FSR and ρ-width IB corrections beyond scalar QED and their impact on muon g-2

Arxiv: 2510.02723

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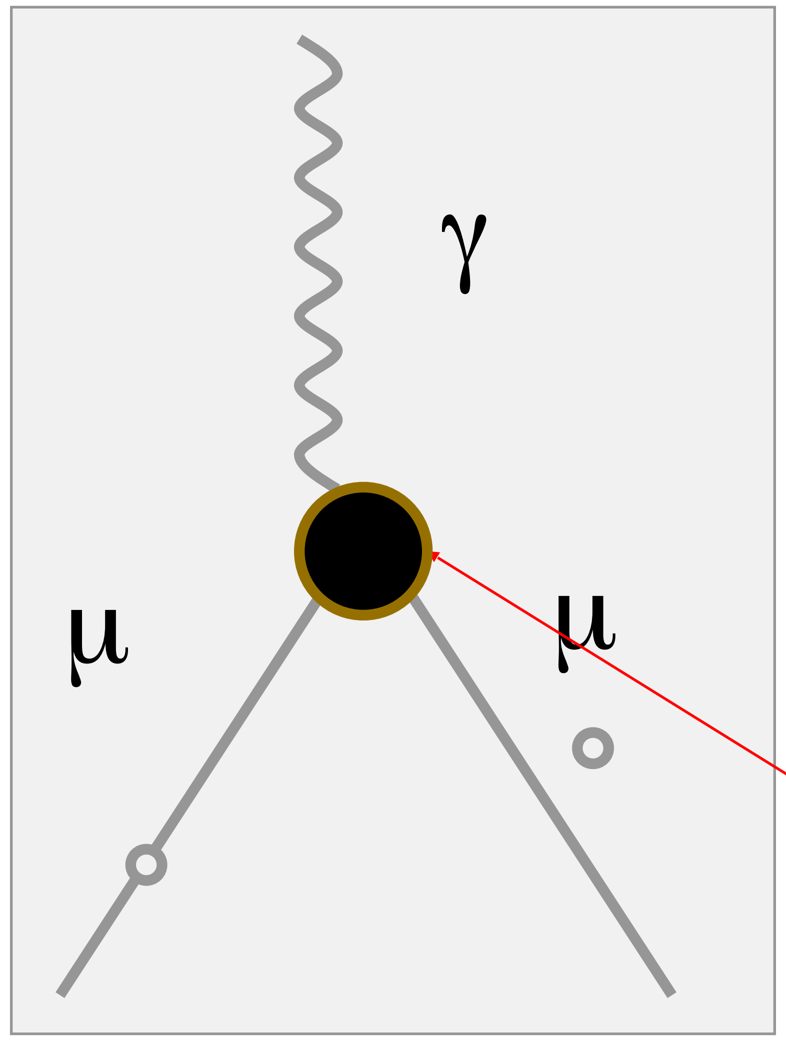
Outline

- Motivation: Muon g-2
- Hadronic vacuum polarization (HVP) from tau data
- Isospin symmetry breaking (IB) corrections
- Neutral-charged rho meson width difference
    - Radiative corrections, sQED and structure dependence
- Structure dependent effects on final state radiation (FSR)
- Impact on muon g-2
- Conclusions

Muon g-2

Magnetic dipole moment is related to the intrinsic spin by the gyromagnetic ratio g



$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\vec{S}$$

Schwinger ‘48

$$g_\mu = 2(1 + \frac{\alpha}{2\pi} + \dots)$$

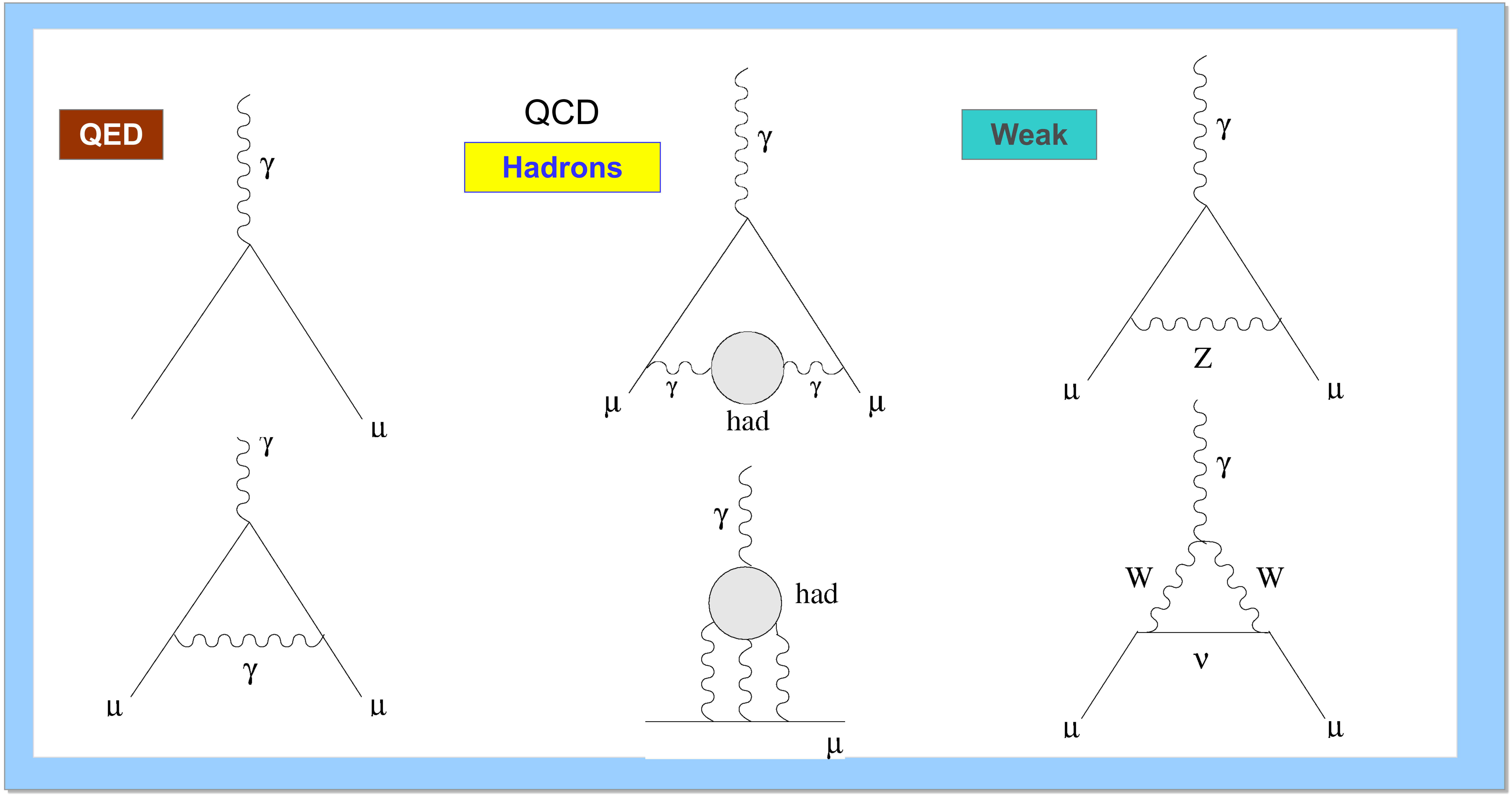
normal anomalous

$$\Gamma_\mu = e\gamma_\mu + a_\ell \frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

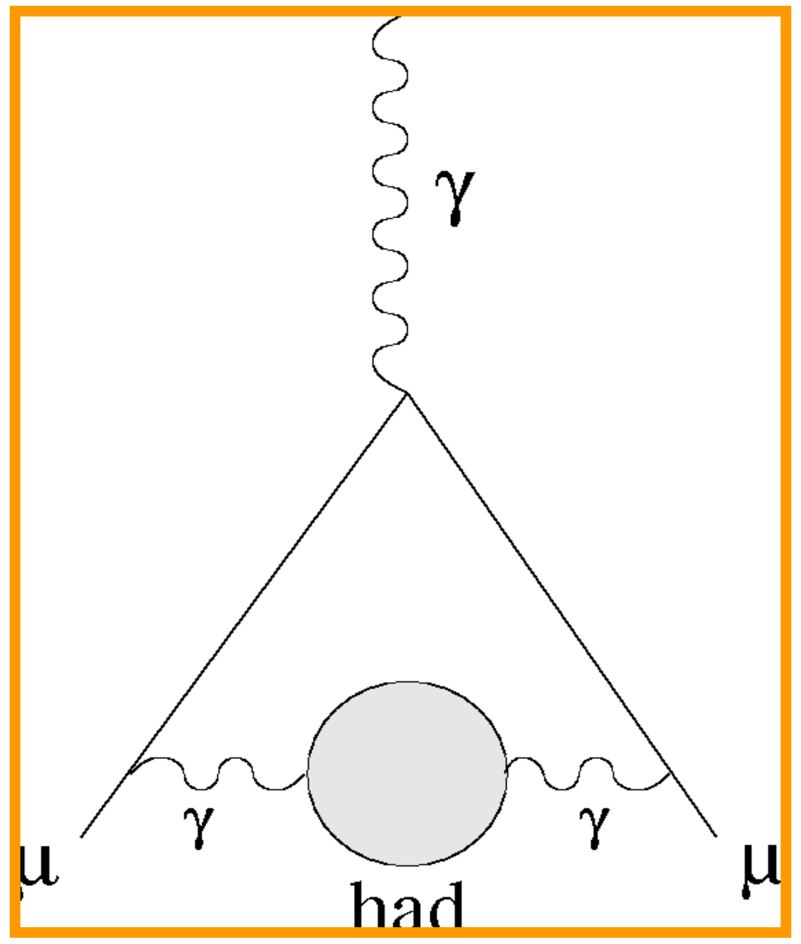
It can be studied throughout the electromagnetic vertex

$$\boxed{a_\mu \equiv \frac{g-2}{2}}$$

Theory



Hadronic Vacuum Polarization

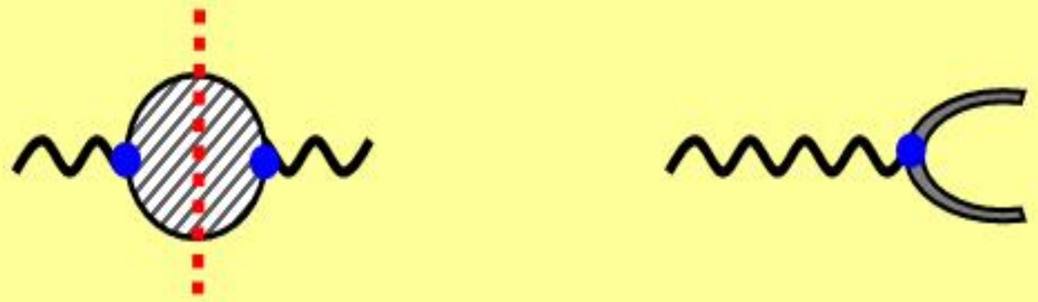


HADRONIC
CORRECTIONS

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \; \frac{K(s)}{s} \; R(s)$$

Decreases with energy,
Low energy part of integral is important

$$12\pi \, \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \mathrm{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

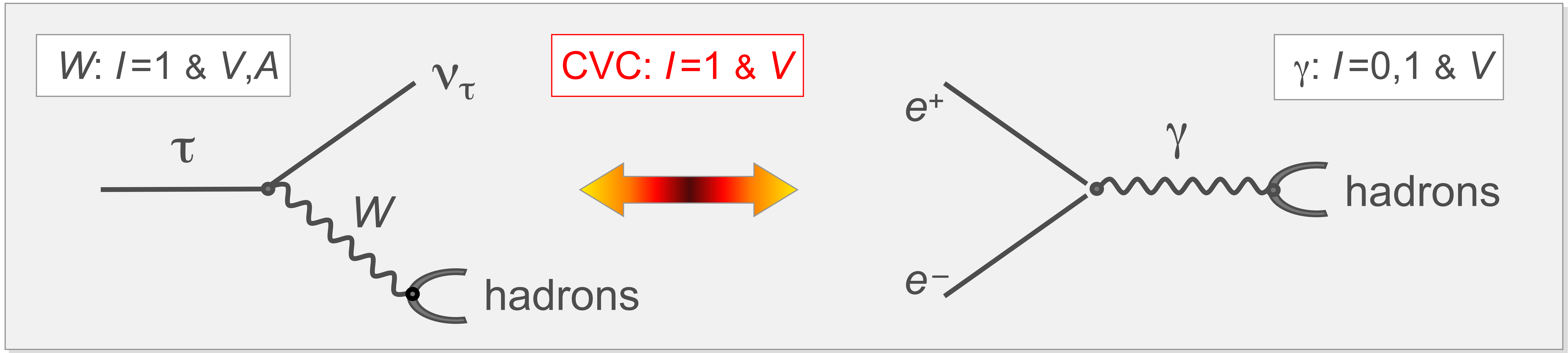


we can use experimental data from e+e- to hadrons!

HVP from τ data

Isospin Symmetry

Assume mu=md connect I=1 e^+e^- cross section with the vector spectral function of the τ



$\tau \longrightarrow \pi\, \pi\, \nu$

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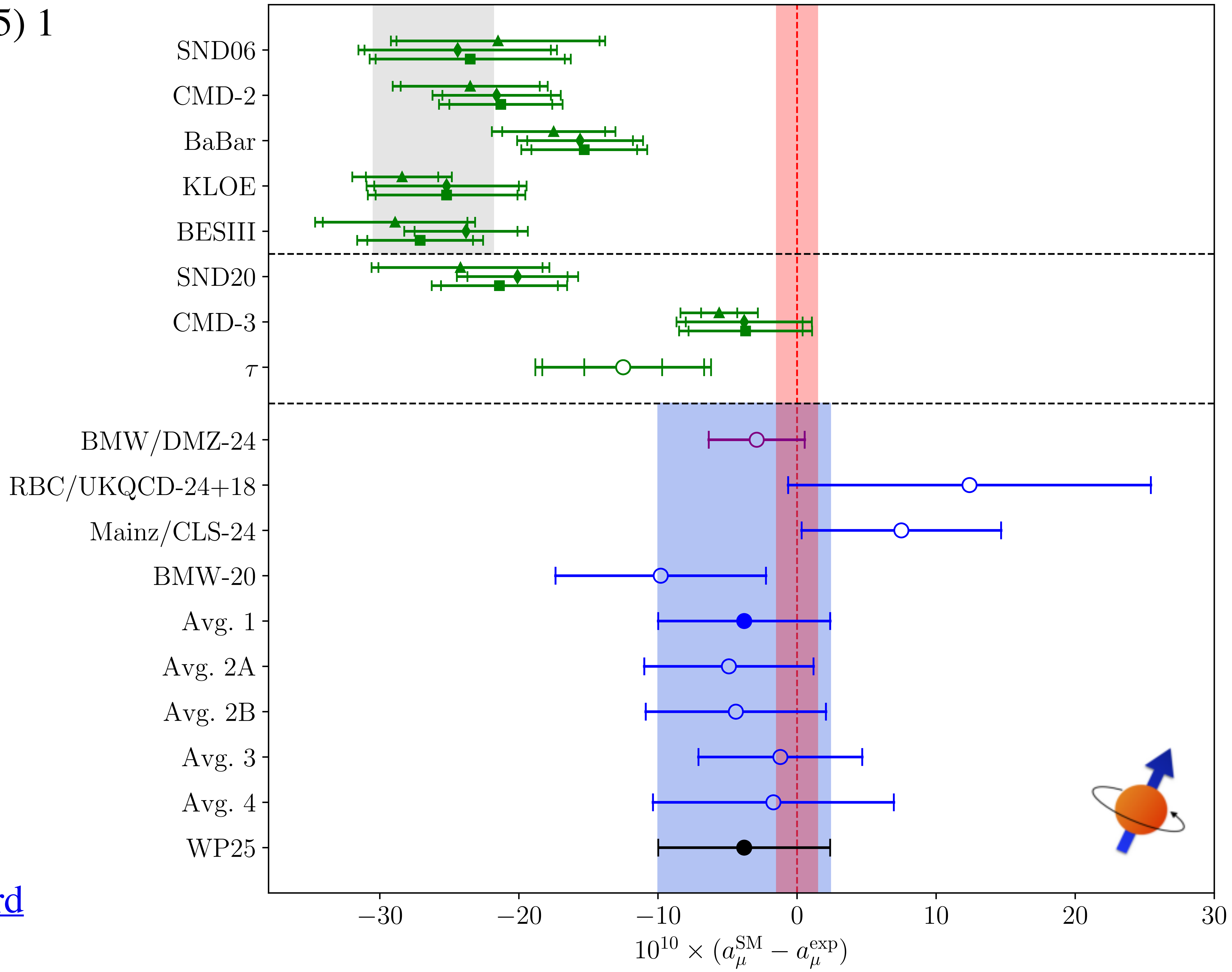
Current status

The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti

- *Phys.Rept.* 1143 (2025) 1

THEORY INITIATIVE



EXPERIMENT

Muon g-2 Collaboration: D. P. Aguillard

arXiv:2506.03069v1 [hep-ex]

The new experimental world average

$$\boxed{a_\mu(\text{exp}) \;=\; 116\,592\,0715(145) \times 10^{-12} \quad (124\,\text{ppb})}$$

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HVP evaluation using tau data

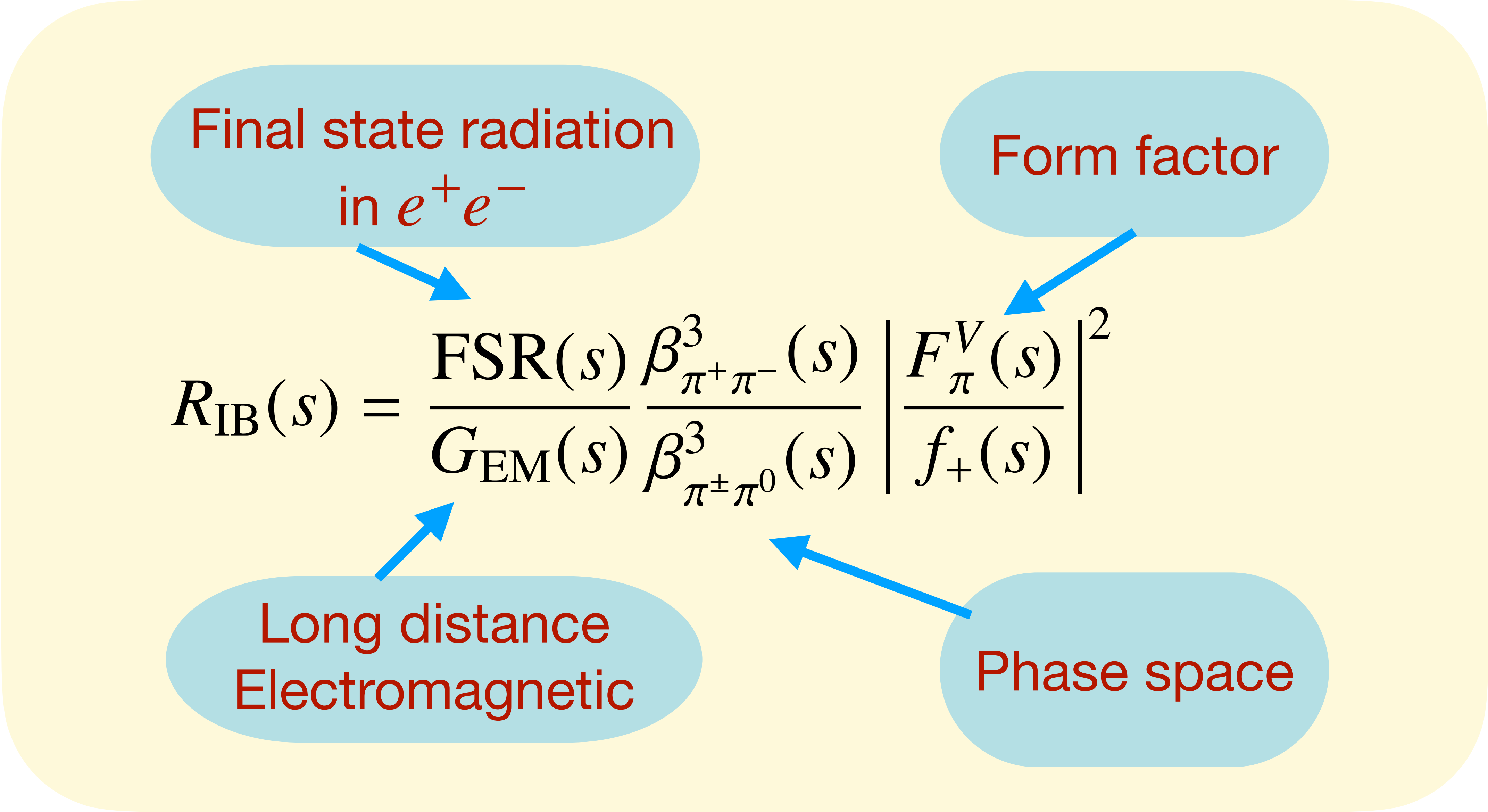
The leading order HVP contribution using tau data requires to account for the IB correction

$$\Delta a_\mu^{HVP}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\quad\frac{dN_{\pi\pi^0}}{N_{\pi\pi^0}ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\rm IB}(s)}{S_{\rm EW}}-1\right]$$

QED Kernel function

Described by data

The **isospin breaking corrections** are given by



Final state radiation in e^+e^-

Form factor

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\frac{\beta^3_{\pi^+\pi^-}(s)}{\beta^3_{\pi^\pm\pi^0}(s)}\left|\frac{F_\pi^V(s)}{f_+(s)}\right|^2$$

Long distance Electromagnetic

Phase space

$S_{\rm EW}$ Short distance EW correction

HVP contribution status

$\Delta a_\mu^{HVP}[\pi\pi, \tau]$ in 10^(-10) units

Phase space	-7.88	-7.52	–	$-7.7(2)$
S_{EW}	$-12.21(15)$	$-12.16(15)$	–	$-12.2(1.3)$
G_{EM}	$-1.92(90)$	$-1.67^{+0.60}_{-1.39}$	–	$-2.0(1.4)$
FSR	$4.67(47)$	$4.62(46)$	$4.42(4)$	$4.5(3)$
ρ–ω mixing	$4.0(4)$	$2.87(8)$	$3.79(19)$	$3.9(3)$

M. Davier, étal EPJC 84, 721 (2024),
M. Davier, étal , EPJC 66, 127 (2010)

Lopez Castro, Miranda, and Roig,
PRD 111, 073004 (2025)

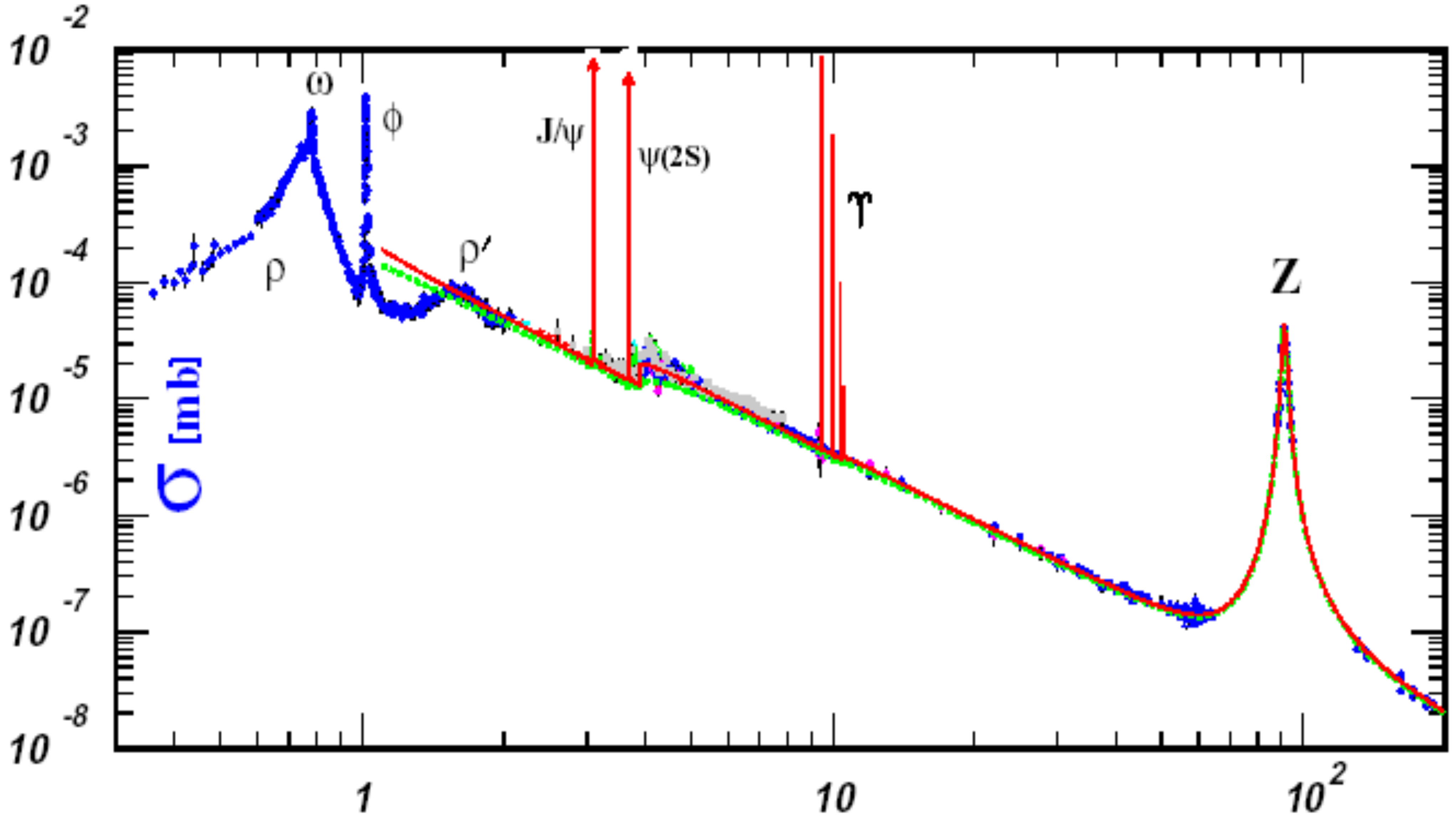
G. Colangelo, etal JHEP 10, 032 (2022)
M. Hoferichter, étal PRL 131, 161905 (2023)

Theory initiative update
Arxiv: 2505.21476

FSR computed using sQED, first introduced in 2010 by M. Davier, étal EPJC 66, 127 (2010).
Uncertainty associated to the missing structure effects

Weak / Electromagnetic form factor correction

ρ is the dominant contribution



The form factor parameterize the distribution for each case

$$F_0(s) \;=\; f_{\rho^0}(s)\left[1-\delta_{\rho\omega}\frac{s}{m_\omega^2 - s - im_\omega\Gamma_\omega(s)}\right]$$

$$F_-(s) \;=\; f_{\rho^-}(s)\,,$$

Phys.Rev.D76:096010,2007

Phys.Rev.D76:033001,2007.

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HVP contribution status. Form Factors

$\Delta a_\mu^{HVP}[\pi\pi,\tau]$ in 10^(-10) units

	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	$4.09(0)(7)$	3.37	
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)(48)$	$-6.66(73)$	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	
	Total	$-1.62(65)(63)$	$(-1.34)^{+1.72}_{-1.71}$	$-1.5(4.7)$

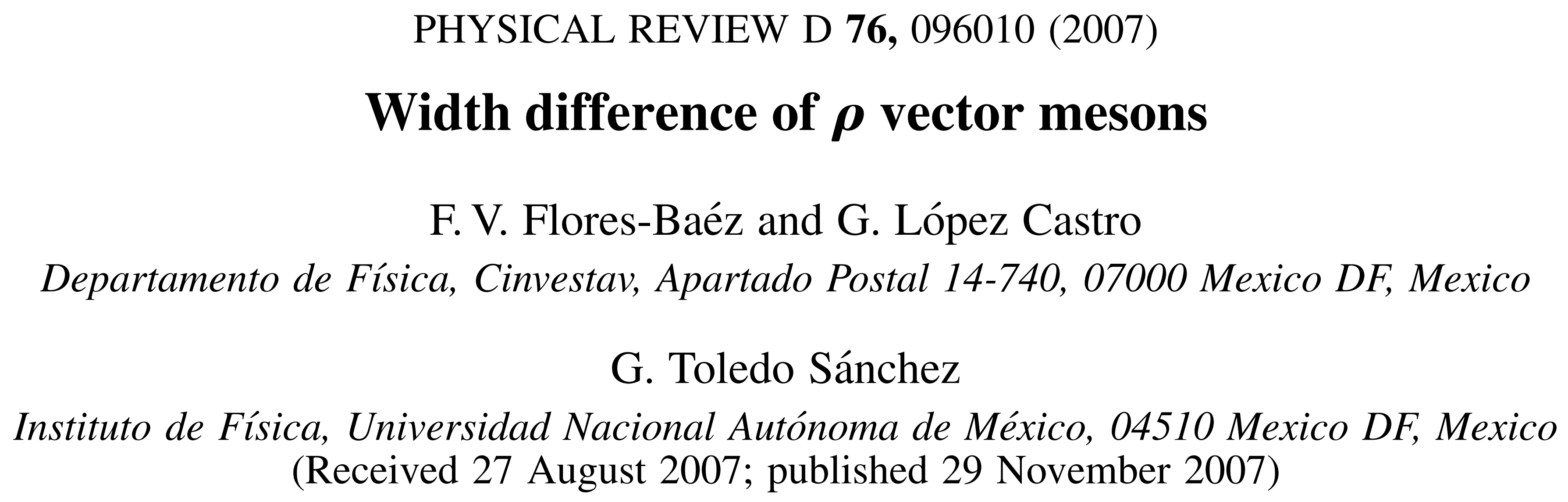
M. Davier, étal EPJC 84, 721 (2024),
M. Davier, étal , EPJC 66, 127 (2010)

Lopez Castro, Miranda, and Roig,
PRD 111, 073004 (2025)

Theory initiative update
Arxiv: 2505.21476

Corrections computed using different FF parameterizations

$\rho \to \pi\pi(\gamma)$ decay, affecting the rho width difference, computed using sQED, structure dependent effects expected, 10% uncertainty assigned



PHYSICAL REVIEW D **76,** 096010 (2007)

Width difference of ρ vector mesons

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(Received 27 August 2007; published 29 November 2007)

The rho width difference gives the most important contribution to FF

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The ρ width difference

$$\Delta\Gamma_\rho \equiv \Gamma_{\rho^0} - \Gamma_{\rho^+} \qquad \Delta m_\rho \equiv m_{\rho^+} - m_{\rho^0}$$

Data based

$\Delta\Gamma_\rho = (0.3 \pm 1.3)$ MeV $\qquad \Delta m_\rho = (+0.7 \pm 0.8)$ MeV

S. Navas et al. [Particle Data Group],
PRD 110 (2024)

$\Delta\Gamma_\rho = (-0.58 \pm 1.04)$ MeV $\qquad \Delta m_\rho = (+0.30 \pm 0.53)$ MeV

Davier, Malaescu, and Zhang
ArXiv:2504.13789v1 (2025)

Theoretical, EM radiative corrections, sQED

$\Delta\Gamma_\rho = (+0.76 \pm 0.20)$ MeV

$+1.82\ (\pm 10\%)$ $\qquad -1.06$ $\qquad$ structure

radiative correction $\qquad \Delta m_\pi$

Flores-Baez, Castro and Toledo
PRD 76, 096010 (2007)

Approximations: Structureless pion and rho photon interactions (sQED)

Only convection-convection terms in virtual corrections

The ρ width difference

The rho width difference can be split into $\Delta\Gamma_\rho = \Delta\Gamma_\rho[\pi\pi(\gamma)] + \Delta\Gamma_\rho(\text{rest})$

where $\Delta\Gamma_\rho(\text{rest})$ includes all the measured channels, except the two pions and the corresponding radiative channel

$$\rho^0 : l^+l^-, \pi^0\gamma, \eta\gamma, 3\pi, 4\pi$$

$$\rho^+ : \pi^+\gamma$$

$$\mathrm{BR}(\rho^0 \to \text{rest}) = (1.04 \pm 0.10) \times 10^{-3}$$

$$\mathrm{BR}(\rho^+ \to \text{rest}) = (4.53 \pm 0.46) \times 10^{-4},$$

S. Navas et al. [Particle Data Group], PRD 110 (2024)

Thus, for a common neutral and charged rho width (for example 150 MeV)

$$\begin{aligned} \Delta\Gamma_\rho(\text{rest}) &= \Gamma_{\rho^0} \times \mathrm{BR}(\rho^0 \to \text{rest}) - \Gamma_{\rho^+} \times \mathrm{BR}(\rho^+ \to \text{rest}) \\ &= (0.088 \pm 0.017)\ \mathrm{MeV}\ , \end{aligned}$$

At the precision level of a few tenths of a percent,
the widths of rho mesons are driven by the $\rho \to \pi\pi(\gamma)$ decay

Contributions to $\Delta\Gamma_\rho[\pi\pi(\gamma)]$

The neutral and charged photon inclusive rho to two pions width are given by

$$\Gamma[\rho^+ \to \pi^+\pi^0(\gamma)] = \frac{g_+^2 m_{\rho^+}}{48\pi}\beta_+^3(1+\delta_+) \qquad \Gamma[\rho^0 \to \pi^+\pi^-(\gamma)] = \frac{g_0^2 m_{\rho^0}}{48\pi}\beta_0^3(1+\delta_0)$$

where δ_+ and δ_0 account for the radiative correction

Thus, the rho width difference can be set, in terms of the IB parameters, as

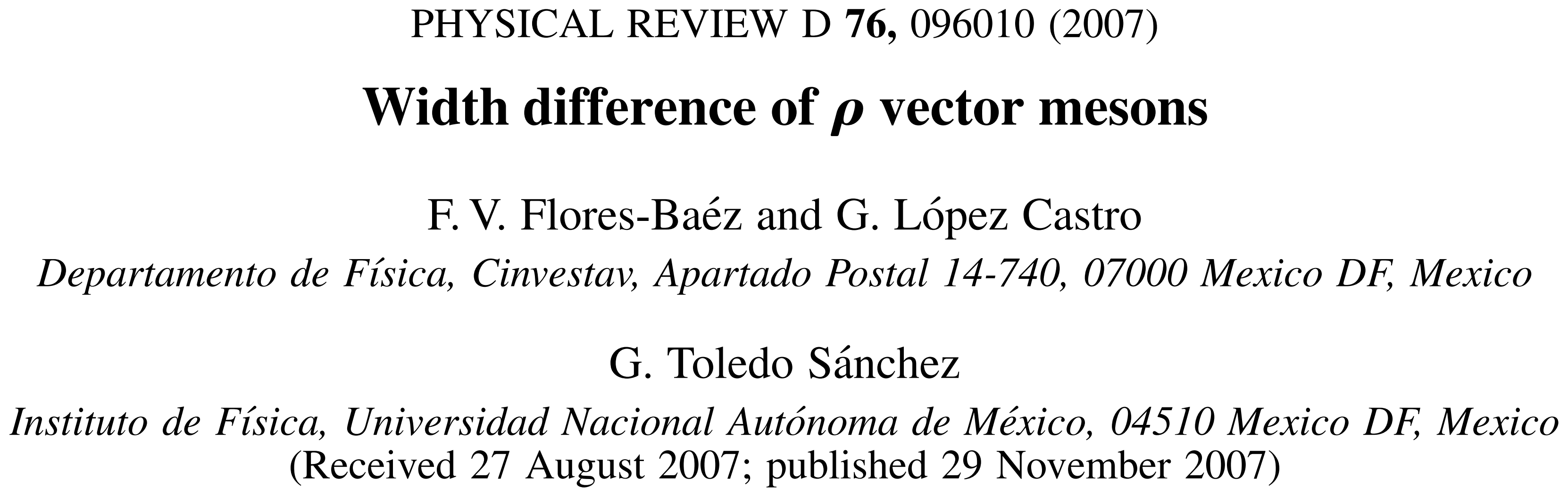
$$\Delta\Gamma_\rho[\pi\pi(\gamma)] = \Gamma(\rho^0 \to \pi^+\pi^-)\left[\delta_0 - \delta_+ - \frac{\Delta m_\rho}{m_{\rho^0}} - \frac{6m_+^2}{m_{\rho^0}^2\beta_0^2}\left(\frac{\Delta}{m_+} + \frac{2\Delta m_\rho}{m_{\rho^0}}\right) - \frac{2\delta g}{g_0}\right]$$

where $\Delta \equiv m_+ - m_0$ $\qquad \Delta m_\rho \equiv m_{\rho^+} - m_{\rho^0}$ $\qquad \delta g \equiv g_+ - g_0$

Thus, the radiative corrections δ_+ and δ_0 for charged and neutral rho are needed

Previous analysis

$$\rho \to \pi\pi(\gamma)$$


PHYSICAL REVIEW D **76,** 096010 (2007)

Width difference of ρ vector mesons

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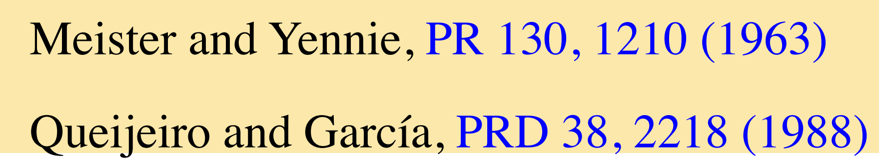
Real photon emission: sQED + Model dependent

Virtual photon emission: $\rho^0 \to \pi^+\pi^-(\gamma)$ sQED

$\rho^+ \to \pi^+\pi^0(\gamma)$ sQED (convection terms only)


Meister and Yennie, PR 130, 1210 (1963)

Queijeiro and García, PRD 38, 2218 (1988)

$\rho\rho\gamma$ vertex $$\Gamma^{\mu\nu\alpha} = (2P-K)^\alpha g^{\mu\nu} + \xcancel{2(k^\mu g^{\nu\alpha} - k^\nu g^{\mu\alpha}) - P^\mu g^{\nu\alpha} - (P-k)^\nu g^{\mu\alpha}}$$

IR Finite
UV Finite with convection

Radiative correction for the neutral and charged processes and their difference, at m_ρ= 775 MeV

$$\delta_+ = -4.15 \times 10^{-3} \qquad \delta_0 = 8.05 \times 10^{-3} \qquad \delta_0 - \delta_+ = 12.2 \times 10^{-3}$$

New analysis

F. V. Flores-Baez, G. L. Castro and G. Toledo
Arxiv: 2510.02723

Improvements

- Structure effect via a form factor ✓
- Full EM vertex of the ρ, not only convection term ✓

Byproduct

- Implications for FSR ✓

Real photon emission not modified

Structure effect

✓ Beyond sQED by modifying the photon propagator in loops $\frac{1}{k^2} \rightarrow \frac{1}{k^2}\left[F_V(k^2)\right]^2$ makes virtual corrections finite

where $F_V(k^2) \equiv \frac{M_V^2}{M_V^2 - k^2}$ and $M_V^2 = m_\rho^2 - im_\rho\Gamma_\rho$

Consider the GVMD by Ignatov et al, where up to three resonances were used to fit the pion form factor and explain pi pi charge asymmetry

Ignatov and Lee, PLB 833,137283(2022)

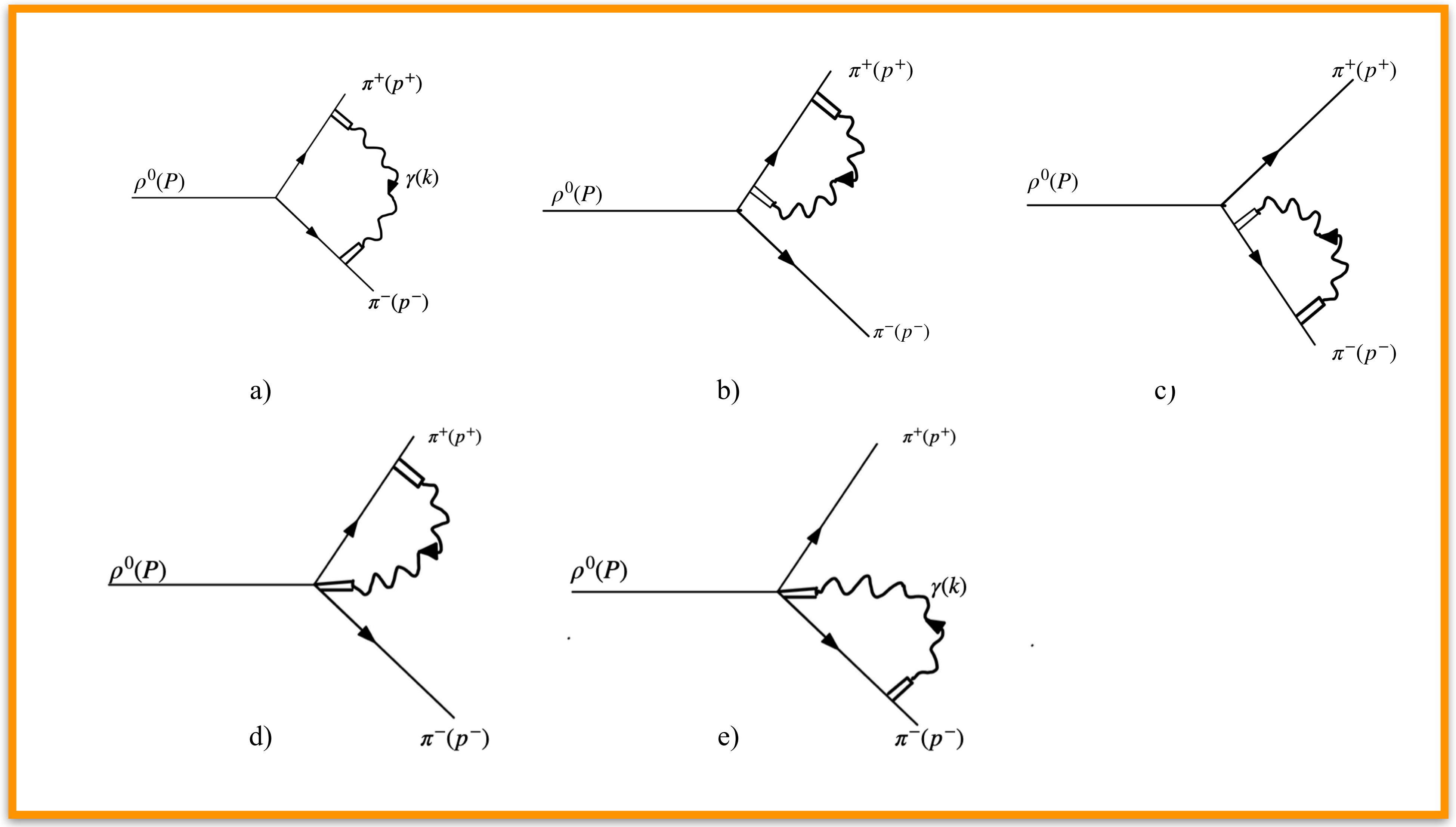
Colangelo, Hoferichter, Monnard, and Ruiz de Elvira, JHEP 08, 295 (2022)

We consider three resonances, the ρ, ρ' , ρ'' with the mass and width as given in the GVMD

Neutral rho meson

$$\rho^0 \to \pi^+\pi^-(\gamma)$$

Structured vertices



$$\delta_0 = 8.05 \times 10^{-3}$$

Flores-Baez, Castro and Toledo
PRD 76, 096010 (2007)

now

$$\delta_0 = 6.04 \times 10^{-3}$$

Contact vertex fixed by gauge invariance

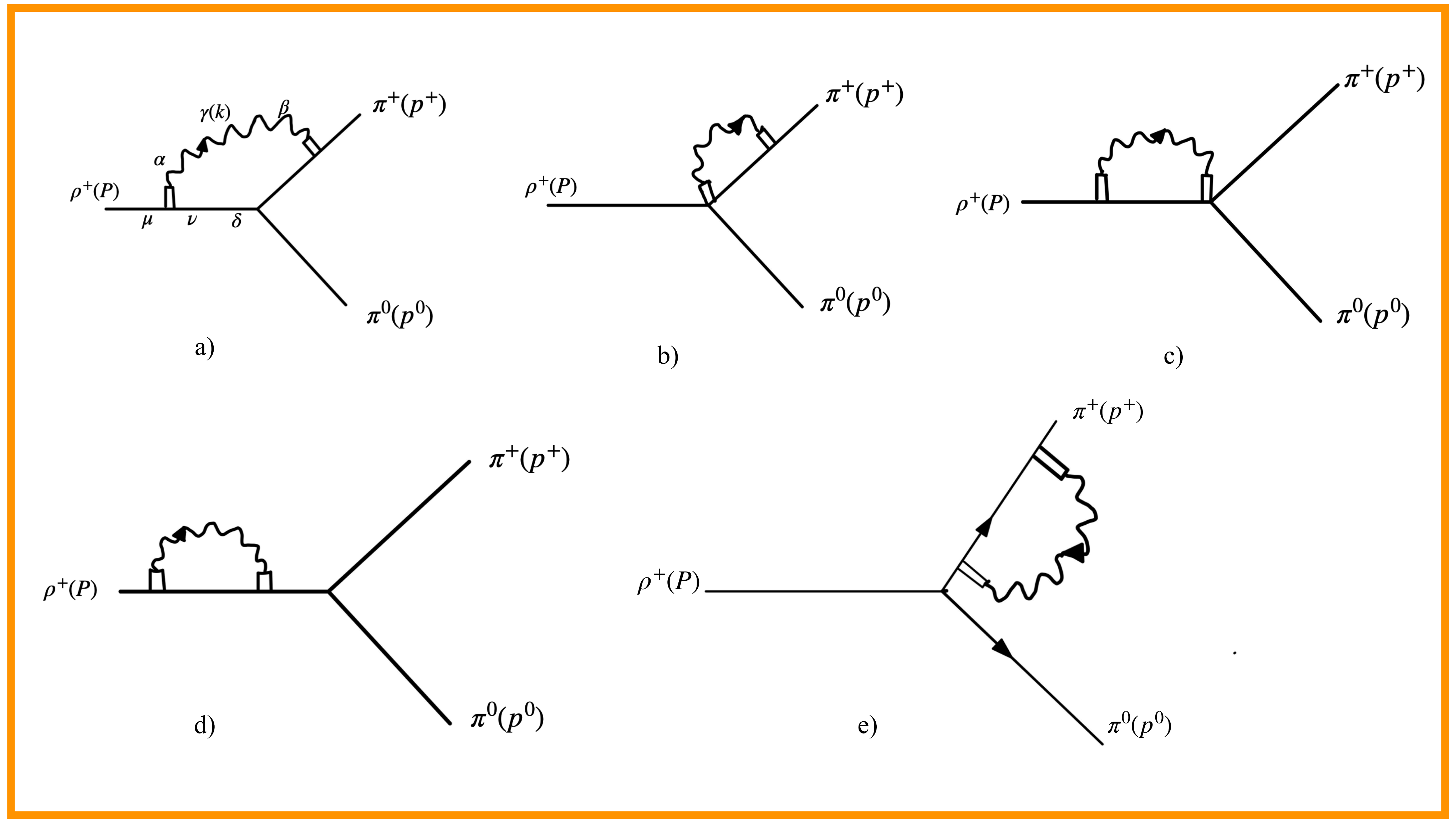
Real photon emission same as in the previous analysis, verified

Infrared and UV finite upon inclusion of real photon emission contribution

Charged rho meson

$$\rho^+ \to \pi^+\pi^0(\gamma)$$

Structured vertices



$\rho\rho\gamma$ vertex
$$\Gamma^{\mu\nu\alpha} = (2P-K)^\alpha g^{\mu\nu} + 2(k^\mu g^{\nu\alpha} - k^\nu g^{\mu\alpha}) - P^\mu g^{\nu\alpha} - (P-k)^\nu g^{\mu\alpha}$$

✓

$$\delta_+ = -4.15 \times 10^{-3}$$

Flores-Baez, Castro and Toledo
PRD 76, 096010 (2007)

now

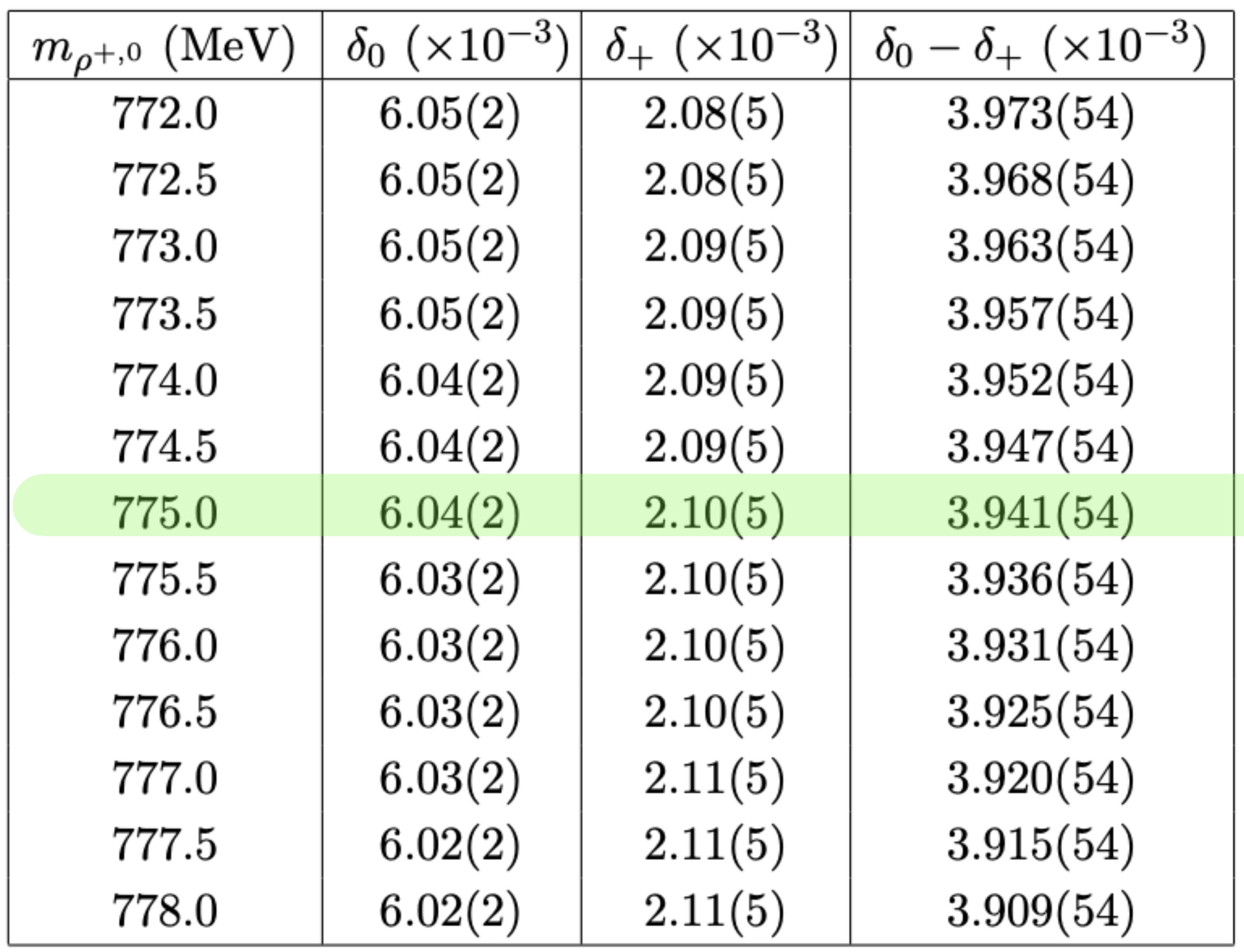
$$\delta_+ = +2.10 \times 10^{-3}$$

Radiative correction as a function of energy
Convection term considered to make UV finite (previous).
VMD (new)

Real photon emission same as in the previous analysis, verified

Infrared and UV finite

Radiative correction



$m_{\rho^{+,0}}$ (MeV)	δ_0 ($\times 10^{-3}$)	δ_+ ($\times 10^{-3}$)	$\delta_0 - \delta_+$ ($\times 10^{-3}$)
772.0	6.05(2)	2.08(5)	3.973(54)
772.5	6.05(2)	2.08(5)	3.968(54)
773.0	6.05(2)	2.09(5)	3.963(54)
773.5	6.05(2)	2.09(5)	3.957(54)
774.0	6.04(2)	2.09(5)	3.952(54)
774.5	6.04(2)	2.09(5)	3.947(54)
775.0	6.04(2)	2.10(5)	3.941(54)
775.5	6.03(2)	2.10(5)	3.936(54)
776.0	6.03(2)	2.10(5)	3.931(54)
776.5	6.03(2)	2.10(5)	3.925(54)
777.0	6.03(2)	2.11(5)	3.920(54)
777.5	6.02(2)	2.11(5)	3.915(54)
778.0	6.02(2)	2.11(5)	3.909(54)

Radiative correction for the neutral and charged processes and their difference, as a function of the ρ mass, in the region around the physical value

Compared to the previous value at the same energy

$$\delta_0 = 8.05 \times 10^{-3} \qquad \delta_+ = -4.15 \times 10^{-3} \qquad \delta_0 - \delta_+ = 12.2 \times 10^{-3}$$

Flores-Baez, Castro and Toledo
PRD 76, 096010 (2007)

Contributions to $\Delta\Gamma_\rho[\pi\pi(\gamma)]$

The rho width difference, in terms of the IB parameters, is

$$\Delta\Gamma_\rho[\pi\pi(\gamma)] = \Gamma(\rho^0 \to \pi^+\pi^-)\left[\delta_0 - \delta_+ - \frac{2\delta g}{g_0} - \frac{\Delta m_\rho}{m_{\rho^0}} - \frac{6m_+^2}{m_{\rho^0}^2\beta_0^2}\left(\frac{\Delta}{m_+} + \frac{2\Delta m_\rho}{m_{\rho^0}}\right)\right]$$

Width difference contribution + 0.5911 MeV + - 0.19 MeV + - 1.1 MeV → $\Delta\Gamma_\rho[\pi\pi(\gamma)]$ = - 0.709 MeV

± 0.22 not included before

For $\Gamma(\rho^0 \to \pi^+\pi^-) = 150$ MeV

$\Delta = 4.5936$ MeV

$\Delta_\rho = (+0.7 \pm 0.8)$ MeV

vs previous $+1.82\ (\pm 10\%)$ + - 1.1 MeV

structure uncertainty now accounted

S. Navas et al. [Particle Data Group], PRD 110 (2024)

Adding the difference from other channels, we obtain the total width difference

$$\Delta\Gamma_\rho = \Delta\Gamma_\rho[\pi\pi(\gamma)] + \Delta\Gamma_\rho(\text{rest}) = -0.621 \pm 0.22 \text{ MeV}$$

This can be compared with $\Delta\Gamma_\rho = (-0.58 \pm 1.04)$ MeV

Davier, Malaescu, and Zhang ArXiv:2504.13789v1 (2025)

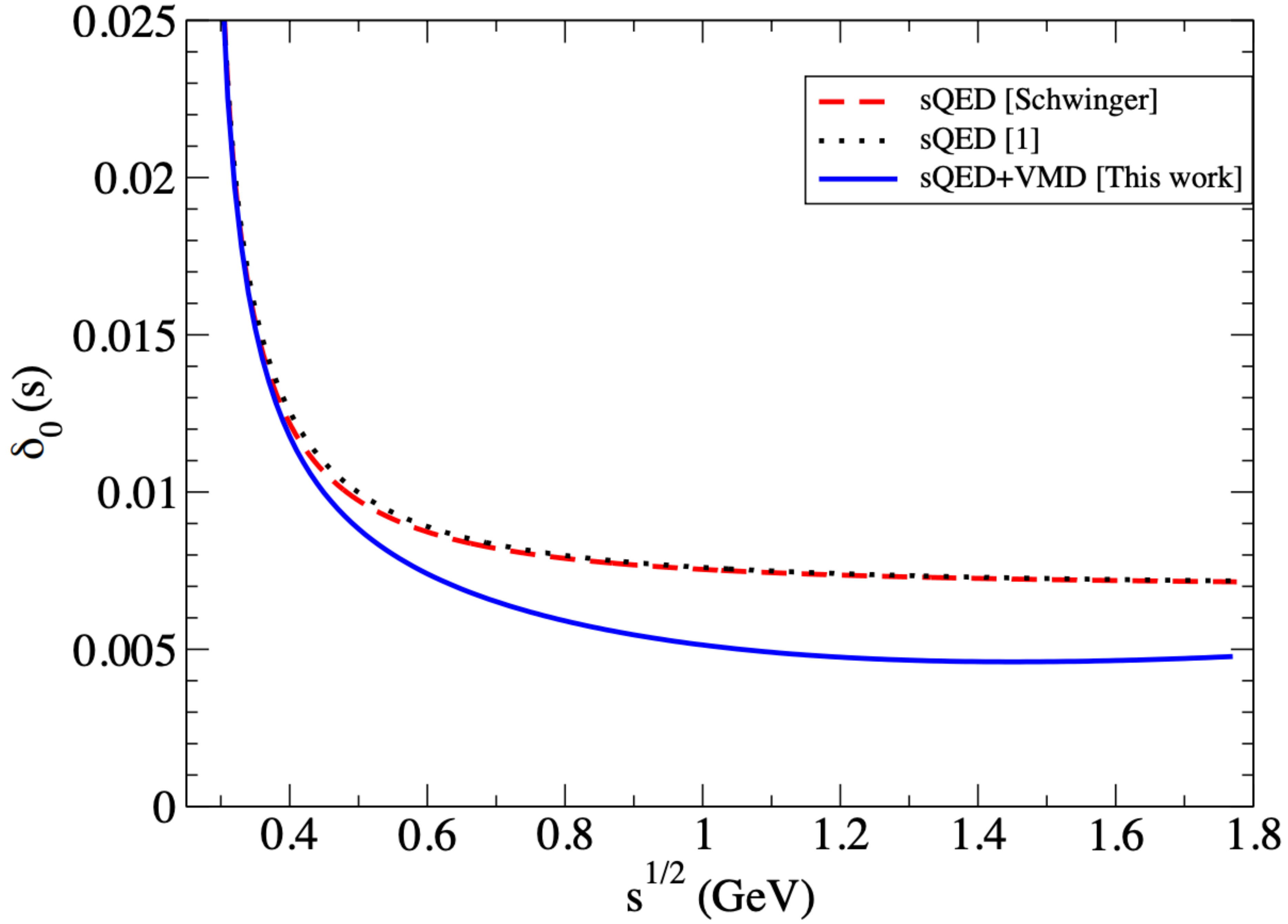
✓

FSR

The $\rho^0 \to \pi^+\pi^-(\gamma)$ decay is related to the FSR contribution in the $e^+e^- \to \pi^+\pi^-$ process.

Thus, our result for $\delta_0(s)$ incorporates the structure on the FSR in the VMD approach

$$FSR(s) = 1 + \delta_0(s)$$



Radiative correction δ_0 as a function of energy

Cross-check: Our result matches the one by Schwinger (Drees-Hikasa) for sQED.
This allows to identify the additional structure effects on VMD

J. S. Schwinger
Particles, Sources and Fields, vol. 3 (AW, 1989)

M. Drees and K. Hikasa
Phys.Lett.B252 127 (1990)

Flores-Baez, Castro and Toledo
PRD 76, 096010 (2007)

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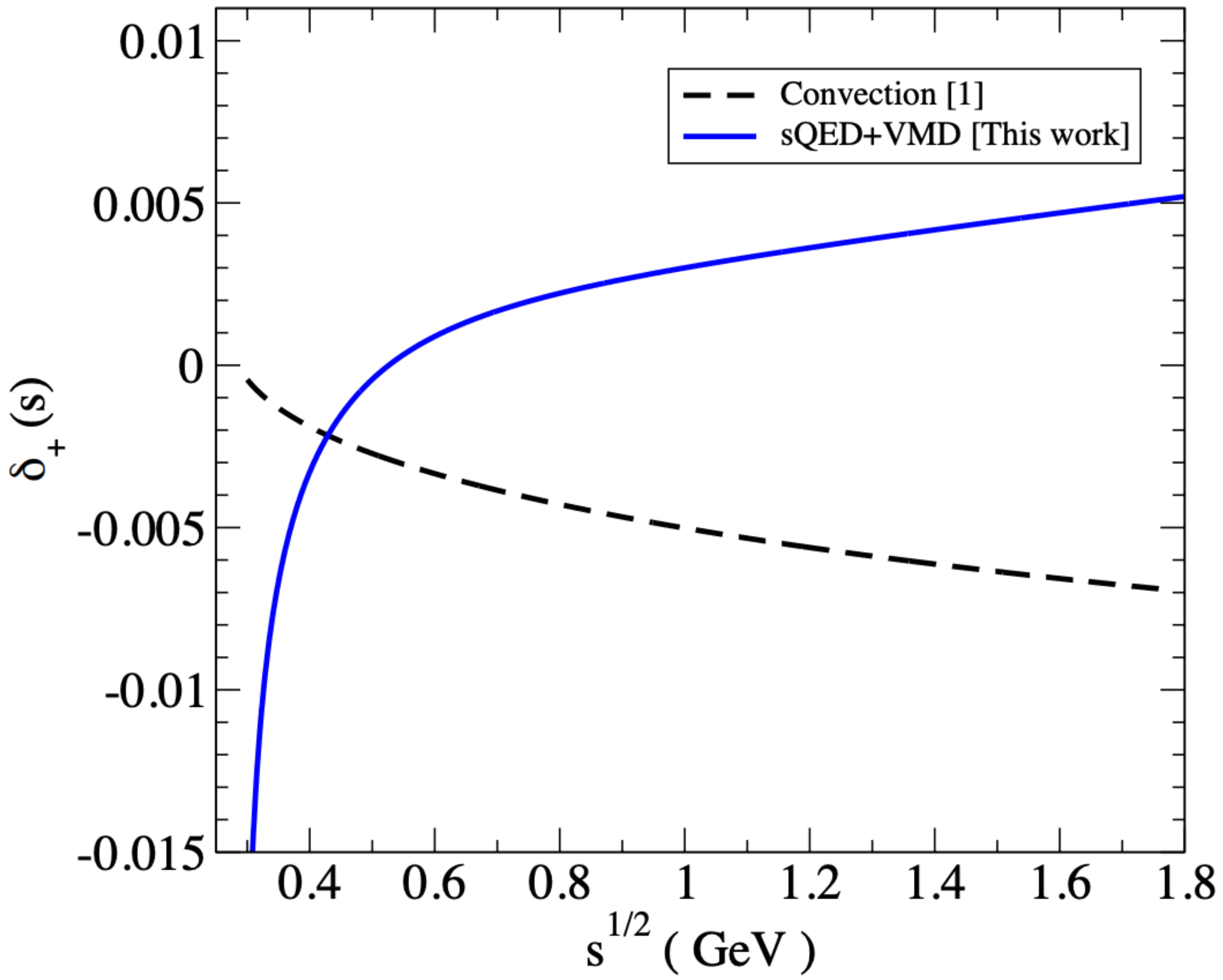
δ_+



Radiative correction δ_+ as a function of energy

Convection term considered to make UV finite (previous).

VMD (new) incorporates the full vertex structure

$\rho\rho\gamma$ vertex

$$\Gamma^{\mu\nu\alpha} = (2P-K)^{\alpha} g^{\mu\nu} + 2(k^{\mu} g^{\nu\alpha} - k^{\nu} g^{\mu\alpha})$$

$$-P^{\mu} g^{\nu\alpha} - (P-k)^{\nu} g^{\mu\alpha}$$

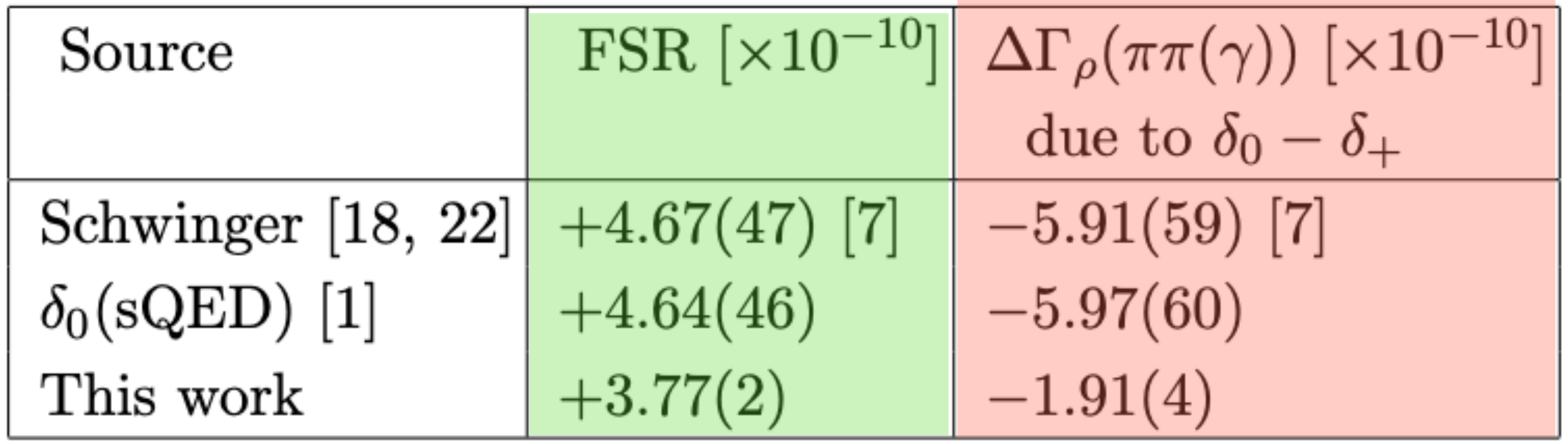
Estimated impact on $\Delta a_\mu^{HVP}[\pi\pi,\tau]$

$$\Delta a_\mu^{HVP}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_{\pi\pi^0}}{N_{\pi\pi^0}ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[FSR(s)\left|\frac{F_\pi^V(s)}{f_+(s)}\right|^2-1\right]$$

We compute the corrections using data (uncorrelated)

For the FF we use Gounaris-Sakurai parameterization. Radiative corrections are taken at m_ρ =775 MeV

$$\Delta a_\mu^{HVP}[\pi\pi,\tau]\ (\times 10^{-10})$$



Source	FSR $[\times 10^{-10}]$	$\Delta\Gamma_\rho(\pi\pi(\gamma))$ $[\times 10^{-10}]$ due to $\delta_0-\delta_+$
Schwinger [18, 22]	$+4.67(47)$ [7]	$-5.91(59)$ [7]
δ_0(sQED) [1]	$+4.64(46)$	$-5.97(60)$
This work	$+3.77(2)$	$-1.91(4)$

For FSR we obtain around 15% reduction respect to sQED

For FF we obtain around 50% reduction respect to sQED

Conclusions

- HVP from tau data requires to properly account for all the IB corrections
- The rho width difference $\Delta\Gamma_\rho$ is a key ingredient in the form factor IB estimates.
- We have computed the $\rho \to \pi\pi(\gamma)$ radiative corrections, including the structure dependence.
- As a byproduct, the structure dependent effects on FSR was obtained
- We estimated the impact on $\Delta a_\mu^{HVP}[\pi\pi, \tau]$. Hadrons structure is now theoretically accounted.

THANK´S

Aún hay otras
Alhóndigas
por incendiar !

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G. Toledo