Pions and nucleons at counterpoint

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Pseudoscalar Mesons and Emergent Mass

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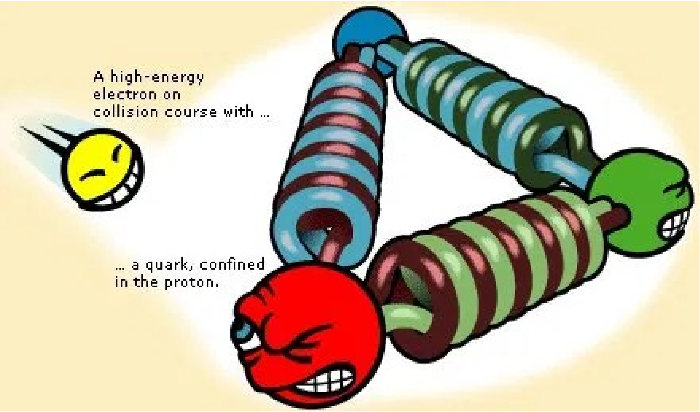
QCD: Emergent Phenomena

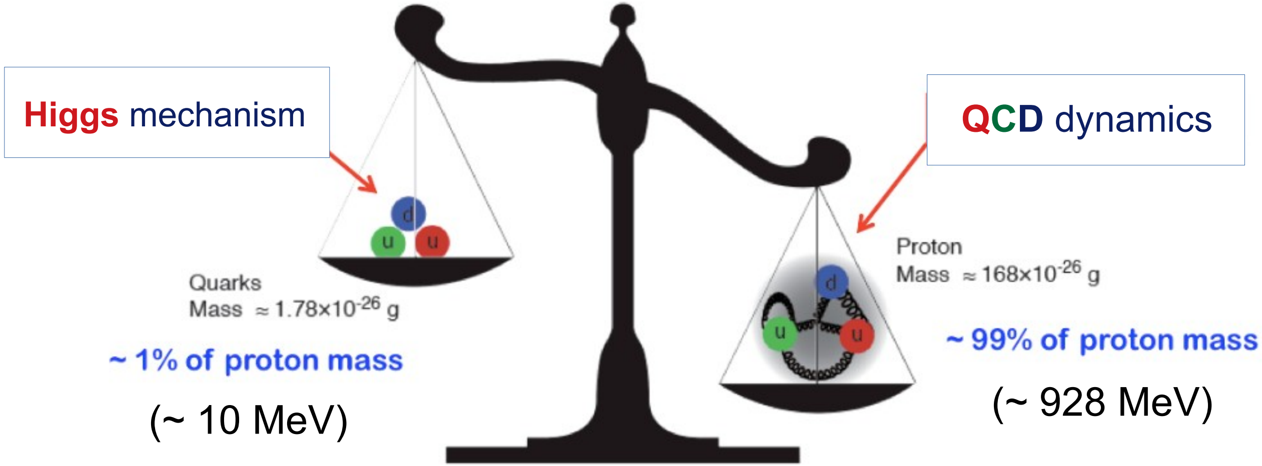
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?



- Emergence of hadron masses (EHM) from QCD **dynamics**



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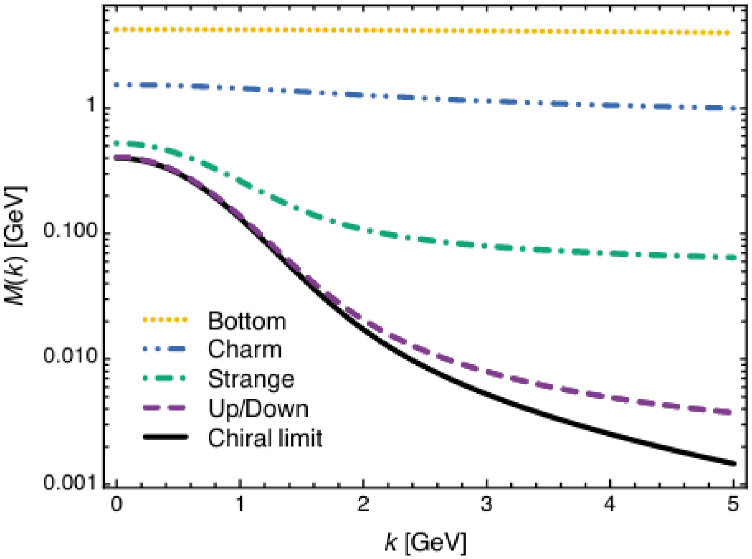
Can we trace them down to fundamental d.o.f?

- Emergence of hadron masses (EHM) from QCD **dynamics**

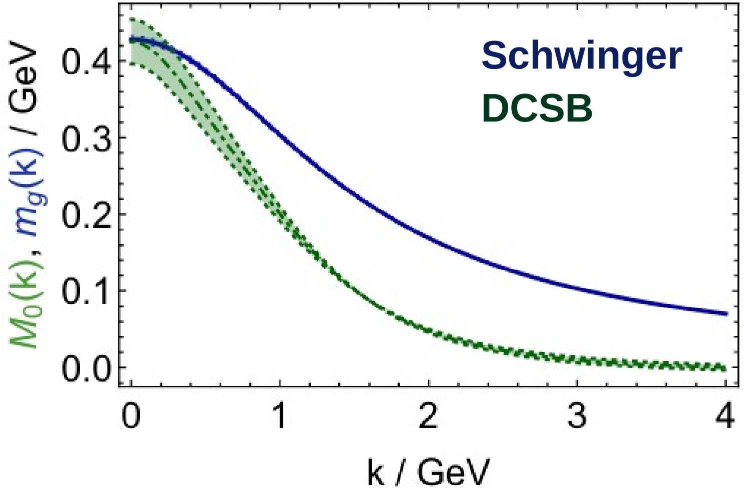
Constituent masses

Dynamical Chiral Symmetry Breaking



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$



Gluon and quark *running masses*

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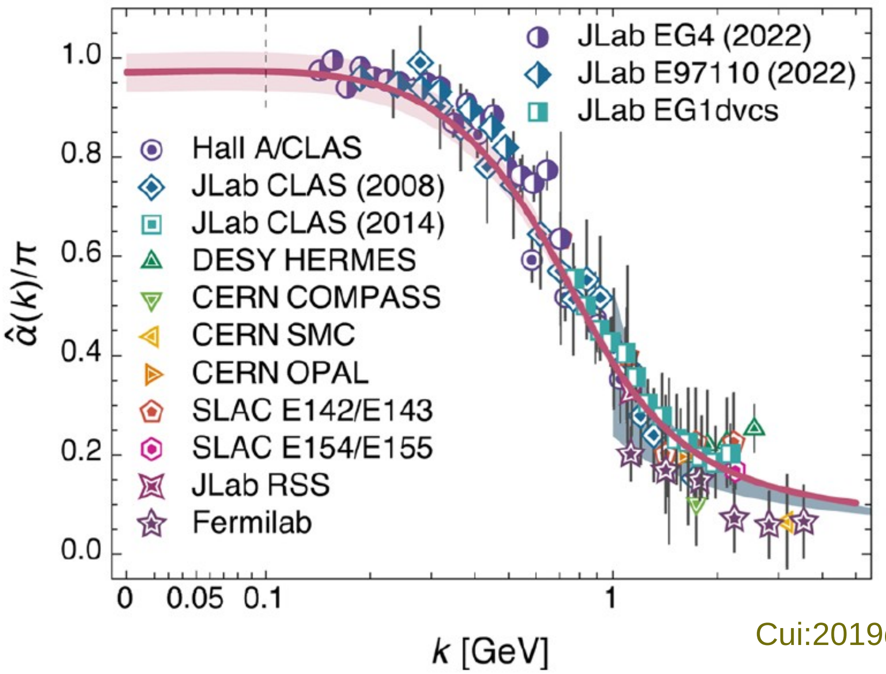
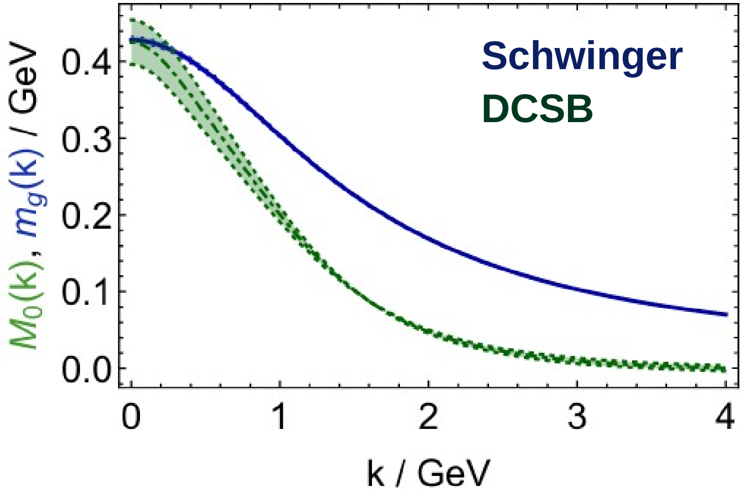


Cui:2019dwv



Gluon and quark *running masses*

Mass **Budgets** $M_{u/d} \approx 0.3\,\text{GeV}$

- What is the origin of **EHM**?

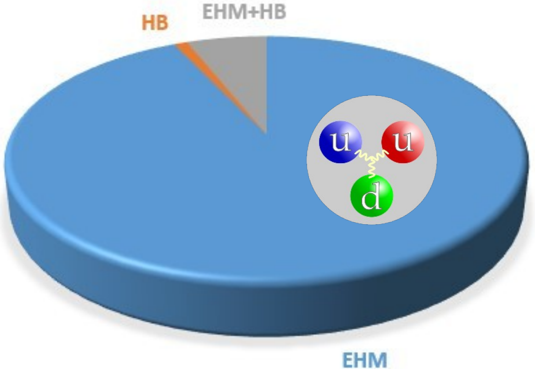
  … its connection with *e.g.* **confinement** and **DCSB**?

> - **Most** of the **mass** in the visible universe is contained within **nucleons**

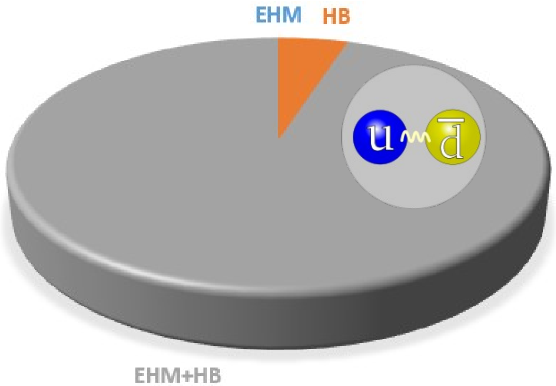
  - Their role in the universe is **crucial**

> - The **Pion** is the most essential **NG boson** of **DCSB**.

  - It's mere **existence** is connected with **mass** generation in the **SM**



Proton mass budget



Pion meson mass budget

$$m_p = 0.938\,\text{GeV} \approx 2M_u + M_d$$
$$m_\pi = 0.14\,\text{GeV} \neq M_u + M_d$$

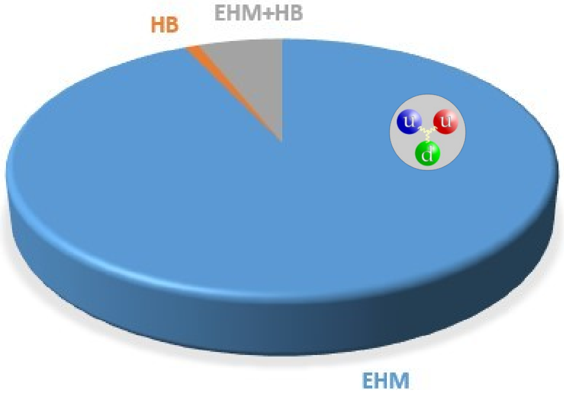
With the same building blocks, Nature makes one **heavy** and one **too light**. What's happening?

Mass **Budgets**

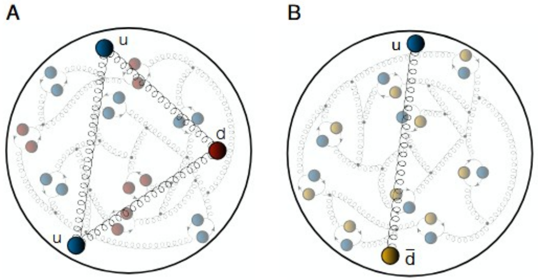
$$m_s/m_u \sim 20$$

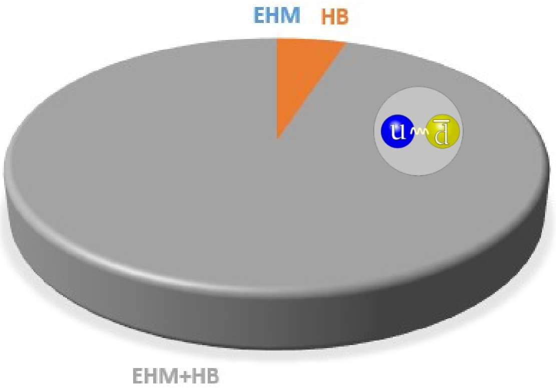
$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

➢ The same **mechanisms** that make the **proton massive**, makes the **pion 'massless'**



Proton mass budget





Pion mass budget

$$m_p = 0.938\,\mathrm{GeV} \approx 2M_u + M_d \qquad m_\pi = 0.14\,\mathrm{GeV} \neq M_u + M_d$$

➢ This **dichotomy** needs to be understood.

EIC, EicC, JLab, Amber...

L. Chang *et al.*,
Phys.Rev.Lett. 110 (2013) 13, 132001

Valence-quark distribution amplitudes (DAs)

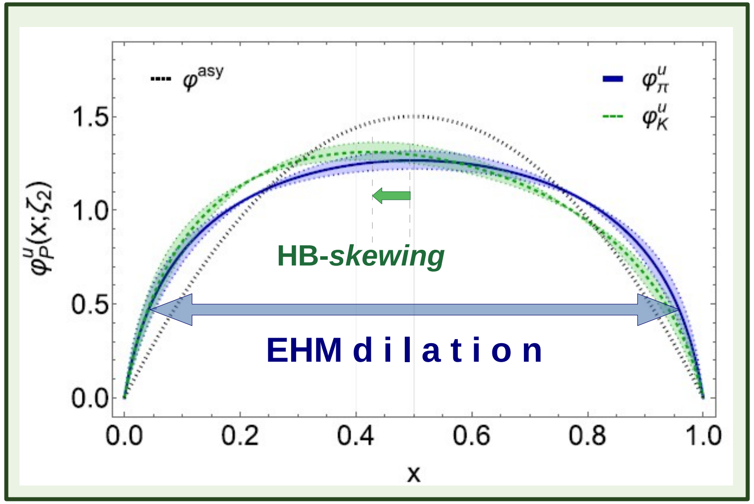
$$f_M \phi_M^q(x) = \mathrm{tr} \int_{dk} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$

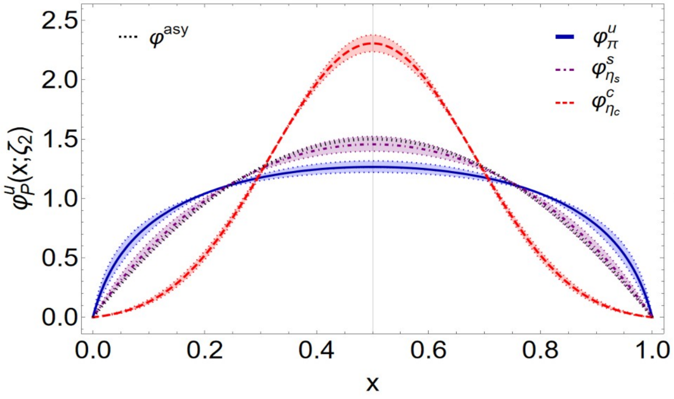
Light-front momentum fraction

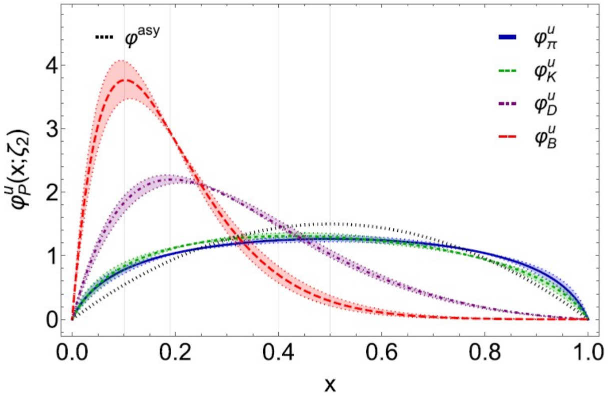
Written in terms of **BSWF**

- **1-dimensional** projection of the **light-front wavefunction**.
- Clear **probe of EHM**, related with hard **exclusive processes**, etc.

Pseudoscalars' DAs







$$f_M \phi_M^q(x) = \mathrm{tr}\int_{dk} \delta_n^x(k_M)\gamma_5\gamma\cdot n\,\chi_M(k_-,P)$$

- **Broad** in the **light-sector**, **narrow** in the **heavy-sector**.
- **Mild *skewness*** for the **Kaon**, striking for **heavy-lights.**

Ssbar in the middle

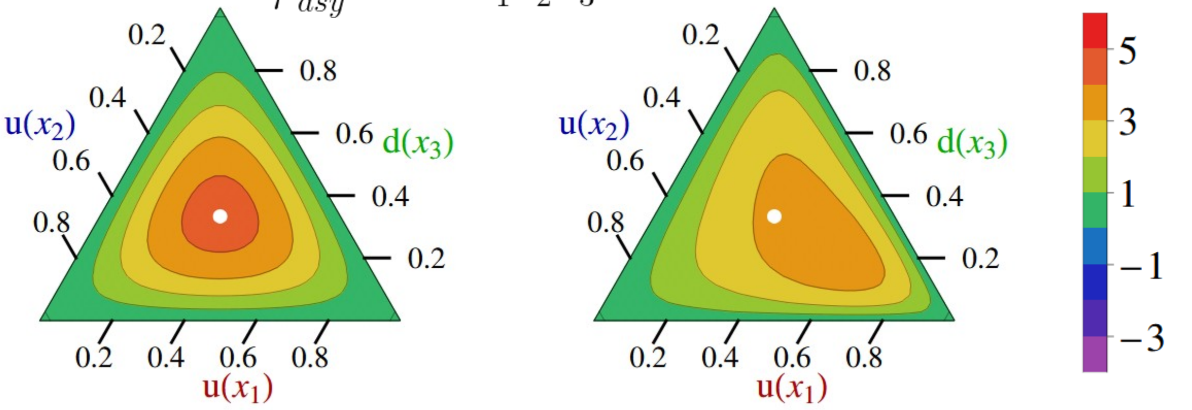
Proton **DAs**

$(M_u = M_d)$

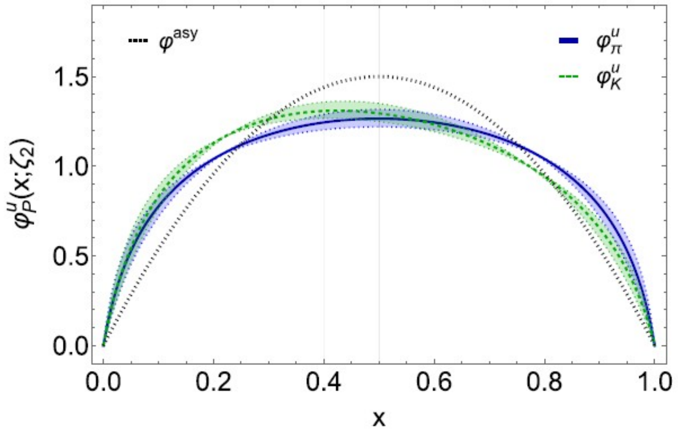
- As the **pion**, the **proton DA** is **broader** than its asympotic counterpart:

Mezrag:2017znp

$$\varphi^p_{asy} = 120 x_1 x_2 x_3$$



Asymptotic **Proton**

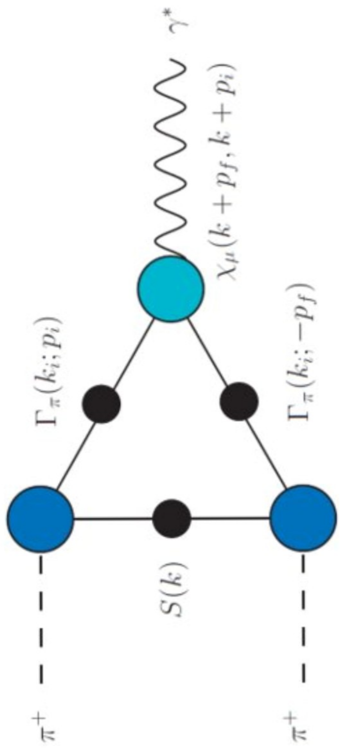


- Nonetheless, even assuming **isospin symmetry**, the proton exhibits a **non-symmetric DA**.
  - This signals the **formation** of non-trivial quark-quark correlations (dynamical **diquarks)**

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like}} \times |F_\pi(q^2)|^2$$

L. Chang *et al.*,
Phys.Rev.Lett. 111 (2013) 14, 141802

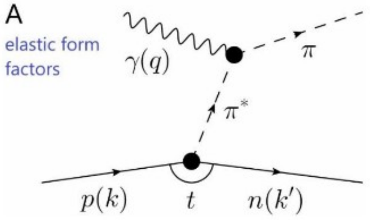
Electromagnetic Elastic Form Factors (EFFs)



$$P_\mu F^q_{\mathbf{P}}(Q^2) = \text{tr}_{\text{CD}} \int_{dk}^{\Lambda} \chi^q_\mu(k+p_o, k+p_i)\Gamma_{\mathbf{P}}(k_i;p_i)\, S_h(k)\, \Gamma_{\mathbf{P}}(k_o;-p_o)$$

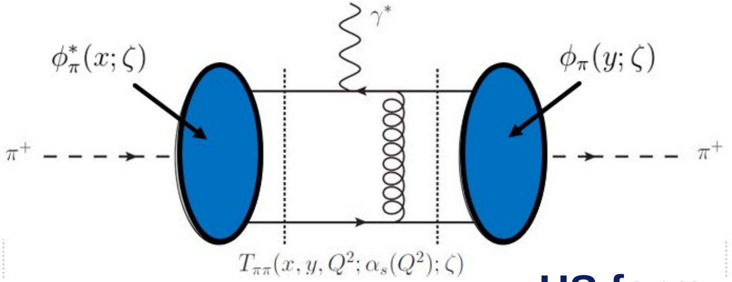
All can be written in terms of **propagators** and **vertices**

- Gives information on **momentum/charge** distribution.
- **Pion EFF** highly relevant for contemporary physics.



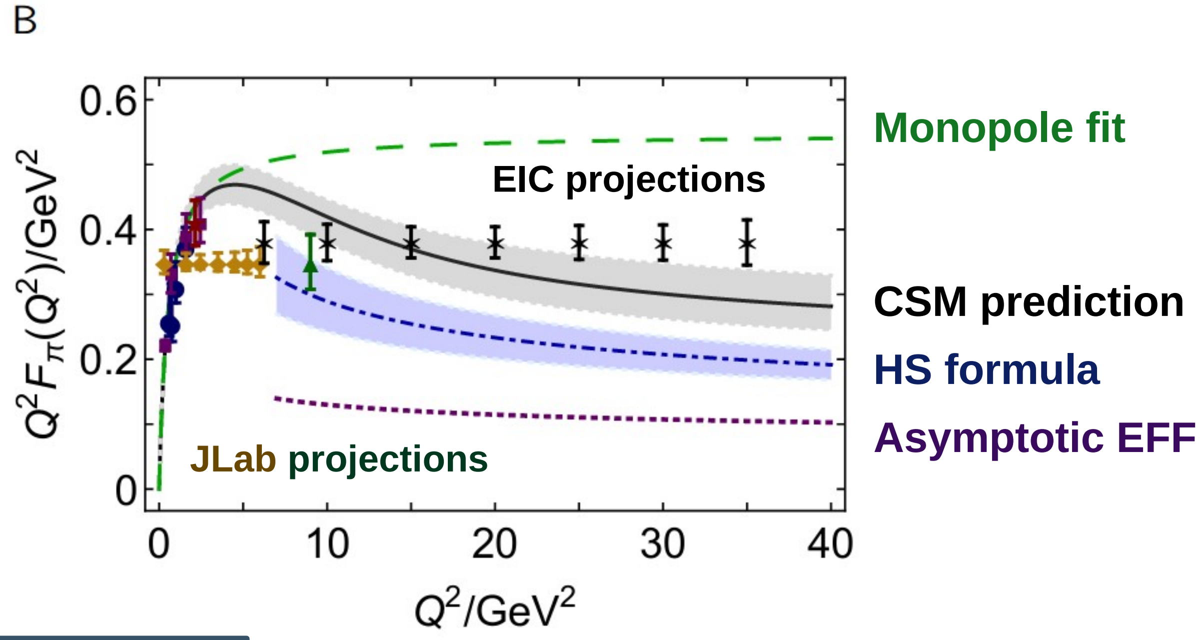
Elastic **Form Factors**

- In the **large-Q^2** regime, **QCD** connects the **EFF** and **DA**:



(at sufficiently large Q^2)

HS formula



- At **leading-order**:

$$Q^2 F_{\mathbf{P}}(Q^2) \overset{Q^2 \gg m_p^2}{\approx} 16\pi\alpha_s(Q^2) f_{\mathbf{P}}^2 w_{\mathbf{P}}^2(Q^2)$$

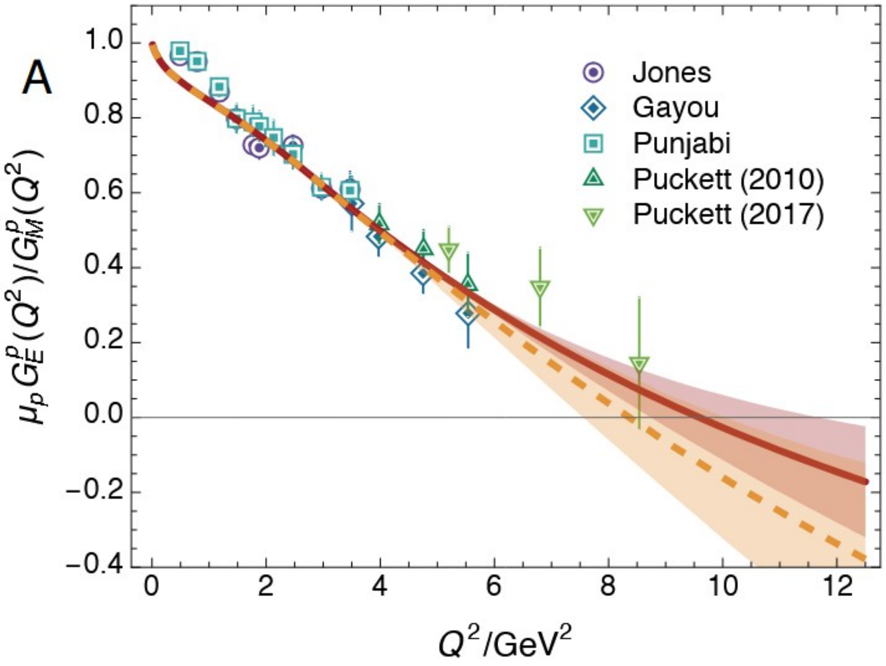
$$w_{\mathbf{P}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_{\mathbf{P}}(x;Q^2)$$ ⟵ **PDA** ⟶

- The asymptotic behavior is weighted by f_P, a measure of **EHM**.
- **Factorization/scaling violations** are proof of the validity of **QCD itself.**

Elastic Form Factors

Cheng:2025yij

- For the **proton**, different theoretical and phenomenological analysis suggest a **zero-crossing** in the **GE/GM** ratio



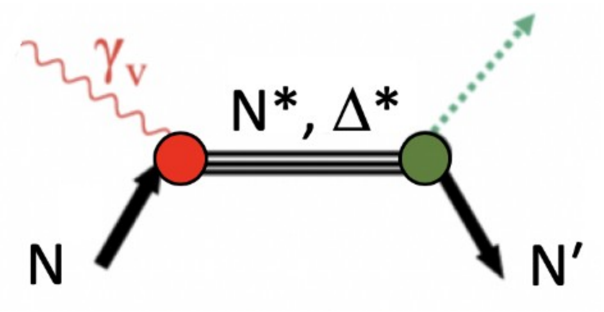
- This reflection on a **destructive interference** between the individual **valence-quark** contributions.
- In fact connected with non-trivial effects coming from orbital **angular momentum**.
- The **location** of the zero is highly sensitive to the **diquark** content of the **nucleon**.

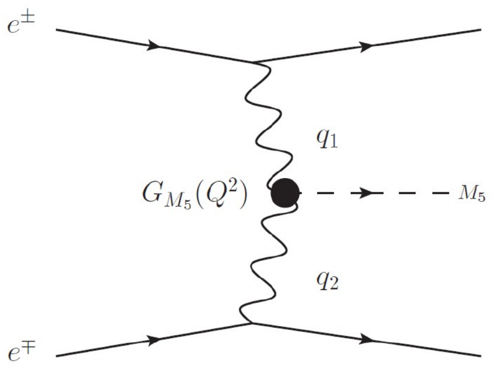
- Modern analyses favor a **non-negligible** component of **axial-vector** diquark:

~ 25 – 35 %

- The rest comes from the expected **scalar diquark**.

Transition Form Factors (TFFs)

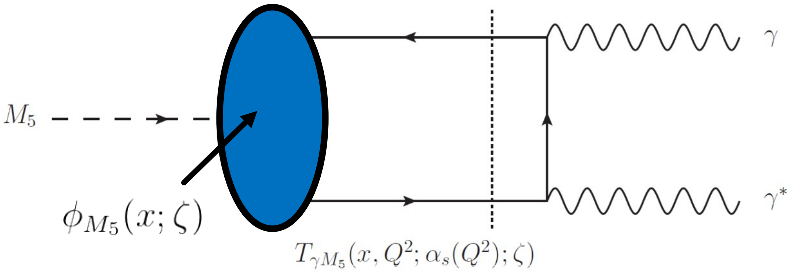




- Gives information on **momentum**/**charge** distribution.
- Highly relevant for contemporaty physics.

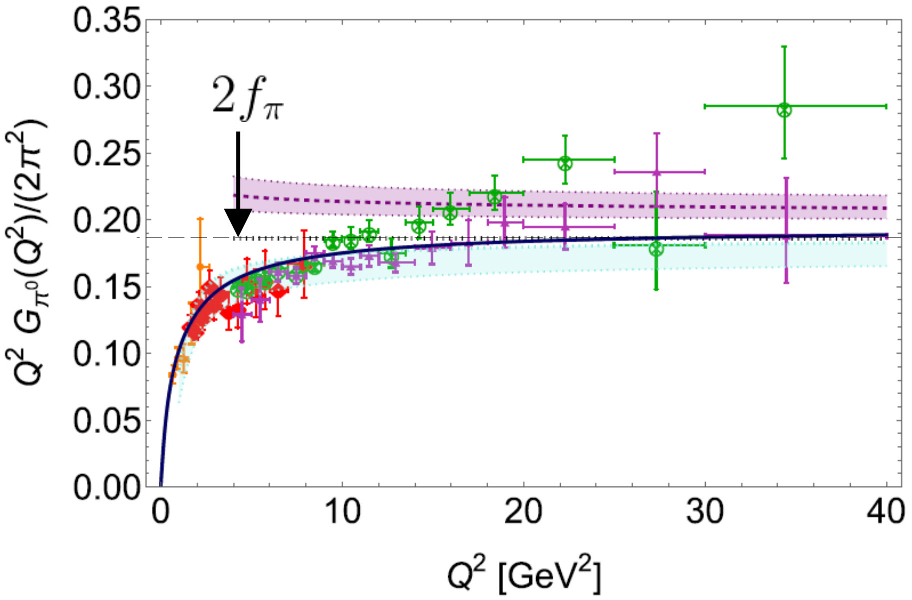
Two-photon TFFs

- In the **large-Q²** regime, **QCD** connects the **TFF** and **DA**:



(at sufficiently large Q^2)

HS formula



HS formula

CSM prediction

Asymptotic TFF

- At **leading-order**:

$$Q^2 G^q_{\mathbf{P}}(Q^2) \overset{Q^2 \gg m_p^2}{\approx} 12\pi^2 \, f^q_{\mathbf{P}} \, e_q^2 \, w_q(Q^2)$$

$$w_{\mathbf{P}} = \frac{1}{3}\int_0^1 dx \, \frac{1}{x} \, \varphi_{\mathbf{P}}(x; Q^2)$$ ← **PDA** →

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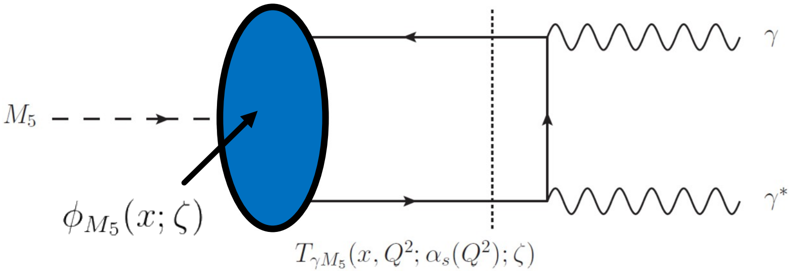

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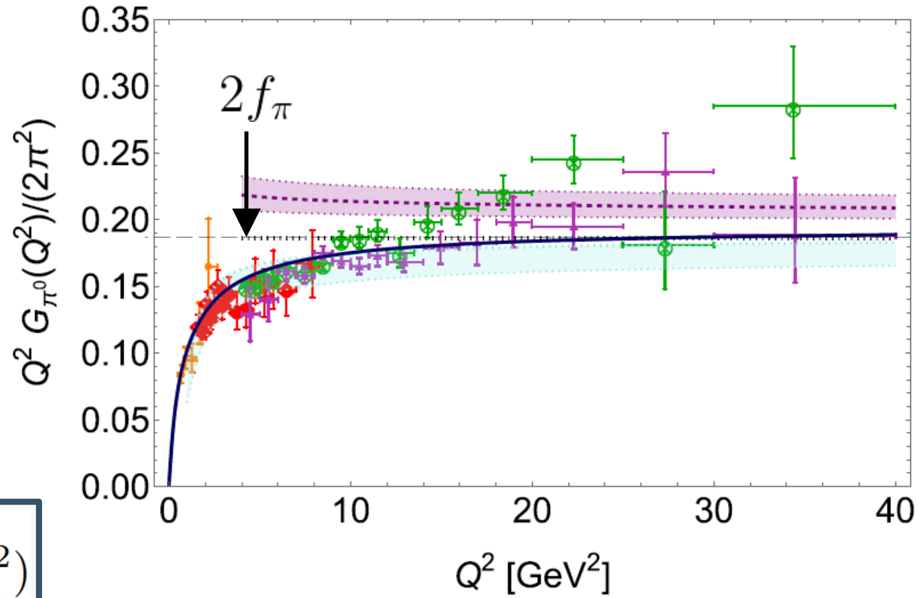


HS formula

CSM prediction

Asymptotic TFF

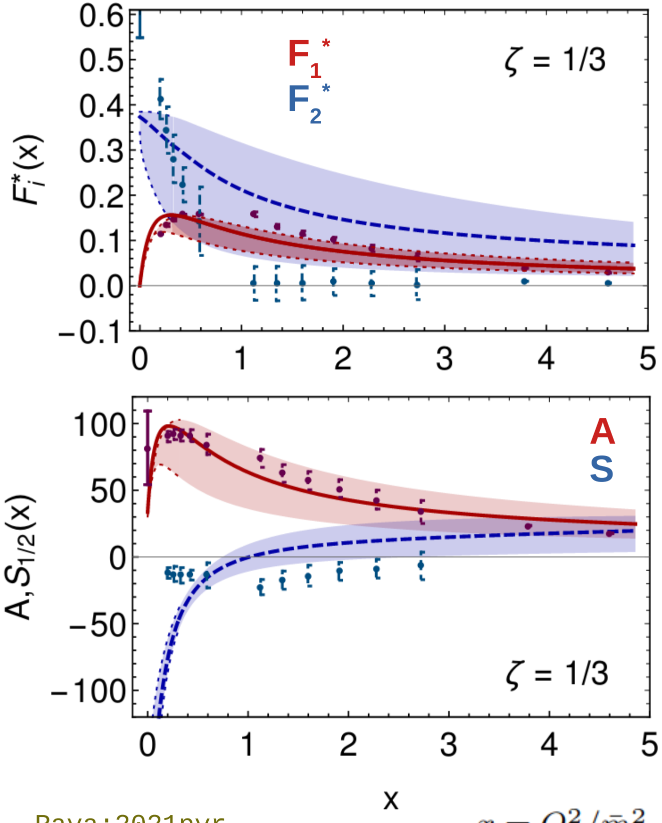
- In the **opposing** end, the **chiral *anomaly*** entails:

$$2 f^0_{\mathbf{P}} G^0_{\mathbf{P}^0}(Q^2 = 0) = 1$$

- Thus **EHM** (via **DCSB**) sets the infrared scale as well.
- Any deviations from this result are a measure of **EHM**+**HB** interplay.

Example: N → N(1535)

- Transition **form factors** and **helicity amplitudes**:



Raya:2021pyr

$x = Q^2/\bar{m}^2$, $\bar{m} = (m_+ + m_-)/2$:

- In the absence of **DCSB**, the **nucleon** and its **parity partner** would be structurally **alike**. This is not the case...

- Their **mass** and internal **structure** is in fact quite **different**. We need to elucidate how and <u>why</u>.
  - → Something not trivial is happening internally.

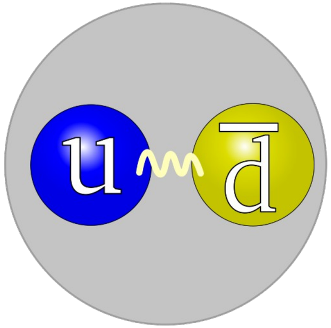
- Nucleon **TFFs** provide stringent **constraints** to the **wavefunctions**.

If one varies $g_{\rm DB} \to g_{\rm DB}(1 \pm 0.5)$, then $m_{N(1535)} = (1.67, 1.82)$ GeV and

$N(1535)\frac{1}{2}^-$	s	a_1^1	a_2^1	p	v_1	v_2
$g_{\rm DB}$ 1.5	0.76	0.27	0.18	0.49	0.12	0.08
$g_{\rm DB}$ 1.0	0.66	0.20	0.14	0.68	0.11	0.09
$g_{\rm DB}$ 0.5	0.35	0.04	0.00	0.92	−0.05	0.18

M. Ding *et al.*, PRD 101 5, 054014 (2020)
Z-F Cui *et al.*, EPJC 80 11, 1064 (2021)

Distribution functions (DFs)

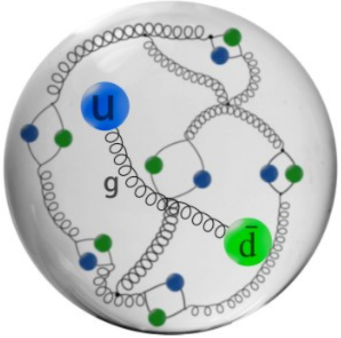


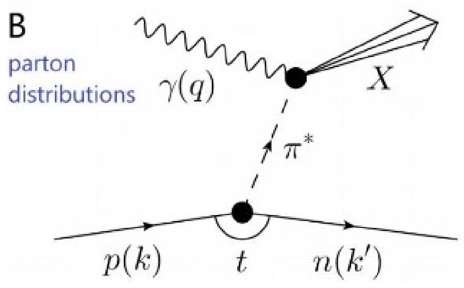
ζ_H

Resolution Scale

Evolution equations

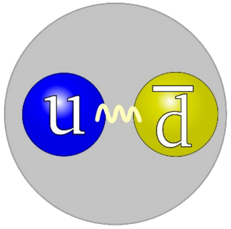
$\zeta > \zeta_H$





- Yields *e.g.* information on **momentum** distribution.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

π-K DFs: hadronic scale



$(M_u = M_d)$

- **Fully-dressed** **valence quarks** (quasiparticles)

ζ_H : hadronic scale

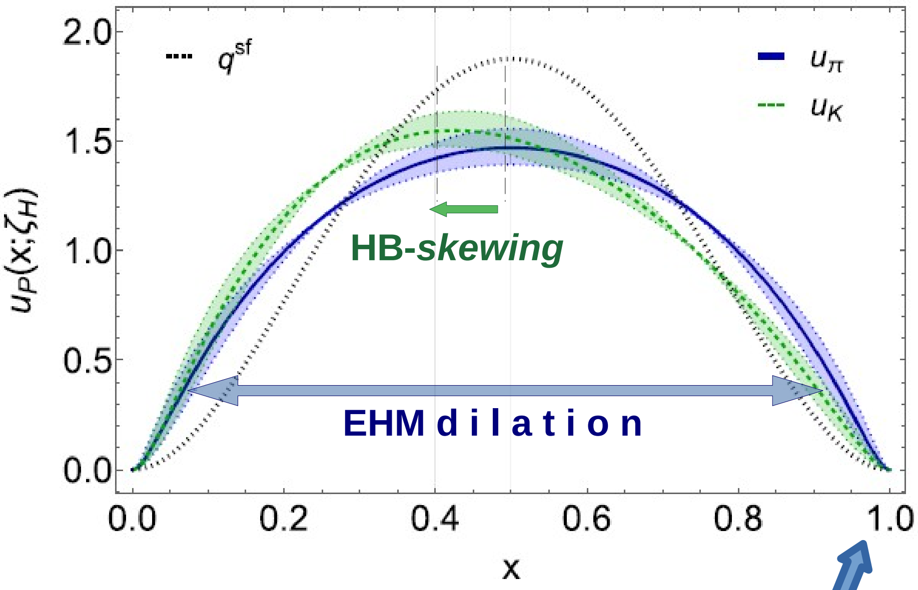
➢ At this scale, **all properties** of the hadron are contained within their valence quarks.

  ➔ Equally massive quarks symmetric distributions and equitable distribution of momentum fraction:

$$<x>_{\pi}^{u} = 0.5\,,\; u_\pi(x;\zeta_H) = u_\pi(1-x;\zeta_H)$$

➢ The **kaon** distributions are only-shifted by a **few-percentage**.

$$<x>_{K}^{u} = 0.48\,,\; <x>_{K}^{\bar{s}} = 0.52$$

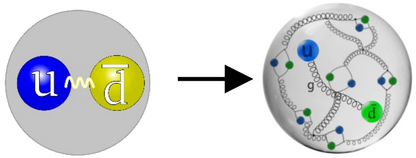


Endpoint **smoothness** is a reflection of the underlying interaction

$$1/(k^2)^\beta \rightarrow (1-x)^{2\beta}$$

Farrar:1975yb Berger:1979du Holt:2010vj

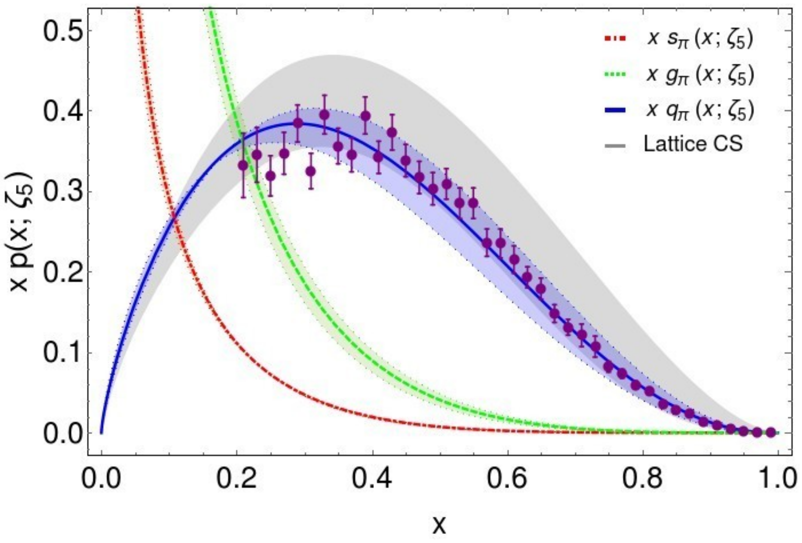
Pion DFs: Lattice & Experiment



- At **5.2 GeV**, the experimental scale, our predictions matches that from Aicher *et al.*

Aicher:2010cb

$$<x_{\text{gluon}}> = 0.45(1)\ ,\ <x_{\text{sea}}> = 0.14(2)$$



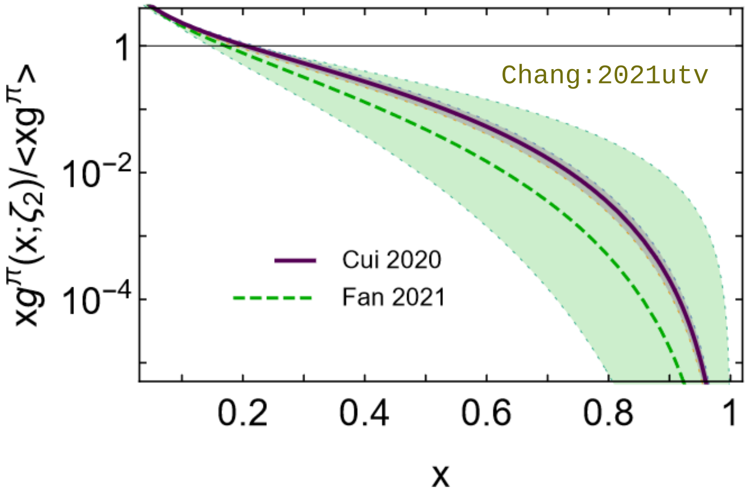
- An agreement with novel **lattice** "Cross Section" results is also obtained.

Sufian:2019bol

- At **2 GeV**, the **valence DF** shows agreement with lattice moments:

ζ_2	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [34]	0.24(2)	0.09(3)	0.053(15)
Ref. [35]	0.27(1)	0.13(1)	0.074(10)
Ref. [36]	0.21(1)	0.16(3)	
Herein	0.24(2)	0.098(10)	0.049(07)

- The **Gluon DF** profiles matches **lattice** expectations:



Pion vs Proton

Y. Lu *et al.*
Phys.Lett.B 830 (2022) 137130

- The (nearly) massless **pion DFs** differs vastly from the massive **proton**. For instance:
  - **Counting rules** entail large-x behaviors **(1-x)²** and **(1-x)³** for the **pion** and **proton**, respectively.
  - The **momentum fractions** at ζ_H: $(M_u = M_d)$

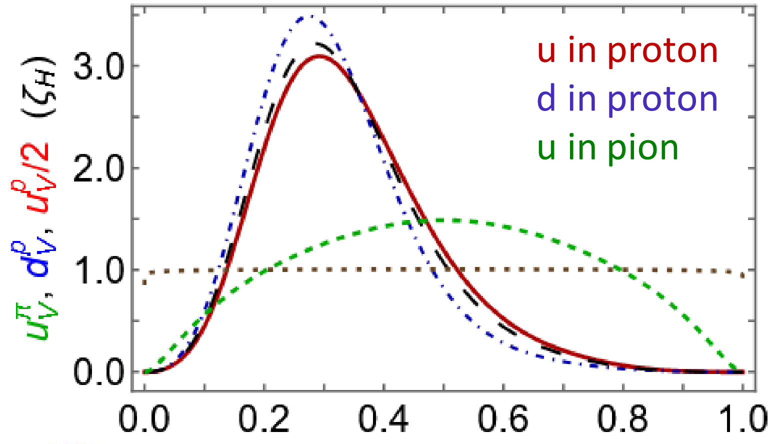
$$\langle x \rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x \rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x \rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5$$

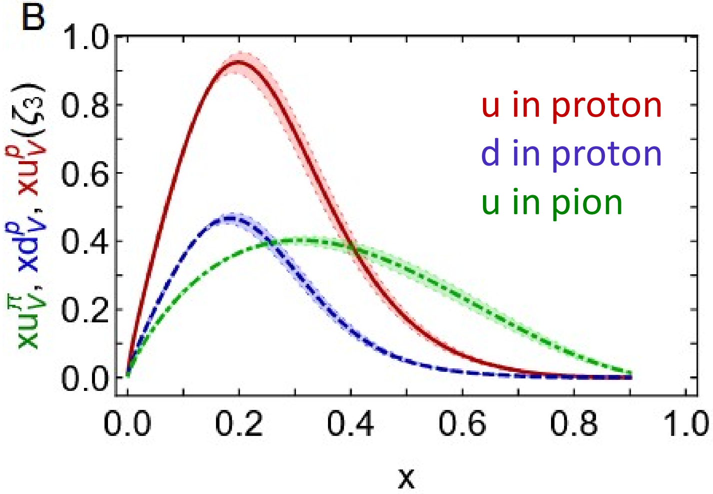
$$\Rightarrow u_V(x) \neq 2 d_V(x)$$

EHM induced diquark correlations inside the proton:

- No equitable distribution of momentum!

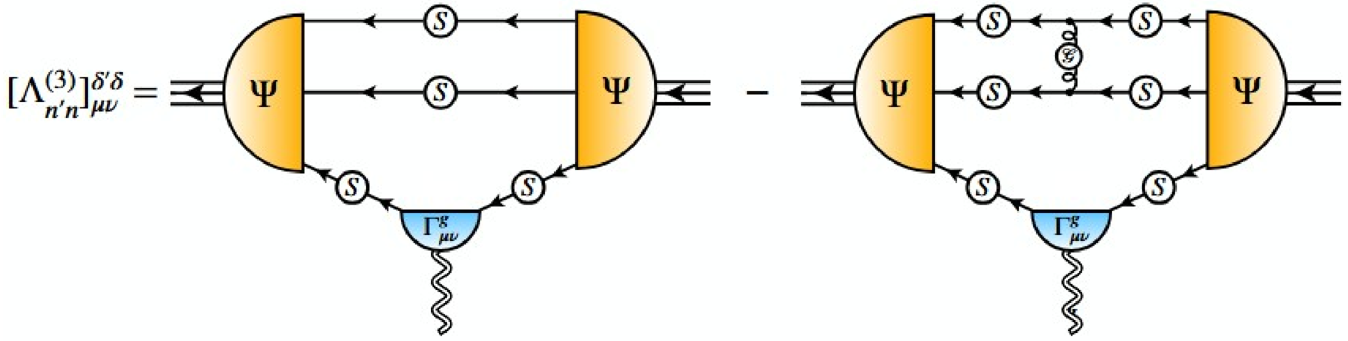
  - Differences are **preserved** after evolution. →





Yin-Zhen Xu *et al.*,
Eur.Phys.J.C 84 (2024) 2, 191

Gravitational Form Factors (GFFs)



- Gives information on **mass**/**pressure** distribution.
- A first step towards **nucleon GFFs**

Gravitational **form factors**

- The expectation value of the energy-momentum tensor (**EMT**) in the **pseudoscalar meson** defines the **gravitational form factors**:

$$\Lambda^{\mathbf{P}}_{\mu\nu}(K,Q) = 2K_\mu K_\nu A^{\mathbf{P}}(Q^2) + \frac{1}{2}[Q_\mu Q_\nu - Q^2\delta_{\mu\nu}]D^{\mathbf{P}}(Q^2) + 2m^2_{\mathbf{P}}\delta_{\mu\nu}\bar{c}^{\mathbf{P}}(Q^2)$$

- Where **symmetry principles** entail:

$$A^{\mathbf{P}}(0) = 1$$

Momentum conservation

$$D^{\mathbf{P}}(0) \overset{m_{\mathbf{P}}=0}{=} -1$$

Soft-pion theorem

$$\bar{c}^{\mathbf{P}}(Q^2) = 0$$

EMT conservation

- The deviation from **D(0)=-1**, is a manifestation of the **interplay** between **Higgs and QCD** mass generation mechanisms.

Polyakov:2018zvc

Gravitational form factors

- The expectation value of the energy-momentum tensor (**EMT**) in the **nucleon** defines the **gravitational form factors**:

$$m_{\mathrm{N}}\Lambda^{\mathrm{N}}_{\mu\nu}(K,Q) = -\Lambda_+(p_f)[K_\mu K_\nu A^{\mathrm{N}}(Q^2) + iK_{\{\mu}\sigma_{\nu\}\rho}Q_\rho J^{\mathrm{P}}(Q^2) + \frac{1}{4}(Q_\mu Q_\nu - Q^2\delta_{\mu\nu})D^{\mathrm{N}}]\Lambda_+(p_i)$$

- Where **symmetry principles** entail:

$$A^{\mathrm{P}}(0) = 1$$

Momentum conservation

$$J^{\mathrm{N}}(0) = 1/2$$

Spin sum-rule

$$\bar{c}^{\mathrm{P}}(Q^2) = 0$$

EMT conservation

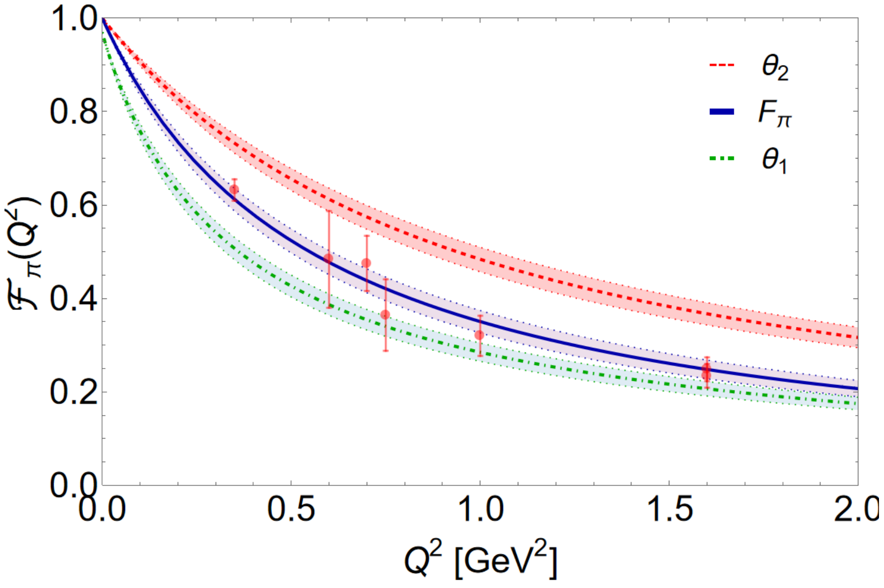
- There is **no constraint on D(0)**, the so called **D-term**, which is often referred to as "**The Last Unknown Global Property of the Nucleon**".

Polyakov:2018zvc

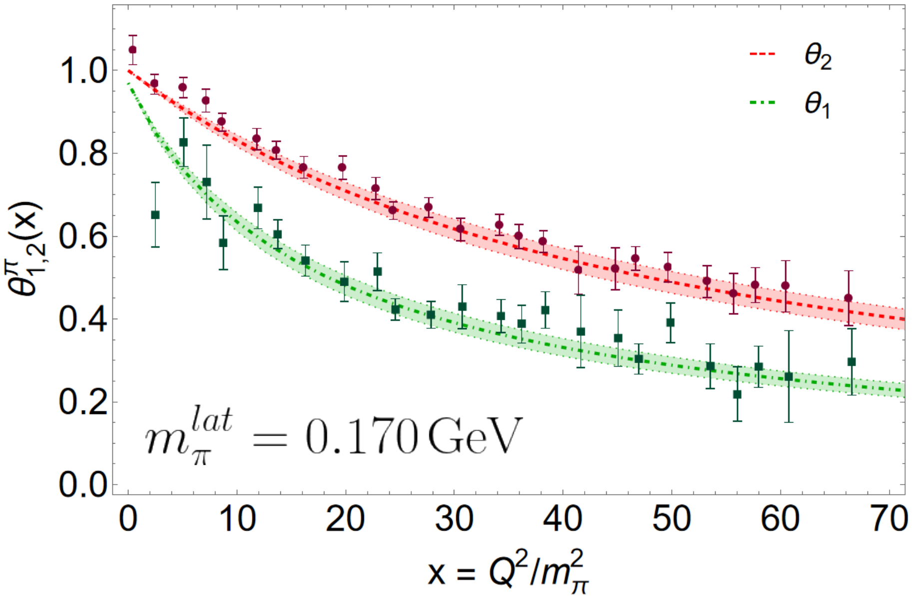
Pion GFFs

$r_{\theta_1}/r_F \approx r_F/r_{\theta_2} \approx 0.75$

- For **π (K)** it is found: $r^{\mathbf{P}}_{\theta_1} = 0.81\,\text{fm}\,(0.63\,\text{fm}) > r^{\mathbf{P}}_{F} = 0.64\,\text{fm}\,(0.58\,\text{fm}) > r^{\mathbf{P}}_{\theta_2} = 0.47\,\text{fm}\,(0.40\,\text{fm})$.
  (mechanical) (charge) (mass)



- Agreement with **Lattice QCD** is also obtained: Hackett:2023nkr



- The **Kaon**, albeit more **compressed**, exhibit similar patters. $\bar{r}_K/\bar{r}_\pi = 0.85(6)$

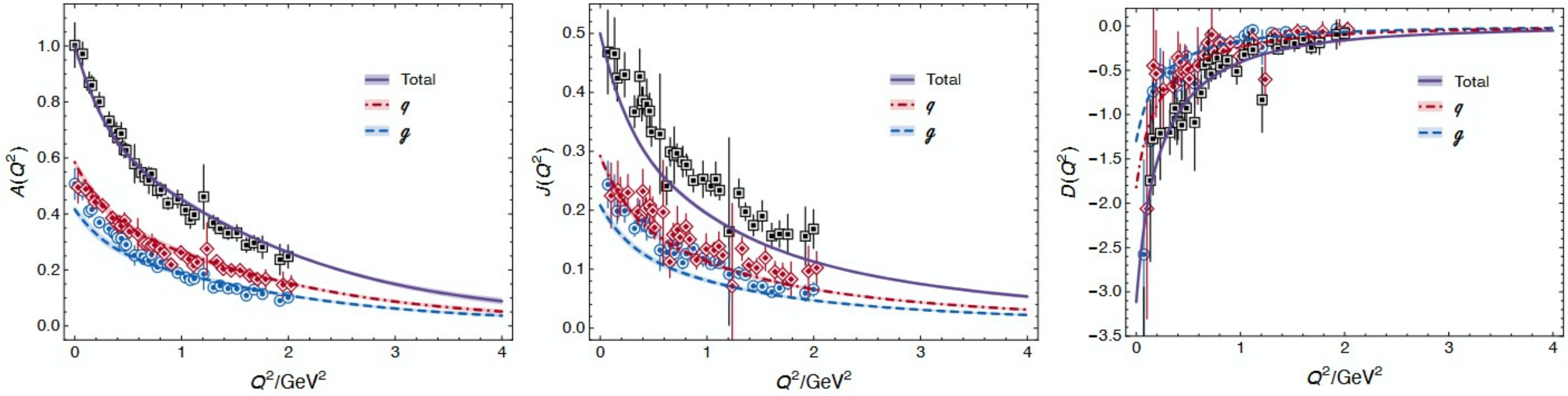
Nucleon **GFFs**

Z-Q Yao *et al.*,
Eur.Phys.J.A 61 (2025) 5, 92

- Our **prediction** is compatible with more recent **lattice** expectations: Hackett:2023rif

herein	$A(0)$	$J(0)$	$-D(0)$
Total	1.00	0.50	$3.114(10)_{\pm}$
q	$0.584(13)_{\pm}$	$0.292(06)_{\pm}$	$1.820(43)_{\pm}$
g	$0.416(13)_{\mp}$	$0.208(06)_{\mp}$	$1.294(33)_{\mp}$
lQCD	$A(0)$	$J(0)$	$-D(0)$
Total	1.011(37)	0.506(25)	3.87(97)
q	0.510(25)	0.251(21)	1.30(49)
g	0.501(27)	0.255(13)	2.57(84)

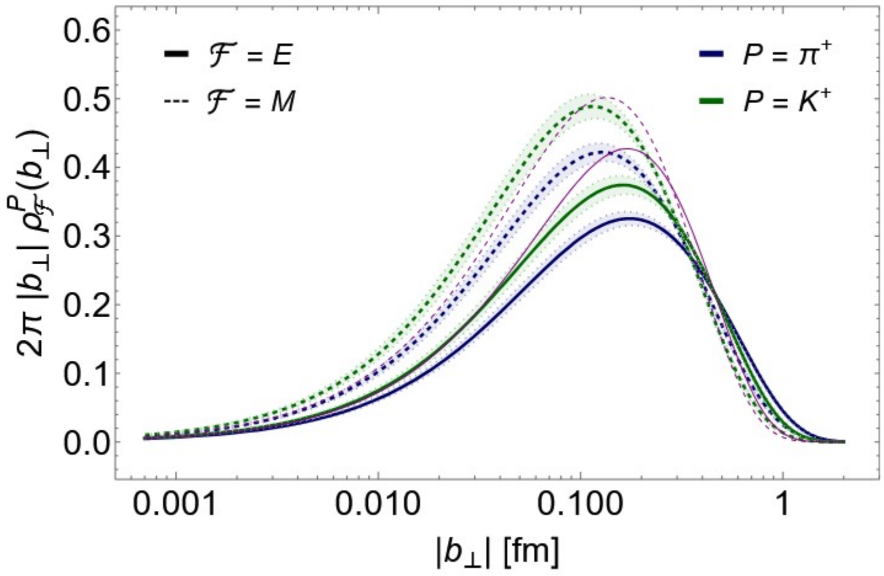
- The **symmetry-preserving** treatment guarantees the **mass and spin** sum rules
- At **2 GeV**, our prediction features slightly smaller gluon contribution
- As with the **π-K**, **charge** effects span over a larger domain that **mass** effects: $r_{\rm mass} = 0.81(5) r_{\rm ch}$

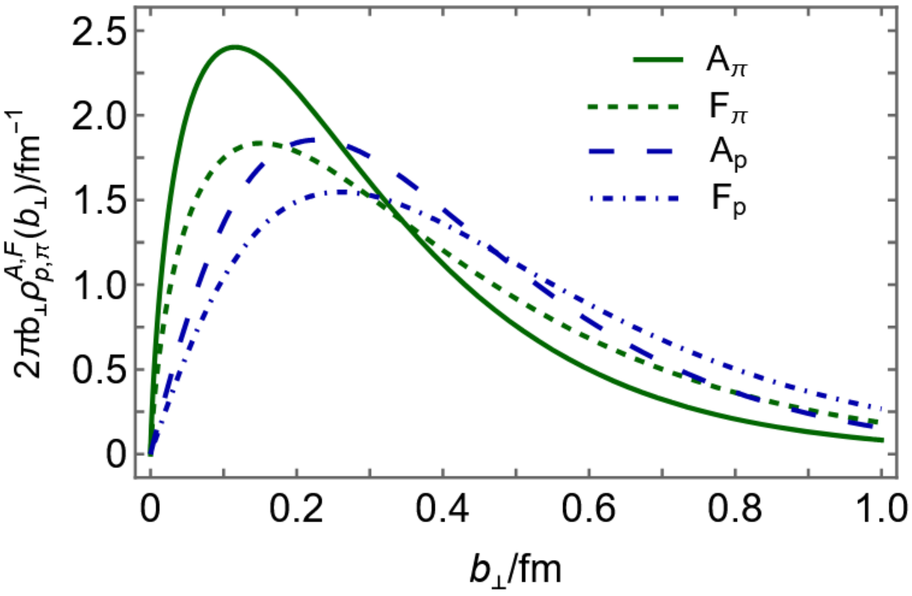


Spatial **Distributions**

- For **π (K)** it is found: $r^{P}_{\theta_1} = 0.81\,\text{fm}\,(0.63\,\text{fm}) > r^{P}_{F} = 0.64\,\text{fm}\,(0.58\,\text{fm}) > r^{P}_{\theta_2} = 0.47\,\text{fm}\,(0.40\,\text{fm})$.

(mechanical) (charge) (mass)





- The **Kaon**, albeit more **compressed**, exhibit similar patters. $\bar{r}_K/\bar{r}_\pi = 0.85(6)$

- The **proton** is spatially more **extended.** $\bar{r}_p/\bar{r}_\pi \approx 1.25$

π-K Pressures

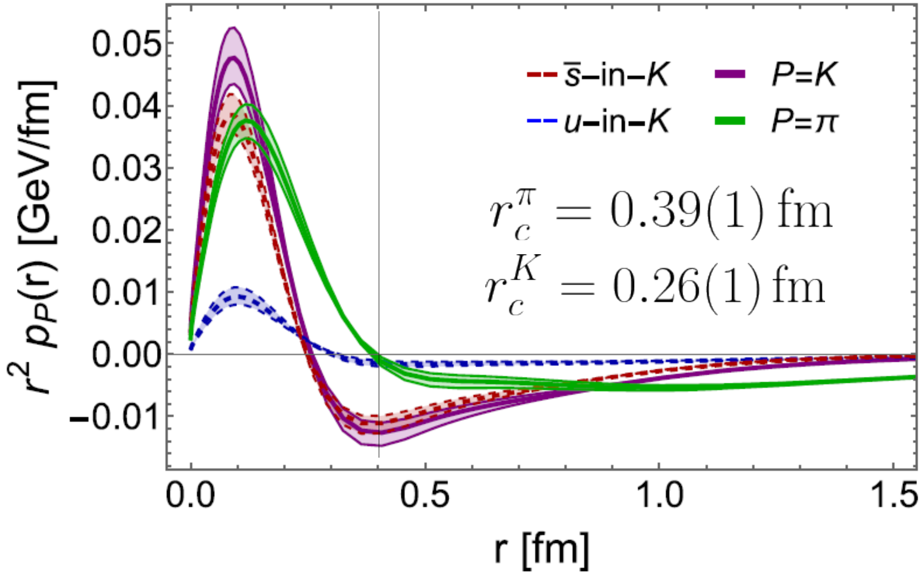
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta \, \frac{\Delta}{2E(\Delta)} \, \sin(\Delta r)[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

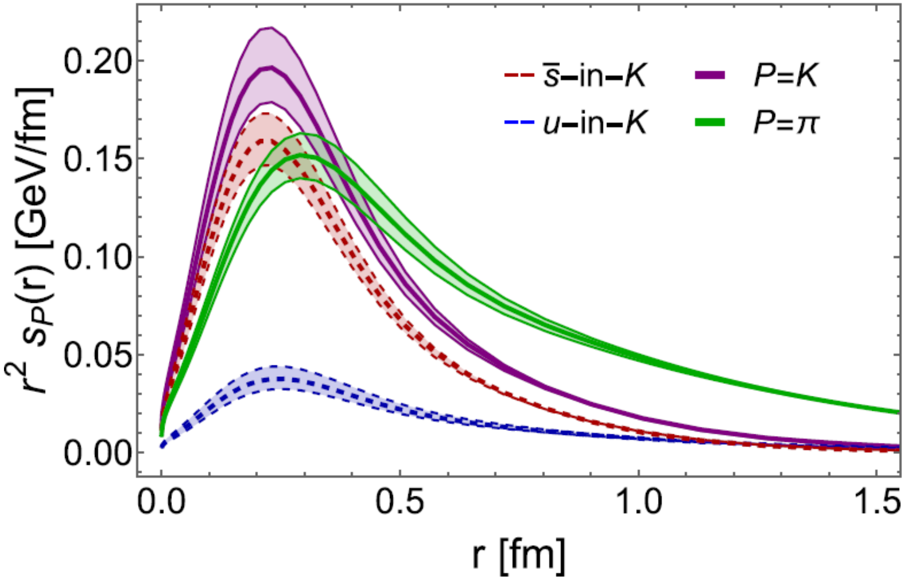
$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta \, \frac{\Delta^2}{2E(\Delta)} j_2(\Delta r)\,[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

"Pressure" Quark attraction/repulsion

CONFINEMENT

"Shear" *Deformation* QCD forces

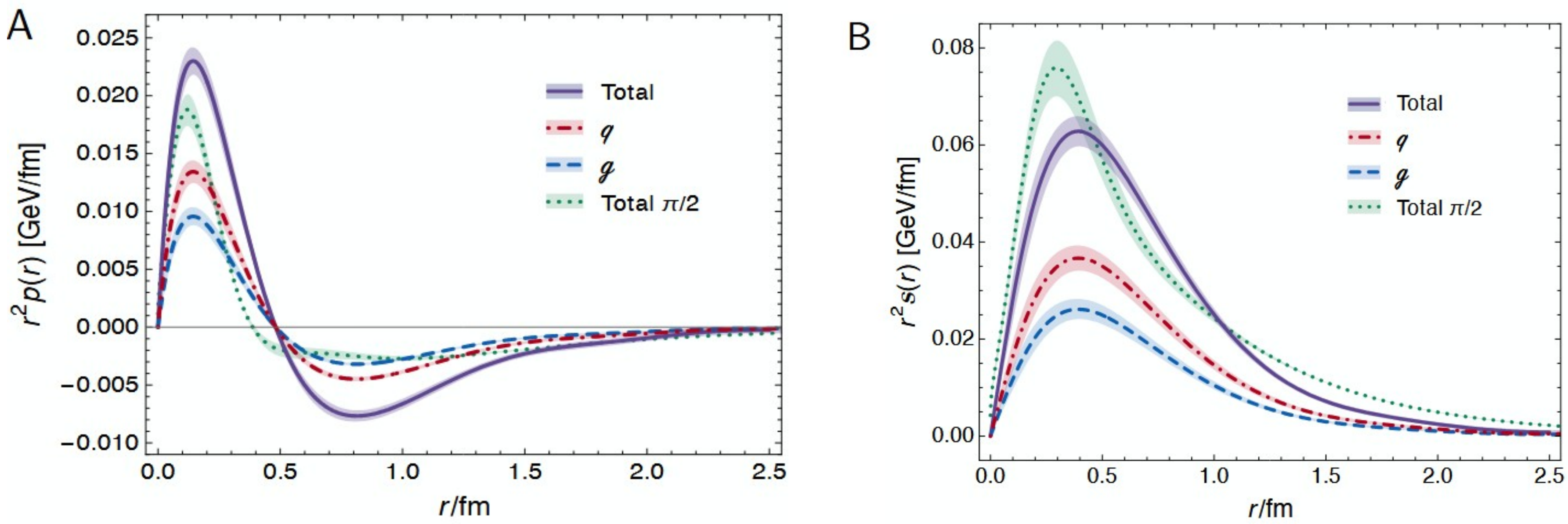




Nucleon **Pressures**

Z-Q Yao *et al.*,
Eur.Phys.J.A 61 (2025) 5, 92

- **Nucleon pressures** follow the same patterns as in the **pion**.



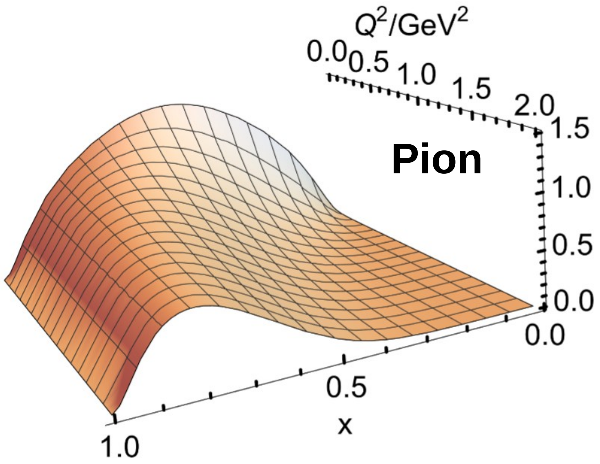
- Although only about **half** as large as those in the **pion**, the **nucleon pressures** are comparable to those found in the core of **neutron stars**.

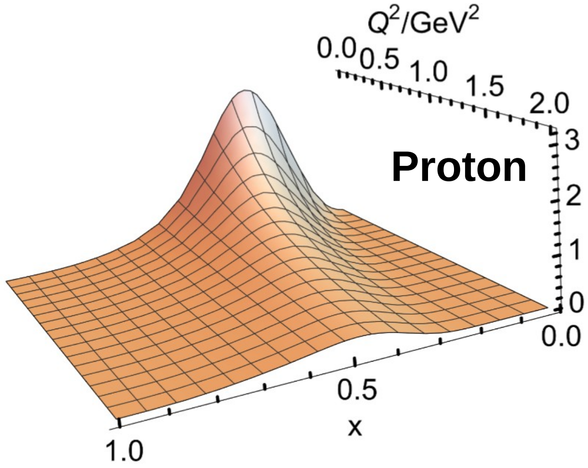
$\sim 10^{35}\,\mathrm{Pa}$

Pion and Proton GPDs

Pion and Proton GPDs

- **Pion** and **Proton** **GPDs,** at the hadronic scale, inherit the **dilation/compression** patterns of the **DF.**





- In the **isospin symmetric** limit, the **pion GPD** is **symmetric** in the forward limit.
  - **Not** the case for the **proton**...

$$\langle x\rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x\rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x\rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5$$

- **GPDs** encode many **aspects** of the hadron internal **structure**.

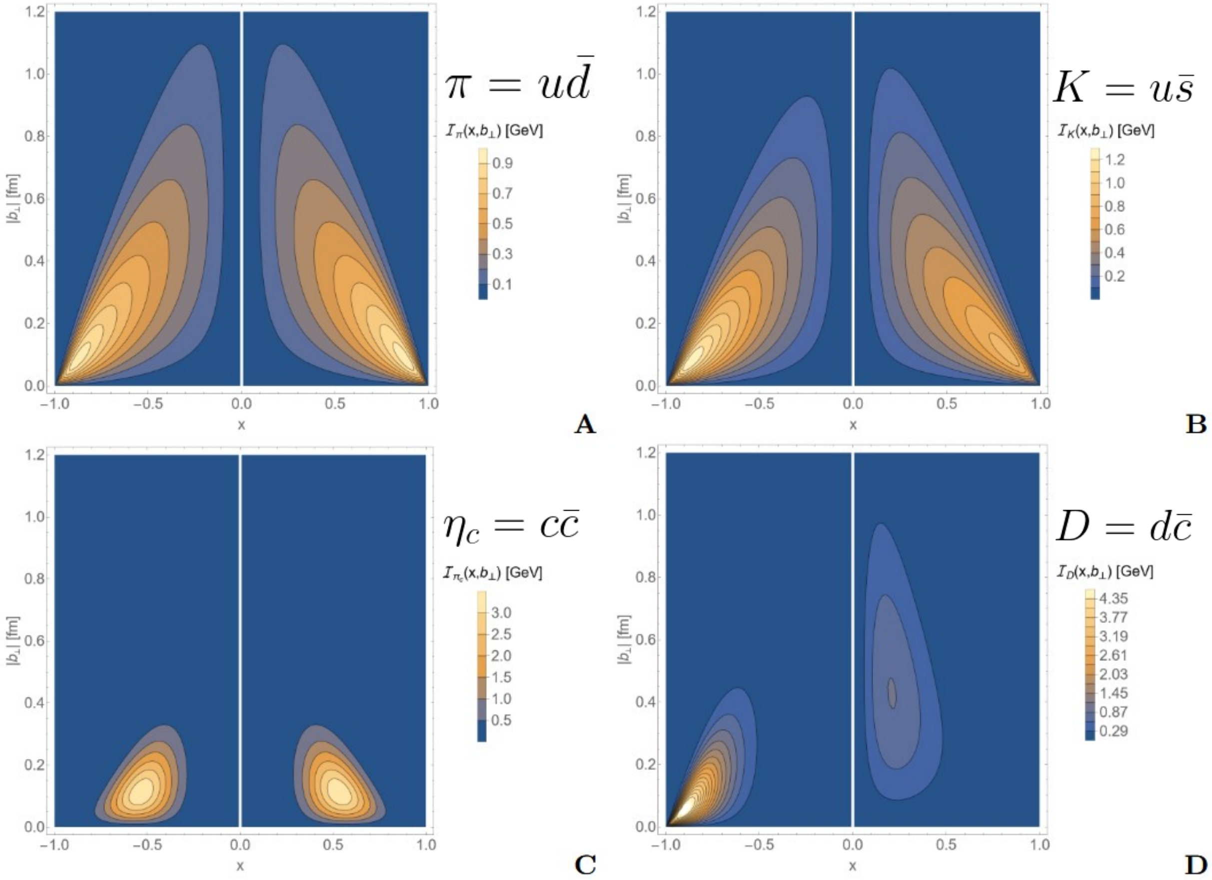
$$q_{\mathbf{P}}(x) = H_{\mathbf{P}}^q(x,0,0) \qquad F_{\mathbf{P}}^q = \int_{-1}^{1} dx\, H_{\mathbf{P}}^q(x,\xi,\Delta^2)$$

$$\theta_2^{\mathbf{P}q}(\Delta^2) + \xi^2\theta_1^{\mathbf{P}q}(\Delta^2) = \int_{-1}^{1} dx\, x H_{\mathbf{P}}^q(x,\xi,\Delta^2)$$

Impact parameter space GPDs

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

- Clearly, the **heavier** system is more **compressed**
  - Narrower and **higher**-peak
  - Tends to the **non-relativistic** limit: **$x^{max} \to 1/2$**
- Marked **asymmetry** in **heavy-light** systems.
  - **Heavy-quark** carries almost **all** the **momentum**
  - **Light-quark** effectively **orbits** the heavy one.



Final Highlights

Final Highlights

- The **emergent** phenomena in **QCD** produces unique outcomes:
  - **Confinement**, dynamical **mass** generation, and a peculiar effective **coupling**.
  - These orchestrate the formation of **hadrons** and their properties, and are responsible for almost all of the **mass** of the **VM**.

- **Nucleons** and **pseudoscalar** mesons take **center** stage in elucidating these aspects
  - **Other** hadrons provide a **complementary** picture.

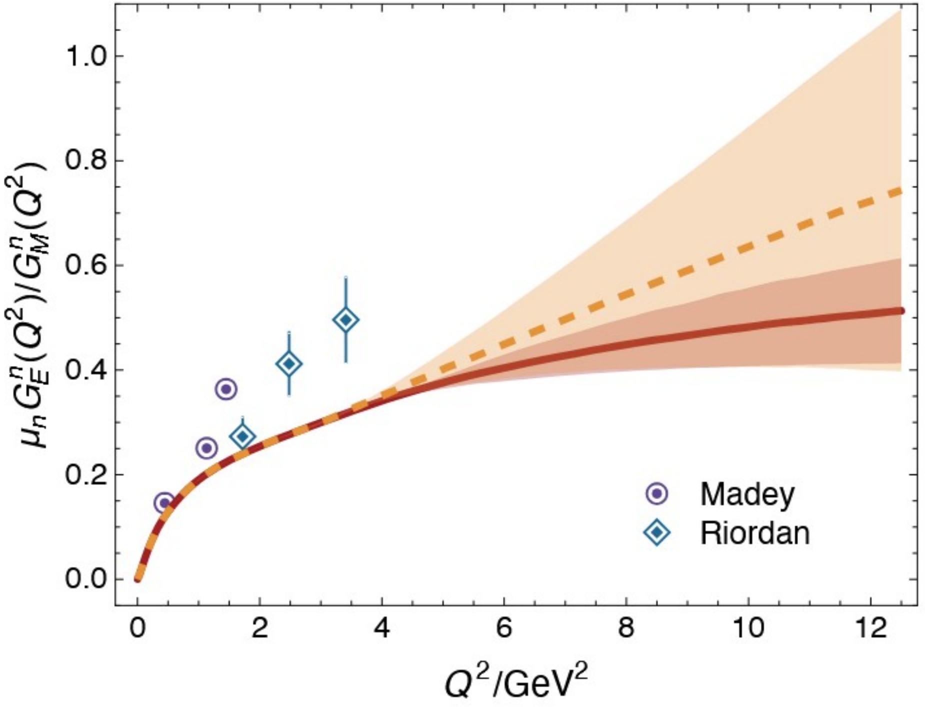
- **Excited and Exotic** states push the limits of our understanding of how the **strong force** binds matter together.

- **Experimental** facilities around the globe are set to examine these aspects at an **unprecedented** level.
  - Convenient.. as **theory** has evolved to the point where all sorts of quantities are **within reach**.

Elastic Form Factors

Yao:2024uej

- For the **proton**, different theoretical and phenomenological analysis suggest a **zero-crossing** in the **GE/GM** ratio



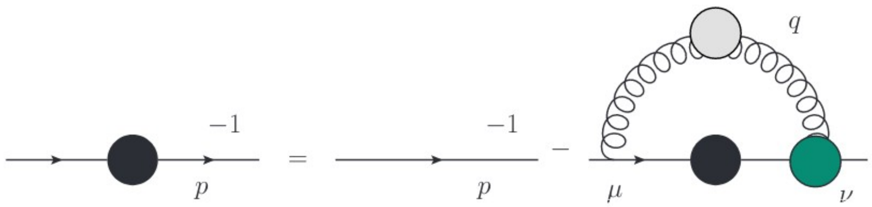
- This reflection on a **destructive interference** between the individual **valence-quark** contributions.
- In fact connected with non-trivial effects coming from orbital **angular momentum**.
- The **location** of the zero is highly sensitive to the **diquark** content of the **nucleon**.

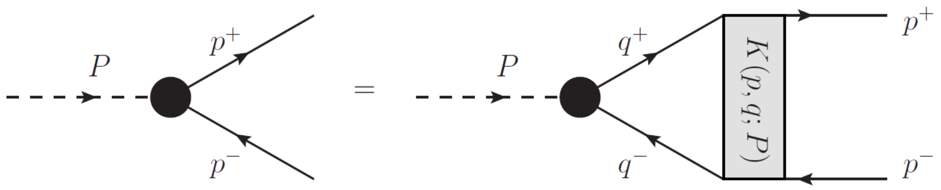
- Modern analyses favor a **non-negligible** component of **axial-vector** diquark:

~ 25 – 35 %

ıe rest comes from the expected **scalar diquark**.

Continuum Schwinger Methods (CSM)





Dyson-Schwinger Equations

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions
  - **Infinite** tower of coupled equations.
  - ✗ Systematic **truncation** required

✓ **No assumptions** on the **coupling** for their derivation.

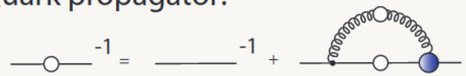
- ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**

✓ **Not limited** to a certain domain of current **quark masses**

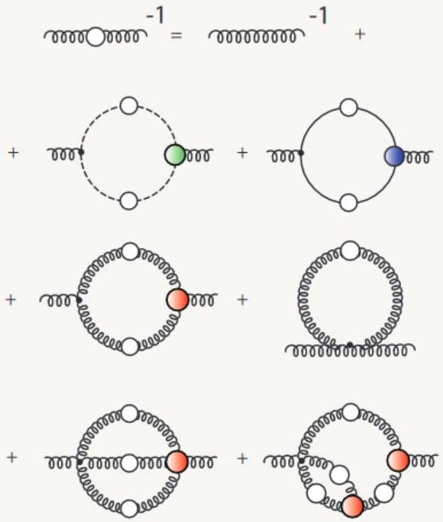
✓ Maintain a **traceable connection** to QCD.

Example DSEs

Quark propagator:



Gluon propagator:



C.D. Robert and A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

G. Eichmann, H. Sanchis-Alipus *et al.*
Prog.Part.Nucl.Phys. 91 (2016) 1-100

Baryons: Faddeev equation

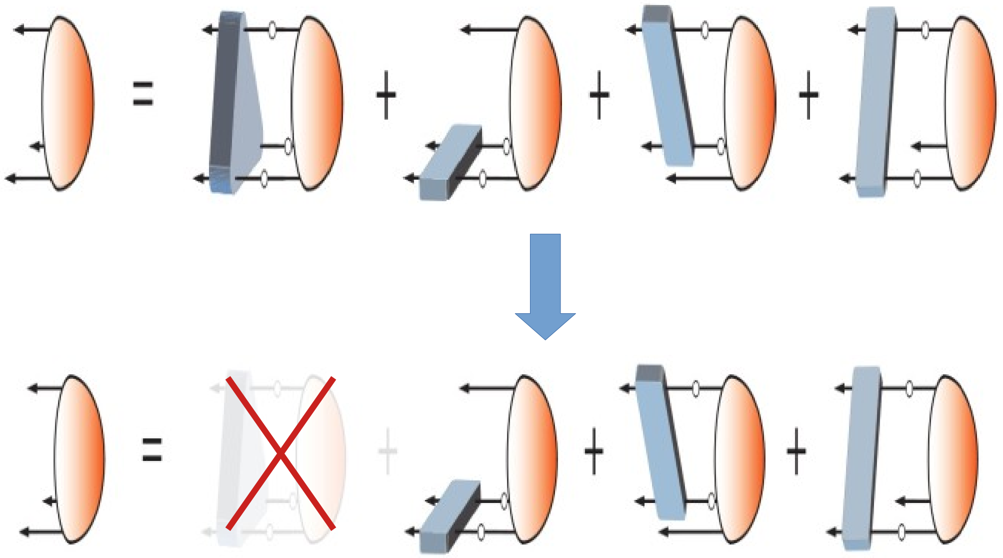
- Strong evidence anticipates the formation of **dynamical** quark-quark correlations (**diquarks**) within **baryons,** for instance:
  - The **primary three-body** force **binding** the quarks within the baryon vanishes when projected onto the color singlet channel.

Eichmann:2016yit

i.e. a 3-gluon vertex attached to each quark once (and only once)

- The dominant 3-gluon contribution is the one attaching twice to a quark
- This produces a strengthening of quark-quark interactions

Barabanov:2020jvn



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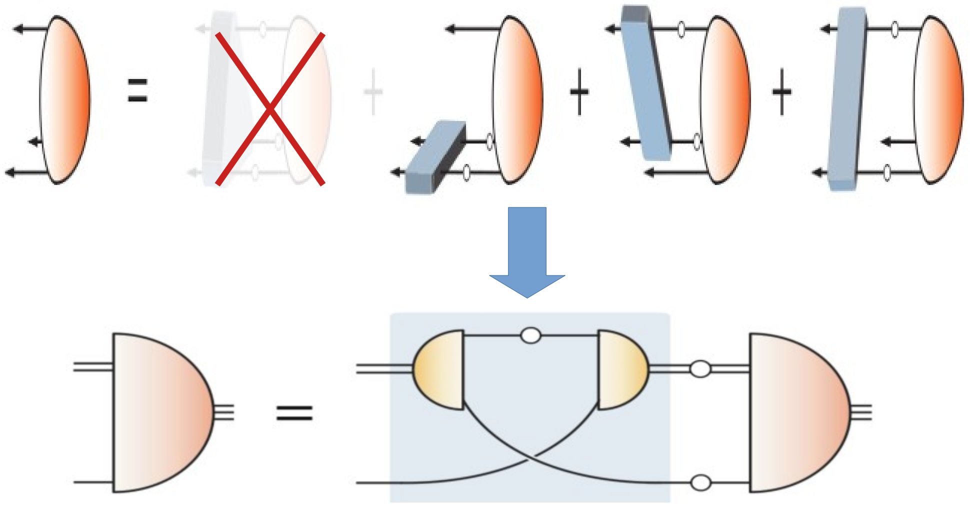
Barabanov:2020jvn

- The **primary three-body** force **binding** the quarks within the baryon vanishes when projected onto the color singlet channel.
- The **attractive** nature of **quark-antiquark** correlations in a color-singlet meson, is also **attractive** for $\bar{3}_c$ **quark-quark** correlations within a color singlet baryon.



Dyamical Quark-diquark picture

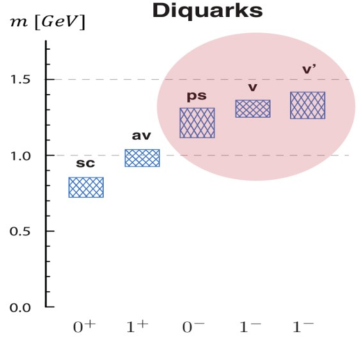
Non-pointlike diquarks:

- Color anti-triplet
- Fully interacting
- Origins related to **EHM** phenomena

Baryons: Quark-diquark picture

Barabanov:2020jvn

- The **attractive** nature of **quark-antiquark** correlations in a color-singlet meson, is also **attractive** for $\bar{3}_c$ **quark-quark** correlations within a color singlet baryon.
  - Due to charge conjugation properties, a ***J^p* diquark** partners with an analogous ***J^-p* meson**.
  - We can thus establish a connection between the **meson** and **diquark** Bethe-Salpeter equations:



$$\Gamma_{q\bar{q}}(p;P) = -\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

Less tightly 'bound'

- Computed '**masses**' should be interpreted as correlation **lengths**:

$$m_{[ud]_{0^+}} = 0.7-0.8\,\text{GeV}, \quad m_{\{uu\}_{1^+}} = 0.9-1.1\,\text{GeV}$$

- Stressing the fact that the **diquarks** have a **finite** size:

$$r_{[ud]_{0^+}} \gtrsim r_\pi, \quad r_{\{uu\}_{1^+}} \gtrsim r_\rho$$

Non-pointlike diquarks:

- Color anti-triplet
- Fully interacting
- Origins related to **EHM** phenomena