Instituto de Ciencias Nucleares UNAM

Energy and centrality dependence of thermodynamical observables from multiplicity in Pb+Pb and Au+Au collisions

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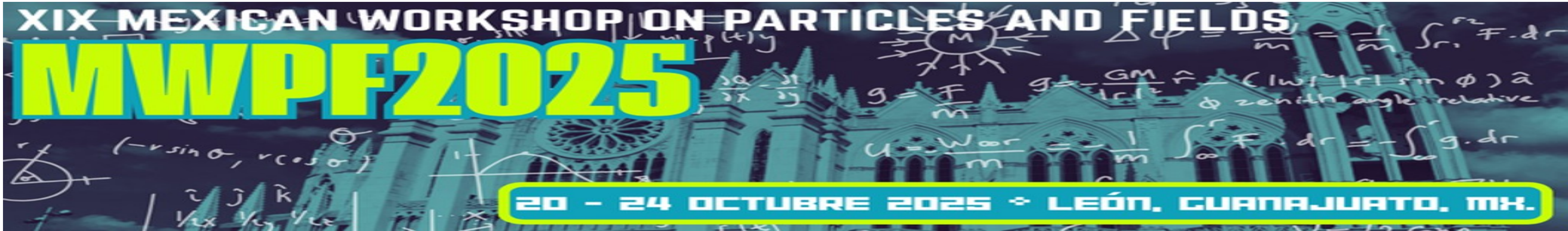
October 20th, 2025

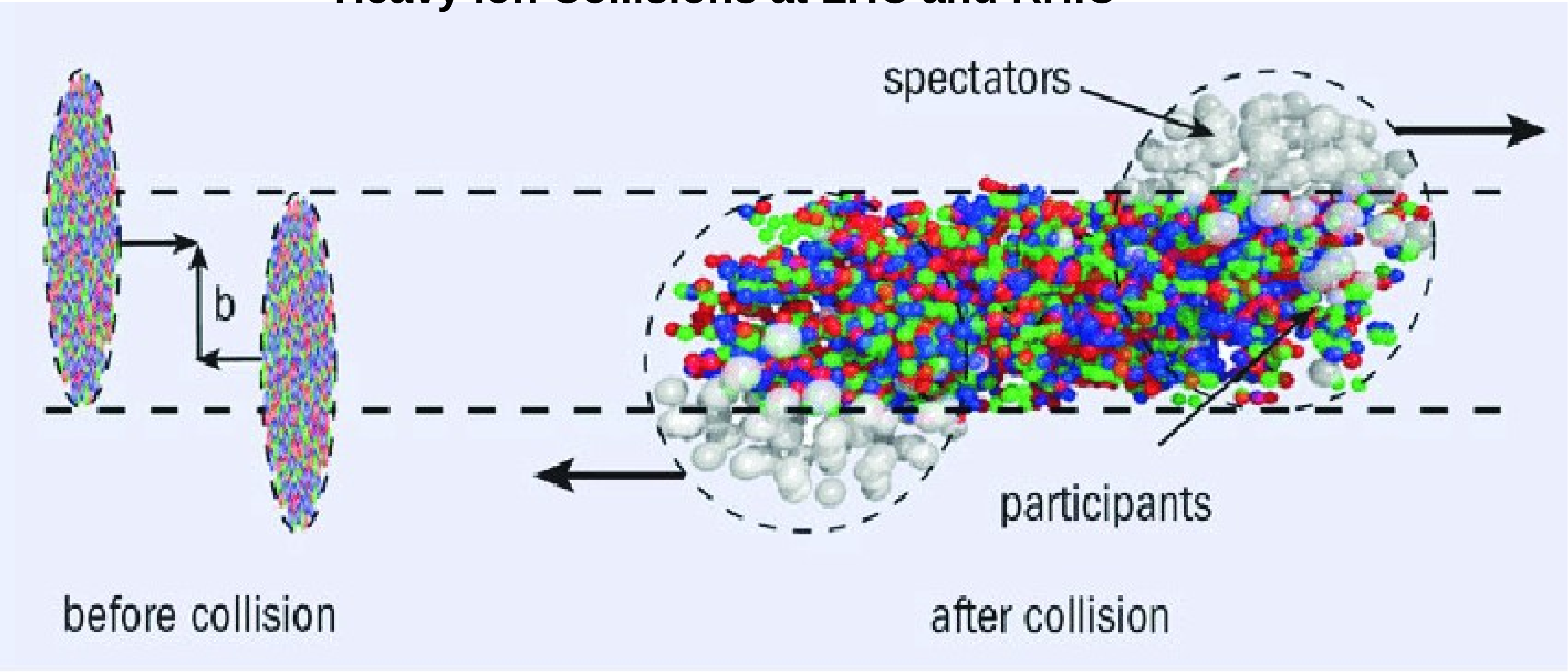


Outline

- ✔ Motivation(strongly interaction matter!)
- ✔ Thermal-FIST model:
  - ✔ Multiplicity and ratios $M^{\pm}/M^{\pm}$, B/M, $\overline{B}/B$
  - ✔ Freeze-out parameters: T_{fo}, μ_B
- ✔ Fluctuation and susceptibilities
- ✔ Conclusions

Motivation & problems

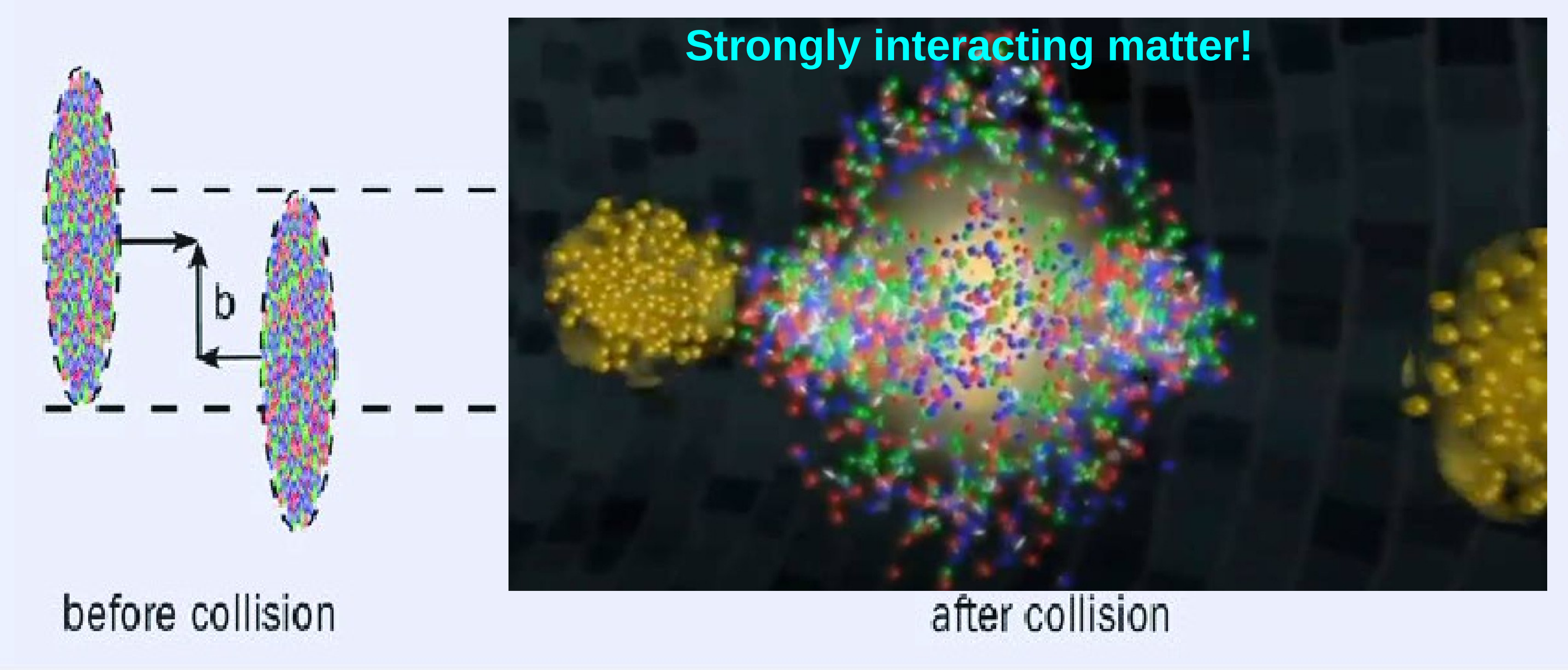
Heavy ion Collisions at LHC and RHIC



Main measurement: **Multiplicity** → **Properties of created matter in HIC**

Motivation & problems

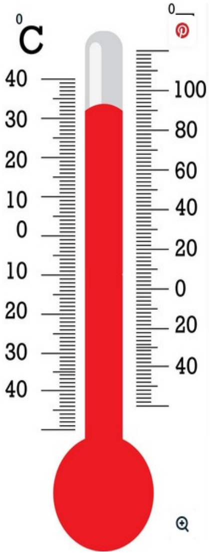
Heavy ion Collisions at LHC and RHIC

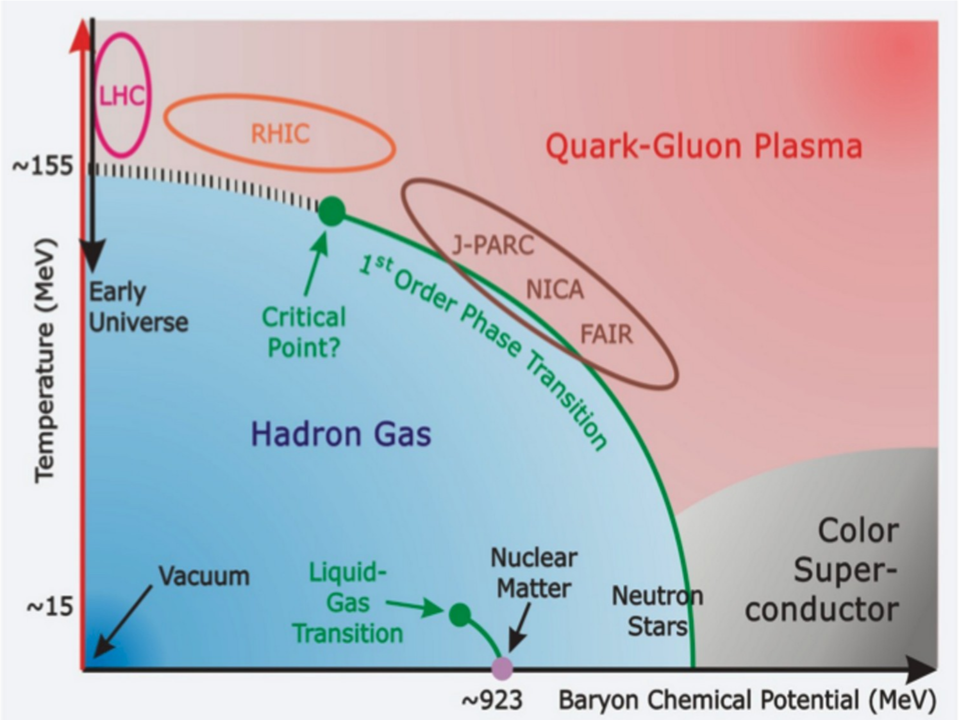


Main measurement: **Multiplicity** → **Properties of created matter in HIC**

Motivation & problems

Thermodynamic properties of the created matter





Main measurement: **Multiplicity** → **Properties of created matter in HIC**

Model used: Statistical thermal model

Thermal-FIST: Model developed to analyze hadrons produced in relativistic HIC. **It simulates a fireball as the starting point**, similar to a sphere formed in HIC. Its **evolution produces hadrons and density for hadron species**, which can be analyzed to characterize the system. **The evolution can be follow by Ideal HRG, QVdW-HRG**

The Grand Canonical Ensemble (**GCE**) is used to describe and compute properties of the system created in HIC (QGP)

$$\ln Z^{GC}(T,V,\mu_i) = \sum_{especies i} \frac{g_i V}{(2\pi)^3} \int d^3p \ln(1 \pm e^{-\beta(E_i-\mu_i)})^{\pm 1}$$

$$\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$$

and the multiplicity is given by:

$$N_i^{GC} = T\frac{\partial \ln Z^{GC}}{\partial \mu_i}$$

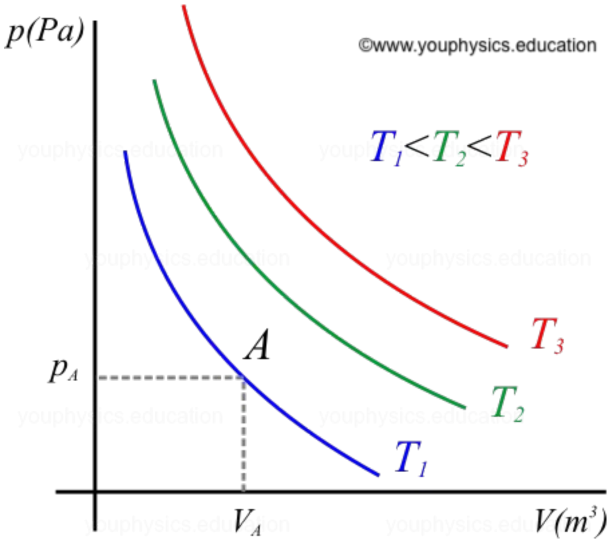
Multiplicity is our input data measured in AGS, NA48, STAR

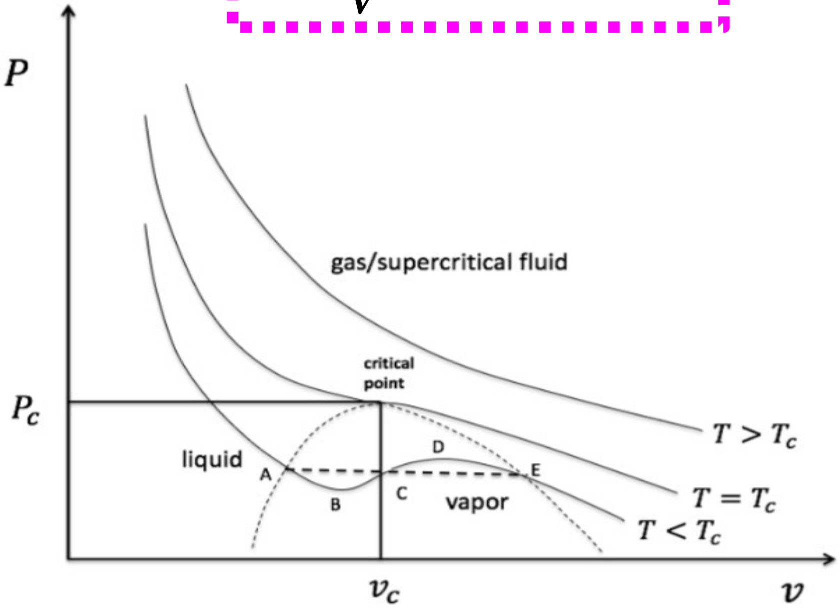
Vovchenko, V., Stoecker, H
Comput. Phys. Commun. 244 295–310 (2019)
Thermal-FIST (Thermal, Fast and Interactive Statistical Toolkit)

Ideal Gas | *Van der Waals*

$$PV = nRT$$

$$\left(p + a\frac{n^2}{V^2}\right)(V - nb) = nRT$$





Hadron resonance gas: Van der Waals

Critical point (P_c, V_c, T_c) using Van der Waals equation
At this point, the system is teetering between liquid and gaseous phases, then it satisfies:

$$\left(p+a\frac{n^2}{V^2}\right)(V-nb) = nRT$$

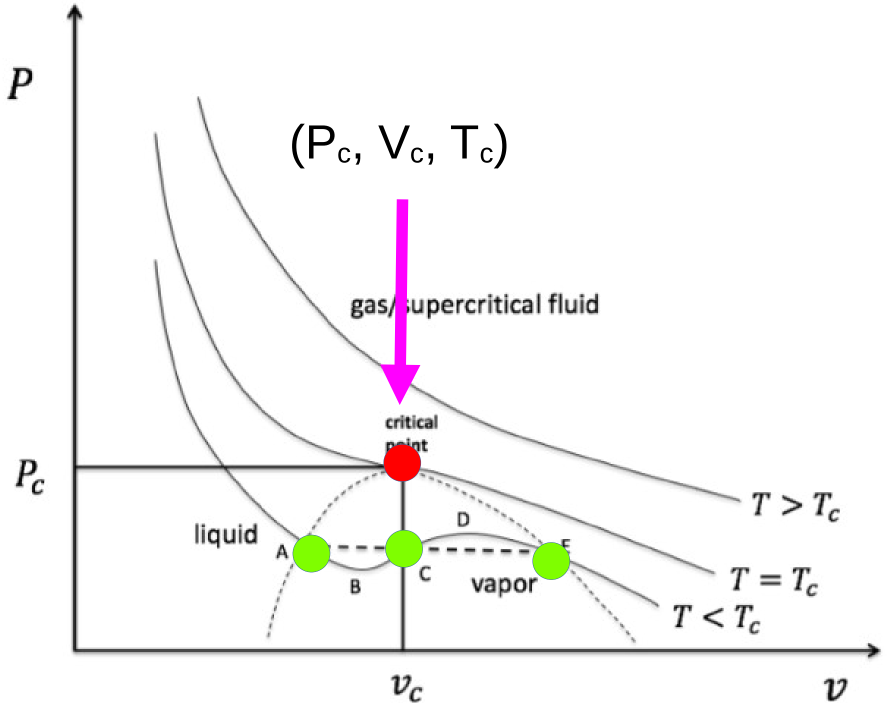
$$\left(\frac{\partial P}{\partial V}\right)_{T_c,V_c}=0$$

$$\left(\frac{\partial^2 P}{\partial V^2}\right)_{Tc,V_c}=0$$

$$(P_c,V_c,T_c)$$

$$\left(\frac{a}{27b^2},3b,\frac{8a}{27bR}\right)$$

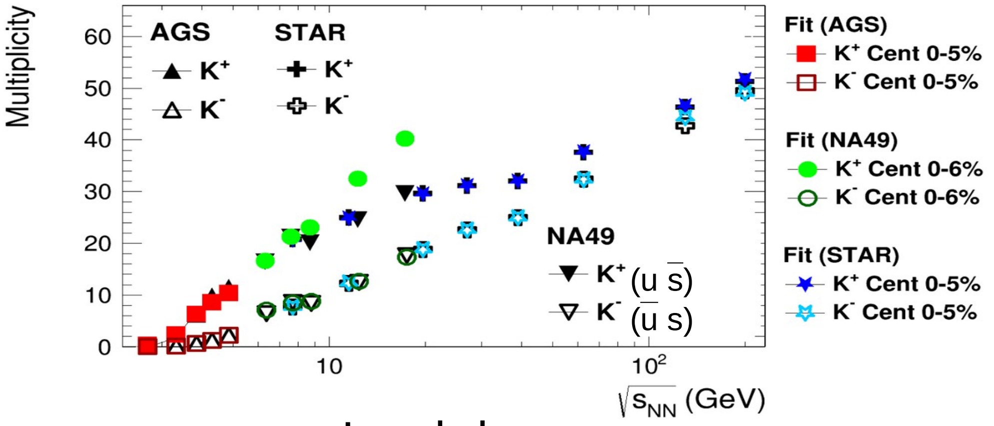
$$\frac{P_cV_c}{T_c}=\frac{3}{8}R$$



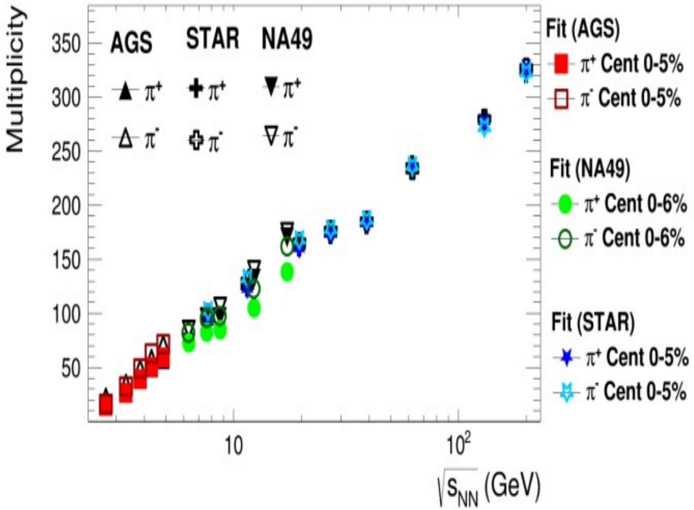
Multiplicity data vs model

Thermal-FIST model describe quantitatively the multiplicity for measured and unmeasured hadrons.
Some examples are as follow:

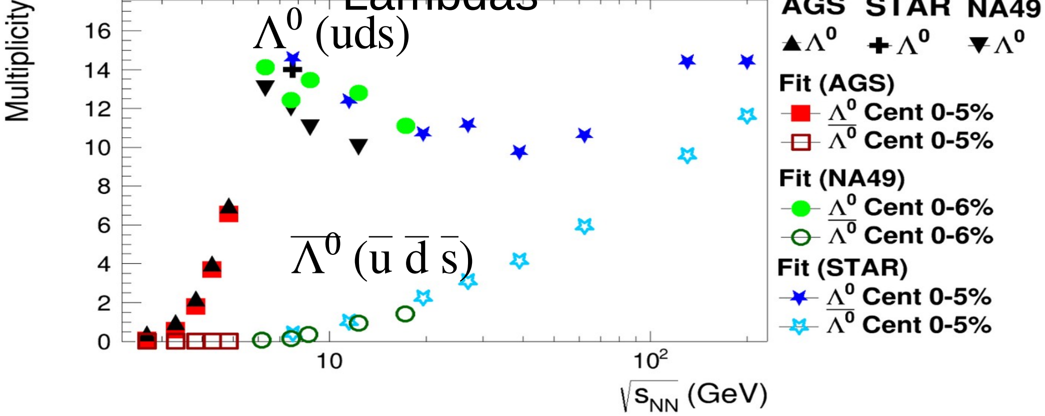
Kaons



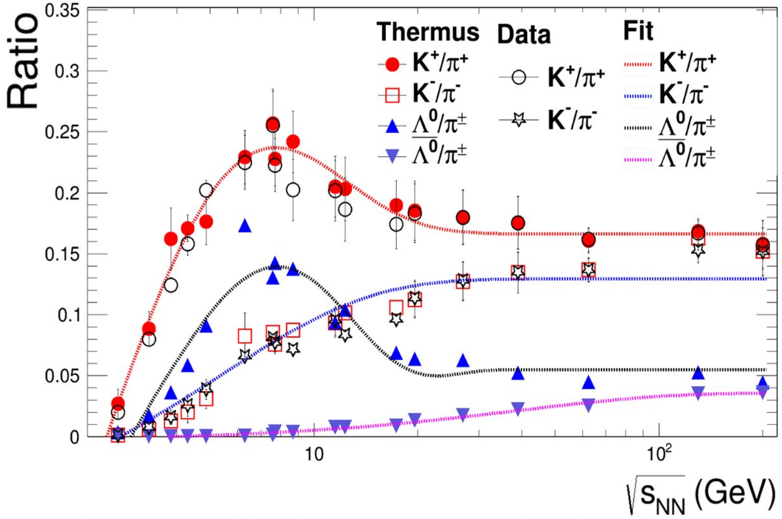
Pions

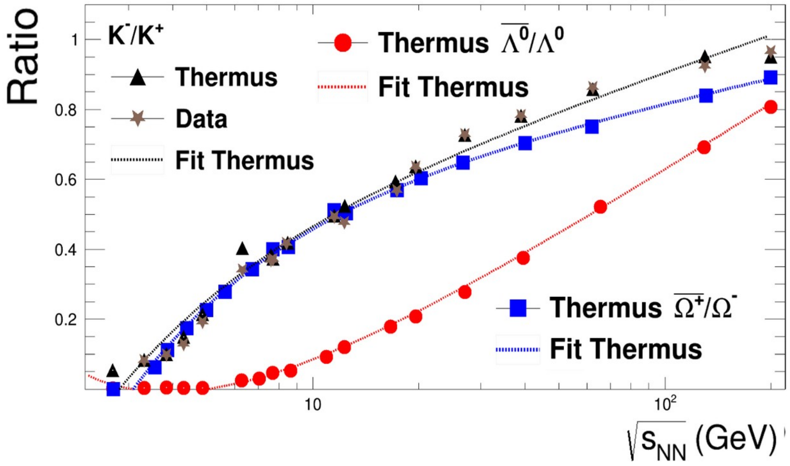


Lambdas



The "horn" in $K^{\pm}/\pi^{\pm}$ and $\Lambda^0/\pi^{\pm}$ ratios





$K^+(u\bar{s})/\pi^+(u\bar{d})$ ~$\bar{s}/\bar{d}$ with a peak ~7 GV

Similar to $\overline{\Lambda^0}/\bar{p}$

$K^-(\bar{u}s)/\pi^-(\bar{u}d)$ monotonous growth

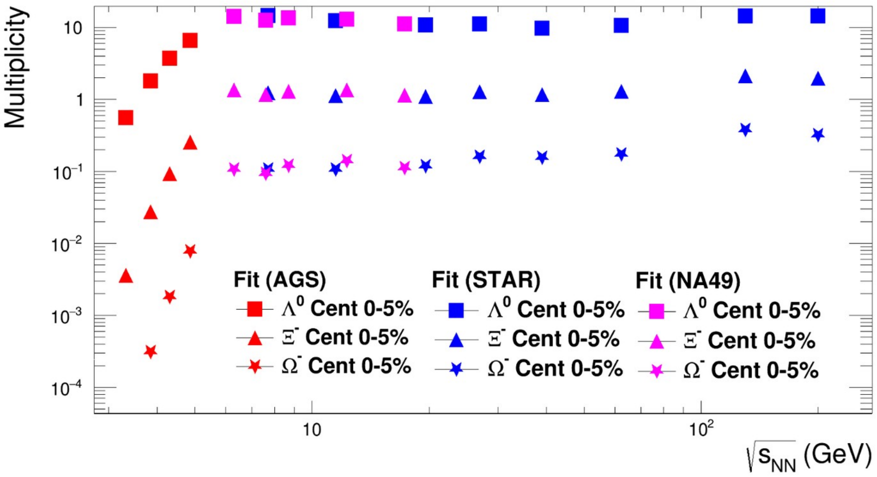
$\Lambda^0(uds)/\pi^{\pm}$ with a peak ~ 7 GeV

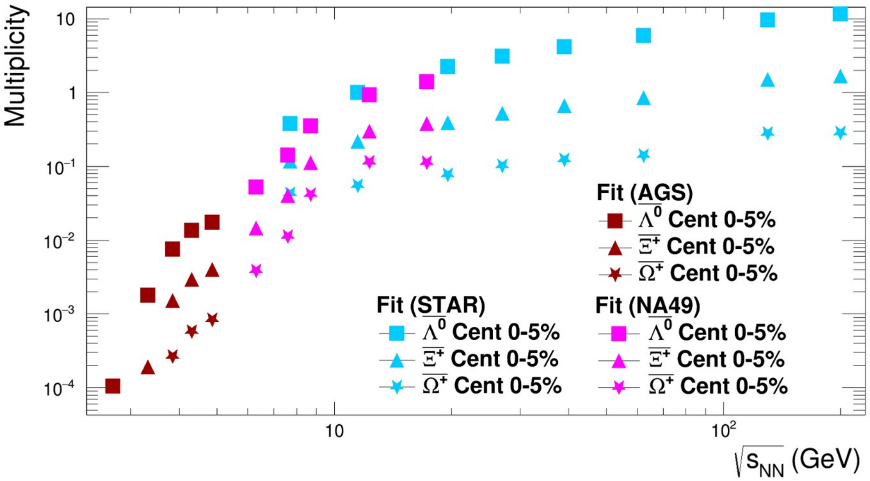
$\overline{\Lambda^0}(uds)/\pi^{\pm}$ monotonous growth

The production of multi-strange (Ω) is lower respect to strange hadrons (K, Λ^0)

NA49, PRC77, 024903, 2008
NA49, PRC66, 054902, 2002
E895, PRC68, 54905, 2003
STAR: arXiv:1007.2613v1 nucl-ex

Multi-strange baryons



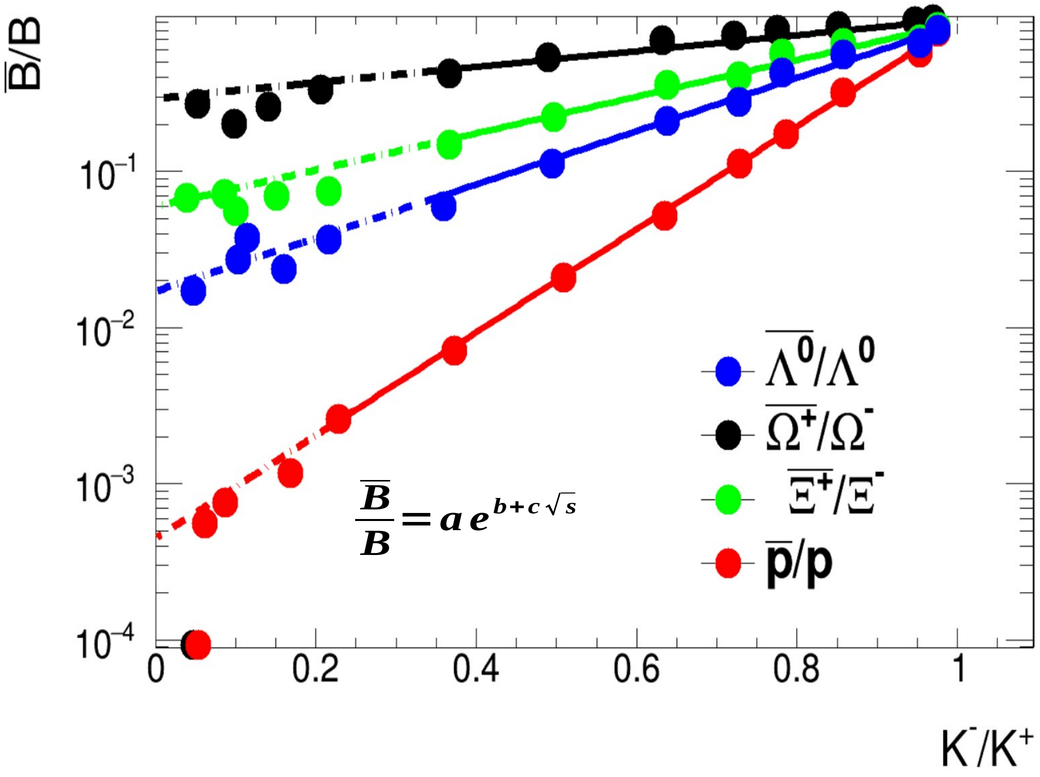


The multiplicity increases faster for hadrons with less strangeness.

The saturation for energy larger than 10 GeV indicates that the system reaches equilibrium of strangeness production.

The antiparticles show a monotonous growth.

Strangeness baryon production vs K^-/K^+



The thermal statistical model produces a **universal behavior of the ratio $\overline{B}/B$** Well description by the Quark-Combination model where:

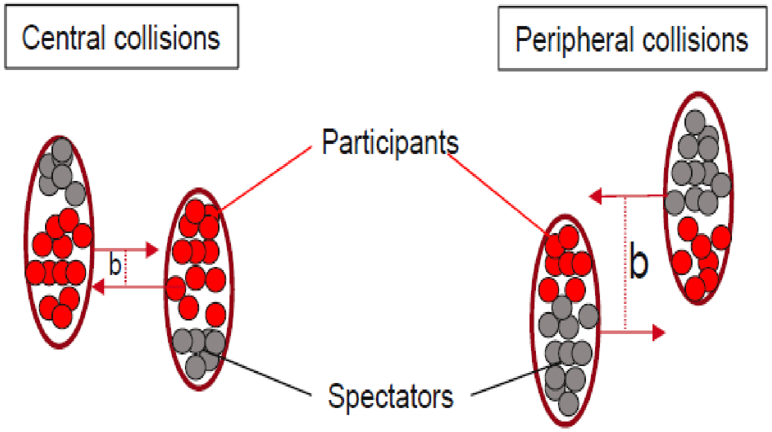
a) The momentum distribution of quarks/anti-quarks is taken to be independent.

b) The probability for the production of different quark-antiquark pairs is taken to be independent of each other

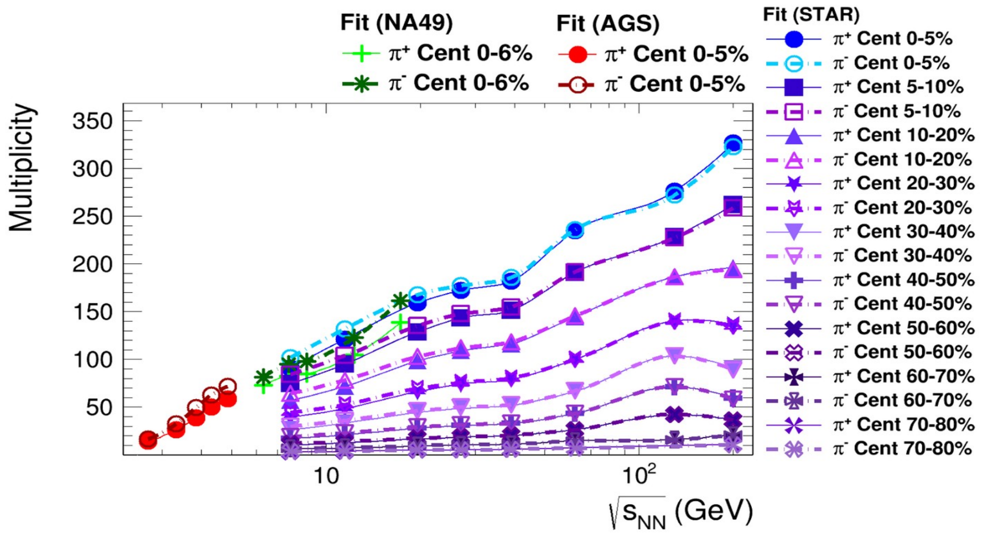
P (uud), Λ^0(uds), Ξ^-(dss), Ω^-(sss), K^-($u\overline{s}$)

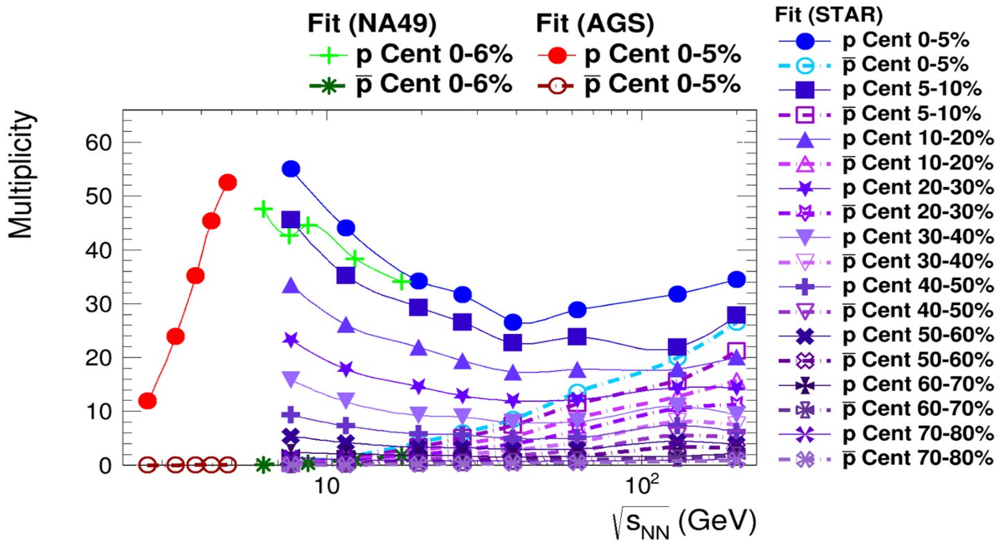
Multiplicity by centrality

Centrality by Glauber model



b ~ 0 b → 2R





Freeze-out parameters:
Temperature and Chemical potential

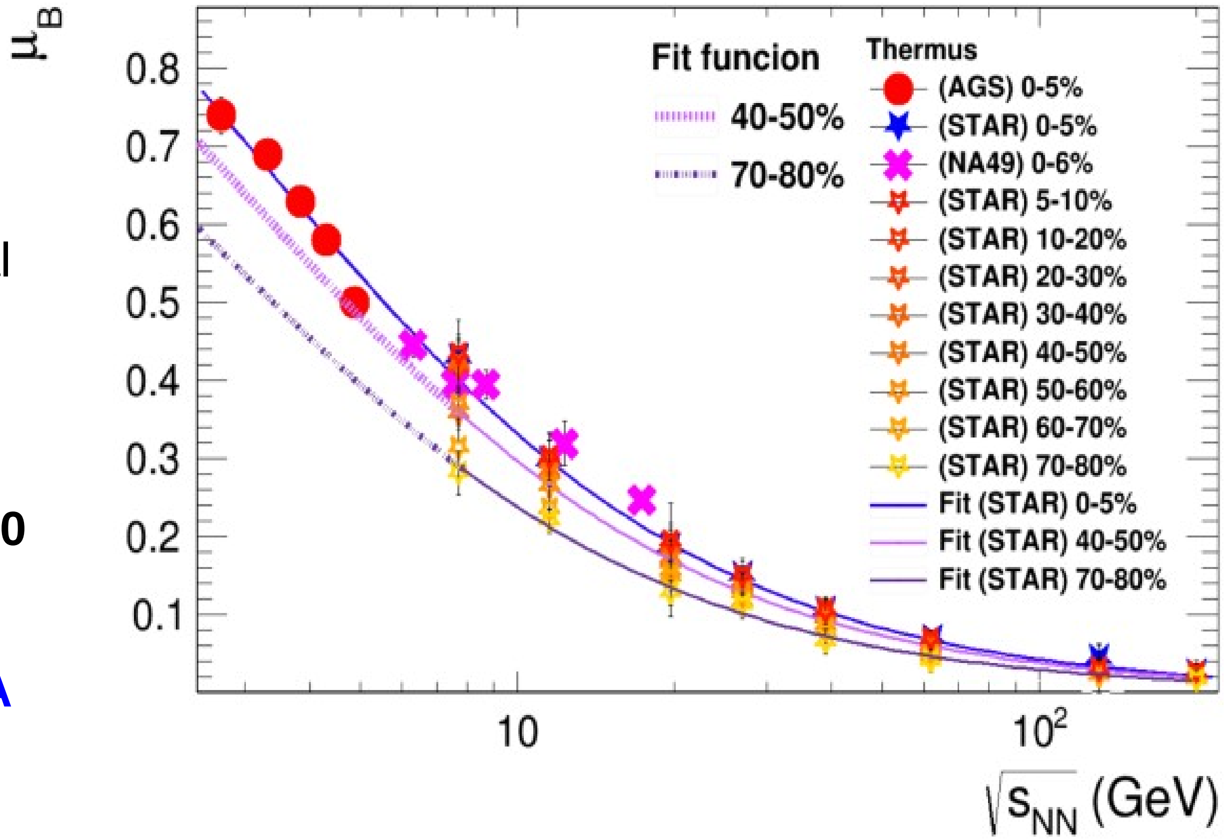
Baryon chemical potential

$$\mu_B(\sqrt{s}) = \frac{d}{1+e\sqrt{s}}$$

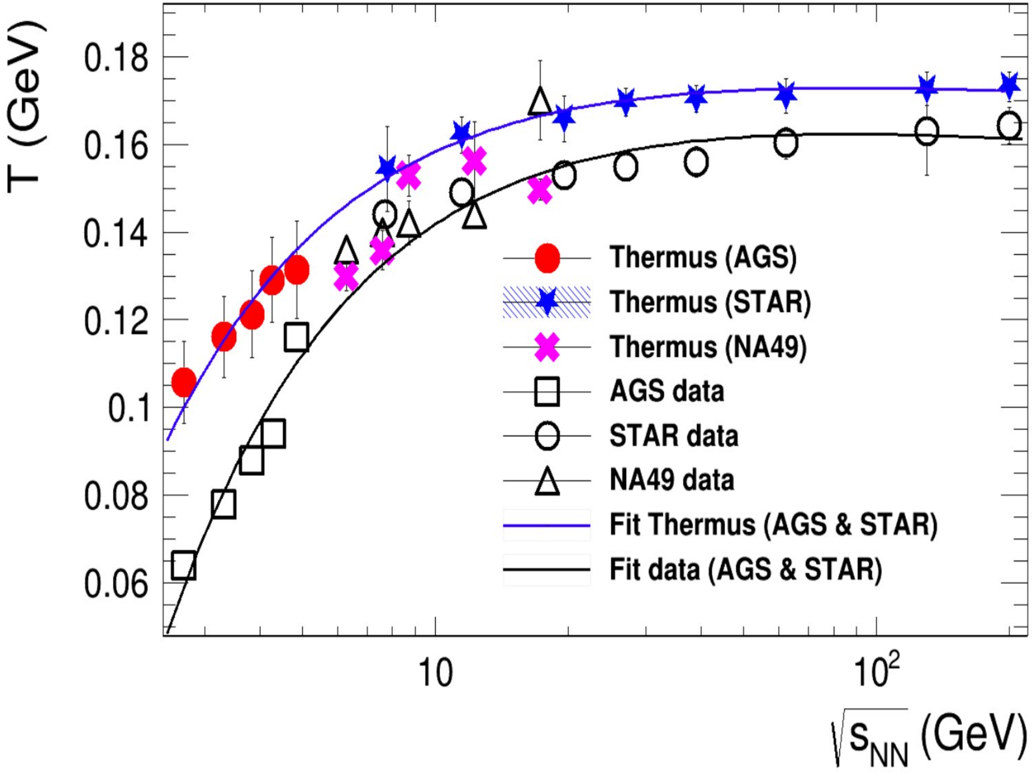
Parametrization of chemical potential for centrality classes measured by STAR experiment.

We observe the limit cases, at high energy and for all centralityies $\mu_B \rightarrow 0$

At lower energy (**4 GeV**) **μ_B** differ up to **18%**, for future, at **FAIR and NICA**



Freeze-out temperature



fit with free parameters: R, γ_s ,

Fit with fixed parameters R, γ_s,

We see differences up to 6%, and 39% for higher and lower energy, respectively.

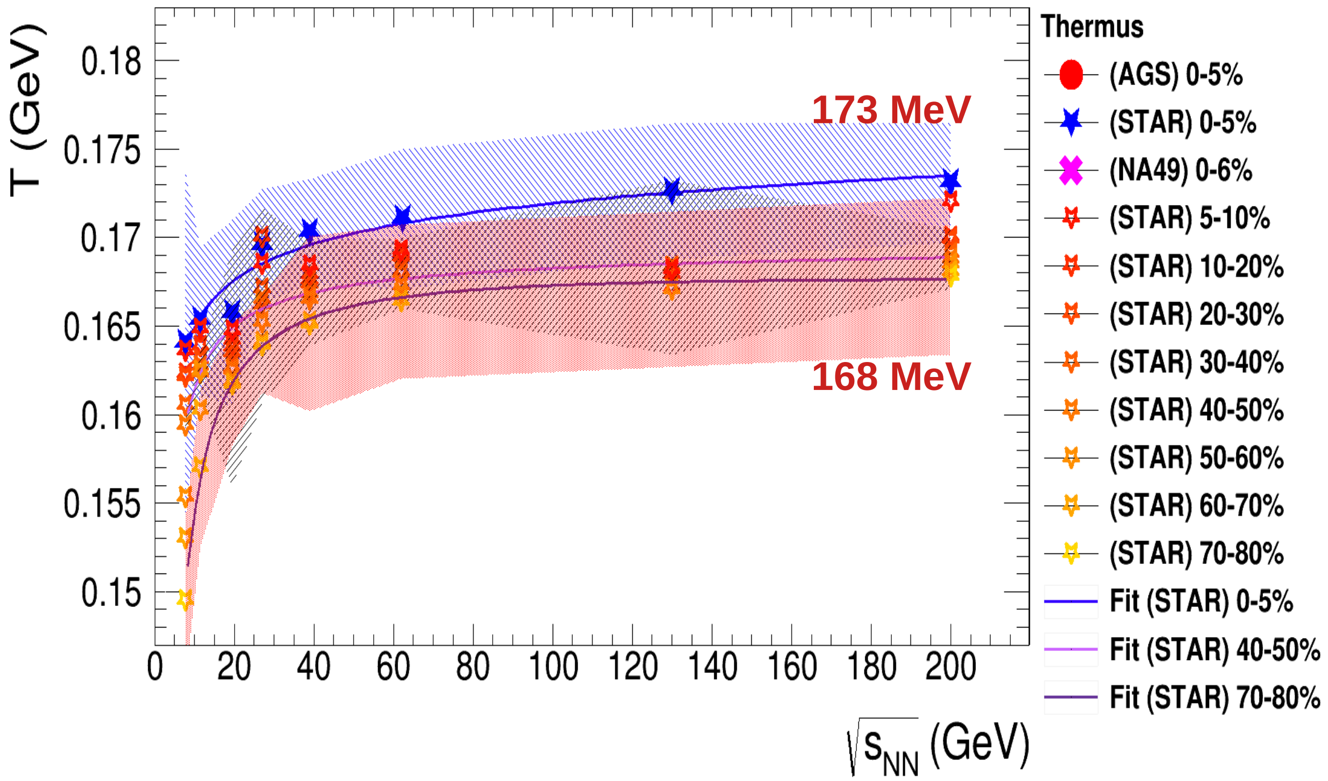
$$T_{\exp} = a + \frac{b}{\sqrt{s_{NN}}} + c\ln\left(s_{NN}^{1/4}\right)$$

Temperature by centrality classes

173 MeV(central) → 168 MeV (Peripheral)

Analyzing multiplicity for centrality classes helps us to characterize the system

Freeze-out Temperature decrease from central to peripheral collisions.



Fluctuation and susceptibilities

Fluctuations → Cumulants→ susceptibilities

Experimental measurements

we measure the fluctuations of quantum numbers Event-by-event, they are given by statistical cumulants

$$C_1 = \langle \Delta N \rangle$$
$$C_2 = \langle (\delta N)^2 \rangle$$
$$C_3 = \langle (\delta N)^3 \rangle$$
$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

Thermodynamic properties

$$C_n = \frac{\partial^n}{\partial(\mu/T)^n}(\log Z(z,V,T))$$

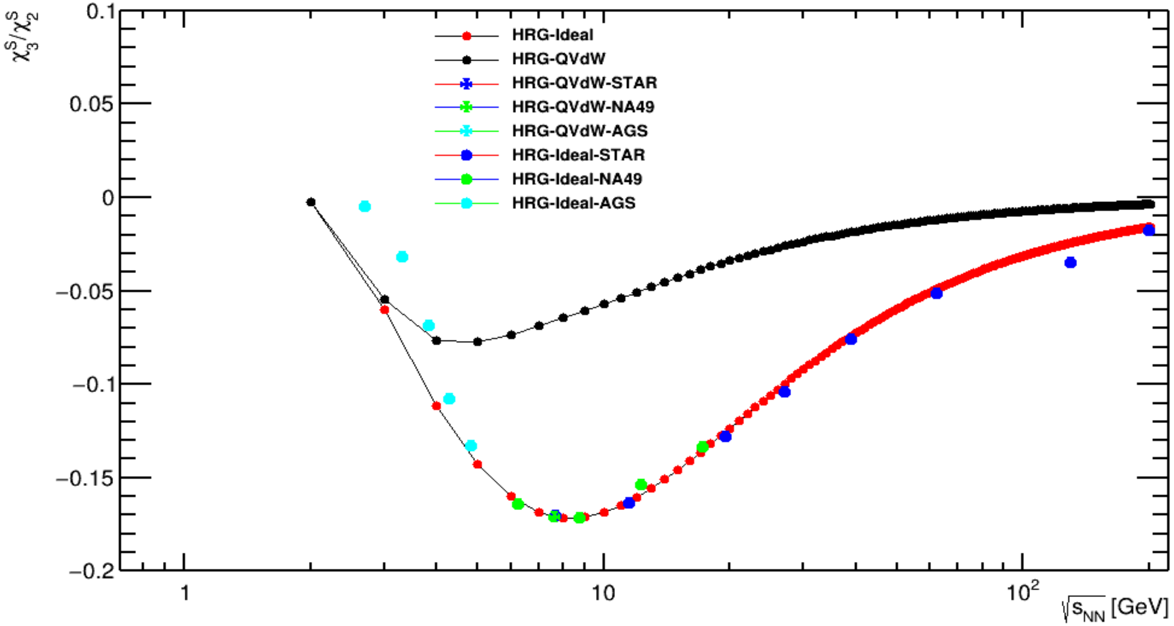
$$\chi_q^{(n)} = \frac{\partial^n p(T,\mu_q)}{\partial(\mu_q/T)^n} = \frac{1}{VT^3}\frac{\partial^n \log Z}{\partial(\mu_q/T)^n}$$

$$C_n = VT^3\chi_q^{(n)}$$

$$\mathit{skewness} = S\sigma = \frac{\chi_3^{Q,B,S}}{\chi_2^{Q,B,S}}$$

$$\mathit{kurtosis} = k\sigma^2 = \frac{\chi_4^{Q,B,S}}{\chi_2^{Q,B,S}}$$

Skewness of strangeness



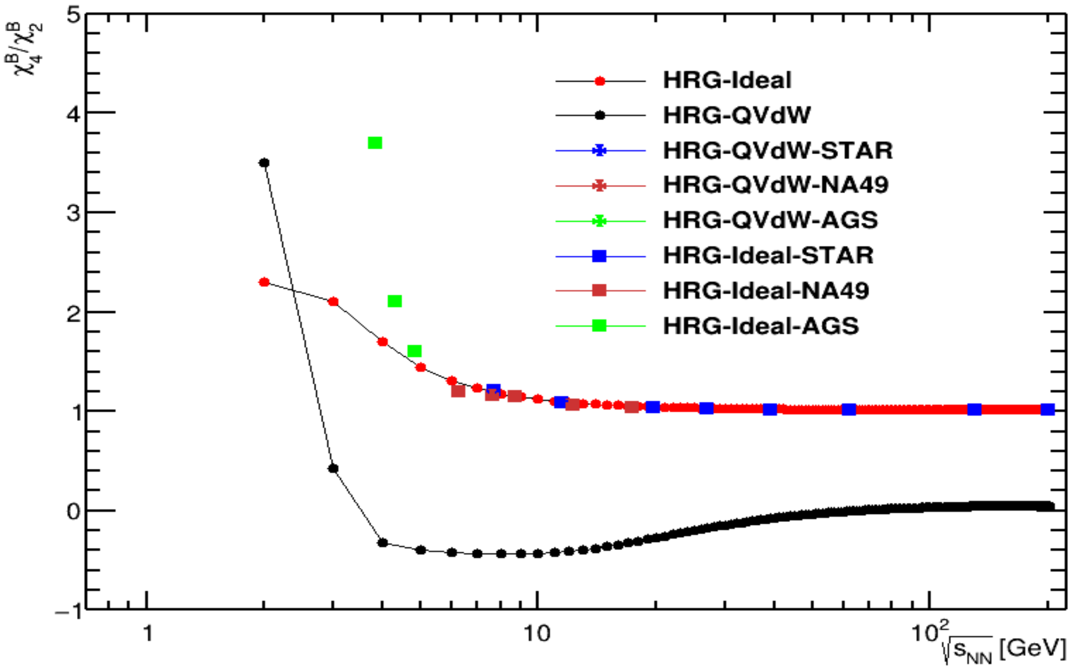
$$skewness = S\sigma = \frac{\chi_3^S}{\chi_2^S}$$

The susceptibility (χ^S) is a thermodynamic observable. It is essential to understand strongly interacting matter. **It measures the collective response of the system, and the behavior of the response reveals the strong correlations between its constituents.**

The χ^S diverges near the phase transition, indicating a possible new state of matter.

The anomaly in the susceptibilities is proof of the strong correlations between constituents of the system

Kurtosis of baryon number



$$kurtosis = k\sigma^2 = \frac{\chi_4^B}{\chi_2^B}$$

The kurtosis is a thermodynamics observable which provide degree of microscopic freedom, and collective behavior close to the transition phase. **It is very sensitive to properties of the strong interacting matter.**

In particular in region close to the phase transition or critical end point in the QCD phase diagram, **the fluctuations of the baryon number are very large, as consequences the kurtosis become negative.**

The **search for a no monotonous behavior** of the χ^B as a function of collision energy is one of the main observables to identify the **critical end point in the QCD phase diagrams**

Conclusions

- Using the Thermal-FIST model, we have presented an analysis of multiplicity from Au+Au and Pb+Pb measured at the energy range from 2.7-200 GeV, obtaining the following results:
- Multiplicity, baryon/meson, anti-baryon/baryon
- Freeze-out temperature and chemical potential (for centrality classes)
- Susceptibilities: Skewness for strangeness and kurtosis: sensitive to freeze-out parameters, mainly at lower energy.

  (*more detailed analysis is required to get CEP, ...*)

Quantum Van der Waals

$$p(T,\mu) = p_B + p_{\bar{B}} + p_M.$$

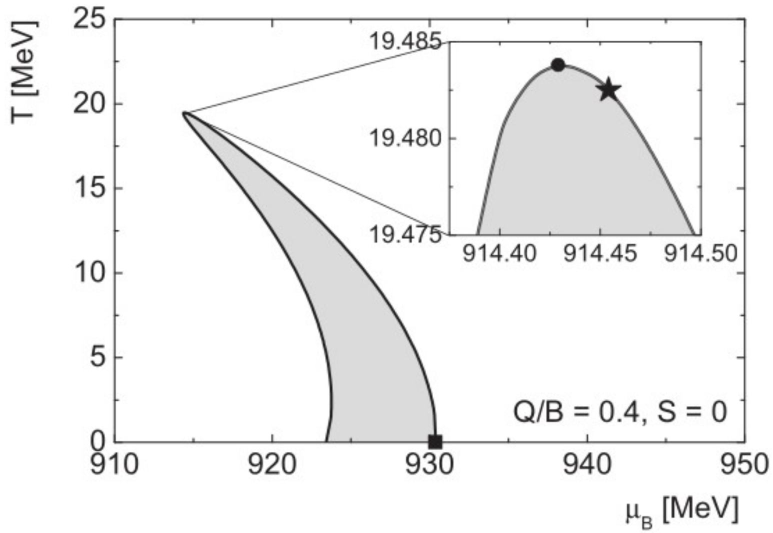
$$p_B(T,\mu) = \sum_{j\in B} p_j^{\text{id}}(T,\mu_j^{B*}) - a n_B^2$$

$$\mu_j^{B*} = \mu_j - b\, p_B - a\, b\, n_B^2 + 2\, a\, n_B$$

a = 329 MeV fm^3 and **b = 3.42** fm^3,
n^{GS} = 0.16 fm^{-3} **E_b = -16** MeV
Of the ground state of the symmetric matter

$$n_B = \left[\frac{\partial p_B}{\partial \mu_B}\right]_T = \sum_{j\in B} n_j = (1 - b n_B)\sum_{j\in B} n_j^{\text{id}}(T,\mu_j^{B*})$$

PRC100, 054904(2019)

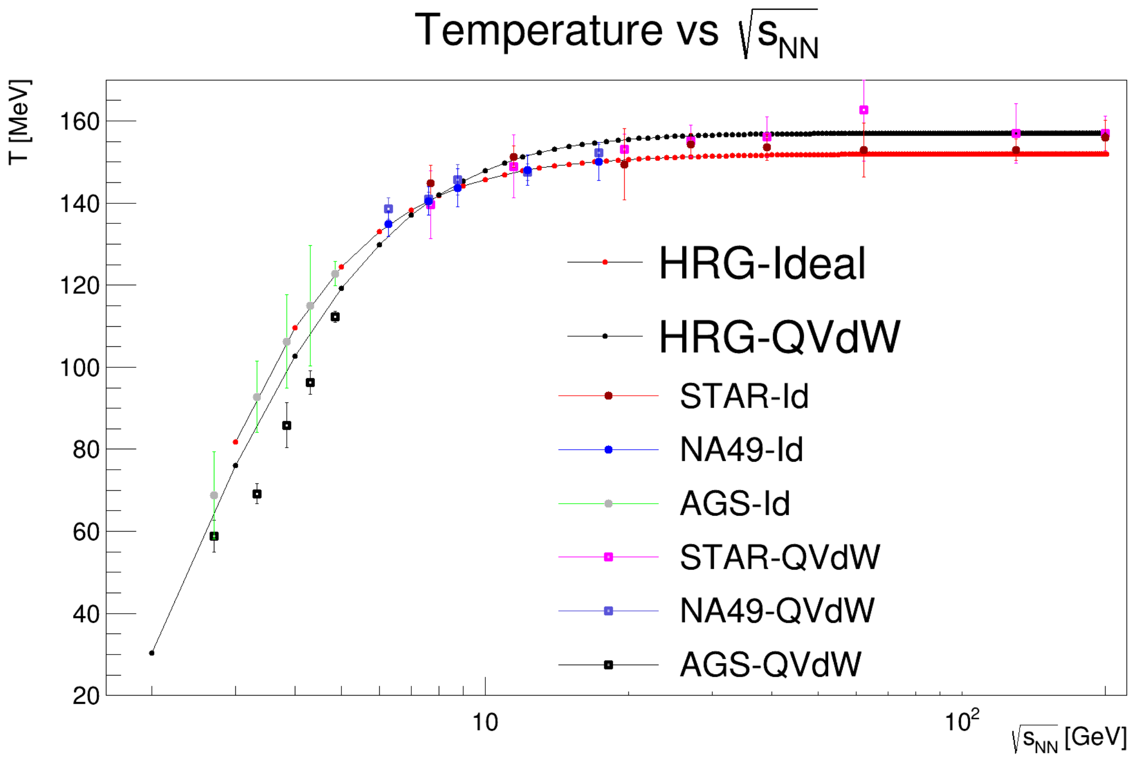


LGPT in QVdW-

The QvdW-HRG model with Q/B = 0.4 exhibits the CP:

$\mu_{B,c}$ = 914.5 MeV, Tc = 19.48 MeV. The LGPT region in the (μB , T)

Temperature from Ideal HRG vs QVdW



Looking the critical end point!