AUTOMATED QUANTUM ALGORITHM FOR EFFICIENT QUERYING OF MULTILOOP CAUSAL CONFIGURATIONS

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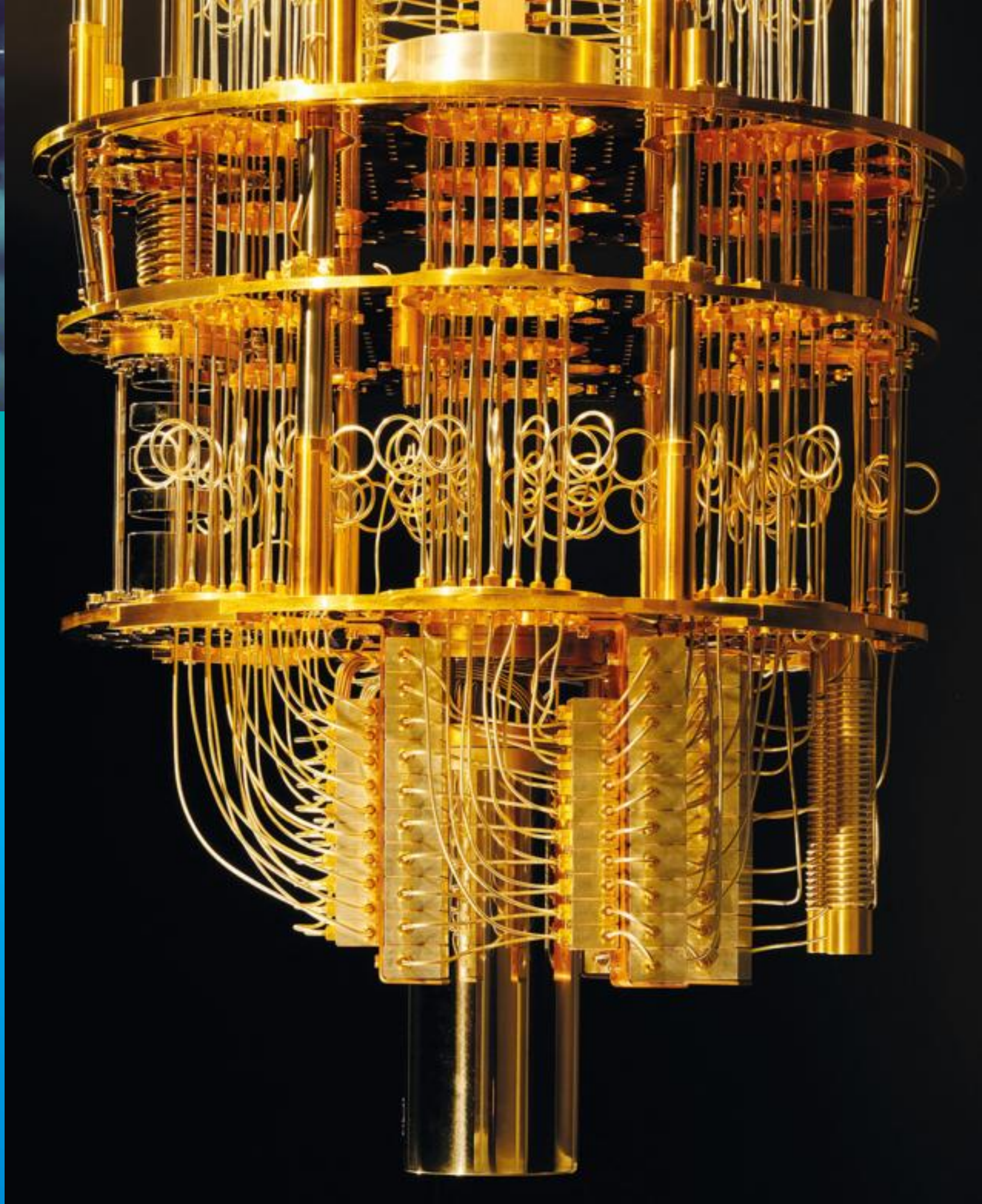

CONTENT

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- Circuit Design
- Results
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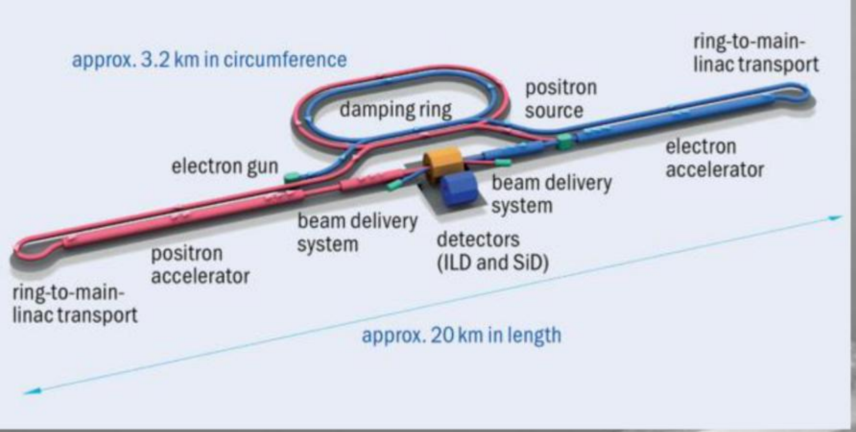
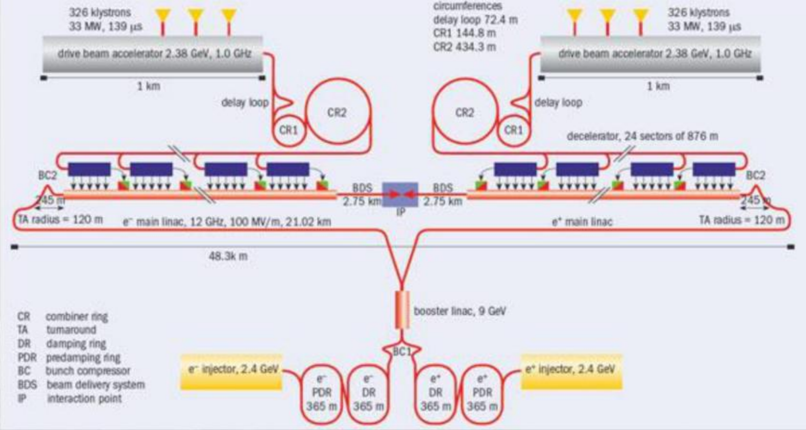
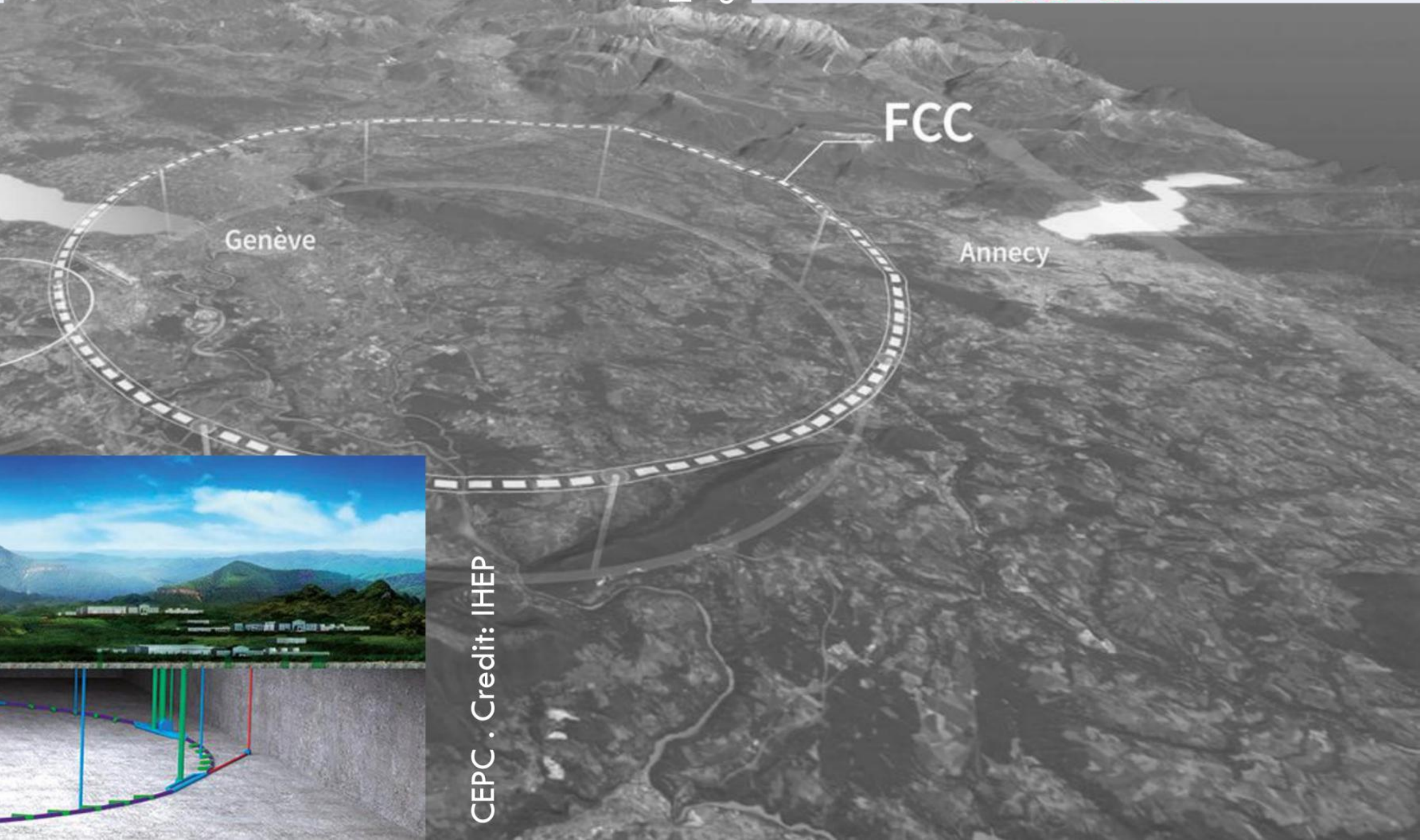
MOTIVATION

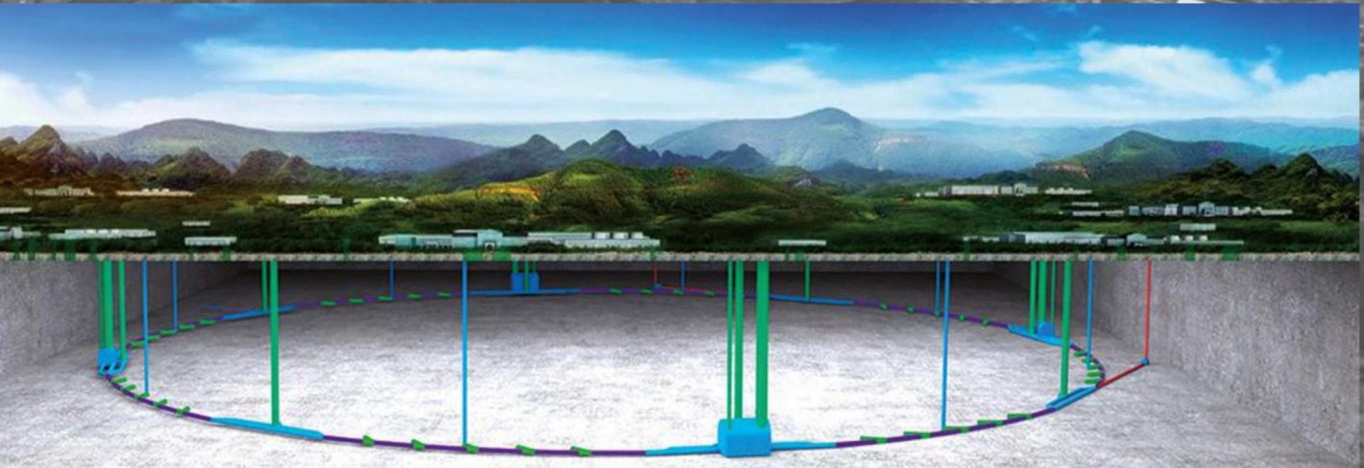


ILC250. Credit: R Hori/KEK



Proposed layout for CLIC
credit: CERN Courier





CEPC . Credit: IHEP

Causality from the loop-tree duality

Thanks to LTD amplitudes can be described only in terms of on-shell propagator.

The integrand has the form:

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} {q^{(+)}}_{i_{s,0}}}$$

$q_{i,0}$ is the energy component of the four momenta q_i

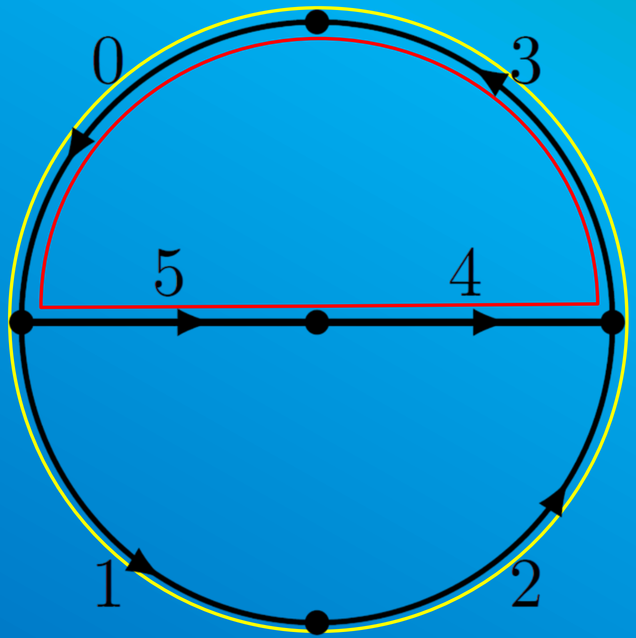
While the propagator:

$$G_F = \frac{1}{\left(q_{i,0} + {q_{i,0}}^{(+)}\right)\left(q_{i,0} - {q_{i,0}}^{(+)}\right)}$$

$$q_{i,0}^{(+)} = \sqrt{\vec{q_i}^{\,2} + m_i^2 - i0}$$

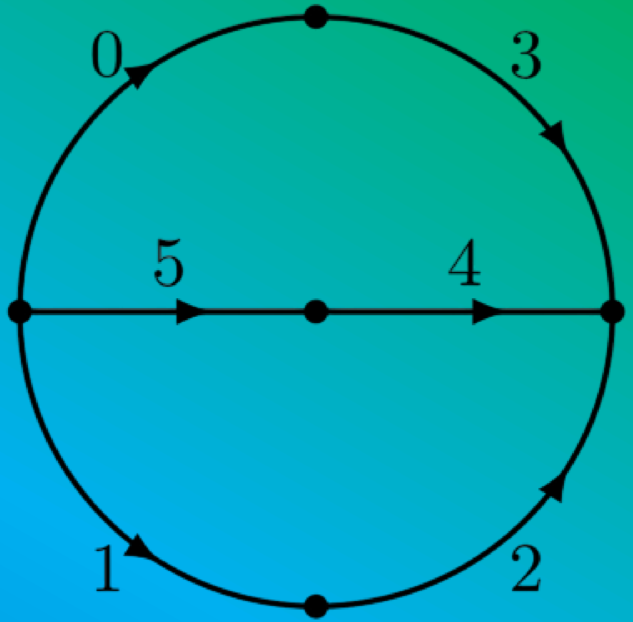
Causality from the loop-tree duality

If a particle returns to the point of emission: it travels back in time, thus breaks causality.

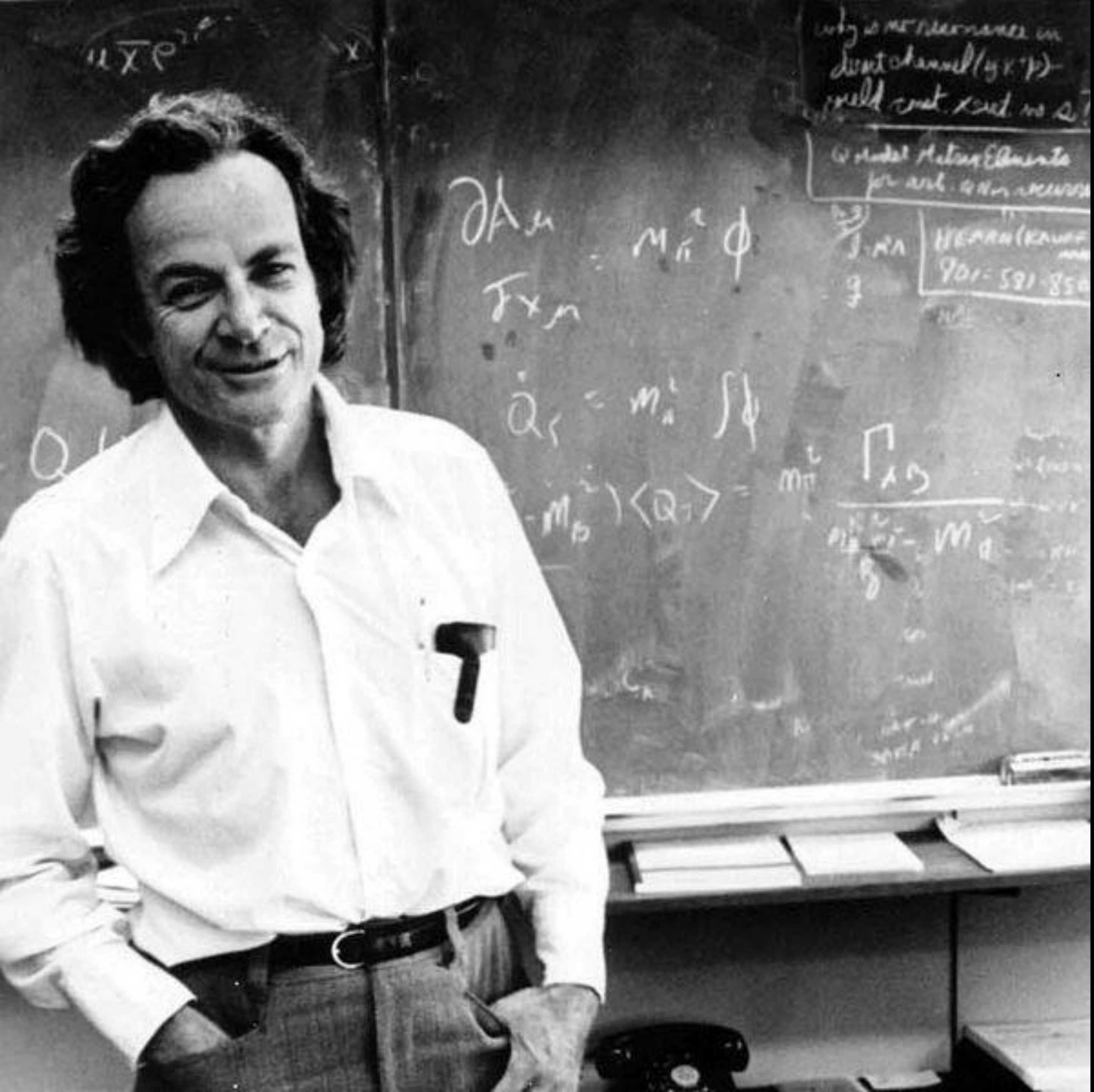


Non-causal configurations are represented by cyclic diagrams

Cyclic Configurations are nonphysical !!!



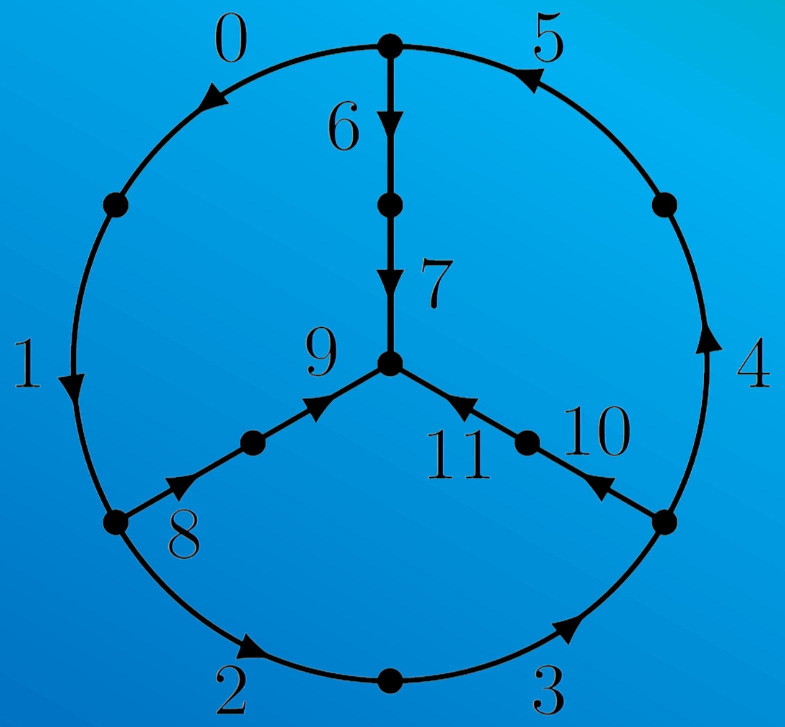
Causal configurations are represented by acyclic diagrams



Nature isn't classical, dammit, and if you want to make a simulation of nature, you better make it quantum.

-Richard P. Feynman

LTD allows to encode vacuum amplitudes in quantum circuits



- A Feynman propagator describes a quantum superposition of propagation in both directions

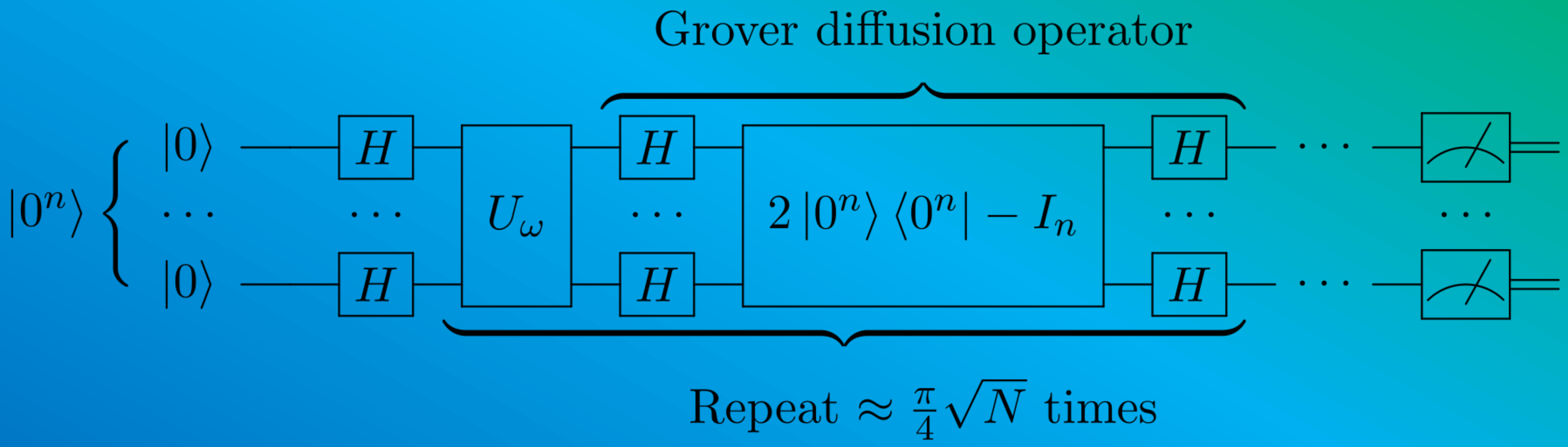
$$G_F = \frac{1}{(q_{i,0}+q_{i,0}{}^{(+)})(q_{i,0}-q_{i,0}{}^{(+)})} \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

- A Feynman diagram is a superposition of 2^n states.

- Causal configurations of Feynman diagrams are Directed Acyclic Graphs (DAG) in graph theory.

GROVERS QUANTUM ALGORTIHM

Is a quantum algorithm for unstructured search that finds the unique input of a black box function that produces a particular output value, using $O(\sqrt{N})$ evaluations, where N is the size of the domain.



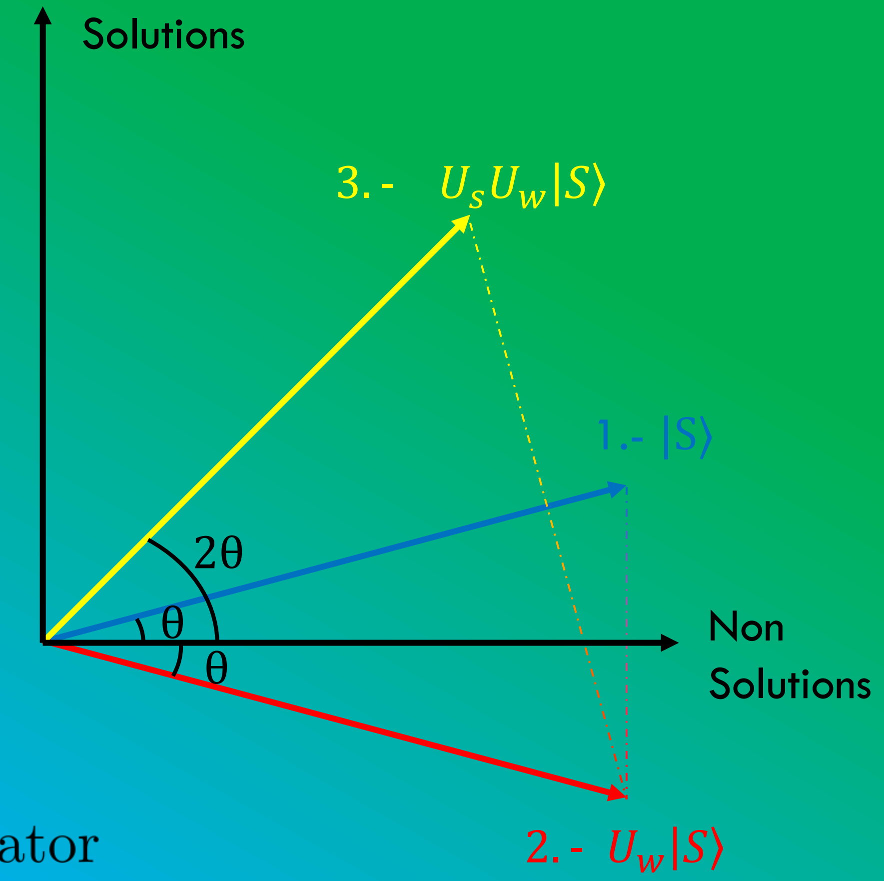
$$\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

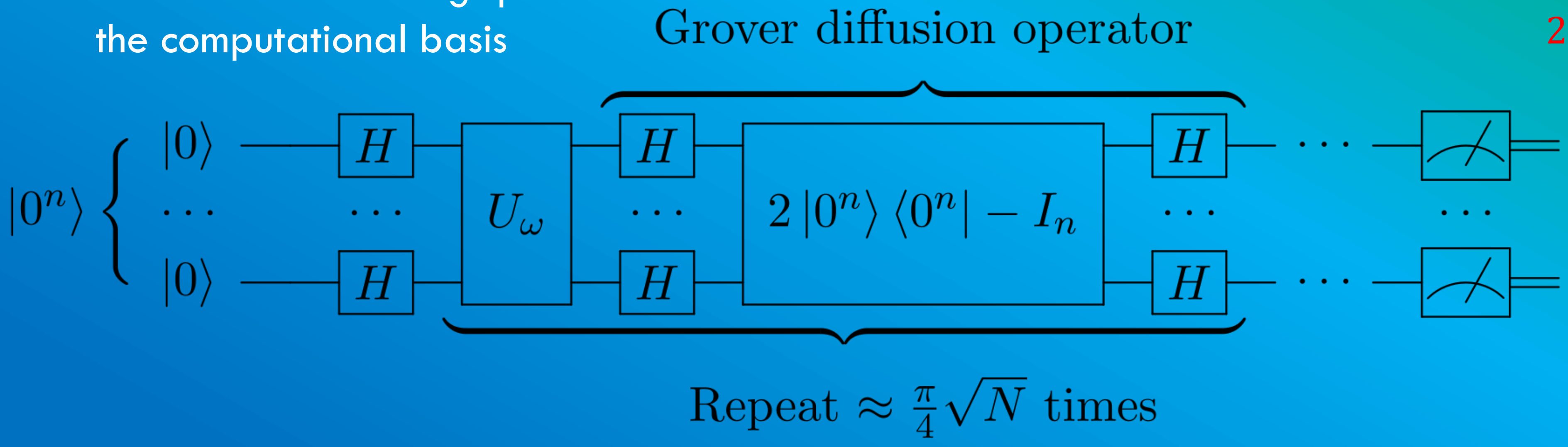
$$\hat{H}\,|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

$$\hat{H}\,|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$

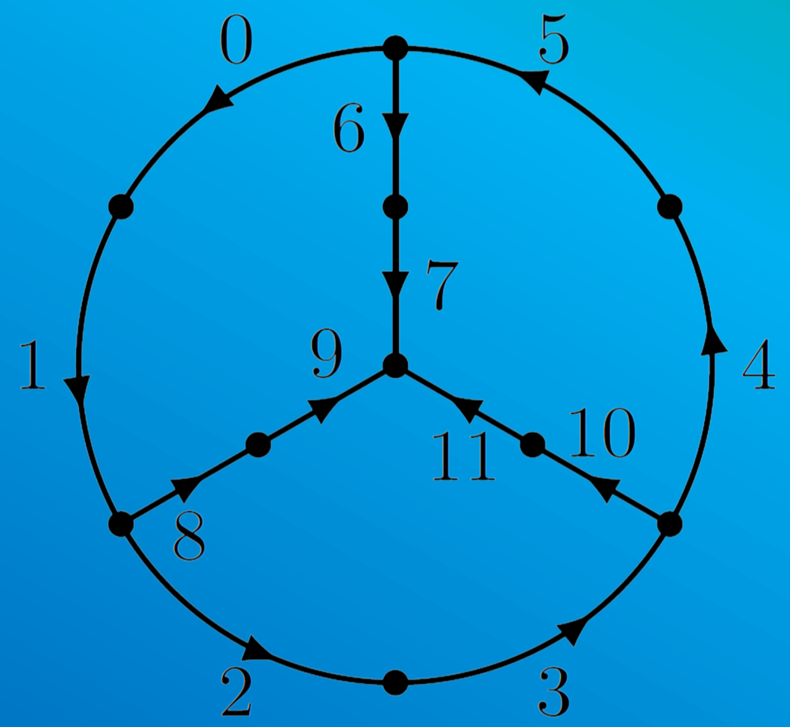
GROVERS QUANTUM ALGORTIHM

1. Initialize the system to the uniform superposition over all states
   $|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$
2. Perform the "Grover iteration" r(N) times:
   1. Apply the operator U_w,
      $U_w|x\rangle = (-1)^{f(x)}|x\rangle$
   2. Apply the Grover diffusion operator
      $U_s = 2|s\rangle\langle s| - I$
3. Measure the resulting quantum state in the computational basis

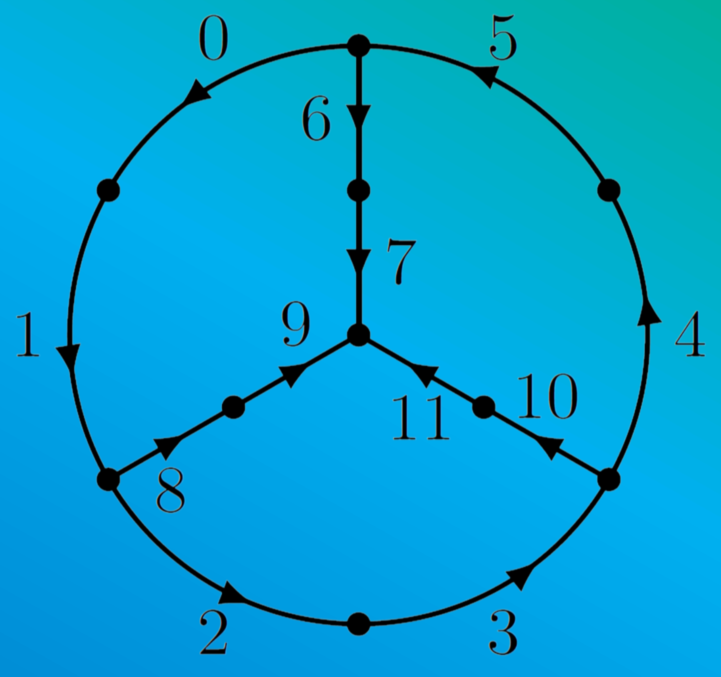


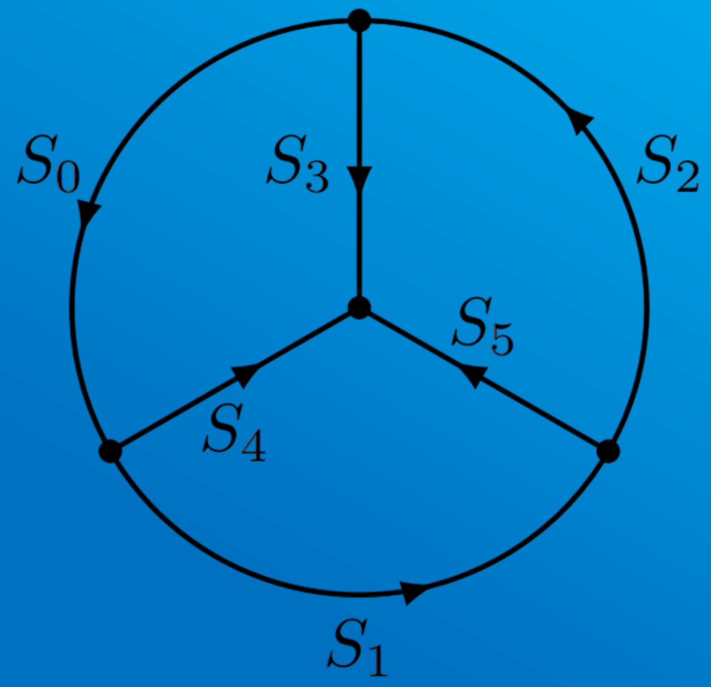


CIRCUIT DESIGN



1. Encoding the edges information in qubits
2. Initializing each qubit in the quantum circuit
3. Designing an oracle operator to identify the acyclic configurations.
4. Applying the diffusion operator to implement amplitude amplifications
5. Measuring the quantum states





CIRCUIT DESIGN

Encoding the edges information in qubits

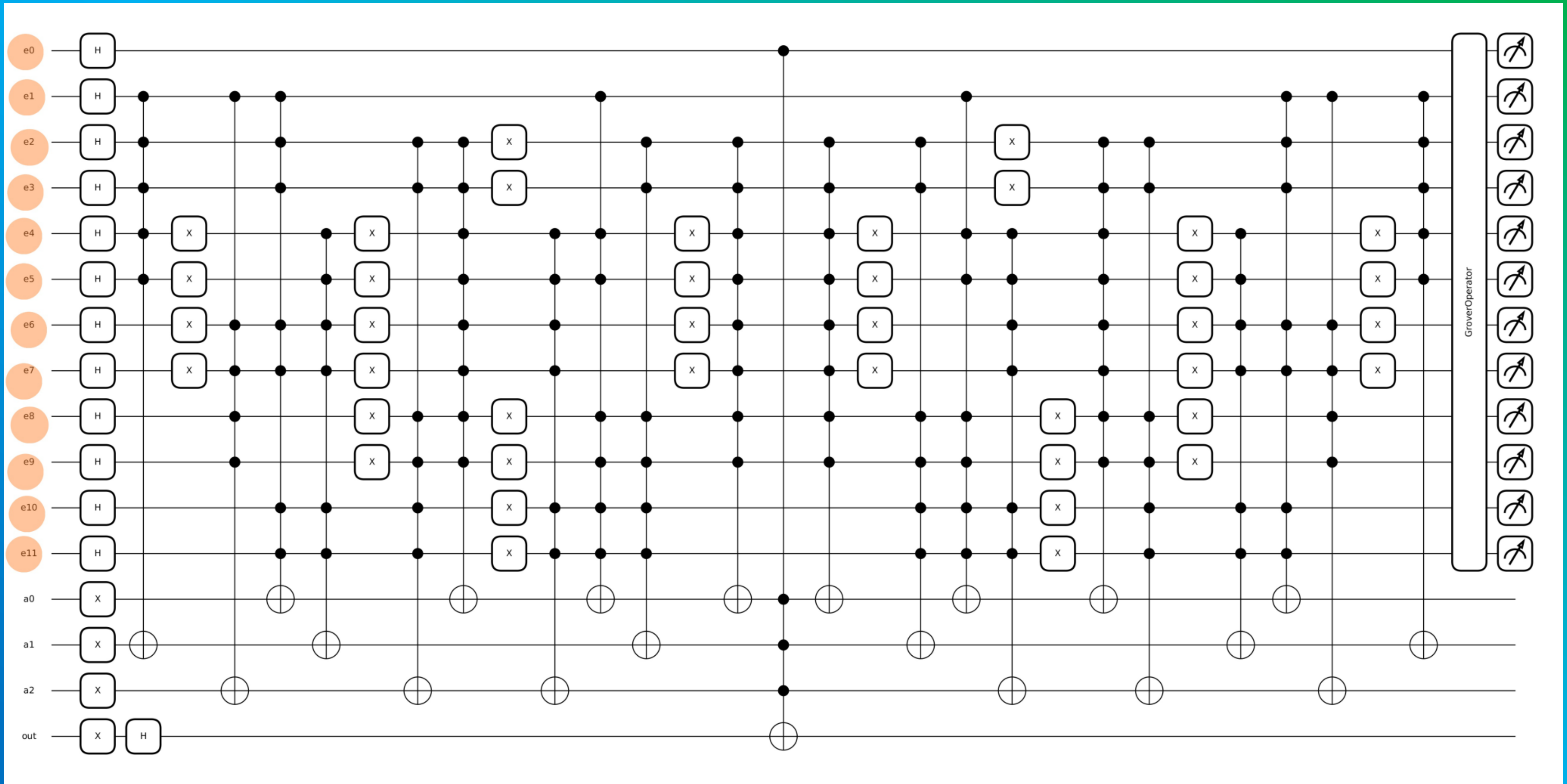
$$s_0 = e_0 \wedge e_1, \quad s_1 = e_2 \wedge e_3,$$
$$s_2 = e_4 \wedge e_5, \quad s_3 = e_6 \wedge e_7,$$
$$s_4 = e_8 \wedge e_9, \quad s_5 = e_1 0 \wedge e_1 \;\; .$$

This formulation represents each edge as a qubit.

Needing 12 qubits to represent every edge in this diagram.

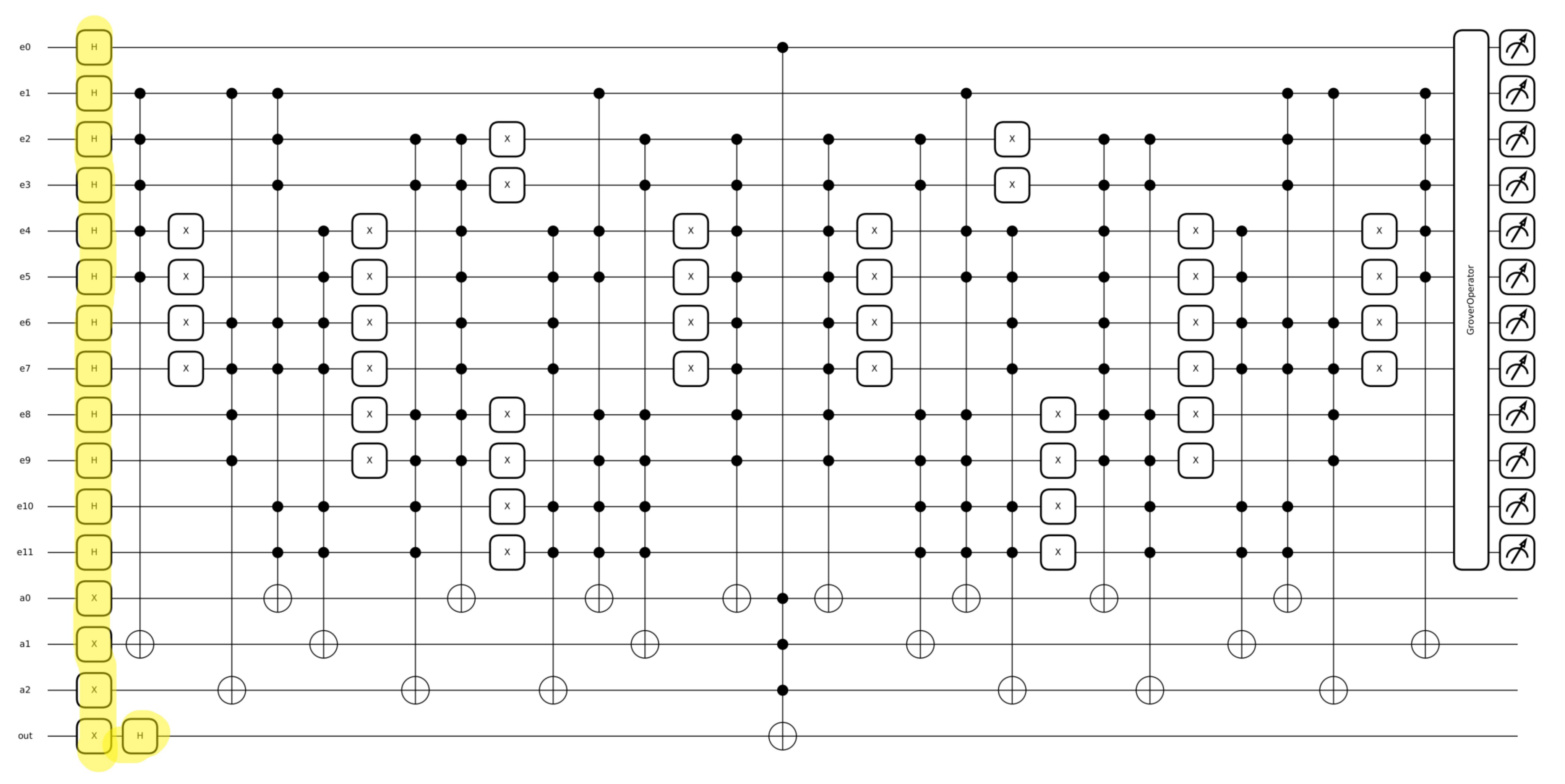
CIRCUIT DESIGN

Encoding edges information in qubits



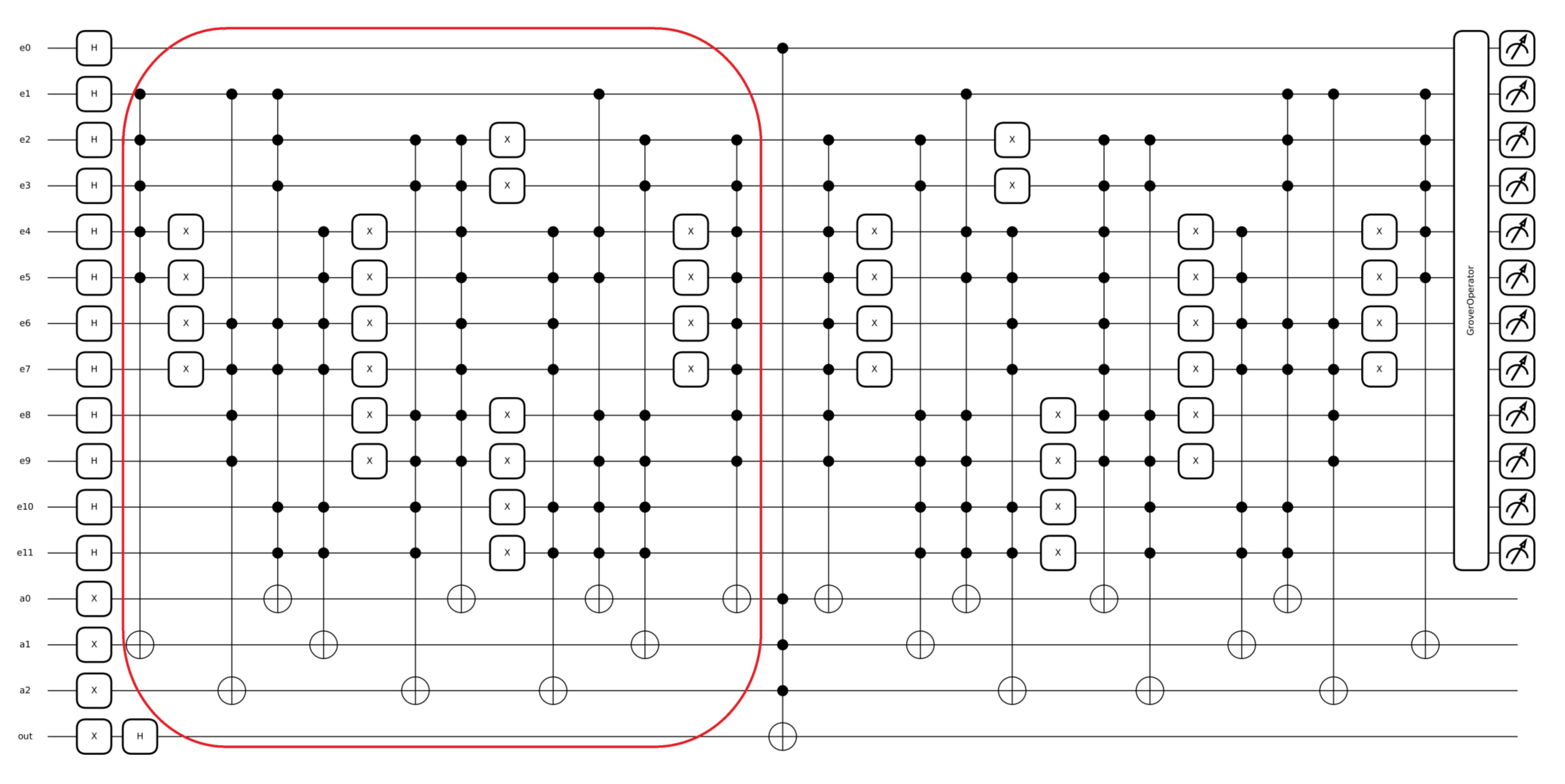
CIRCUIT DESIGN

Initialization



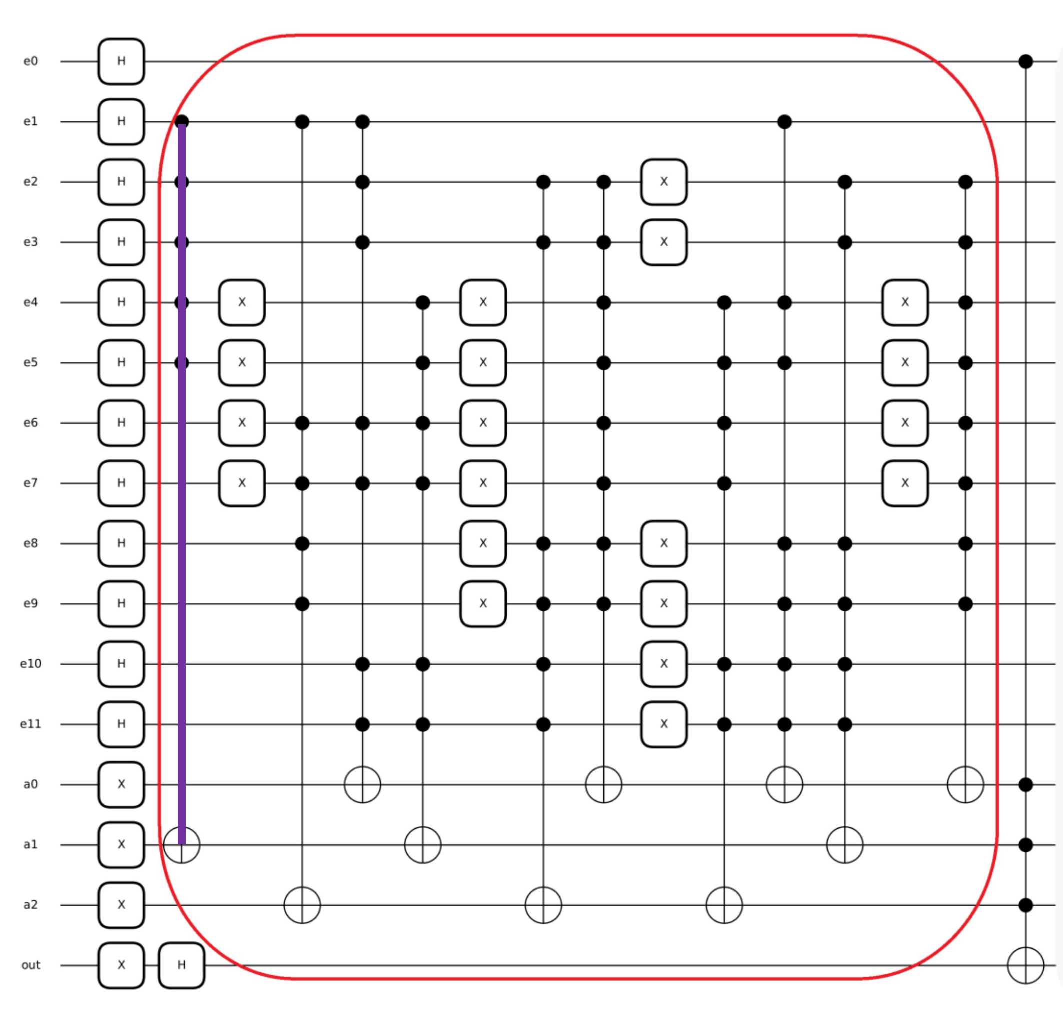
CIRCUIT DESIGN

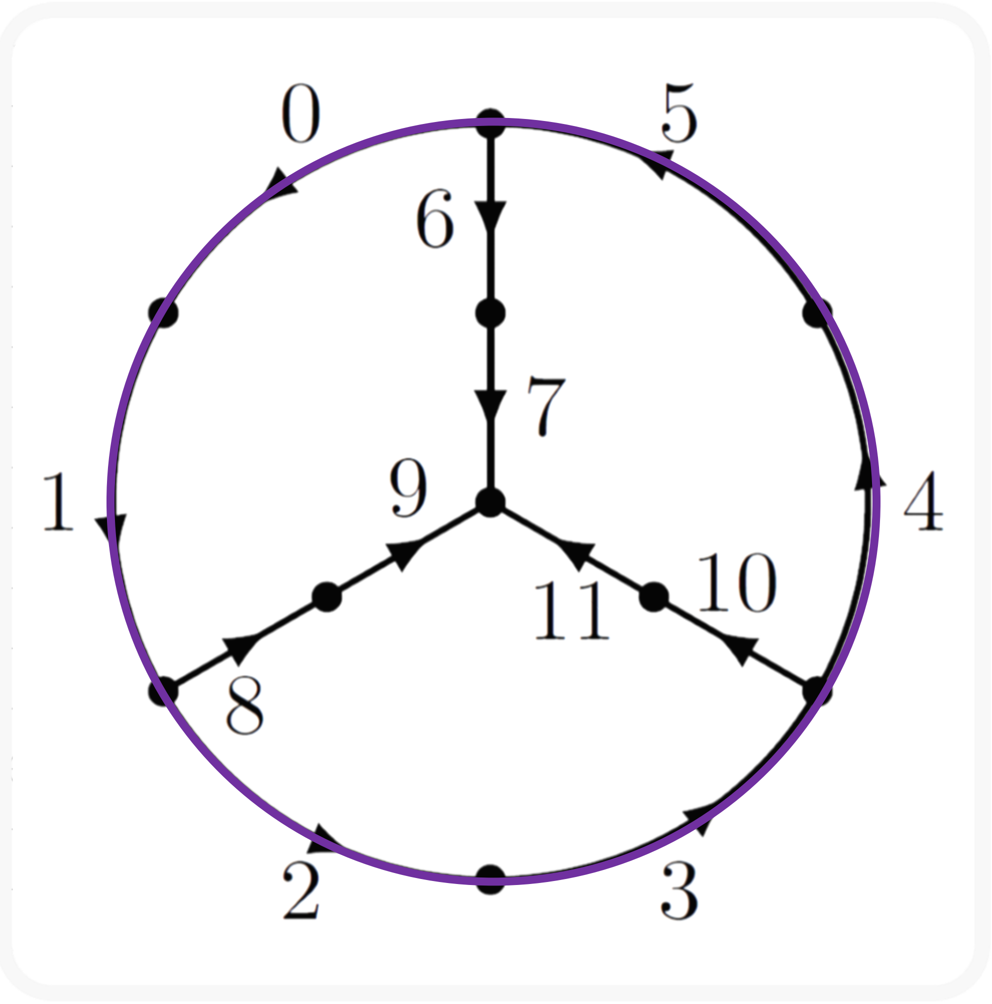
Oracle Design



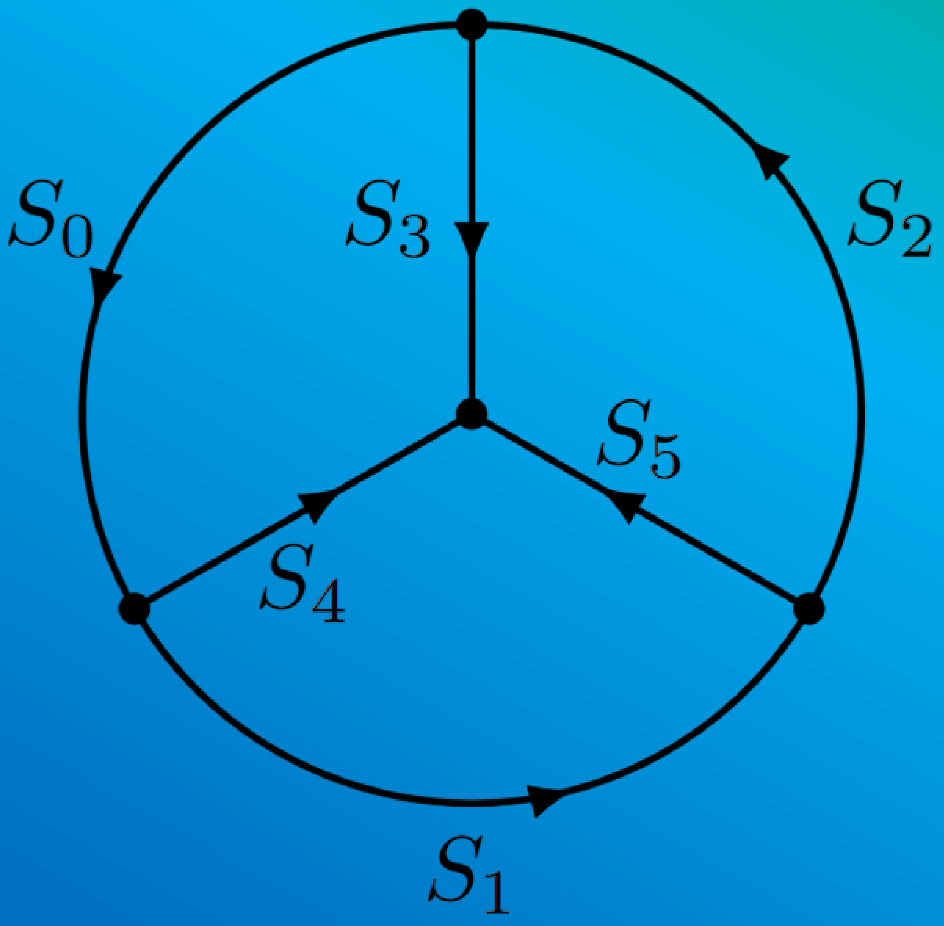
CIRCUIT DESIGN

Oracle Design





CIRCUIT DESIGN



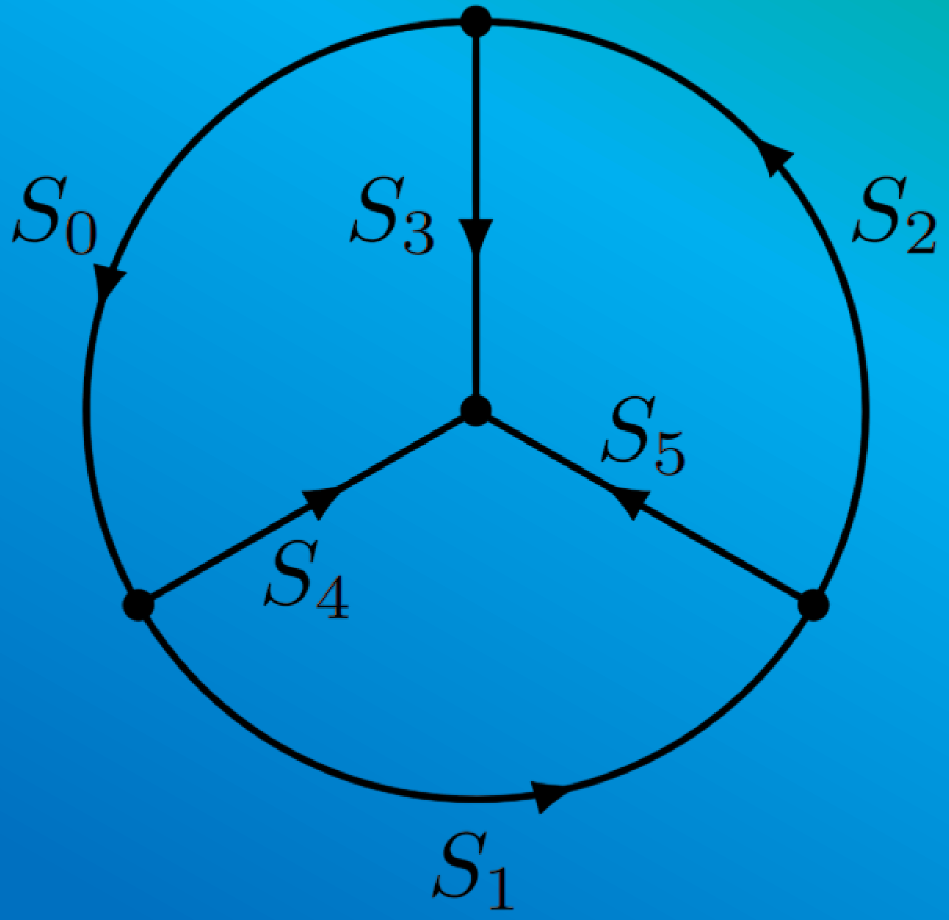
$c_0 = s_0 \wedge s_1 \wedge s_2,$
$c_1 = s_0 \wedge \overline{s_3} \wedge s_4,$
$c_2 = s_1 \wedge \overline{s_4} \wedge s_5,$
$c_3 = s_2 \wedge s_3 \wedge \overline{s_5},$
$c_4 = s_0 \wedge s_1 \wedge \overline{s_3} \wedge s_5,$
$c_5 = s_1 \wedge s_2 \wedge s_3 \wedge \overline{s_4},$
$c_6 = s_0 \wedge s_2 \wedge s_4 \wedge \overline{s_5},$
$c_7 = \overline{c_2},$
$c_8 = \overline{c_3}$
$c_9 = \overline{c_5}$

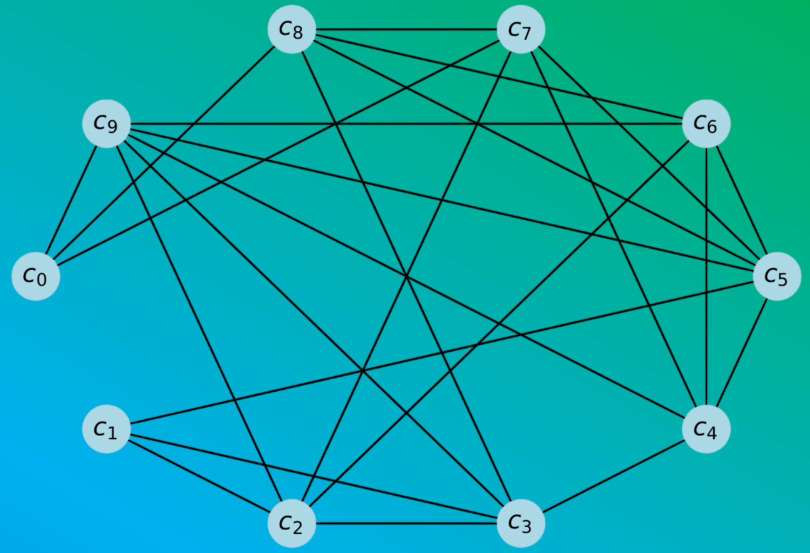
A first reduction on the number of qubits needed is achieved by clustering "mutually exclusive clauses". $c_i \wedge c_j = 0$

Needing only 7 instead of 10.

The information on mutually exclusive clauses can be stored in a single qubit because: $c_i \vee c_j = c_i \underline{\vee} c_j$, $\forall c_i \neq c_j$

CIRCUIT DESIGN



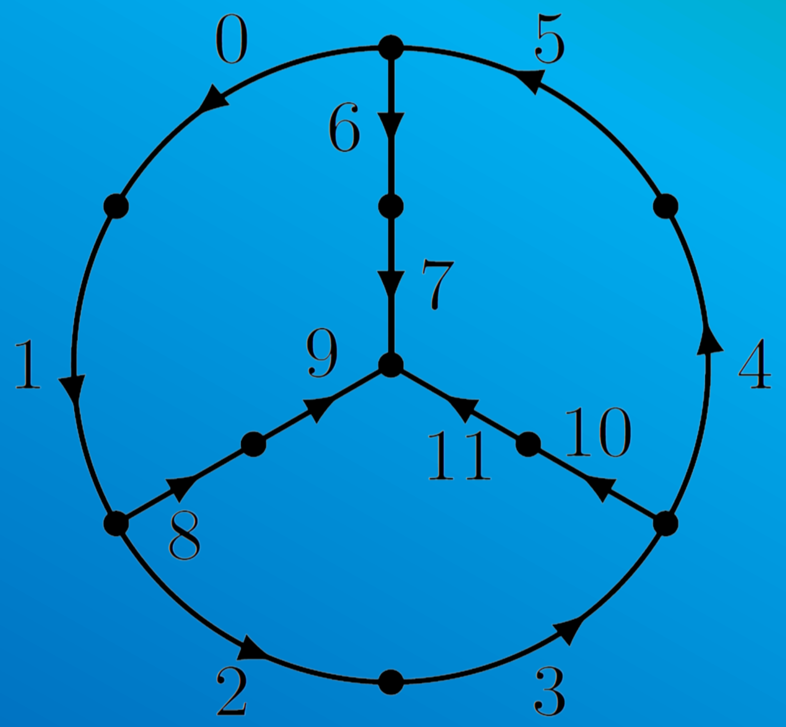


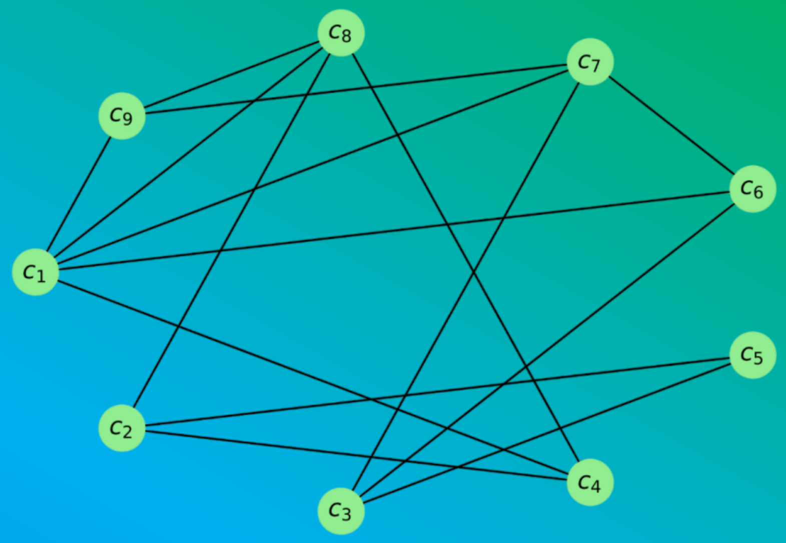
Graph representing the adjacency matrix of mutually exclusive clauses

A first algorithm is applied to solve the Minimum Clique Partition problem of the adjacency matrix

$$\mathrm{MAUXC}^{(3,12)} = \{\{c_4, c_5, \overline{c_5}, c_6\}, \{c_0, \overline{c_2}, \overline{c_3}\}, \{c_1, c_2, c_3\}\}$$

CIRCUIT DESIGN





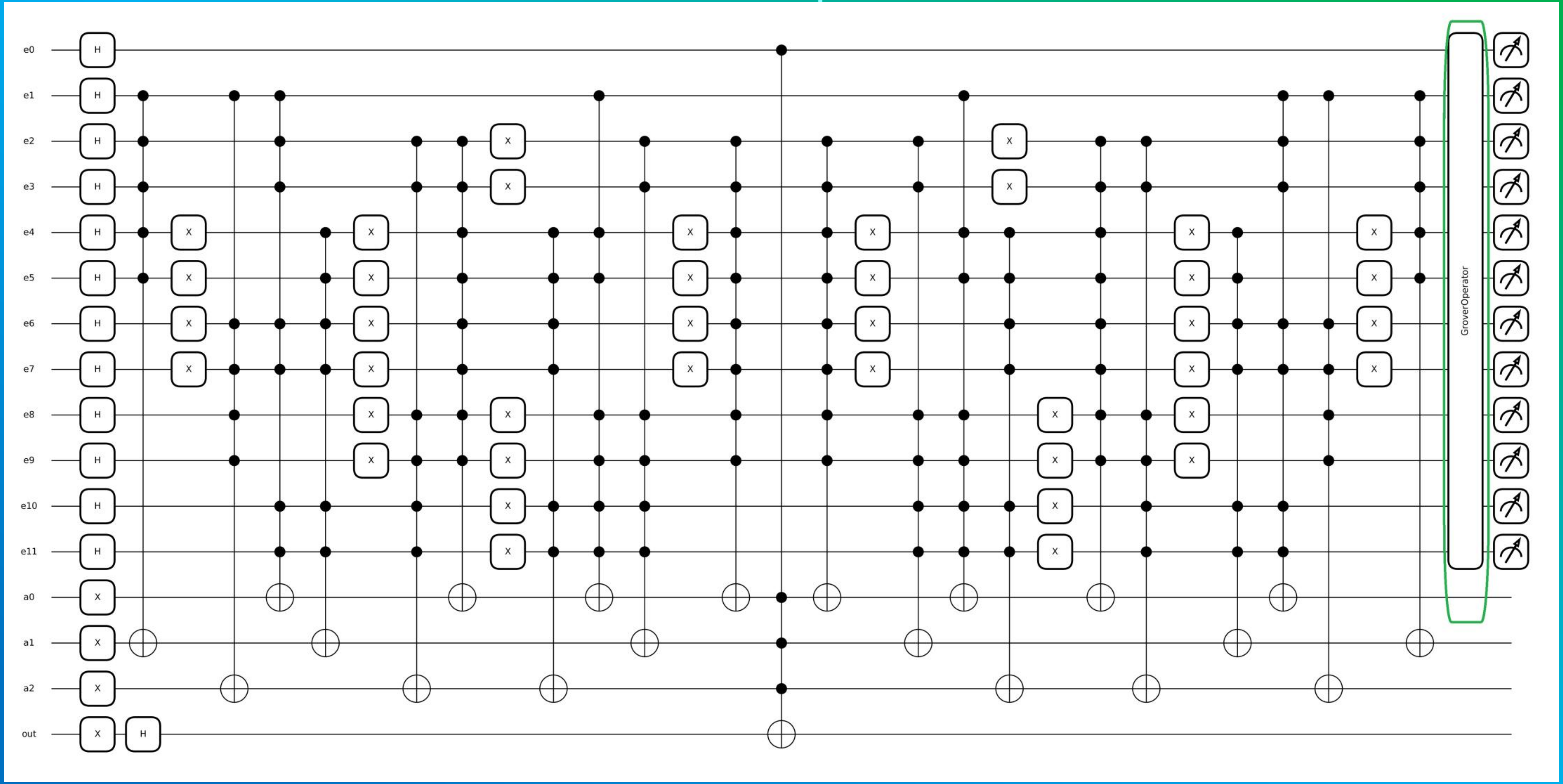
Graph representing the adjacency matrix of equally oriented edges

A second algorithm is implemented to determine the optimal order of the gates

$$\mathrm{OMUTc}^{(3,12)} = \{\{c_0\}, \{c_1, c_4, \overline{c_3}\}, \{c_2, c_5\}, \{c_3, c_6, \overline{c_2}\}, \{\overline{c_5}\}\}$$

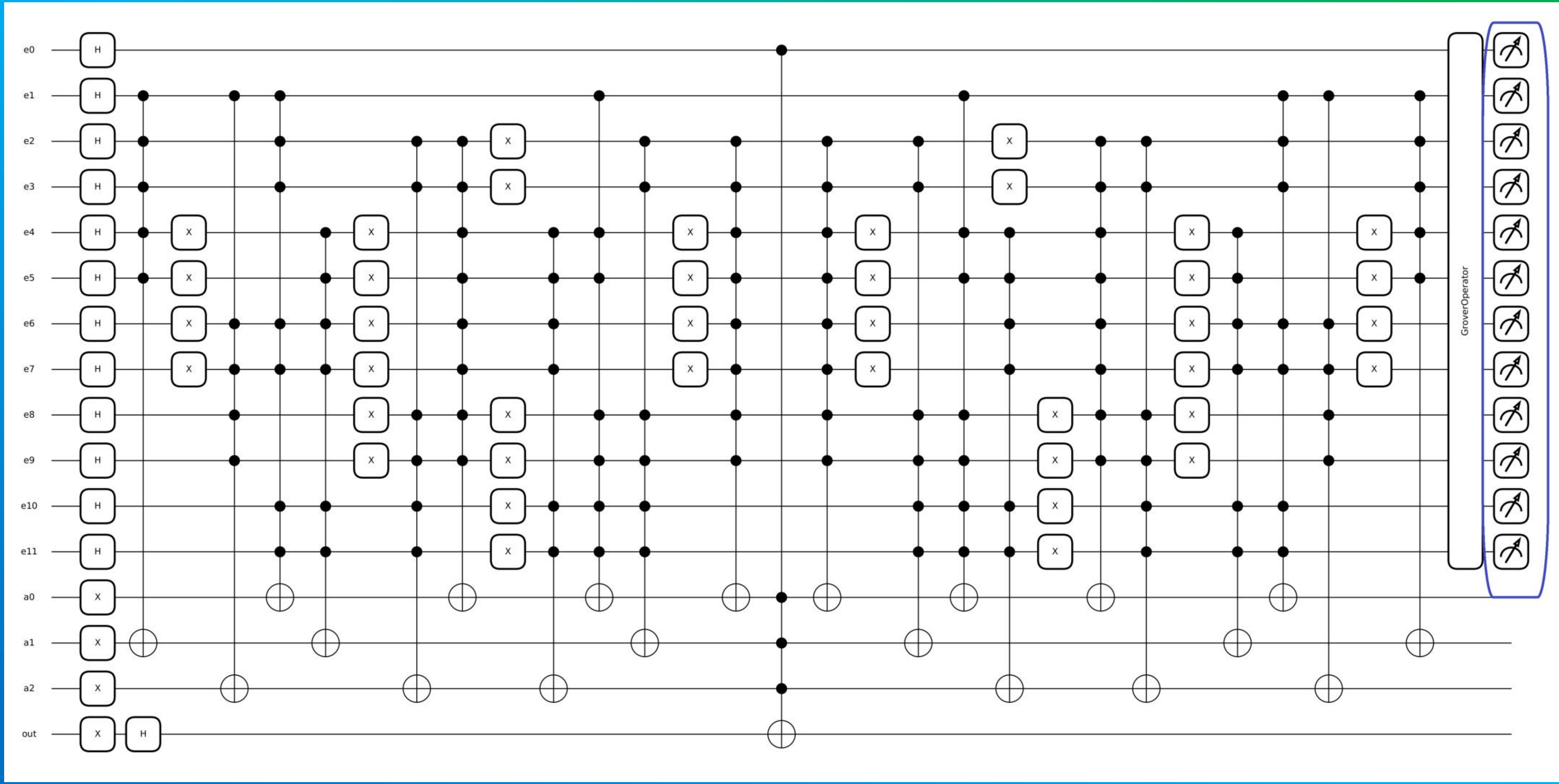
CIRCUIT DESIGN

Diffusion Operator



CIRCUIT DESIGN

Measurements



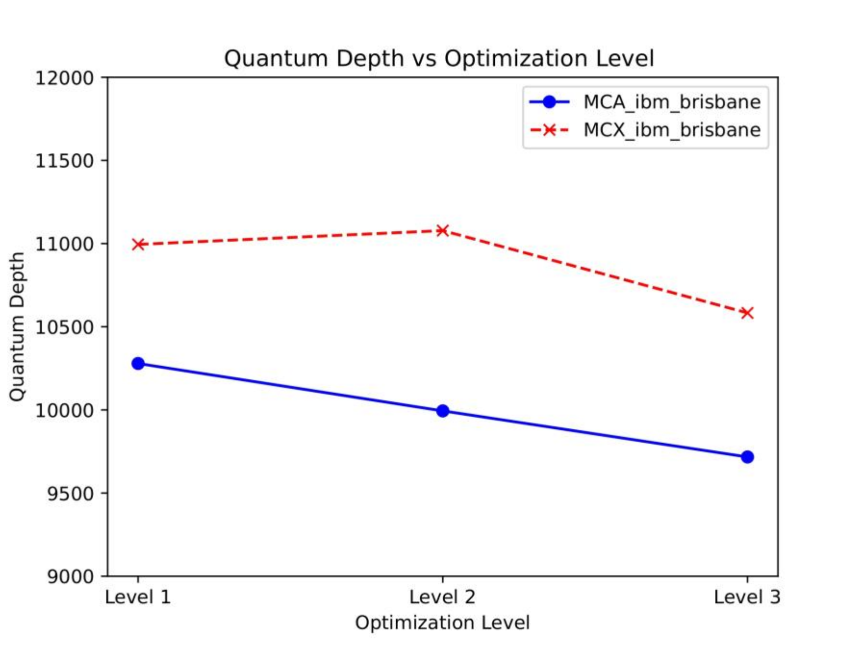
RESULTS

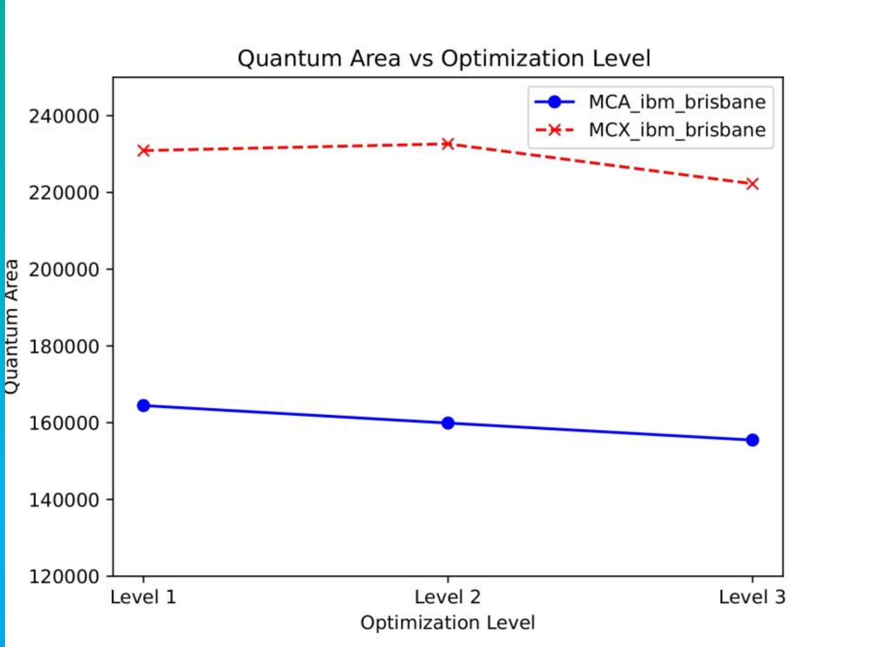
eloops (edges)	$\vert e\rangle$	$\vert a\rangle$	Total Qubits	Quantum Depth	Total states
three (9)	9	2 | 4	12 | 14	15 | 17	512
three (12)	12	3 | 7	16 | 21	23 | 31	8192
four$^{(c)}$ (12)	12	4 | 5	17 | 18	15 | 15	4096
four$^{(c)}$ (16)	17	6 | 13	24 | 31	39 | 45	131072

Quantum resources required and theoretical quantum circuit depth of the quantum algorithms used to analyze the three-eloop topology with nine and twelve edges, and the four-eloop topology with twelve and sixteen edges. The first number in the third, fourth and fifth columns are from the MCA quantum algorithm, whereas the second number corresponds to MCX quantum algorithm

CIRCUIT DESIGN

Transpilation behavior





Quantum circuit depth (left) and quantum circuit area (right) for the three-eloop topology with twelve edges implementing the MCX and MCA quantum algorithms for different optimization levels

CONCLUSIONS

- By mapping Feynman propagators to qubits and using graph theory its possible to automize the design of an oracle circuit
- It is possible to reduce the qubits needed for the quantum circuit design by clustering mutually exclusive clauses in the same ancilla qubit
- The quantum depth its reduced by a proper arrange of the quantum gates.

REFERENCES

- Ochoa-Oregon, S. A., Uribe-Ramírez, J. P., Hernández-Pinto, R. J., Ramírez-Uribe, S., & Rodrigo, G. (2025). Graph theory-based automated quantum algorithm for efficient querying of acyclic and multiloop causal configurations [2508.04019]
- Ramírez-Uribe, S., Rentería-Olivo, A. E., & Rodrigo, G. (2025). Quantum querying based on multicontrolled Toffoli gates for causal Feynman loop configurations and directed acyclic graphs. [2404.03544]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