}\hat{Y}_1(\tau)\\ \hat{Y}_2(\tau)\end{pmatrix} \;:=\; \begin{pmatrix} -3\sqrt{2}\frac{\eta^3(3\tau)}{\eta(\tau)} \\ 3\frac{\eta^3(3\tau)}{\eta(\tau)}+\frac{\eta^3(\tau/3)}{\eta(\tau)}\end{pmatrix}$$

[4] Baur, Nilles, Ramos-Sanchez, Trautner, Vaudrevange: 2405.2037

Low energy effective field theory

At low energies we have a 4D supergravity theory with $N = 1$. The most important functions are given by[5]

The superpotential

$$W = -\frac{1}{\sqrt{2}}\tilde{\lambda}_1 \hat{Y}_1(\tau) + \tilde{\lambda}_2\, \hat{Y}_2(\tau) + \frac{\Omega_1(S)H_1(\tau)}{\eta^2(\tau)} + \frac{\Omega_2(S)H_2(\tau)}{\eta^2(\tau)}$$

The Kähler potential

$$K = -\log\left[S + \overline{S} - \frac{1}{8\pi^2}\delta_{GS}\log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)\right] - \log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)$$

Some definitions:

$$H_i(\tau) = \left(\frac{E_4(\tau)}{\eta^8(\tau)}\right)^{n_i}\left(\frac{E_6(\tau)}{\eta^{12}(\tau)}\right)^{m_i} P(j(\tau))$$

$$j(\tau) = 1728\frac{E_4(\tau)^3}{E_4(\tau)^3 - E_6(\tau)^2}, \qquad \Omega_i(S) = A_i e^{-\alpha_i S}$$

[5] Nilles, Ramos-Sanchez, Vaudrevange: 2004.05200
Cvetic, Font, Ibañez, Lust, Quevedo 1991

Scalar potential

The scalar potential is given by[6]

$$V = e^{K}\left(K^{A\overline{B}}D_AWD_{\overline{B}}\overline{W} - 3|W|^2\right)\,,$$

where $D_AW = \partial_AW + \partial_AKW$.

For our particular model:

$$V = \mathrm{e}^{K}\left[|YW_S - W|^2 + \frac{Y}{Y - \frac{\delta_{\mathrm{GS}}}{8\pi^2}}\left|\frac{\delta_{\mathrm{GS}}}{8\pi^2}\left(\frac{2W}{Y} - W_S\right) + \mathrm{i}(\mathrm{i}\overline{\tau} - \mathrm{i}\tau)W_\tau - W\right|^2 - 3|W|^2\right]$$

with $Y := S + \overline{S} - \frac{1}{8\pi^2}\delta_{\mathrm{GS}}\log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)$.

[6] F. Quevedo: 1011.1491

Refined de-Sitter conjecture

The scalar potential V in any EFT consistent with quantum gravity must satisfy[7]

$$|\nabla V| \geq \frac{c}{M_p} V \,, \quad \text{or,} \quad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_p^2} V \,.$$

where c and c' are positive constants of $\mathcal{O}(1)$, and $\min(\nabla^i \nabla_j V)$ denotes the smallest eigenvalue of the Hessian in an orthonormal frame.

According to this conjecture, our theory does not admit any de-Sitter minimum.

[7] Ooguri, Palti, Shiu, Vafa: 1810.05506

1 Motivation

2 Modular-invariant effective action

3 De-Sitter landscape

4 Modular quintessence

5 Conclusions

Objectives

- To explain current cosmic acceleration (quintessence or cosmological constant).
- To search for relevant vacua: de-Sitter minima or saddle points.
- To analyze the dynamics of saddle points as candidates for quintessence.
- To test the de Sitter conjecture.

We choose the parameters

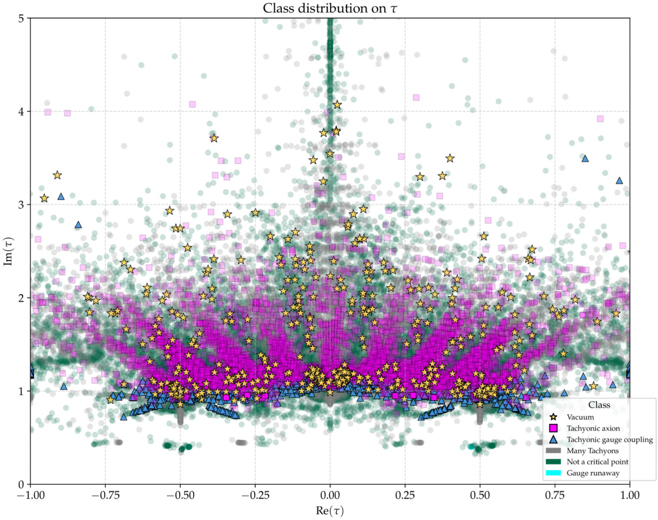
$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	c_1	c_2	b_1	b_2	b_1^0	b_2^0	M_d	k_1	k_2	δ_{GS}
$3/25000$	$3/5000000$	1	$8\pi^2\mathrm{e}$	5	-26	-9	-12	$1/65$	1	1	-1

The initial values for our implemented search lie within the ranges

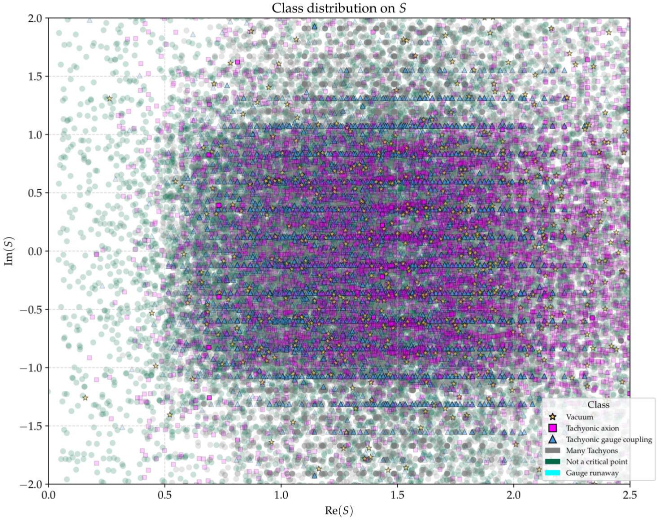
$$-0.6 \leq \mathrm{Re}(\tau) \leq 0.6\,, \qquad \sqrt{1-(\mathrm{Re}\tau)^2} \leq \mathrm{Im}(\tau) \leq 1.3\,,$$
$$0.5 \leq \mathrm{Re}(S) \leq 2.0\,, \qquad -1 \leq \mathrm{Im}(S) \leq 1\,.$$

with values of $\{n_1, m_1, n_2, m_2\}$ from 0 to 24.

Results for τ



Results for S



The Landscape

By scanning over 400,000 random points, we find the following types of vacua:

- Stable AdS vacua ☺
- AdS critical points with one unstable direction.
- dS critical points with one unstable direction ☺
- Critical points with more than one unstable direction.
- No dS vacua found ☹

These de Sitter saddle points could serve as promising candidates for quintessence

1 Motivation

2 Modular-invariant effective action

3 De-Sitter landscape

4 Modular quintessence

5 Conclusions

Modular quintessence from a saddle point

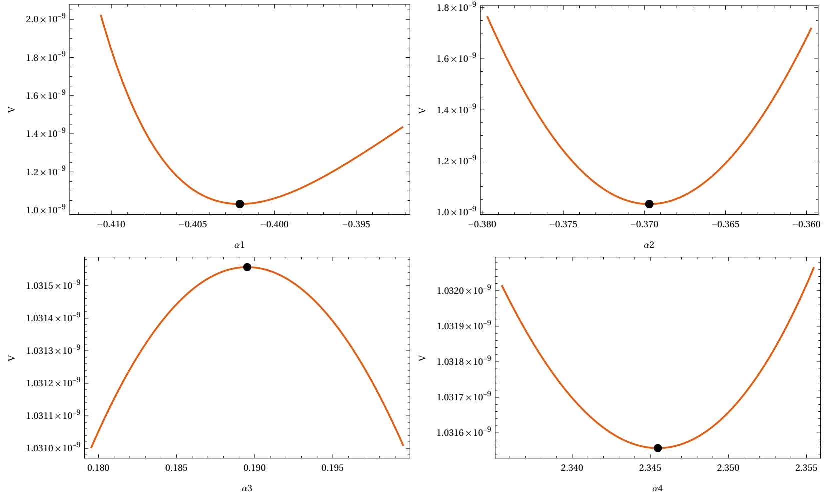
- In quintessence scenarios, a field (or several fields) slowly rolls down towards a minimum such that its energy density remains nearly constant throughout most of the cosmological history.
- Its potential energy drives the accelerated expansion we observe today.
- An interesting vacuum of this kind is found at:

$$\tau = -0.004063 + 1.297\,\mathrm{i}$$

$$S = 1.390 + 0.1272\,\mathrm{i}$$

with $\{n_1 = 0,\ m_1 = 12,\ n_2 = 3,\ m_2 = 10\}$.

De-Sitter saddle point



α_i are the eigenstates of the Hessian matrix.

Cosmological equations

EOM for modular fields:

$$
\begin{aligned}
3\left(\frac{\dot{\mathrm{a}}}{\mathrm{a}}\right)^2 &= \frac{1}{2} g_{ij}\dot{\varphi}^i\dot{\varphi}^j + V + 3H_0^2\Omega_{m,0}\mathrm{a}(t)^{-3} + 3H_0^2\Omega_{r,0}\mathrm{a}(t)^{-4}\,, \\
0 &= \ddot{\varphi}^i + 3\frac{\dot{\mathrm{a}}}{\mathrm{a}}\dot{\varphi}^i + \Gamma^i_{jk}\dot{\varphi}^j\dot{\varphi}^k + g^{ij}\frac{\partial V}{\partial \varphi^j}\,,
\end{aligned}
$$

where φ^i are the real scalar fields $\{\tau_r, \tau_i, S_r, S_i\}$ and $\Omega_{m,0} = 0.3111$ and $\Omega_{r,0} = 0.0001$.

Easier to solve in terms of the e-folds number

$$
\mathrm{a} = e^N\,, \qquad \frac{d}{dt} = H\frac{d}{dN}\,.
$$

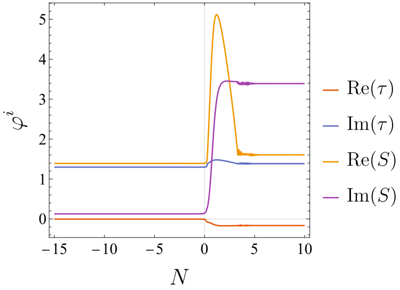
Taking $N = 0$ as today.

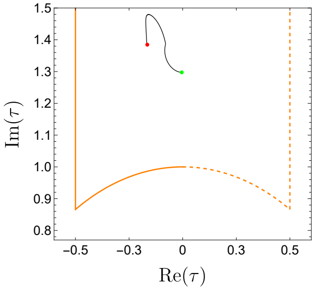
Modular fields evolution

- Fields at the SP $\Rightarrow$ remain static

Modular fields evolution

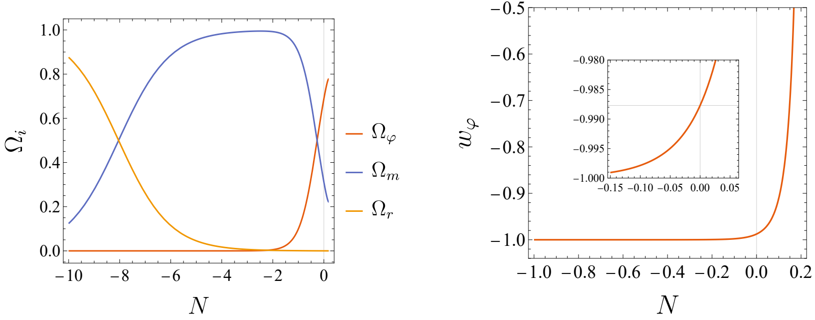
- Fields at the SP ⇒ remain static
- Slightly displaced from SP ⇒ evolve towards an AdS minimum





Equation of state and density parameter

$$\Omega_\varphi = \frac{\rho_\varphi}{3H^2}\,, \quad \rho_\varphi = \frac{1}{2} g_{ij}\dot\varphi^i\dot\varphi^j + V\,, \quad w_\varphi = \frac{\frac{1}{2} g_{ij}\dot\varphi^i\dot\varphi^j - V}{\rho_\varphi}$$



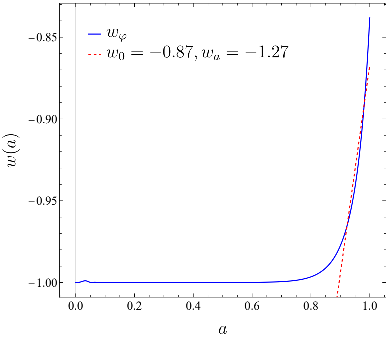
Today we have:

$$w_{\varphi,0} \simeq -0.9877\,, \qquad \Omega_{\varphi,0} \simeq 0.6883\,,$$

which is consistent with ΛCDM.

DESI comparison

CPL parameterization[8]: $w_{\mathrm{DE}}(a) \;=\; w_0 + (1-a)\, w_a$



Comparing with DESI+CMB+Union3[9] $w_0 \;=\; -0.752 \pm 0.057$ and $w_a \;=\; -0.86^{+0.23}_{-0.20}$ we get $\chi^2 \simeq 5.6$.

[8] Chevallier, Polarski: gr-qc/0009008, Linder: astro-ph/0208512
[9] DESI: 2503.14738

1. Motivation
2. Modular-invariant effective action
3. De-Sitter landscape
4. Modular quintessence
5. Conclusions

Conclusions

- String-theory compactifications give rise to new fields that can serve as cosmological models.
- It is not possible to provide a counterexample to the de Sitter conjecture within our model.
- The dynamics of the moduli can act as quintessence.
- It is possible to obtain values close to the current observations.
- Further work: Extend the model to more realistic compactifications with additional moduli, perform a detailed analysis of the *penacho* structure, test swampland constraints using modular symmetries, and investigate the possibility of phantom crossing within this framework.

Thank you

Explicit model

We take the heterotic string $E8 \times E8$ compactified in an orbifold $\mathbb{Z}_6 - \mathrm{II}$. The shift vector and Wilson lines are:

$$
\begin{aligned}
v &= \frac{1}{6}(0,1,2,-3),\\
V_1 &= \left(-\frac{5}{12},-\frac{5}{12},-\frac{5}{12},-\frac{5}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}\right),\ \left(-\frac{5}{12},-\frac{1}{4},-\frac{1}{4},-\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}\right),\\
A_5 &= \left(-\frac{5}{4},\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3}{4},\frac{1}{4},\frac{3}{4},\frac{5}{4}\right),\left(-\frac{1}{2},1,1,-\frac{3}{2},-1,0,1,1\right),\\
A_6 &= \left(-\frac{3}{2},-\frac{1}{2},1,1,\frac{1}{2},0,-\frac{1}{2},1\right),\left(-\frac{1}{4},\frac{3}{4},\frac{3}{4},\frac{5}{4},-\frac{1}{4},\frac{7}{4},\frac{1}{4},\frac{7}{4}\right).
\end{aligned}
$$

The 4-dimensional gauge group is:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \left[\mathrm{SU}(3)^3 \times \mathrm{U}(1)^6\right].$$

Yukawa couplings

$$W_{\mathrm{Yuk}} \supset \left(\hat{Y}^{(1)}_{\mathbf{2''}}(\tau) \otimes \Phi^{(-2/3)}_{\alpha} \otimes \Phi^{(-2/3)}_{\beta} \otimes \Phi^{(-2/3)}_{\gamma} \right)_{\mathbf{1}}$$

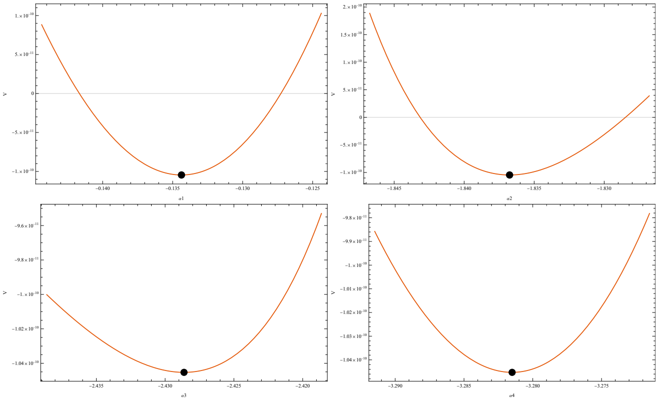
$$\begin{aligned} W_{\mathrm{Yuk}} \supset \lambda\, \chi_1\, \chi_2 \Big[-\frac{\hat{Y}_1(\tau)}{\sqrt{2}} \big(&\Phi_{1,3}\Phi_{2,2}\Phi_{3,1} + \Phi_{1,3}\Phi_{2,1}\Phi_{3,2} + \Phi_{1,2}\Phi_{2,3}\Phi_{3,1} \\ &+ \Phi_{1,1}\Phi_{2,3}\Phi_{3,2} + \Phi_{1,2}\Phi_{2,1}\Phi_{3,3} + \Phi_{1,1}\Phi_{2,2}\Phi_{3,3} \big) \\ &+ \hat{Y}_2(\tau) \left(\Phi_{1,1}\Phi_{2,1}\Phi_{3,1} + \Phi_{1,2}\Phi_{2,2}\Phi_{3,2} + \Phi_{1,3}\Phi_{2,3}\Phi_{3,3} \right) \Big] \end{aligned}$$

The scalar potential

$$
\begin{aligned}
V = {} & \frac{1}{\mathrm{Sr}\,\left(i\,(-i\,\mathrm{T2i}+\mathrm{T2r}) - i\,(i\,\mathrm{T2i}+\mathrm{T2r})\right)} \\
& 9.1612\times 10^{-71}\left(-3\left(\frac{1}{100\,000\,000}11\left(\frac{9\,e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(1-e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-30i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-42i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^3}{500\,000\,000\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)}\right.\right.\right. + \\
& \frac{1}{100\,000\,000\,000}3\left(\frac{3\,e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(1-e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-30i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-42i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^3}{1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}}\right. + \\
& \left.\left.\left.\frac{\left(1+e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(-1-e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-\frac{8}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)\right)^3}{1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}}\right)\right)\right) - \left(3\,e^{-1-\frac{8}{3}\pi^2(-i\,\mathrm{Si}+\mathrm{Sr})+\frac{5}{6}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right. \\
& \left(1+240\left(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+9\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+28\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+73\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+252\right.\right. \\
& \left.\left.\left. e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+344\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+585\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+757\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1134\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)\right)\right) \Big/ \\
& \left(3200\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^{10}\pi^2\right) - \left(3\,e^{-1-52.6379(-i\,\mathrm{Si}+\mathrm{Sr})+\frac{65}{6}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right. \\
& \left(1+240\left(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+9\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+28\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+73\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+\right.\right. \\
& \left.\left. 252\,e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+344\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+585\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+757\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1134\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)\right)^{10} \\
& \left(1-504\left(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+33\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+244\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1057\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+3126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+8052\right.\right. \\
& \left.\left.\left. e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+16\,808\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+33\,825\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+59\,293\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+103\,158\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)\right)^4\right) \Big/ \\
& \left.\left(320\,000\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^{130}\pi^2\right)\right)
\end{aligned}
$$

...

AdS minimum


V
0
−0.140
−0.135
−0.130
−0.125
−1.845
−1.840
−1.835
−1.830
−2.435
−2.430
−2.425
−2.420
−3.290
−3.285
−3.280
−3.275