FROM TOP-DOWN TO BOTTOM-UP: THE RÔLE OF SYMMETRIES IN SOME EXTENSIONS OF THE SM

Myriam Mondragón
IFUNAM

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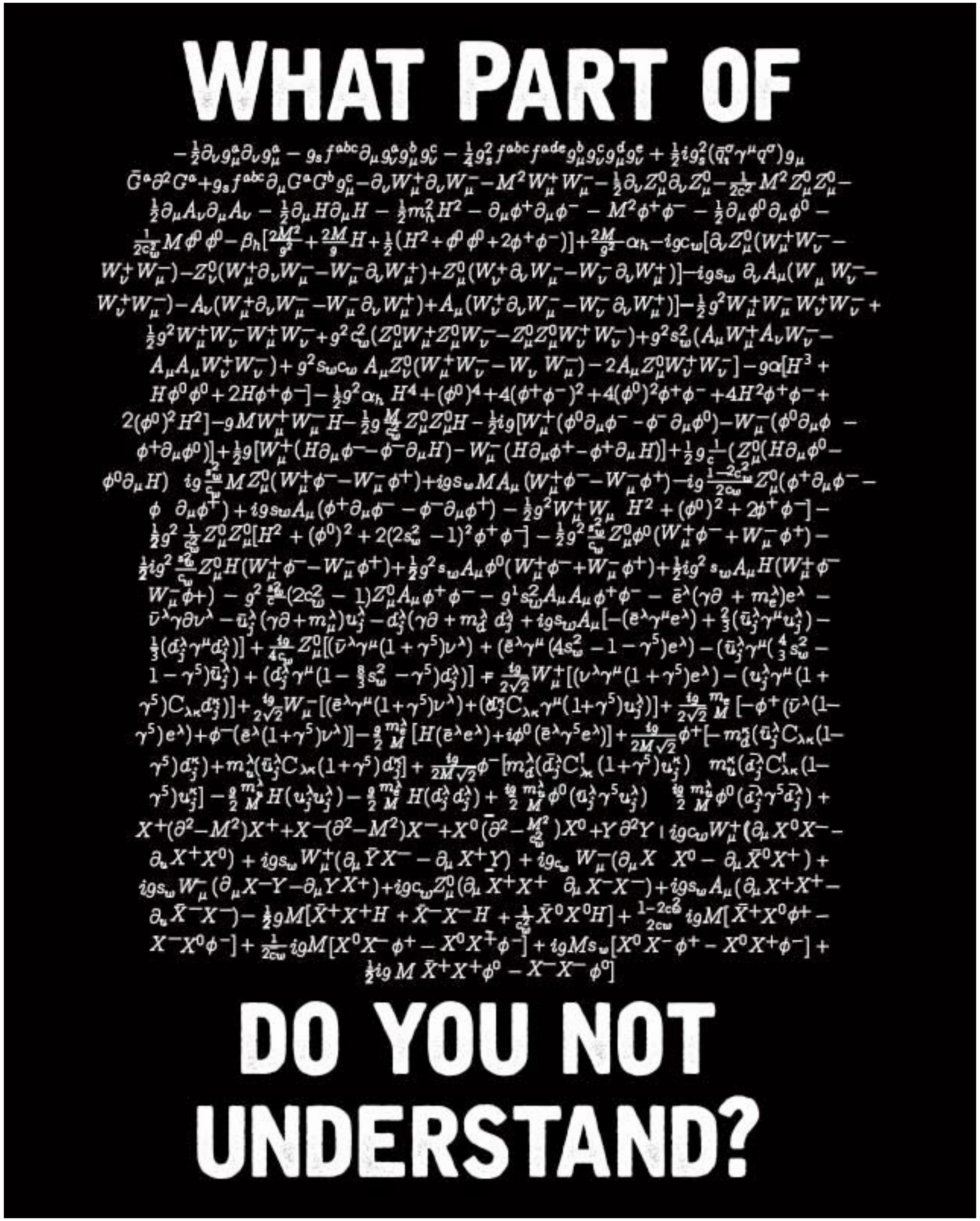


$+\ \Lambda CDM\ldots$

WHAT'S GOING ON?

- What happens as we approach the Planck scale?
- What happened at the early Universe?
- How do we go from an effective theory like the SM to a more fundamental one?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- Why/how are the elementary particle masses so different?
- Is there more than one Higgs, more scalars?
- What about flavor?

- **Where is the new physics?**

WHY GO BEYOND THE SM?

- The hierarchy problem
- Neutrino masses
- All masses
- Origin of gauge interactions
- Dark matter
- Matter over anti-matter abundance
- Cosmological constant
- Inflation
- …

Higgs sector not natural

Fermion masses vastly different

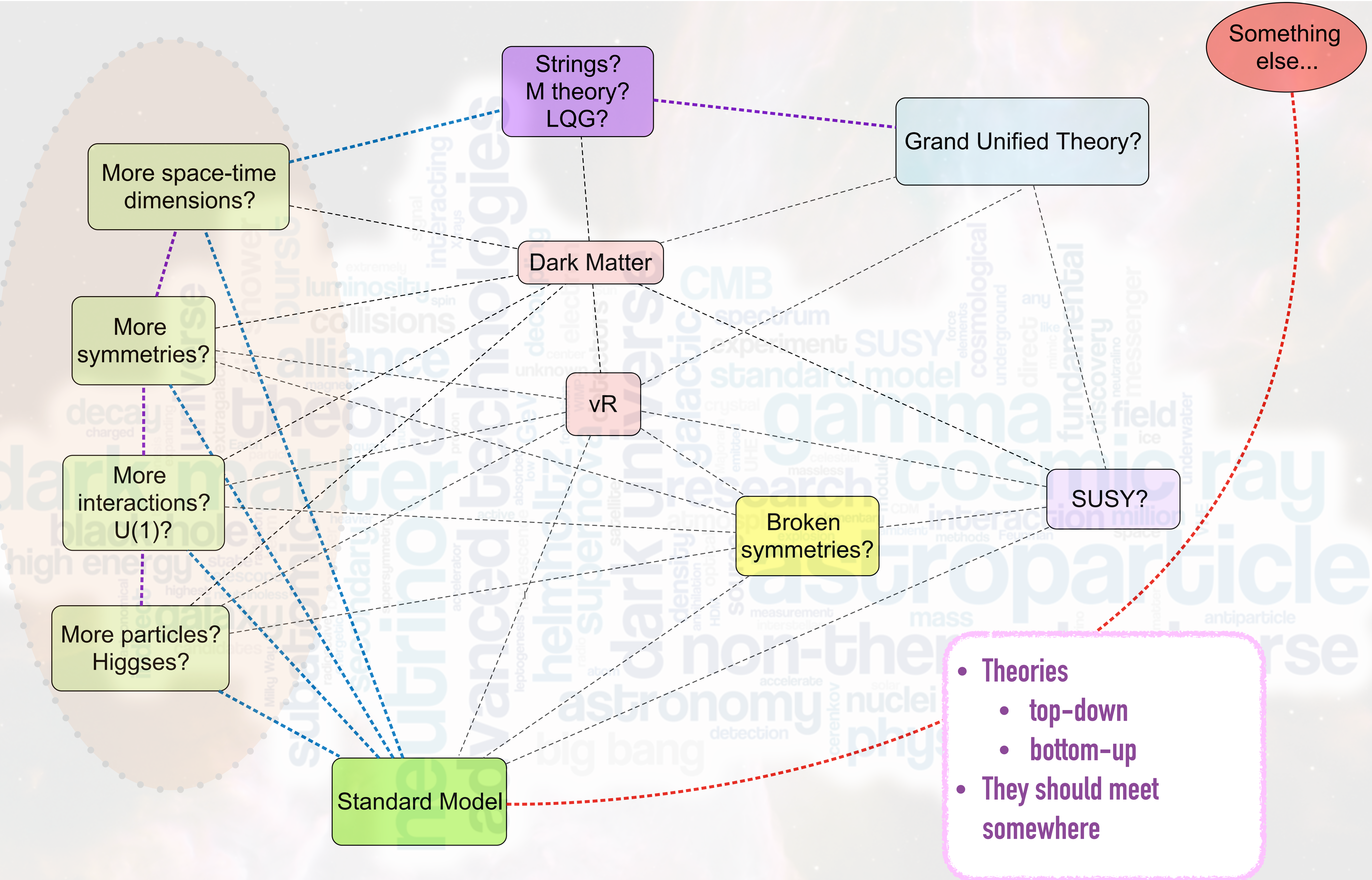
Origin of electroweak symmetry breaking unknown

Dirac or Majorana neutrinos

Strong CP problem

Not enough CP in SM for Baryogengesis
Value of cosmological constant
Inflation inconsistent with non-zero baryon number
Is DM a particle, then which, is it only one

HOW DO WE GO BEYOND THE SM?



Something else...

Strings?
M theory?
LQG?

Grand Unified Theory?

More space-time dimensions?

Dark Matter

More symmetries?

vR

More interactions?
U(1)?

SUSY?

Broken symmetries?

More particles?
Higgses?

Standard Model

- Theories
  - top-down
  - bottom-up
- They should meet somewhere

SYMMETRIES

- Modern physics is built on the observation that there are symmetries in Nature (exact or broken)

- Symmetry is a transformation that leaves the system invariant

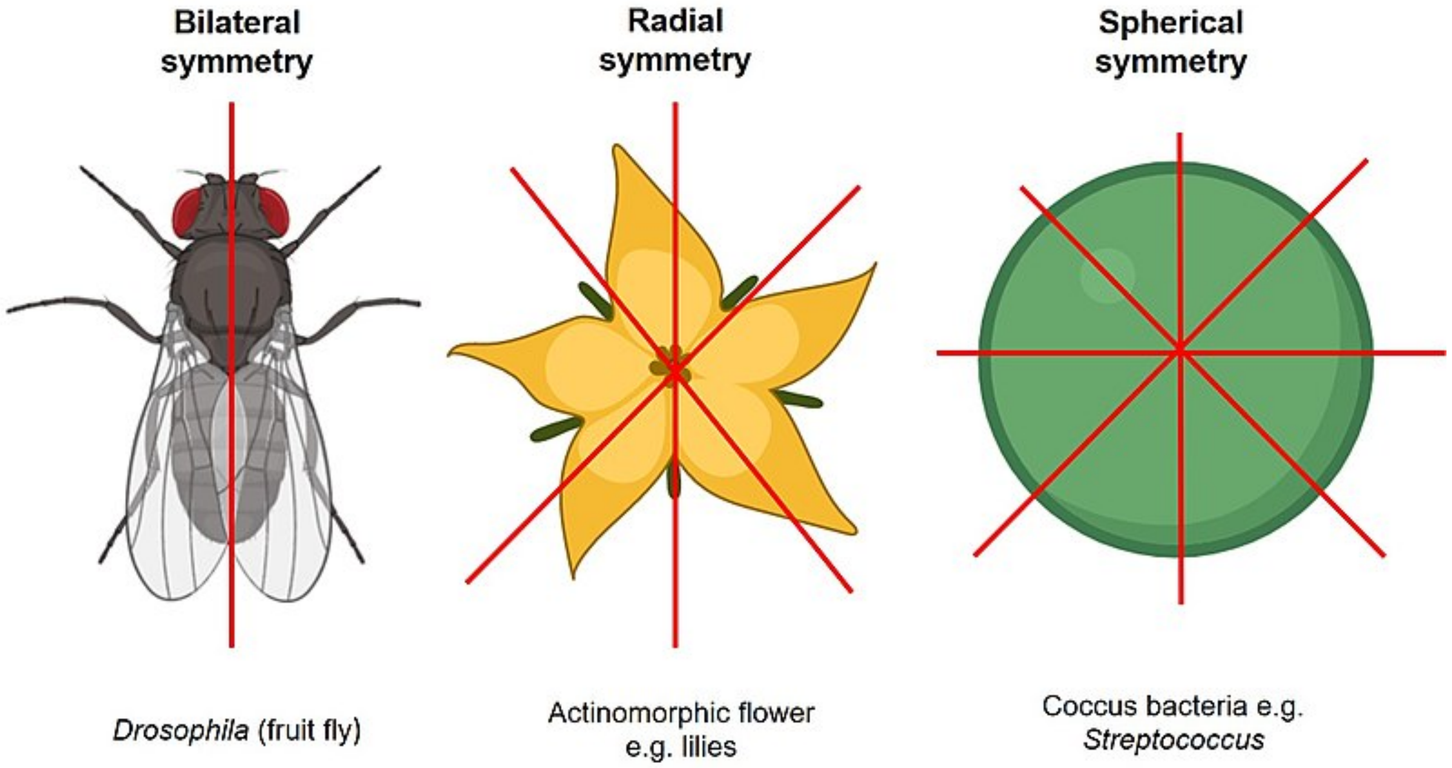
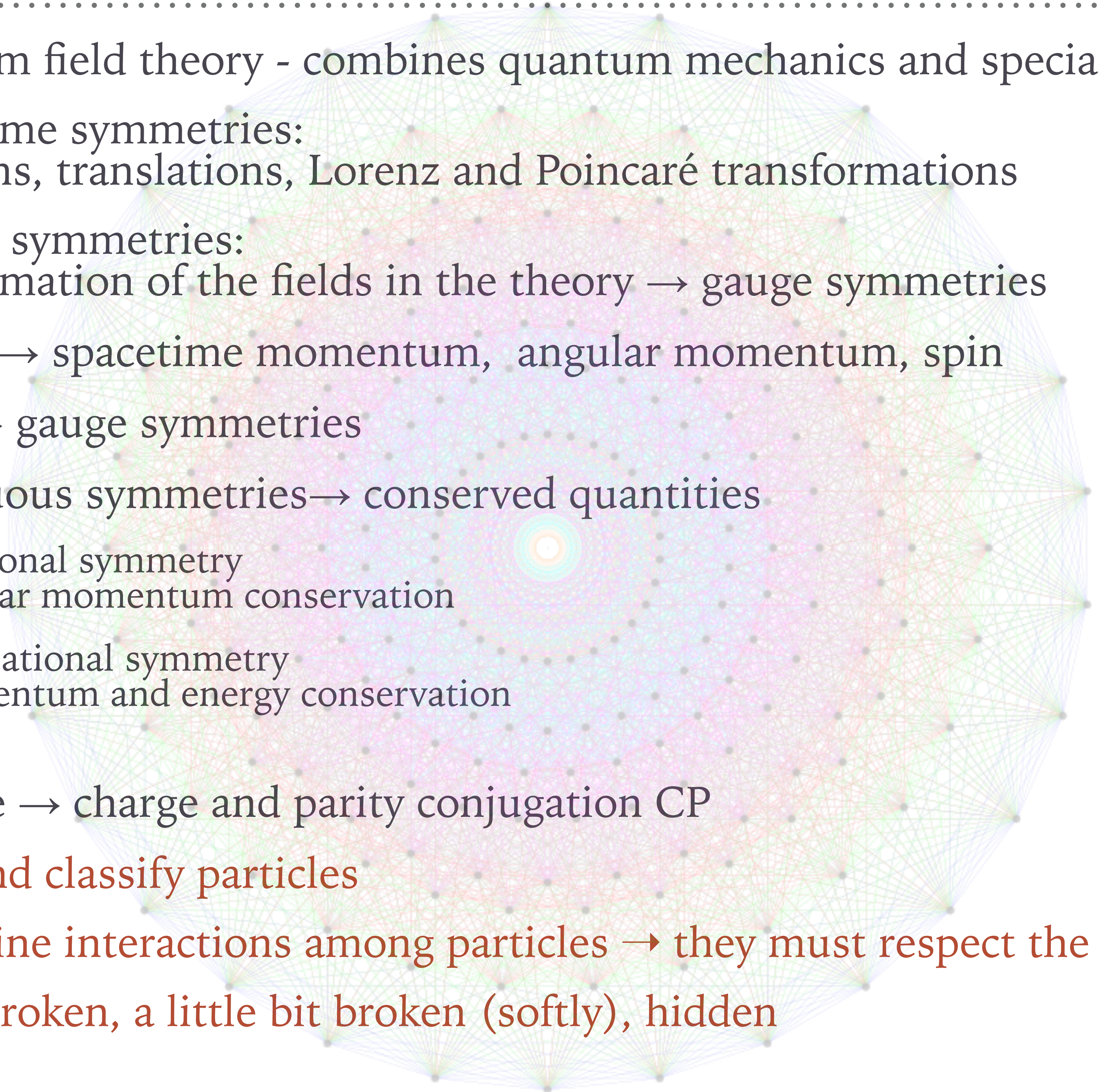


Fig: Wikipedia

SYMMETRIES



- Quantum field theory - combines quantum mechanics and special relativity
- Space-time symmetries:
  rotations, translations, Lorenz and Poincaré transformations
- Internal symmetries:
  transformation of the fields in the theory → gauge symmetries
- Global → spacetime momentum, angular momentum, spin
- Local → gauge symmetries
- Continuous symmetries→ conserved quantities
  - rotational symmetry
    angular momentum conservation
  - translational symmetry
    momentum and energy conservation
- Discrete → charge and parity conjugation CP
- Label and classify particles
- Determine interactions among particles → they must respect the symmetries
- Exact, broken, a little bit broken (softly), hidden

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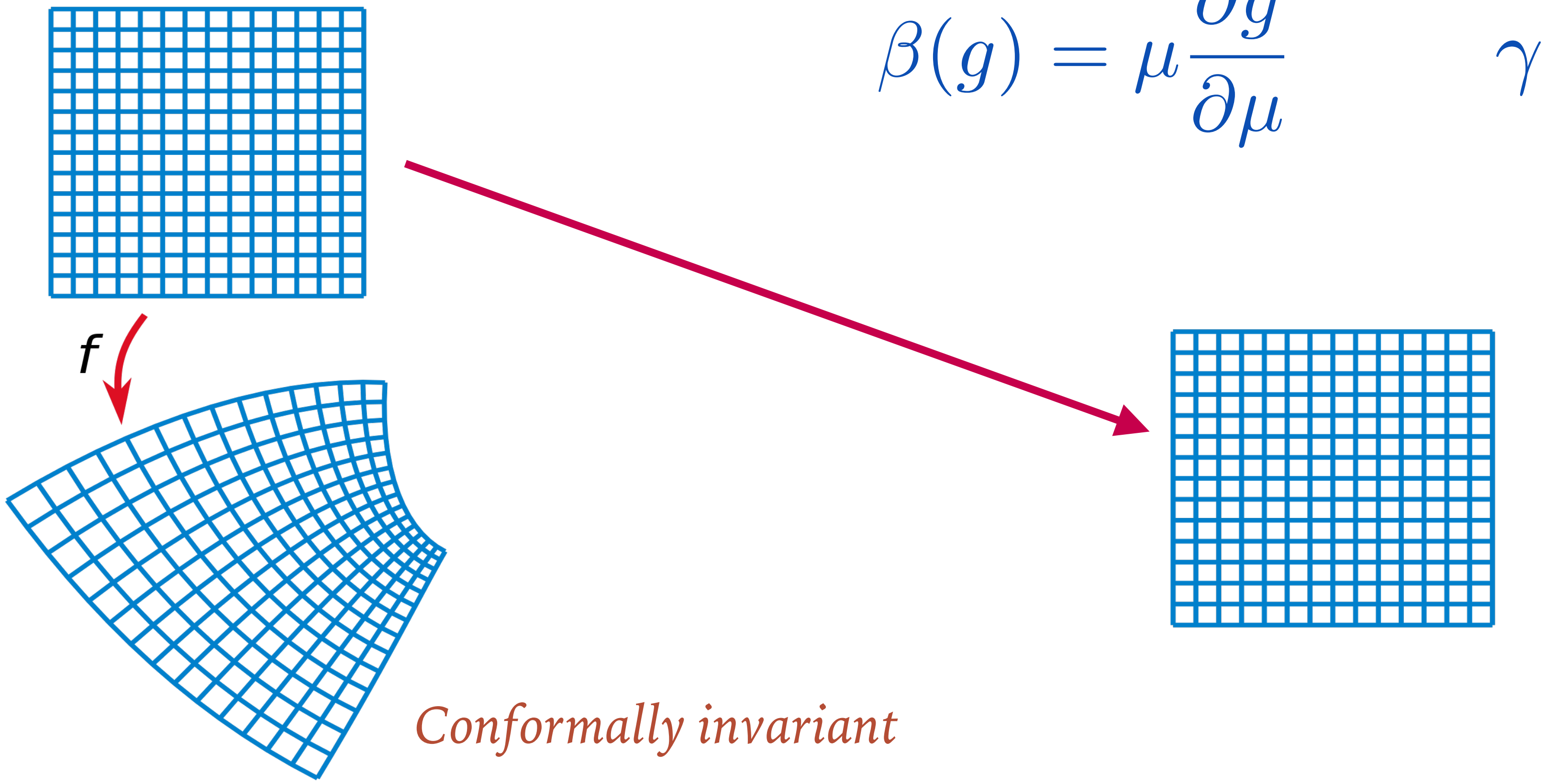
HOW DO WE MOVE UP (OR DOWN) IN ENERGY?

- We know how a QFT behaves at different scales through the renormalization group RG
- The theory has the same structure at different energy scales, but the parameters — couplings and masses — change with energy
- Related to scale invariance and conformal invariance

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} \qquad \gamma(\phi) = \mu \frac{\partial \ln Z}{\partial \mu}$$



Conformally invariant

Scale invariant

Scale invariance appears in many physical systems

HOW TO GO BEYOND THE STANDARD MODEL (BSM)?

- Traditional way ⇒ addition of symmetries

N=1 SUSY

- Very effective, but too many free parameters

Can get messy…

- Complementary approach
Look for renormalization group invariant relations at high energies

GUT ⇒ Planck

- Resulting theory has few free parameters ∴ very predictive

Relates gauge and Yukawa sector
Predictions for 3rd generation masses

RENORMALIZATION GROUP INVARIANTS RGI

- Search for more fundamental theory ⇒ less parameters

  Renormalization Group Invariants (RGI)

$$\Phi(g_1, \ldots, g_N) = 0$$

$$\mu\, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i = 0$$

- Equivalent to solve reduction equations

$$\beta_g\,(dg_i/dg) = \beta_i$$

$$i = 1, \ldots, N$$

- Reduced theory has only one coupling and its beta function
- Reduction → power series solution
- Uniqueness of solution can be studied at one-loop

Zimmermann (1985); Zimmermann, Oehme, Sibold (1984-1985)

REDUCTION OF COUPLINGS

- Couplings related to a primary coupling
  totally reduced — all couplings depend on one
  partially reduced — some couplings depend on one
- Can be applied to SUSY and non-SUSY models
- SM analyzed — results now ruled out, still impressive

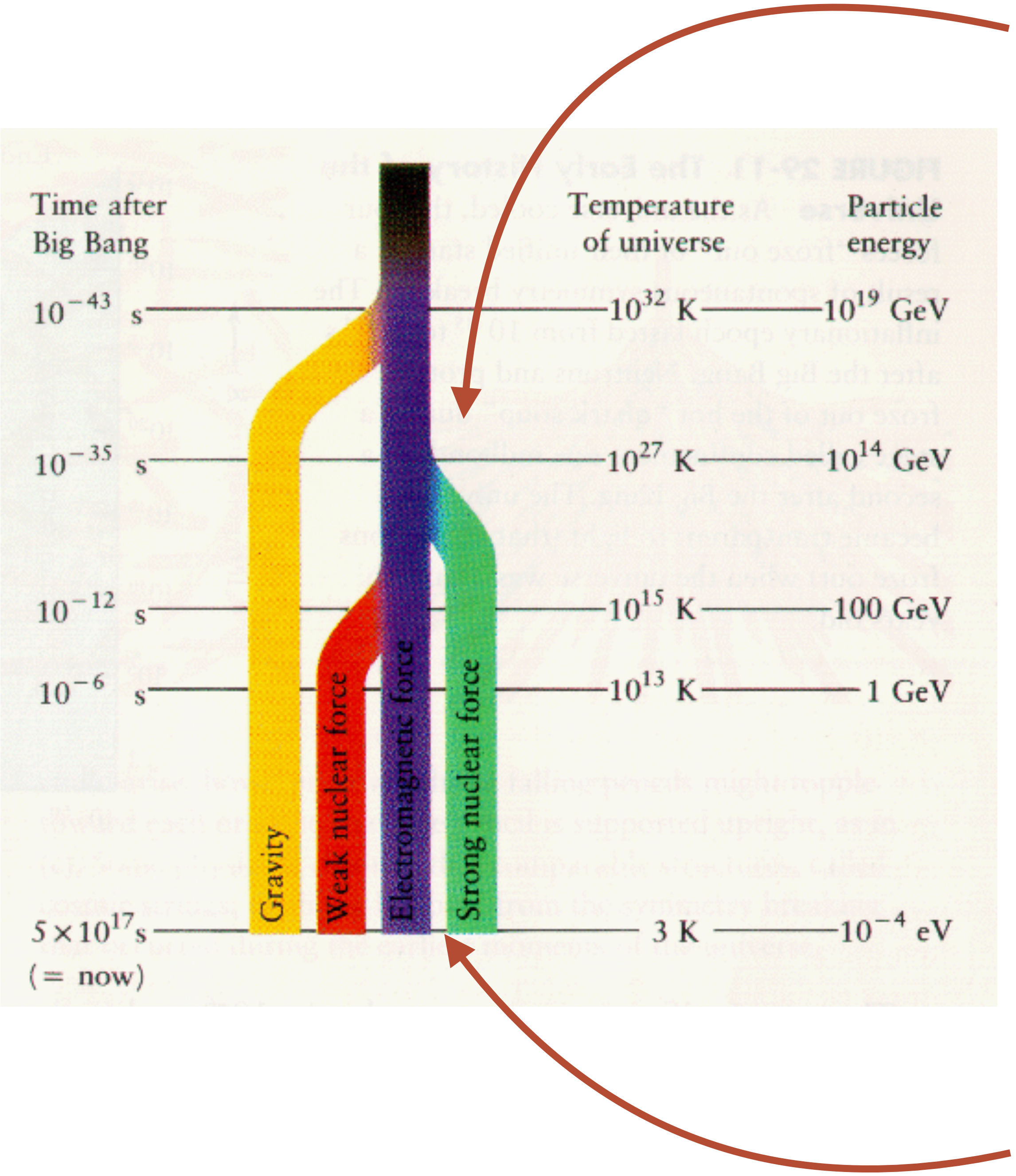
Kubo, Sibold, Zimmermann (1984-1987)

- 2HDM analyzed Denner (1990) — now re-analysed:
  possible to have one-loop reduced equations in type II 2HDM at a high-scale boundary

May Pech, MM, Patellis, Zoupanos (2023)

- Under some conditions SUSY unification models might be **finite**

FINITENESS = SCALE/CONFORMAL INVARIANCE



- All-loop finiteness ⇒ $\beta = 0$

  to all orders in perturbation theory

- Scale/conformal invariance

  Conformal and scale invariant = Yukawa couplings
  Scale invariant = Soft breaking terms
  Do not depend on energy scale
  Based on RGI and reduction of couplings

- Gives UV completion of the QFT

- Reduces greatly the number of free parameters

  ⇒ new symmetries

- Partial reduction ⇒ predictions for 3rd generation masses

FINITE SU(5) THEORIES — THIRD GENERATION

- Prediction for top mass — very clean

$M_{top}^{th} \sim 178$ GeV 1993 Kapetanakis, M.M., Zoupanos

m_bot also predicted, large tan beta

$M_{top}^{exp} = 176 \pm 18$ GeV 1995

$M_{top}^{th} \sim 172.5$ GeV 2007 Heinemeyer, M.M.,Zoupanos

$M_{top}^{exp} = 173.1 \pm .09$ GeV 2013

- Prediction for Higgs mass — depends on soft breaking terms, also very restricted

$M_{Higgs}^{th} \sim 121 - 126$ GeV 2008, 2013 Heinemeyer, M.M., Zoupanos

$M_{Higgs}^{exp} = 126 \pm 1$ GeV 2013

FINITESS $\Longrightarrow$ GAUGE YUKAWA UNIFICATION

Grand Unified SUSY N=1, no gauge anomalies:

$$W = \frac{1}{2} m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6} C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k$$

$$\beta_g^{(1)} = 0 = \gamma_i^{j\,(1)}$$

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)$$

T Dynkin index of irrep, C_2 Casimir invariant of group *C_{ijk} Yukawa couplings, g gauge coupling*

- Restricts the gauge group
- Relates gauge and Yukawa couplings
- If finite to all orders $\Rightarrow$ Conformal invariance
- May imply extra symmetries, in this case discrete

- Just analyze one-loop solution
- One-loop finite $\Rightarrow$ two-loop finite
- Isolated and non-degenerate solution $\Rightarrow$ all-loop finite Lucchesi, Piguet, Sibold

$\beta = 0$ non-renormalization of coupling constants, not complete UV finiteness where field renormalization is absent

SUSY BREAKING SSB

- Explicit/soft breaking >100 new free parameters 😫

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

- SSB can also be restricted through RGI ⇒ β = 0
- Leads to a sum rule among scalars and gauging masses

$$(\, m_i^2 + m_j^2 + m_k^2 \,)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4)$$

- Breaks conformal invariance BUT remains scale invariant!

- one- and two-loop finiteness conditions known
- all-loop finiteness possible

Kazakov, Jack, Jones, Pickering…

- Depends on the gaugino mass scale M
- Scale invariant but not conformal

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

SU(5) FINITE UNIFIED MODELS

The one- and two-loop finiteness conditions imply following matter content:

$$3\ \overline{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \overline{\mathbf{5}}) + \overline{\mathbf{24}}$$

3 generations, 4 pairs of Higgs doublets one field in the adjoint

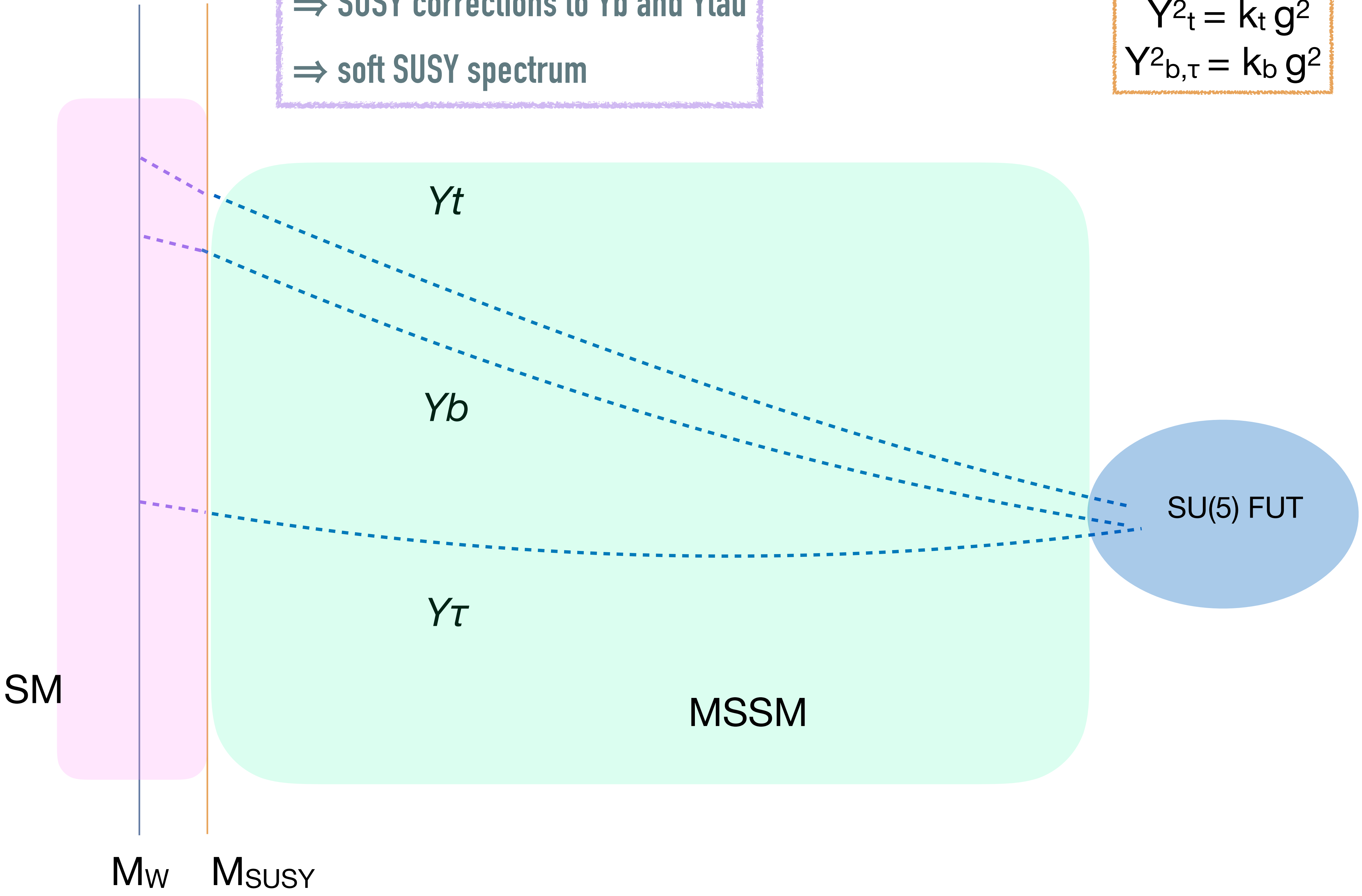
- Soft scalar masses obey sum rule
- No proton decay
- At GUT scale finiteness is broken ⇒ MSSM finiteness broken
- Rotation of FUT Higgs sector ⇒ 2 Higgs doublets of MSSM maximally coupled to third generations

Finite soft breaking terms included

⇒ SUSY corrections to Yb and Ytau

⇒ soft SUSY spectrum

$$Y^2_t = k_t\, g^2$$
$$Y^2_{b,\tau} = k_b\, g^2$$



Results confronted to experimental constraints ⇒

gives available parameter space

$m_t = Y_t\, v_u$ $\qquad v_u / v_d = \tan\beta$

$m_{b,\tau} = Y_{b,\tau}\, v_d$ $\qquad v_d = m_\tau^{exp} / Y_\tau$

INTERPLAY HIGH-LOW ENERGIES: SEARCHES AT FUTURE COLLIDERS

GUT scale, Finiteness gives:

- Relations between gauge-Yukawa couplings
- Sum rule for soft breaking terms
- ⇒ Very few free parameters

Require:

- Absence of proton decay
- Proper unification of gauge couplings
- MSSM

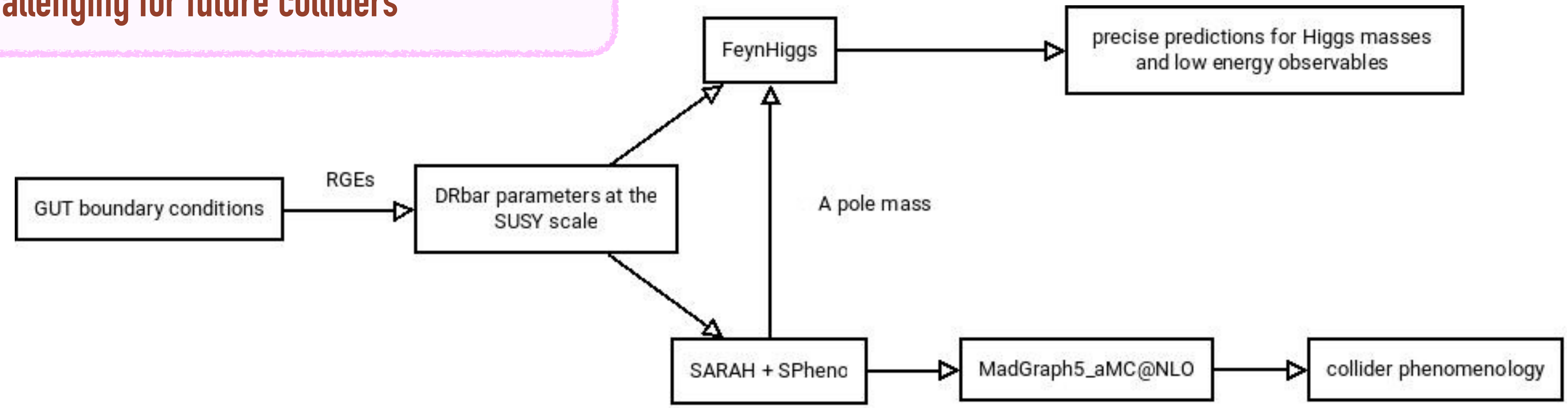
Low energies:

- Radiative eW symmetry breaking
- Include SUSY radiative corrections
- Quark and Higgs masses in experimental range
- Compliance with B physics (not trivial)

Large tan β
High SUSY spectrum > 1 TeV
Challenging for future colliders

B constraints:

- BR ($b \to s\gamma$)
- BR ($B_s \to \mu^+\mu^-$)
- BR ($B_u \to \tau\upsilon$) B_s
- $\Delta M_{B_s\,SM/MSSM}$



GUT boundary conditions → RGEs → DRbar parameters at the SUSY scale

DRbar parameters at the SUSY scale → FeynHiggs

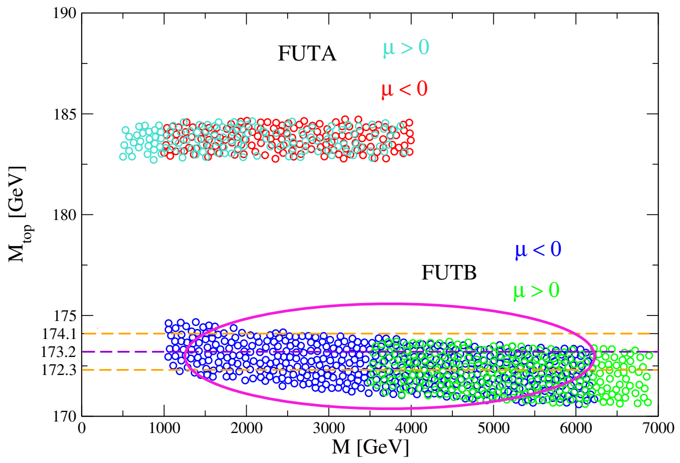
DRbar parameters at the SUSY scale → SARAH + SPheno

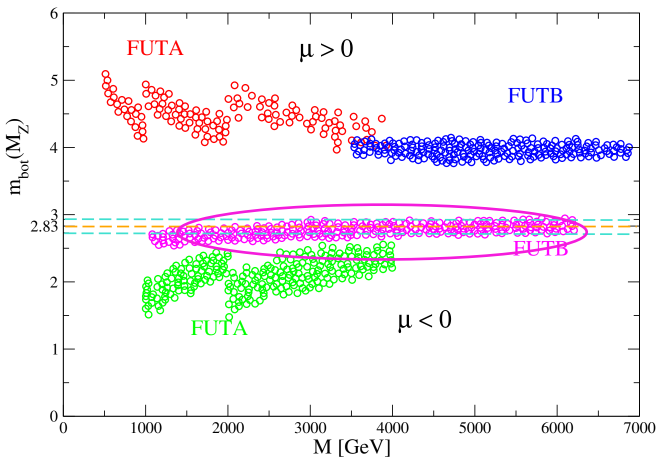
SARAH + SPheno → A pole mass → FeynHiggs

FeynHiggs → precise predictions for Higgs masses and low energy observables

SARAH + SPheno → MadGraph5_aMC@NLO → collider phenomenology

Heinemeyer, Kalinowski, Kotlarski , Mondragon, Patellis, Tracas, Zoupanos (2021)





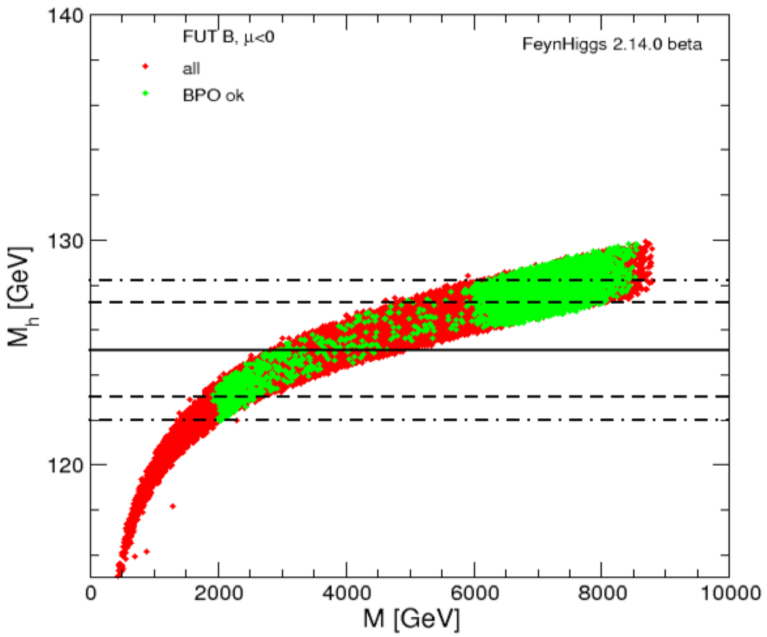
FUTB — 3rd generation

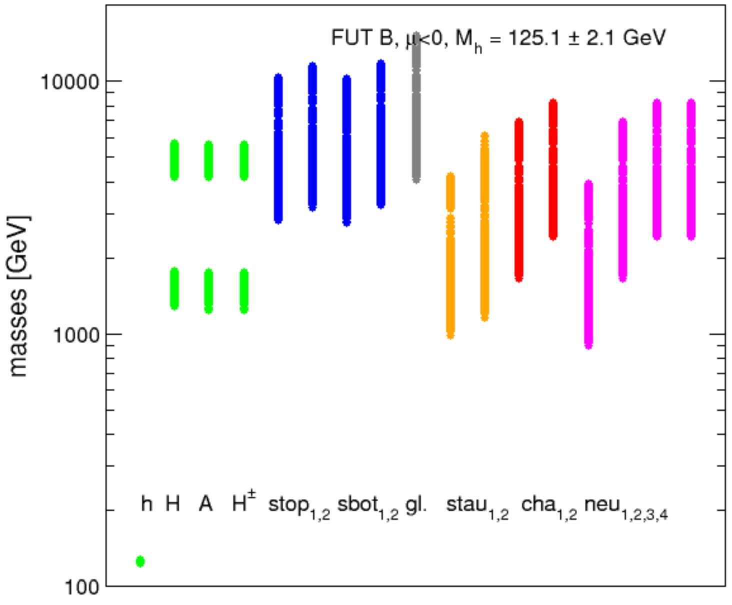
1 free parameter in gauge-Yukawa sector

2 free parameters in soft SUSY breaking

Higgs mass range determined by finiteness, sum rule, B physics constraints and radiative top contributions to Higgs mass ⇒ heavy spectrum

These are now related!





MANY ASPECTS OF FINITENESS STUDIED

- SU(5) models extensively studied Rabi et al; Kazakov et al; Quirós et al; MM, Zoupanos et a
- One coincides with a non-standard Calabi-Yau MM, Zoupanos
- Finite string theories and criteria for branes Ibáñez
- Models with three generations Babu, Enkhbat, Gogoladze; MM & Jiménez; Estrada, MM, Patellis, Zoupanos
- $SU(N)^k$ models finite $\Longleftrightarrow$ 3 generations
  only $SU(3)^3$ compatible with phenomenology MM, Ma, Zoupanos
- Relations non-commutative theories and finiteness Jack, Jones
- Proof of conformal invariance (dimensionless part) Kazakov, Bork; MM & Reyes
- Relation between finiteness and QFT in curved space-time & inflation

  Elizalde, Odintsov, et al
- Recent reviews Heinemeyer, M.M, Tracas, Zoupanos, Phys.Rept. 814 (2019); Fortsch.Phys. 68 (2020)

RECENT DEVELOPMENTS

- Updated phenomenological analysis still consistent with 3rd generation masses, large tanβ and **very heavy SUSY spectrum**
  S. Heinemeyer, J. Malinowski, W. Kotlarski, M. Mondragón, N. Tracas, G. Zoupanos 2018 and 2022
- Finiteness implies conformal invariance and phase transition
  L.E. Reyes Rodríguez, Lic. Thesis (2018)
- Three generation analysis SU(5):
  Diagonal quark mass matrix compatible with data and proton decay
  Luis Odín Estrada, M.Sc. Thesis (2018)
- Three generation solution for SU(5) with Z symmetries compatible with good textures at high energies
  Luis Odín Estrada,Ph.D. Thesis (2025)
- Finiteness in Soft breaking terms lead to anomaly mediated type breaking L.E. Reyes Rodríguez, M.Sc.Thesis (2021)
- $SU(3)^3$ finite split susy model in progress L.E. Reyes Rodríguez, Ph. D. Thesis

SUPERPOTENTIAL

- The SU(5) superpotential of possible finite models is

$$\bar{\mathscr{H}}_{ai} = \bar{\mathbf{5}}\,, \quad \mathscr{H}^{i}_{\ a} = \mathbf{5}\,, \quad \bar{\Psi}_{a'i} = \bar{\mathbf{5}}\,, \quad X^{ij}_{a'} = \mathbf{10}\,, \quad \Sigma^{i}_{\ j} = \mathbf{24}$$

3 generations, 4 pairs of Higgs doublets and one field in the adjoint

$$3\ \bar{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \bar{\mathbf{5}}) + \overline{\mathbf{24}}$$

$$\mathcal{W}_{SU(5)-R} = \bar{g}_{a'b'a} \bar{\Psi}_{b'i} X^{ij}_{a'} \bar{\mathcal{H}}_{aj} + \frac{1}{2} g_{a'b'a} \epsilon_{ijklm} X^{ij}_{a'} X^{kl}_{b'} \mathcal{H}^{m}_{a} + f_{ab} \bar{\mathcal{H}}_{ai} \Sigma^{i}{}_{j} \mathcal{H}^{j}_{b} + \frac{1}{3!} p \Sigma^{i}{}_{j}\ \Sigma^{j}{}_{k}\ \Sigma^{k}{}_{i} + \frac{1}{2} \lambda^{(\Sigma)} \Sigma^{i}{}_{j} \Sigma^{j}{}_{i} + m_{ab} \bar{\mathcal{H}}_{ai} \mathcal{H}^{i}_{b}\ .$$

$\bar{g}_{ijk}$ = *down Yukawa couplings*, g_{ijk} = *up Yukawa couplings*

WHAT ABOUT FLAVOR? 3 GENERATIONS

Classification of SU(5) FUT with off-diagonal γ done already

Coupled to 3 Higgs doublets

$$V_3^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{123}\langle\mathcal{H}_3^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{213}\langle\mathcal{H}_3^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_3^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(4)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & 0 & 0 \\ 0 & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Coupled to 4 Higgs doublets

$$V_4^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{124}\langle\mathcal{H}_4^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{214}\langle\mathcal{H}_4^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ 0 & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_4^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(4)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Top and down mass matrices with same structure

Babu, Enkhbat, Gogoladze (2003)

2-LOOP FINITE MODEL — $V_4{}^1$

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_8	4	3	5	0	7	1	0	2	6	1	4	6	2	5	0

We find the following symmetries ⇒

parametric relations among couplings ⇒ 2-loop solution

up-type Yukawa

$$|g_{124}|^2 = |g_{214}|^2 = \frac{4}{5}g_5^2 \quad , \quad |g_{222}|^2 = \frac{2}{5}g_5^2 \quad , \quad |g_{231}|^2 = |g_{321}|^2 = \frac{1}{10}\left(8g_5^2 - 5\,|g_{111}|^2\right) \quad ,$$

$$|g_{333}|^2 = \frac{6}{5}g_5^2 \quad , \quad |\bar{g}_{111}|^2 = |\bar{g}_{124}|^2 = \frac{3}{20}\left(8g_5^2 - 5\,|g_{111}|^2\right) \quad ,$$

down-type Yukawa

$$|\bar{g}_{214}|^2 = \frac{3}{4}\,|g_{111}|^2 \quad , \quad |\bar{g}_{222}|^2 = |\bar{g}_{231}|^2 = \frac{3}{10}g_5^2 \quad , \quad |\bar{g}_{321}|^2 = -\frac{3}{20}\left(2g_5^2 - 5\,|g_{111}|^2\right) \quad ,$$

$$|\bar{g}_{333}|^2 = \frac{9}{10}g_5^2 \quad , \quad |f_{22}|^2 = \frac{3}{4}g_5^2 \quad , \quad |f_{33}|^2 = \frac{g_5^2}{4} \quad , \quad |p|^2 = \frac{15}{7}g_5^2 \quad ,$$

$$|g_{132}|^2 = |g_{312}|^2 = |\bar{g}_{132}|^2 = |\bar{g}_{312}|^2 = |f_{11}|^2 = |f_{44}|^2 = 0 \quad .$$

By imposing the positivity conditon to the squared norm of the couplings, we find the following constraint for $|g_{111}|^2$:

$$\frac{2}{5}g_5^2 \leq |g_{111}|^2 \leq \frac{8}{5}g_5^2 \; .$$

evaluating at the end points
implies more symmetry = more zeroes

ALL-LOOP FINITE MODEL — V_4^2

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_3	0	2	0	0	2	0	1	1	0	0	1	1	0	0	0
Z_4	3	3	2	3	3	2	2	3	0	2	2	3	0	2	0

- We find the following symmetries ⇒ isolated solution

  unique relation among couplings ⇒ all-loop finite solution

$$|g_{114}|^2 = |g_{121}|^2 = |g_{211}|^2 = |g_{232}|^2 = |g_{322}|^2 = |g_{333}|^2 = \frac{4}{5}g_5^2$$

$$|\bar{g}_{114}|^2 = |\bar{g}_{121}|^2 = |\bar{g}_{211}|^2 = |\bar{g}_{232}|^2 = |\bar{g}_{322}|^2 = |\bar{g}_{333}|^2 = \frac{3}{5}g_5^2 \quad ,$$

$$|f_{33}|^2 = |f_{44}|^2 = \frac{1}{2}g_5^2 \quad , \quad |p|^2 = \frac{15}{7}g_5^2 \quad .$$

- For the SSB ⇒ sum rule ⇒ 3 free parameters

$$m^2_{\tilde{\psi}_1} = m^2_{\tilde{\psi}_3} = \frac{1}{6}\left(-MM^\dagger + 9m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\psi}_2} = \frac{1}{6}\left(-MM^\dagger - 6m^2_{H_1} + 15m^2_{H_3}\right) ,$$

$$m^2_{\tilde{\chi}_1} = m^2_{\tilde{\chi}_3} = \frac{1}{2}\left(MM^\dagger - m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\chi}_2} = \frac{1}{2}\left(MM^\dagger - 2m^2_{H_1} + m^2_{H_3}\right) ,$$

$$m^2_{\bar{H}_1} = m^2_{\bar{H}_2} = \frac{1}{3}\left(2MM^\dagger + 3m^2_{H_1} - 6m^2_{H_3}\right) \quad , \quad m^2_{\bar{H}_3} = m^2_{\bar{H}_4} = \frac{1}{3}\left(2MM^\dagger - 3m^2_{H_3}\right) ,$$

$$m^2_{H_2} = m^2_{H_1} \quad ; \quad m^2_{H_4} = m^2_{H_3} \quad , \quad m^2_{\phi_\Sigma} = \frac{1}{3}MM^\dagger \quad . \tag{89}$$

ALL-LOOP FINITE MASS MATRICES

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

- It is possible to find the minimum amount of phases — rephasing invariants
- The mass matrices are then:

$$M_u = \begin{pmatrix} g_{114}\langle\mathcal{H}_4^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & 0 & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \langle\mathcal{H}_4^5\rangle & \langle\mathcal{H}_1^5\rangle & 0 \\ \langle\mathcal{H}_1^5\rangle & 0 & \langle\mathcal{H}_2^5\rangle \\ 0 & \langle\mathcal{H}_2^5\rangle & e^{i\phi_3}\langle\mathcal{H}_3^5\rangle \end{pmatrix},$$

$$M_d = \begin{pmatrix} \bar{g}_{114}\langle\bar{\mathcal{H}}_{45}\rangle & \bar{g}_{121}\langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ \bar{g}_{211}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \bar{g}_{232}\langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & \bar{g}_{322}\langle\bar{\mathcal{H}}_{25}\rangle & \bar{g}_{333}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix} = \sqrt{\tfrac{3}{5}}g_5 \begin{pmatrix} \langle\bar{\mathcal{H}}_{45}\rangle & \langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ e^{i\bar{\phi}_1}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & e^{i\bar{\phi}_2}\langle\bar{\mathcal{H}}_{25}\rangle & e^{i\bar{\phi}_3}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix}.$$

- After the rotation in the Higgs sector to the MSSM basis:

$$M_u = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \widetilde{\alpha}_4 & \widetilde{\alpha}_1 & 0 \\ \widetilde{\alpha}_1 & 0 & \widetilde{\alpha}_2 \\ 0 & \widetilde{\alpha}_2 & e^{i\phi_3}\widetilde{\alpha}_3 \end{pmatrix} \langle\mathcal{K}_3^5\rangle,$$

$$M_d = \sqrt{\frac{3}{5}}g_5 \begin{pmatrix} \widetilde{\beta}_4 & \widetilde{\beta}_1 & 0 \\ e^{i\bar{\phi}_1}\widetilde{\beta}_1 & 0 & \widetilde{\beta}_2 \\ 0 & e^{i\bar{\phi}_2}\widetilde{\beta}_2 & e^{i\bar{\phi}_3}\widetilde{\beta}_3 \end{pmatrix} \langle\bar{\mathcal{K}}_{35}\rangle.$$

Same solution as FUTB
for 3rd generation!
we know it works…

α_i, β_i refer to the rotation angles in up and down sectors respectively, *$\Sigma\beta_i = \Sigma\alpha_i = 1$*

FINALLY, HOW MANY FREE PARAMETERS?

GUT scale 89 free parameters

Yukawa couplings, soft breaking terms, phases, vev's of the Higgs fields

After Finiteness solutions

33 free parameters

Require doublet-triplet splitting, rotation to MSSM basis with constraints over angles, rephasing invariants

Low energies:

radiative electroweak breaking, fix m_τ^{exp} and SM vev give $\tan\beta$

⇒ 12 parameters left:

The soft breaking terms, the phases, and the rotation angles

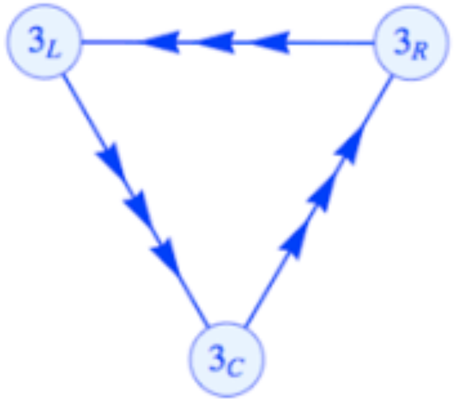
$$\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \beta_1,\ \beta_2,\ \beta_3,\ \mathrm{M},\ \mu$$

Only one phase is observable

$$\Rightarrow \phi_{obs},\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \beta_1,\ \beta_2,\ \beta_3,\ \mathrm{M},\ \mu$$

only 9 parameters left to fit masses and mixing angles

SU(3)³ — TRINIFICATION MODEL



- Trinification model beta function

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,.$$

- Finite $\Longleftrightarrow$ 3 generations

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3),$$

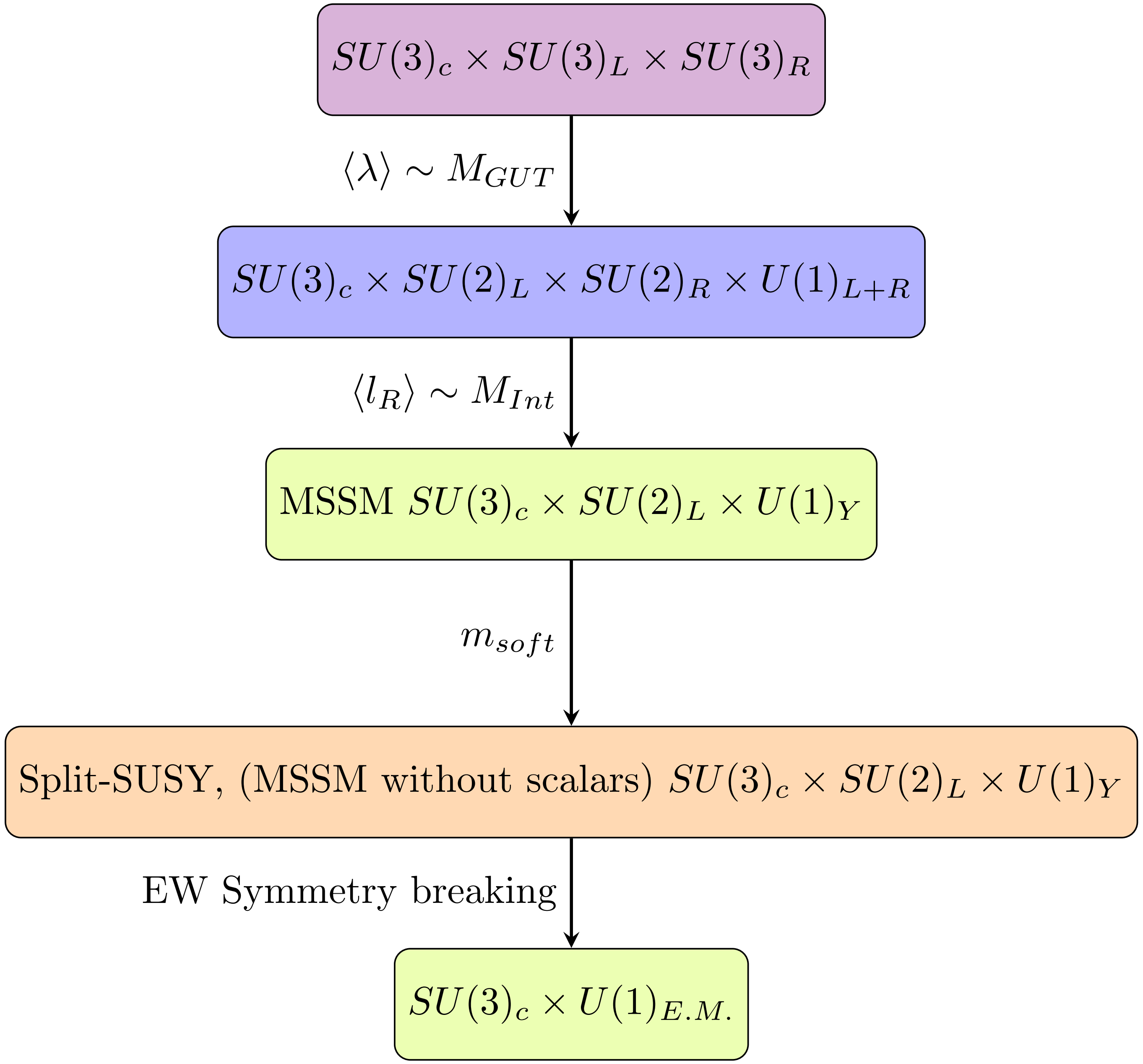
$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*).$$

- Among the “exotics”: right-handed neutrinos, down-like quarks

2-LOOP SU(3)^3 — OUT OF SEVERAL POSSIBILITIES

- Finite to two-loops, parametric solution depends on r
- Previously studied:
    - Possible to find values of r which simultaneously fit top and bottom quark masses
    - Heavy SUSY spectrum > 6.4 TeV
      Testable at the FCC
    - Possible to get DM candidate
- Now: Implement it in a split-SUSY scenario
    - Decouple SUSY scalars at a high energy scale m_s, except for the Standard Model Higgs, while keeping the fermions light through the existence of global symmetries

SPLIT SUSY IN SU(3)3



$SU(3)_c \times SU(3)_L \times SU(3)_R$

$\langle \lambda \rangle \sim M_{GUT}$

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{L+R}$

$\langle l_R \rangle \sim M_{Int}$

MSSM $SU(3)_c \times SU(2)_L \times U(1)_Y$

m_{soft}

Split-SUSY, (MSSM without scalars) $SU(3)_c \times SU(2)_L \times U(1)_Y$

EW Symmetry breaking

$SU(3)_c \times U(1)_{E.M.}$

PRELIMINARY ANALYSIS

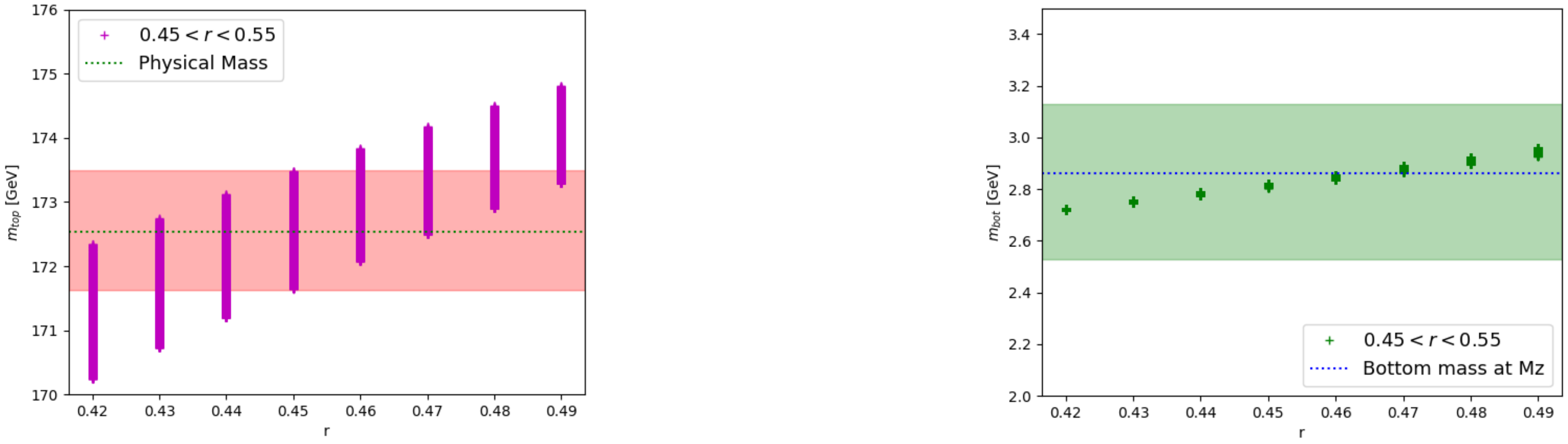
- We start by assuming a first breaking of the LR model close to the GUT scale
- Very small splitting between fermions and scalars using the sum rule
- Requiring that top and bottom lie within experimental bounds gives a lower bound on M and constrains

  $$0.45 < r < 0.55$$

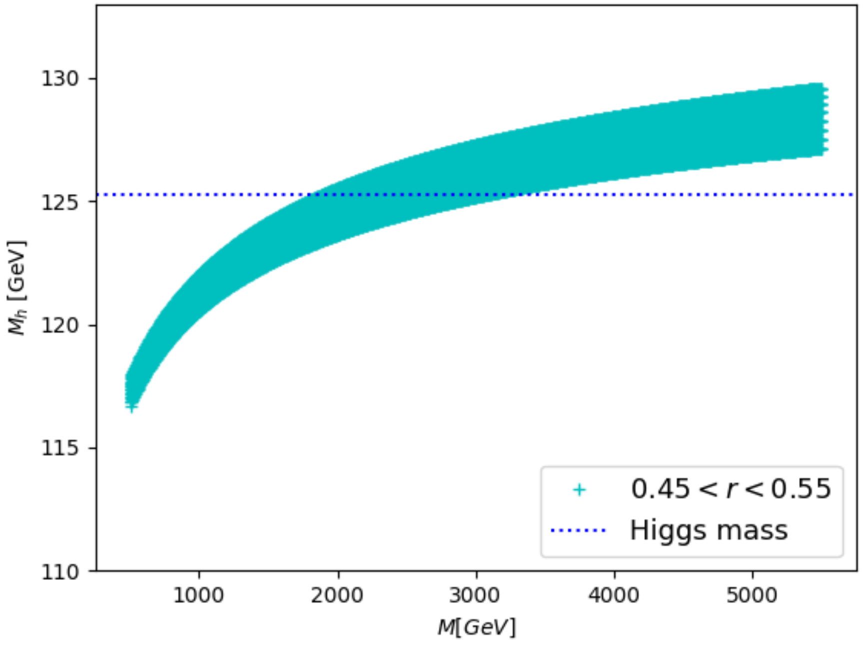
- $\Rightarrow$ Higgs mass estimation, only partial corrections, within experimental bounds for $2 \text{ TeV} < M < 4 \text{ TeV}$.

- No B physics constraints yet, so results bound to change
- Heavy spectrum
- Ph.D. Thesis Luis Enrique Reyes

PRELIMINARY RESULTS

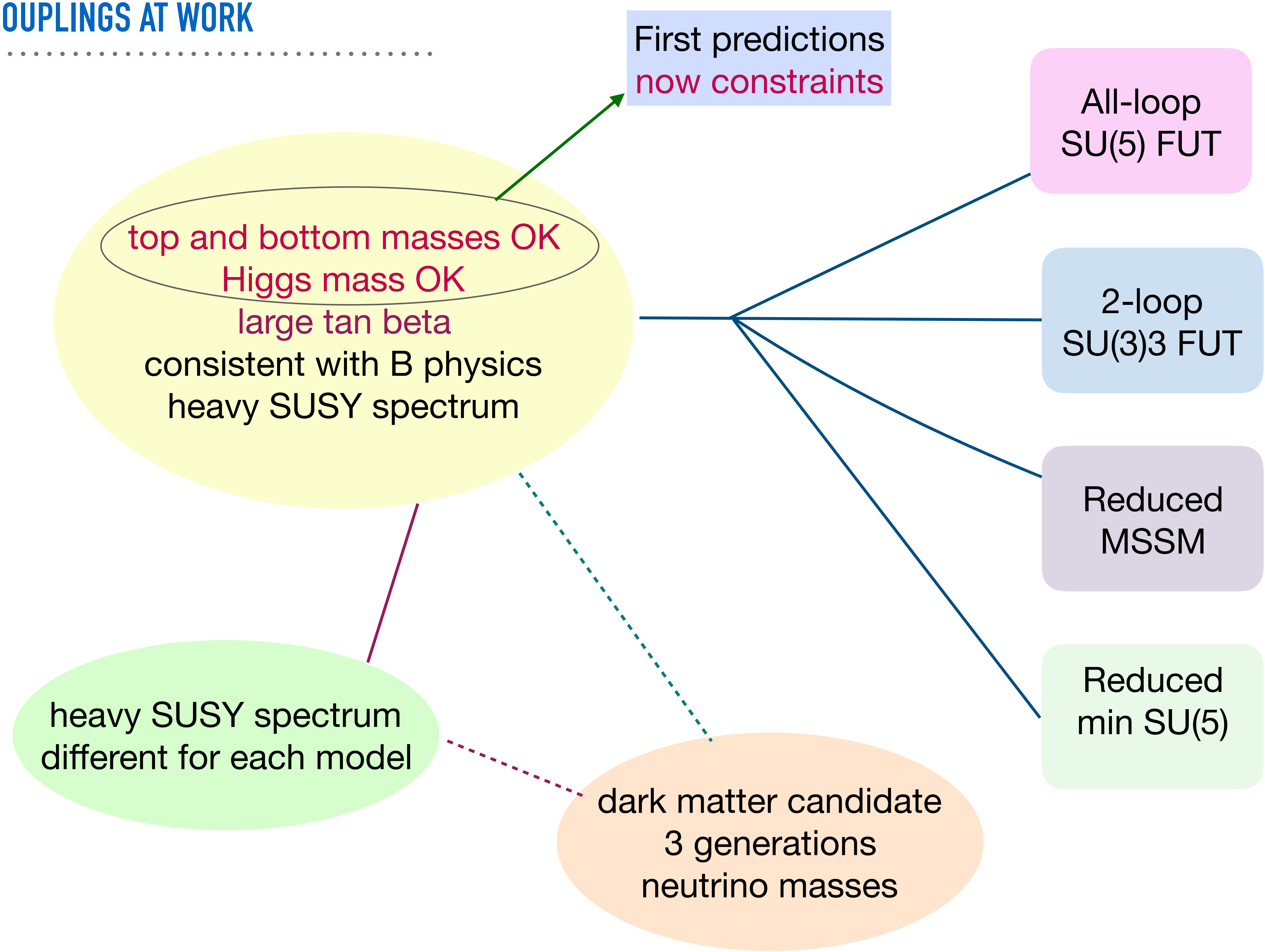


top and bottom quark masses sensitive to r and M



Higgs mass mainly sensitive to M and indirectly to m_{top}

GYU FROM REDUCTION OF COUPLINGS AT WORK



First predictions
now constraints

top and bottom masses OK
Higgs mass OK
large tan beta
consistent with B physics
heavy SUSY spectrum

All-loop
SU(5) FUT

2-loop
SU(3)3 FUT

Reduced
MSSM

Reduced
min SU(5)

heavy SUSY spectrum
different for each model

dark matter candidate
3 generations
neutrino masses

WHAT ABOUT NEUTRINO MASSES, DARK MATTER, ETC?

- **SU(5) models:**
  Cold DM
  LSP is neutralino
  ⇒ overabundance
- Neutrino masses may be incorporated by breaking R symmetry ⇒

  gravitino Dark Matter
- Other mechanisms?

  thermal inflation?
- g-2 like in SM

- **SU(3)^3 models:**
  ν_R are present
- Neutrino masses may be generated by seesaw or radiatively
- Depending on the breaking of SU(3)^3
  DM may be neutralino
  (or scalar?)
- Neutralino DM overabundance

Flavor Structure may change the above!

OTHER SYMMETRIES? — MODULAR SYMMETRIES

- Related to moduli spaces, geometric spaces: solutions of geometric classification problems. Objects are identified (isomorphic) if they are the same geometrically.

- Using modular symmetries as flavor symmetries:
  Inspiration from supersymmetric theories, initially with extra dimensions Feruglio, Altarelli (2006-2022); Petcov et al (2019, 2021, 2022)
  Magnetized branes, superstring theories
  Cremades et al (2004); Kobayashi et al (2018)
  Superstring compactifications, especially from orbifold compactifications
  e.g. Kobayashi et al (2018, 2019); Chen, Ramos-Sánchez, Ratz (2022)

- Usually applied in supersymmetric models, but now also in non-supersymmetric models e.g. Nomura, Okada et al, (2019,2020)

MODULAR GROUPS AS FLAVOR GROUPS

- Isomorphism between some finite modular groups and some groups associated to polygons (invariance under rotations and reflections)

$$\begin{array}{ccc} \Gamma_2 & \simeq & S_3 \\ \Gamma_3 & \simeq & A_4 \\ \Gamma_4 & \simeq & S_4 \\ \Gamma_5 & \simeq & A_5 \end{array}$$

- Yukawa couplings expressed in terms of modular forms, i.e. functions of a complex scalar field

$$Y(\alpha, \beta, \gamma|\tau) = \frac{d}{d\tau}\left(\alpha \log \eta\left(\frac{\tau}{2}\right) + \beta \log \eta\left(\frac{\tau+1}{2}\right) + \gamma \log \eta\,(2\tau)\right)$$

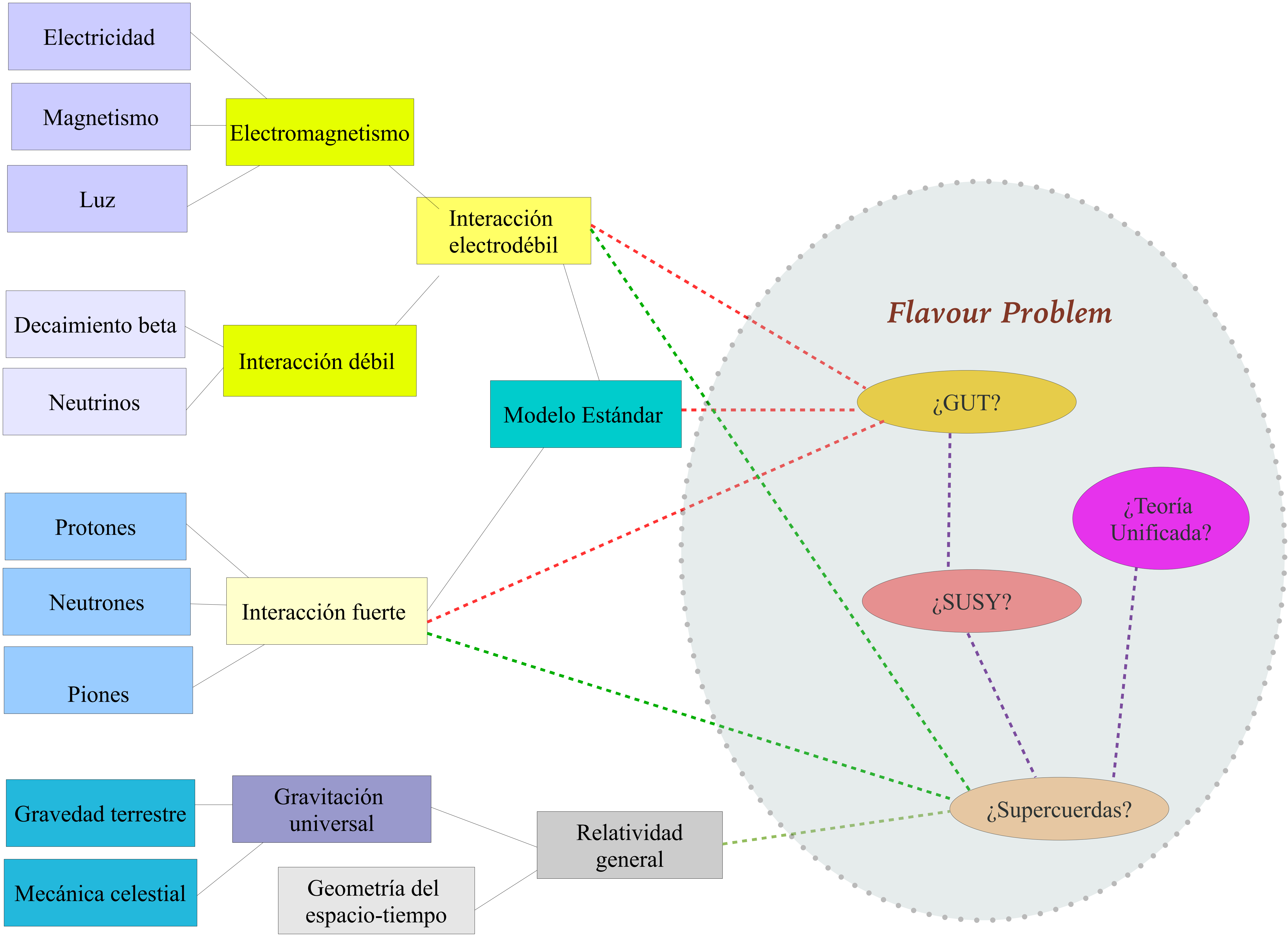
with τ acquiring a vev (spurion field, measures breaking of modular symmetry) on the upper half of complex plane

- Fermions and scalar fields transform with a weight

$$\phi \to (c\tau + d)^{k_\phi}\phi,$$

SU(5) Modular S3
See poster by Antonio Samaniego
Ph.D. Thesis

AND AT LOWER ENERGIES….?


Electricidad
Magnetismo
Luz
Electromagnetismo
Interacción electrodébil
Decaimiento beta
Neutrinos
Interacción débil
Modelo Estándar
Protones
Neutrones
Piones
Interacción fuerte
Gravedad terrestre
Mecánica celestial
Gravitación universal
Geometría del espacio-tiempo
Relatividad general
Flavour Problem
¿GUT?
¿Teoría Unificada?
¿SUSY?
¿Supercuerdas?

SOME ASPECTS OF THE FLAVOR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

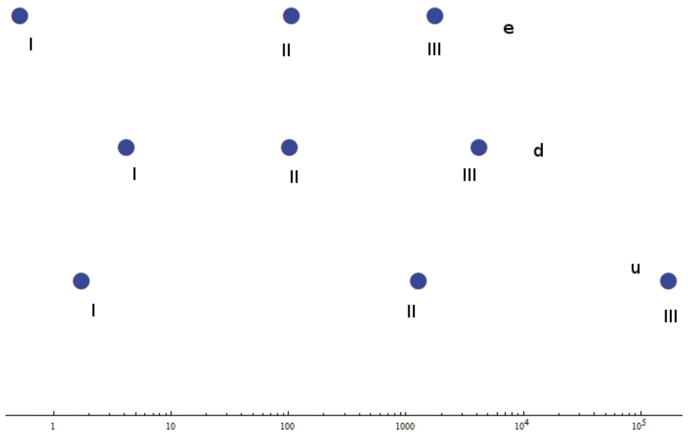
$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

- Neutrino mixing angles

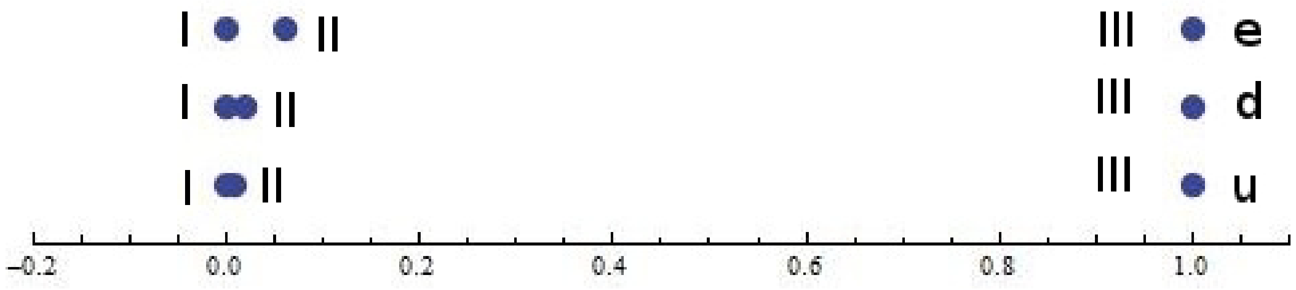
$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

?



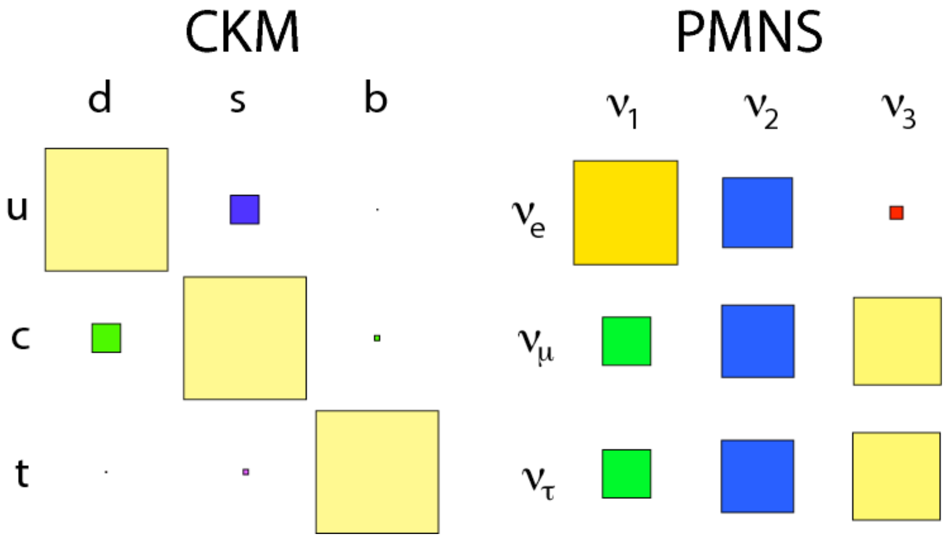
Plot of mass ratios



Logarithmic plot of quark masses

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \left[\begin{array}{cc|c} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ \hline 0.009 & 0.040 & 0.999 \end{array}\right],$$

Suggests a 2⊕1 structure for quarks… for leptons?

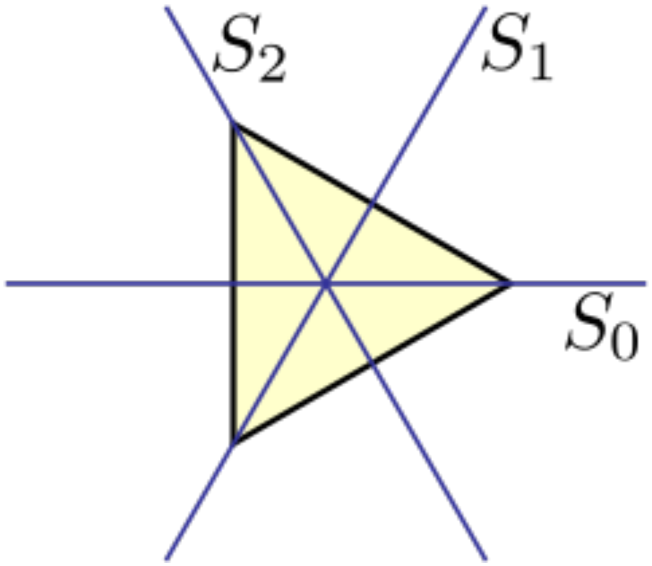


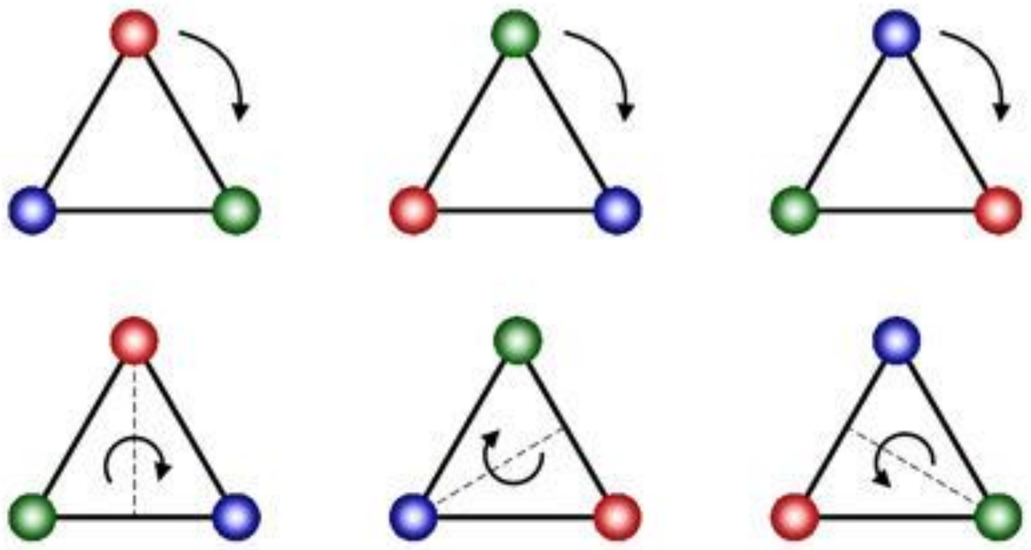
HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs, L-R models, etc
- Look at low energy phenomenology
- At some point they should intersect…
- In here, look at low energy phenomenology:
  - Try a flavor symmetry with 2+1 structure
  - Explore how generally it can be applied
  - Lots of scalars…quarks ok, what about neutrinos?
  - Compare it with the data — prospects for dark matter
- See how predictive it turns out

WE WILL EXPLORE S3 AND Q6

- S3 is the smallest non-abelian group
- Has irreducible representations **$2, 1_S, 1_A$**
- Permutations of three objects or rotations and reflections that leave invariant an equilateral triangle





- Q6 is double covering of S3, has irreps

$2_1, 2_2, 1_{++}, 1_{--}, 1_{+-}, 1_{-+}$

A sample of S3 models

S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)

E. Derman, Phys. Rev. D19, 317 (1979)

D. Wyler, Phys. Rev. D19, 330 (1979)

R. Yahalom, Phys. Rev. D29, 536 (1984)

Y. Koide, Phys. Rev. D60, 077301 (1999)

A. Mondragon et al, Phys. Rev. D59, 093009, (1999)

J. Kubo, A. Mondragon, et al, Prog. Theor. Phys. 109, 795 (2003)

J. Kubo et al, Phys. Rev. D70, 036007 (2004)

S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)

A. Mondragon et al, Phys. Rev. D76, 076003, (2007)

S. Kaneko et al, hep-ph/0703250, (2007)

S. Chen et al, Phys. Rev. D70, 073008 (2004)
T. Teshima et al, Phys.Rev. D84 (2011) 016003 Phys.Rev. D85 105013 (2012)

F. Gonzalez Canales, A&M. Mondragon Fort. der Physik 61, Issue 4-5 (2013)

H.B. Benaoum, Phys. RevD.87.073010 (2013)

E. Ma and B. Melic, arXiv:1303.6928

F. Gonzalez Canales, A. &M Mondragon, U. Saldaña, L. Velasco, arXiv:1304.6644

R. Jora et al, Int.J.Mod.Phys. A28 (2013),1350028

A. E. Cárcamo Hernández, E. Cataño Mur, R. Martinez, Phys.Rev. D90 (2014) no.7, 073001

A.E. Cárcamo, I. de Medeiros E. Schumacheet, Phys.Rev. D93 (2016) no.1, 016003

A.E. Cárcamo, R. Martinez, F. Ochoa, Eur.Phys.J. C76 (2016)

D Das, P Pal, Pays Rev D98 (2018)

C Espinoza, E Garcés, MM, H. Reyes, PLB 788 (2019)

JC Gómez Izquierdo, MM, EPJC 79 (2019)

O. Felix-Beltran, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)

A. Dicus, S Ge, W Repko, Phys. Rev D82 (2010)

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)

D. Meloni, JHEP 1205 (2012) 124

S. Dev et al, Phys.Lett. B708 (2012) 284-289

S. Zhou, Phys.Lett. B704 (2011) 291-295

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

E. Ma and B. Melic, Phys.Lett. B725 (2013)

E. Barradas et al, 2014

P. Das et al, PhyrRev D89 (2014,) 2016

ZZ Zhing, D Zhang JHEP 03 2019)

S Pramanick, Phys Rev D100 (2019)

Emmanuel-Costa et al, JHEP (2016)

Kuncinas et al, PRD (2020)

M. Gómez-Bock, A. Pérez, MM, EPJC81 (2021)

T. Kobayashi, K. Tanaka, T.H. Tatsuishi, PRD98 (2018)

D. Meloni, M. Parriciatu, JHEP (2023)

H. Okada, Y. Orikasa, arXiv:2501.15748

Just a sample, there are many more… I apologize for those not included

2+1 STRUCTURE

- 2+1 structure works well for quarks:
  S3, S4, Q6
- Neutrinos?
  Mixing angles known
  Masses? Only difference of squared masses
- Type I seesaw works well, other ways?
- Scalar sector? More Higgses?
  Residual symmetries possible once full minimization of scalar potential is done
- Dark Matter?

- Explore non-minimal models in scalars
- Neutrino masses: ISSM or radiatively
- Quarks:
  2+1 works well
- Scalars, allow for complex vev's
- Phenomenology

S3 AND Q6 MODELS (UNDER CONSTRUCTION)

- S3 x Z2 $U(1)_{B\text{-}L}$ flavored model:
  - 2+1 in quarks
  - 1+2 in leptons ⟹ mu-tau symmetry
  - ISSM mechanism for neutrino masses
  - Cobimaximal mixing in neutrinos, with deviations from leptonic sector ⟹ we can fit observables
  - Many scalars, some with complex vev’s (new in quarks)
  - DM sector not yet worked out
- Q6 x Z4 xZ2
  - 2+1 in quarks, leptons and Higgs
  - Many scalars, some with complex vev’s
  - Neutrino masses radiatively generated at two-loops
  - Cobimaximal mixing, with deviations from leptonic sector
  - Higgs sector and DM worked out,
consistent with known phenomenological constraints

S3 X Z2 FLAVORED U(1)$_{B-L}$ MODEL

- Usual U(1)$_{B-L}$ has 3N$_R$ and 3S and 3s fermionic singlets and one Higgs

  Khalil PRD 2010

- In our version we have 3N$_R$, 3S and 3s neutrinos, plus 3 Higgs doublets and 3 singlets (lots of exotics!)

  J.C. Gómez-Izquierdo, C. Espinoza, L. Gutiérrez-Luna, M.M, NPB 1018 (2015)

- Additional Z2 to forbid some Yukawa couplings

Matter	$Q_I, d_{IR}, u_{I,R}, H_I, L_J, e_{JR}, N_{J,R}, S_{JL}, s_{JL}$	$L_1, e_{1R}, N_{1R}, S_{1L}, s_{1L}$	$Q_3, d_{3R}, u_{3R}, H_3, \phi_3$	ϕ_I
$\mathbf{S}_3$	2	1_S	1_S	2
$\mathbf{Z}_2$	1	-1	1	-1

TABLE II. Flavored $B-L$ model. Here, $I = 1, 2$ and $J = 2, 3$.

Φ_i, S$_i$, s$_i$ at high energies (TeV), H$_i$ at low energies (eW)

SCALAR POTENTIAL

- Scalar potential is of the form:

$$V = V(H) + V(\phi) + V(H,\phi),$$

- V(H) usual S3-3H potential, V(*Φ*) of B-L sector, V(H,*Φ*) mixing term

$$V(H) = M^2(H^\dagger H) + \frac{a}{2}(H^\dagger H)^2.$$

$$V(\phi) = \mu_{BL}^2(\phi^\dagger\phi) + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$V(H,\phi) = -L\left(H^\dagger H\right)\left(\phi^\dagger\phi\right) \ ,$$

Vev's for eW sector complex:

$\langle H_1\rangle = v_1,\ \langle H_2\rangle = iv_2$ and $\langle H_3\rangle = v_3$

no residual symmetry

Vev's for B-L sector real, no mixing:

$w_1 = w_2$ or $\langle\phi_2\rangle = \langle\phi_1\rangle.$

QUARKS

- Quark mass matrices after minimizing potential

$$\mathbf{M}_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y_2^d v_3 + i y_1^d v_2 & y_1^d v_1 & y_3^d v_1 \\ y_1^d v_1 & y_2^d v_3 - i y_1^d v_2 & i y_3^d v_2 \\ y_4^d v_1 & i y_4^d v_2 & y_5^d v_3 \end{pmatrix}, \mathbf{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_2^u v_3 - i y_1^u v_2 & y_1^u v_1 & y_3^u v_1 \\ y_1^u v_1 & y_2^u v_3 + i y_1^u v_2 & -i y_3^u v_2 \\ y_4^u v_1 & -i y_4^u v_2 & y_5^u v_3 \end{pmatrix}.$$

- $M_{23} = M_{32}$ via appropriate rotation and assume hierarchical structure in complex Yukawas and vevs, in polar form

Fritzsch and Xing (1999)

$$\bar{\mathbf{M}}_q = \begin{pmatrix} |\tilde{A}_q| & |\tilde{b}_q| & 0 \\ |\tilde{b}_q| & |\tilde{B}_q| & |\tilde{C}_q| \\ 0 & |\tilde{C}_q| & |\tilde{h}_q| \end{pmatrix}.$$

$|\langle H_1 \rangle| < |\langle H_2 \rangle| < |\langle H_3 \rangle|$ and $y_3^q, y_4^q \ll y_1^q$ $\qquad$ $\tilde{m}_{q_i} = m_{q_i}/m_{q_3},$ $\qquad$ $1 > |\tilde{h}_q| > \tilde{m}_{q_2} > |\tilde{m}_{q_1}| > |\tilde{A}_q|$

- 10 parameters + 2 phases = too many, but we reparametrize 3 in terms of the mass ratios $\Rightarrow$ 6 parameters in total with these assumptions:

$$|h_q|, \ |A_q| \ (q = u, d) \qquad \bar{\beta} = \beta_q - \alpha_q \text{ and } \bar{\gamma} = \gamma_q - \alpha_q$$

Still too many parameters

GATTO–SARTORI–TONIN LIMIT

- We take a particular benchmark

$$|\tilde{h}_q| \approx 1 - \tilde{m}_{q_2} \text{ and } |\tilde{A}_q| \approx 0$$

- We obtain Gatto-Sartori-Tonin relations as limiting case, 4 free parameters, fit everything correctly
- If Yukawa couplings considered real not possible to fit Jarlskog invariant
- We fit all 6, BFP goes to Gatto-Sartori-Tonin (as expected):

$$\mathcal{L}_{ij} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2} \frac{(v_{ij} - v_{ij}^{\exp})^2}{\sigma^2} \right\}$$

$$\Delta\chi^2 \equiv \chi^2 - \chi^2_{\min}$$

$$\Delta\chi^2 = -2 \log\left(\mathcal{L}/\mathcal{L}_{\max}\right)$$

$$|\tilde{h}_u| = 0.8919159, \qquad |\tilde{A}_u| = 6.85834 \times 10^{-6}, \qquad \bar{\beta} = 1.005637;$$

$$|\tilde{h}_d| = 0.8693808, \qquad |\tilde{A}_d| = 4.85842 \times 10^{-4}, \qquad \bar{\gamma} = 0.956388.$$

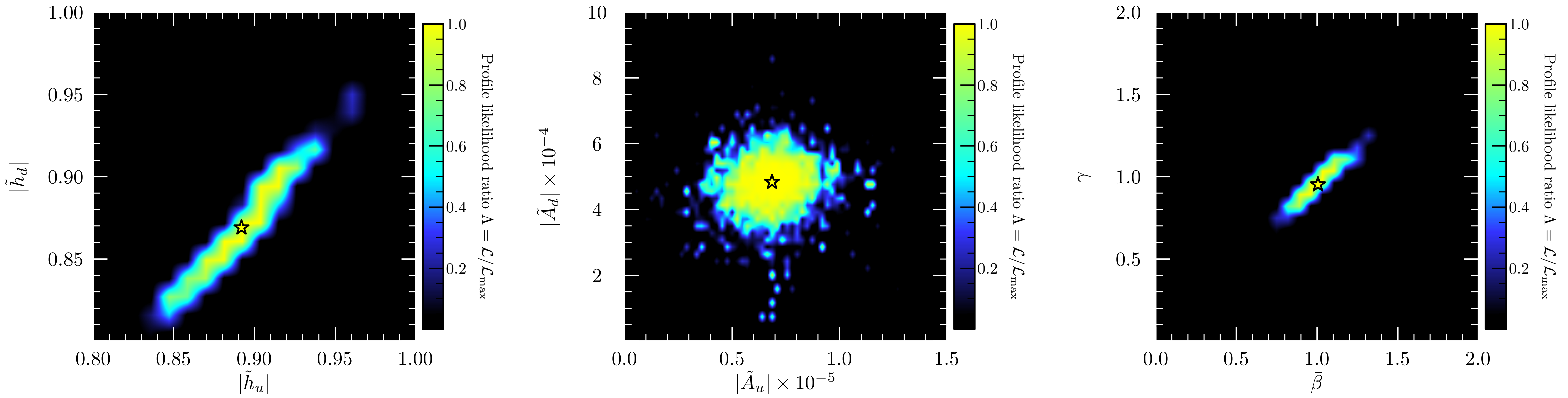


FIG. 1. Regions of the free parameter space where the model can fit accurately the experimental observations regarding the CKM matrix. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

NEUTRINOS – ISSM AND CONTRIBUTION FROM LEPTONS

- Light neutrinos acquire mass through inverted seesaw

$$-\mathcal{L} = \frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \overline{(N_R)^c} & \overline{(S)^c}\end{pmatrix} \overbrace{\begin{pmatrix} 0 & \mathbf{M}_D & 0 \\ \mathbf{M}_D^T & 0 & \mathbf{M}_R \\ 0 & \mathbf{M}_R^T & \mathbf{M}_2 \end{pmatrix}}^{\mathcal{M}_\nu} \begin{pmatrix} (\nu_L)^c \\ N_R \\ S \end{pmatrix} + h.c.$$

$$\mathcal{U}_\nu \approx \begin{pmatrix} \left(\mathbf{1}_{3\times3} - \frac{1}{2}\mathcal{A}\mathcal{A}^\dagger\right) & \mathcal{A} \\ -\mathcal{A}^\dagger & \left(\mathbf{1}_{6\times6} - \frac{1}{2}\mathcal{A}^\dagger\mathcal{A}\right) \end{pmatrix} \begin{pmatrix} \mathbf{U}_\nu & 0 \\ 0 & \mathbf{V}_R \end{pmatrix}$$

$$\mathcal{U}_\nu^\dagger \mathcal{M}_\nu \mathcal{U}_\nu^* = \hat{\mathcal{M}}_\nu \text{ with } \hat{\mathcal{M}}_\nu = \text{Diag.}\left(\hat{\mathbf{M}}_\nu, \hat{\mathbf{M}}_R\right)$$

$$\mathbf{U}_\nu^\dagger \mathbf{M}_\nu \mathbf{U}_\nu^* = \hat{\mathbf{M}}_\nu \qquad \mathbf{V}_R^\dagger \mathcal{M}_R \mathbf{V}_R^* = \hat{\mathbf{M}}_R.$$

$$\mathcal{A} = \left((\mathbf{M}_D(\mathbf{M}_R^T)^{-1}\mathbf{M}_2\mathbf{M}_R^{-1})_{3\times3} \quad (\mathbf{M}_D(\mathbf{M}_R^T)^{-1})_{3\times3}\right) \approx \left((0)_{3\times3} \quad (\mathbf{A})_{3\times3}\right)$$

$$\mathbf{M}_\nu = \mathbf{M}_D(\mathbf{M}^T)_R^{-1}\mathbf{M}_2\mathbf{M}_R^{-1}\mathbf{M}_D^T$$

- Then in the PMNS, η is mixing between light and heavy scalars:

$$\mathbf{U} = \mathbf{U}_l^\dagger\,(1-\eta)\,\mathbf{U}_\nu \qquad\qquad \eta = \mathcal{A}\mathcal{A}^\dagger/2$$

- Neutrinoless double beta decay and charged LFV get extra contributions from heavy scalars

NEUTRINOS

- In our model, assume neutrino Yukawas are real, then:

$$\mathbf{M}_\nu = \mathbf{M}_D (\mathbf{M}^T)_R^{-1} \mathbf{M}_2 \mathbf{M}_R^{-1} \mathbf{M}_D^T$$

$$\mathbf{A} \equiv \begin{pmatrix} a_1 & -a_2 & -a_2 \\ a_3 & a_4^* & a_5^* \\ a_3^* & a_5 & a_4 \end{pmatrix}, \qquad \eta = \begin{pmatrix} a_\nu & b_\nu & b_\nu^* \\ b_\nu^* & c_\nu & d_\nu \\ b_\nu & d_\nu^* & c_\nu \end{pmatrix}, \qquad \mathbf{M}_\nu = \begin{pmatrix} A_\nu & B_\nu & B_\nu^* \\ B_\nu & C_\nu^* & D_\nu \\ B_\nu^* & D_\nu & C_\nu \end{pmatrix}$$

- M_ν has cobimaximal texture, 4 parameters, $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$ are fixed

- Parameters can be expressed in terms of masses and mixing angles

- The contribution from the leptonic sector breaks this texture ⟹ deviations from cobimaximal with 2 lepton parameters

  ⟹ possible to fit correctly all neutrino parameters

$$\sin^2\theta_{13} = |(\mathbf{U})_{13}|^2 = \sin^2\rho_{13};$$

$$\sin^2\theta_{23} = \frac{|(\mathbf{U})_{23}|^2}{1-|\mathbf{U}_{13}|^2} = \frac{1}{2}\left[1+\sin 2\theta_e \cos\eta_\ell\right];$$

$$\sin^2\theta_{12} = \frac{|(\mathbf{U})_{12}|^2}{1-|\mathbf{U}_{13}|^2} = \sin^2\rho_{12}.$$

$$\sin\delta_{CP} = -\frac{\cos 2\theta_e}{\sqrt{1-\sin^2 2\theta_e \cos^2\eta_\ell}}.$$

LEPTON FLAVOR VIOLATION

- BR charged lepton flavor violating decays, depends on η

$$BR\,(\ell_\alpha \to \ell_\beta \gamma) \approx \frac{\alpha_W^3 \sin^2\theta_W}{256\pi^2}\frac{m_{\ell_\alpha}^4}{m_W^4}\frac{m_{\ell_\alpha}}{\mathit{\Gamma}_{\ell_\alpha}}\left|\sum_{i=1}^{3}\mathbf{U}_{\alpha i}\mathbf{U}_{\beta i}^{*}G_\gamma\left(\frac{m_i^2}{m_W^2}\right)+\sum_{j=1}^{6}\mathcal{K}_{\alpha j}\mathcal{K}_{\beta j}^{*}G_\gamma\left(\frac{m_{jR}^2}{m_W^2}\right)\right|^2$$

- We define the likelihood function like

$$\log\mathcal{L} = \log\mathcal{L}_{\delta_{CP}} + \log\mathcal{L}_{\theta_{23}} + \log\mathcal{L}_{\Delta m_{21}^2} + \log\mathcal{L}_{\Delta m_{31}^2} + \log\mathcal{L}_{m_{\rm tot}} + \log\mathcal{L}_{\mu\to e\gamma} + \log\mathcal{L}_{\tau\to e\gamma} + \log\mathcal{L}_{\tau\to\mu\gamma}$$

- We check that m$_{ee}$ is within GERDA limits

$$\begin{aligned}\left|m_{ee}\right| &\approx \left|\sum_{i=1}^{3}(\mathbf{U})_{ei}^{2}\,m_i + \sum_{i=1}^{6}(\mathcal{K})_{ei}^{2}\,p^2\frac{m_{iR}}{p^2-m_{iR}^2}\right|;\\ &\approx \left|\sum_{i=1}^{3}(\mathbf{U})_{ei}^{2}\,m_i - p^2\sum_{i=1}^{6}\frac{(\mathcal{K})_{ei}^{2}}{m_{iR}}\right|,\end{aligned}$$

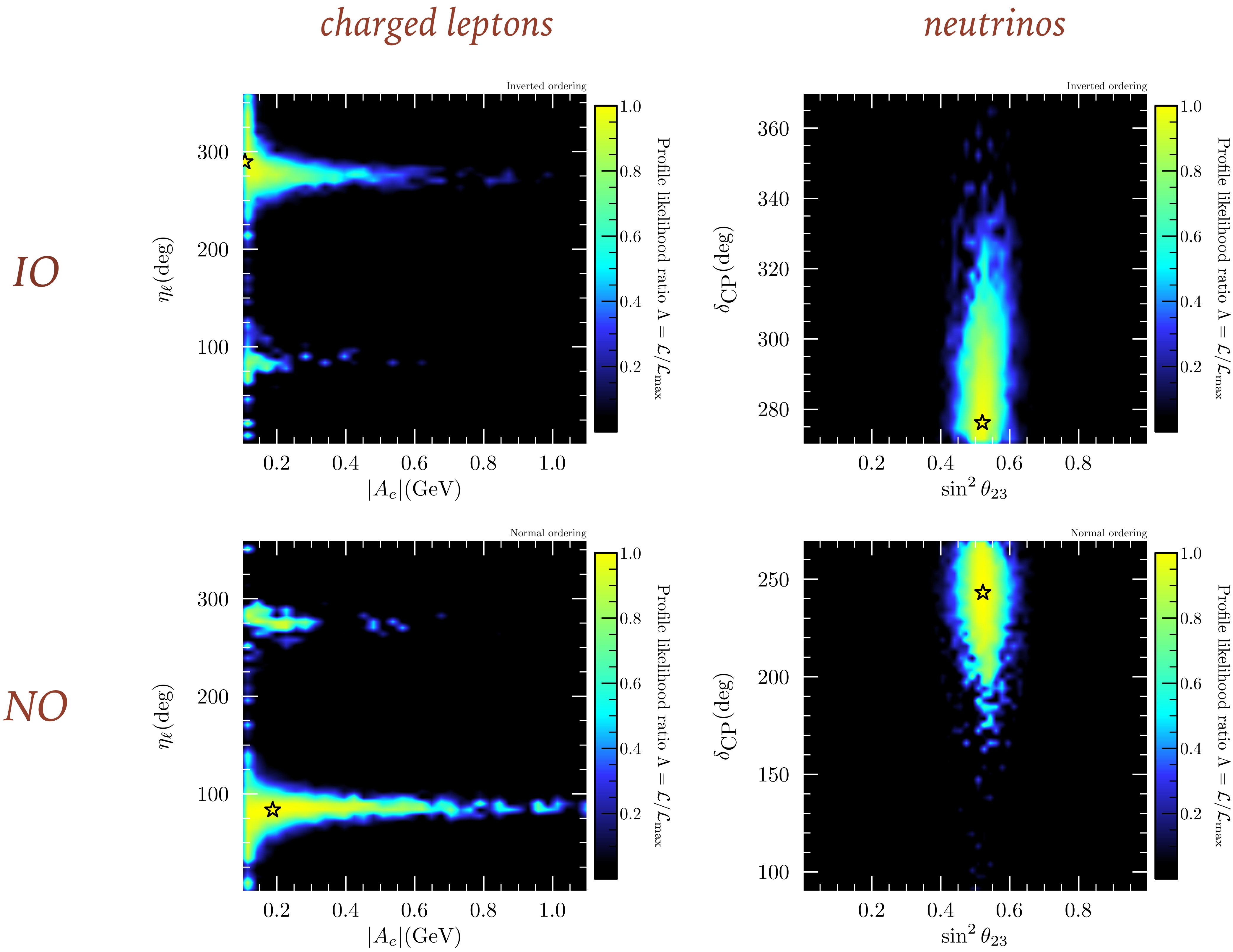


FIG. 2. Regions of the free parameters where the model can fit accurately the experimental observation regarding the observables in the lepton sector (left panels), the right panels show the values of $\sin^2\theta_{23}$ and δ_{CP} predicted by the model that are most compatible with current observations. Top (bottom) panel is for Inverted (Normal) Hierarchy. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

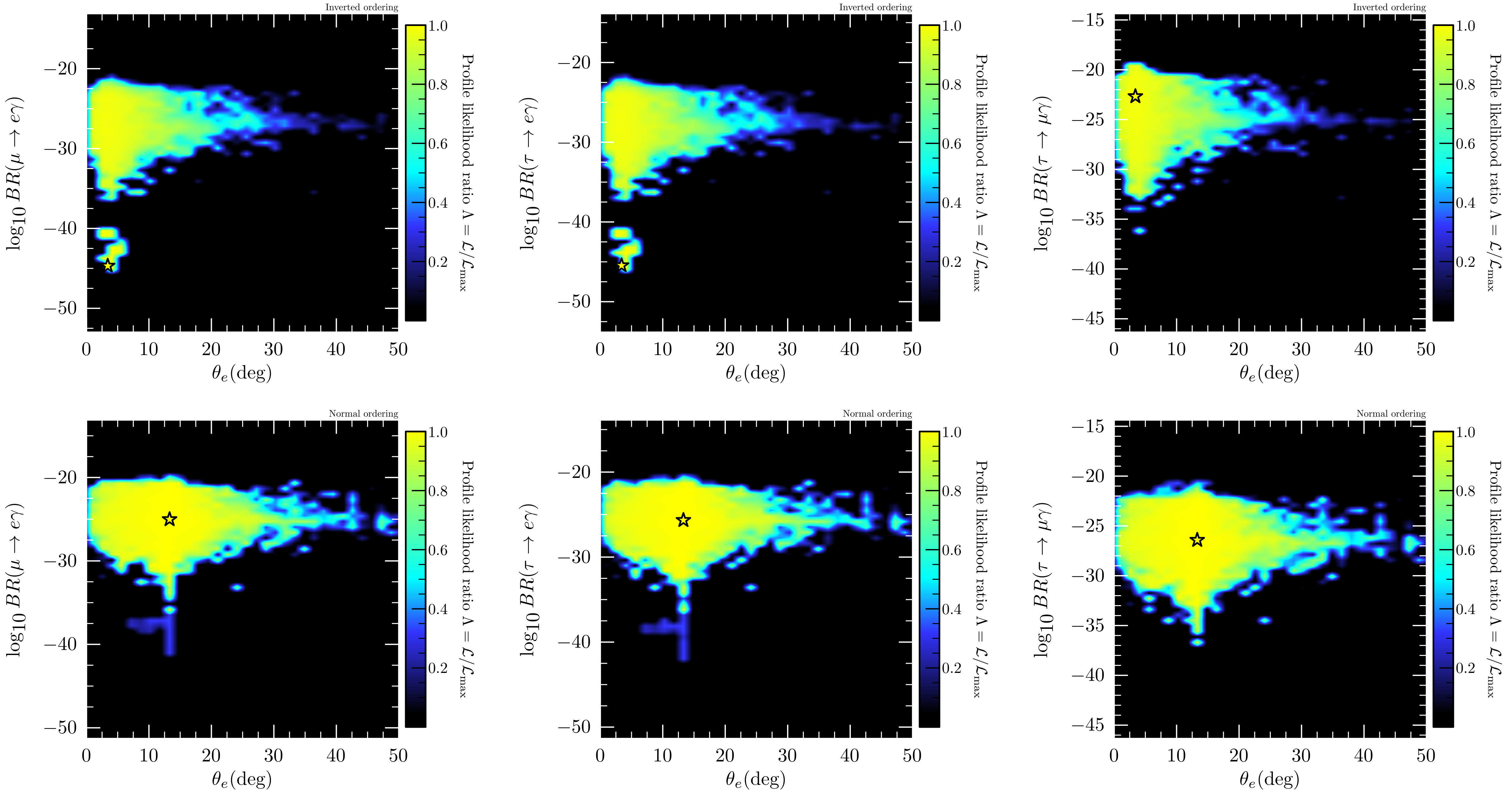


FIG. 3. Predicted branching fractions as functions of θ_e. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

RECAP S3 B-L

- 2+1 + complex vev's still works for quarks (new)
  - Gatto-Sartori-Toni relations as a limit
- Leptons: complex vev's and 2+1 assignment $\Longrightarrow$ we recover cobimaximal plus contribution from leptons
- Despite the large amount of extra scalars, effective number of parameters is reduced greatly by symmetry (6)
- We can get values for LFV and m_{ee}, consistent with data and also BFPs values

  $\Longrightarrow$ testable or falsifiable
- To do, Higgs and DM sectors, leptogenesis also possible

Q6, LOTS OF SCALARS, COBIMAXIMAL MIXING AND DM

Cárcamo, Espinoza, Gómez-Izquierdo, Merchant, MM (2025)

$$\begin{aligned} \mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_4 \times Z_2 &\xrightarrow{v_\sigma, v_\rho, v_\xi} \\ SU(3)_C \times SU(2)_L \times U(1)_Y \times \widetilde{Z}_2 \times Z_2 &\xrightarrow{v_1, v_2, v_3} \\ SU(3)_C \times U(1)_Q \times \widetilde{Z}_2 \times Z_2, & \end{aligned}$$

Z2,Z2 will allow for
3 DM candidates

3 H_i doublets + 1H_4 inert doublet + 6 scalar neutral singlets + 3N_R

← *High energies*

→

	q_L	q_{3L}	u_R	u_{3R}	d_R	d_{3R}	l_{1L}	l_L	e_{1R}	e_R	N_{1R}	N_R
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	0	0
Q_6	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$
Z_2	0	0	0	0	0	0	0	0	0	0	1	1
Z_4	0	0	0	0	0	0	0	0	0	0	1	1

Table I: Fermion content with the $SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_2 \times Z_4$ assignments.

← *High energies*

→

	H	H_3	H_4	φ_1	φ_2	σ	ξ	ρ
$SU(3)_C$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0
Q_6	$\mathbf{2}_1$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{2}_1$	$\mathbf{1}_{--}$
Z_2	0	0	1	0	1	0	0	0
Z_4	0	0	1	1	2	2	2	2

Table II: Scalar content with the $SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_2 \times Z_4$ assignments.

QUARKS

- General form of mass matrices

$$\mathbf{M}_d = \begin{pmatrix} 0 & y_1^d\langle H_3^0\rangle & y_2^d\langle H_1^0\rangle \\ -y_1^d\langle H_3^0\rangle & 0 & y_2^d\langle H_2^0\rangle \\ y_3^d\langle H_1^0\rangle & y_3^d\langle H_2^0\rangle & y_4^d\langle H_3^0\rangle \end{pmatrix}, \qquad \mathbf{M}_u = \begin{pmatrix} 0 & y_1^u\langle \tilde{H}_3^0\rangle & y_2^u\langle \tilde{H}_1^0\rangle \\ -y_1^u\langle \tilde{H}_3^0\rangle & 0 & y_2^u\langle \tilde{H}_2^0\rangle \\ y_3^u\langle \tilde{H}_1^0\rangle & y_3^u\langle \tilde{H}_2^0\rangle & y_4^u\langle \tilde{H}_3^0\rangle \end{pmatrix}.$$

- We minimize the scalar potential, with one inert vev:

$$\left(\langle H_1^0\rangle, \langle H_2^0\rangle\right) = \left(0, v_2/\sqrt{2}\right) \text{ and } \langle H_3^0\rangle = v_3/\sqrt{2}$$

$$\mathbf{M}_q = \begin{pmatrix} 0 & A_q & 0 \\ -A_a & 0 & b_q \\ 0 & c_q & F_q \end{pmatrix}$$

NNI

- Reparameterizing in terms of mass ratios leaves

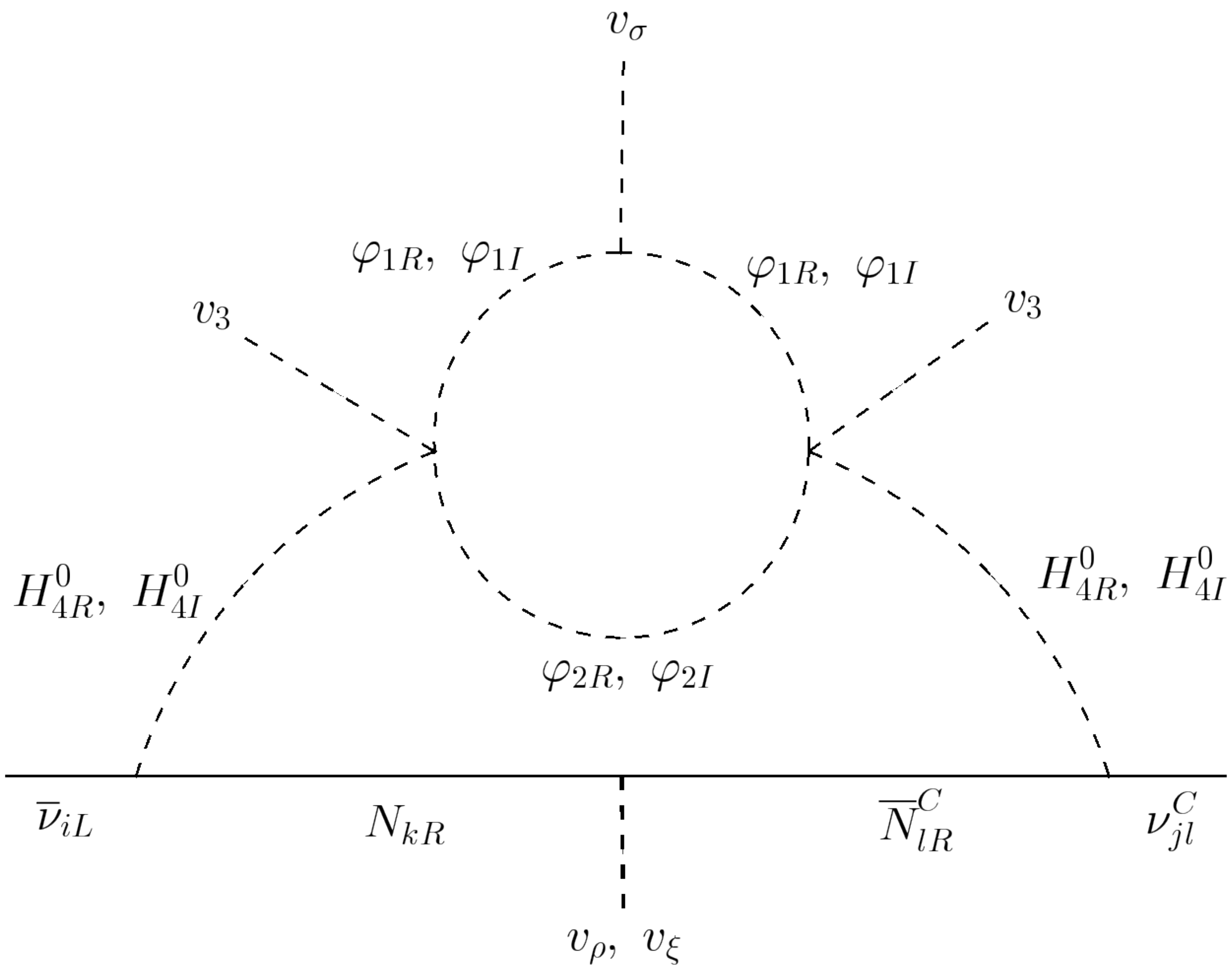
$$y_q \equiv |F_q|/m_{q3}$$

$$1 > y_q > \tilde{m}_{q_2} > \tilde{m}_{q_1}$$

- 4 free parameters: yu, yd, phi_u, phi_d
  VCKM can be fitted exactly (work in progress)

NEUTRINOS

- Active neutrino masses generated at two-loops



NEUTRINO MASS MATRICES

- After minimization of potential

$$M_\nu = \frac{1}{16\pi^2}\widetilde{Y}_{\nu D}\begin{pmatrix} m_{N_1}f_1 & 0 & 0 \\ 0 & m_{N_2}f_2 & 0 \\ 0 & 0 & m_{N_3}f_3 \end{pmatrix}\widetilde{Y}^T_{\nu D}, \qquad \widetilde{Y}_{\nu D} = Y_{\nu D}R_N,$$

$$Y_{\nu D} = \begin{pmatrix} y_1^{(\nu)} & 0 & 0 \\ 0 & 0 & y_2^{(\nu)} \\ 0 & -y_2^{(\nu)} & 0 \end{pmatrix}, \qquad (M_N)_{diag} = \begin{pmatrix} m_{N_1} & 0 & 0 \\ 0 & m_{N_2} & 0 \\ 0 & 0 & m_{N_3} \end{pmatrix} = R_N^T M_N R_N$$

$$f_k = \frac{m_R^2}{m_R^2 - m_{N_k}^2}\ln\left(\frac{m_R^2}{m_{N_k}^2}\right) - \frac{m_I^2}{m_I^2 - m_{N_k}^2}\ln\left(\frac{m_I^2}{m_{N_k}^2}\right), \qquad k = 1, 2, 3$$

$$m_R = m_{\mathrm{Re}\,H_4^0},\ m_I = m_{\mathrm{Im}\,H_4^0}$$

$$M_N = \begin{pmatrix} y_{1N}\frac{v_\sigma}{\sqrt{2}} & y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} \\ y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{2N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & y_{3N}\frac{v_\rho}{\sqrt{2}} \\ y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & y_{3N}\frac{v_\rho}{\sqrt{2}} & y_{2N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} \end{pmatrix}$$

$$\Downarrow$$

$$M_\nu \simeq \frac{m_R^2 - m_I^2}{8\pi^2\left(m_R^2 + m_I^2\right)}\begin{pmatrix} y_{1\nu}^2 y_{1N}\frac{v_\sigma}{\sqrt{2}} & y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} \\ y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{2\nu}^2 y_{2N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & -y_{2\nu}^2 y_{3N}\frac{v_\rho}{\sqrt{2}} \\ y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & -y_{2\nu}^2 y_{3N}\frac{v_\rho}{\sqrt{2}} & y_{2\nu}^2 y_{2N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} \end{pmatrix},$$

$$\mathbf{M}_\nu = \begin{pmatrix} A_\nu & \tilde{B}_\nu & \tilde{B}_\nu^* \\ \tilde{B}_\nu & \tilde{C}_\nu^* & D_\nu \\ \tilde{B}_\nu^* & D_\nu & \tilde{C}_\nu \end{pmatrix}$$

CHARGED LEPTONS

- Form of mass matrix after minimization of potential

$$M_l = \begin{pmatrix} a_l & 0 & b_l \\ 0 & 0 & d_l \\ c_l & -d_l & 0 \end{pmatrix}$$

- After reparameterization only 1 free parameter

$$|m_\tau| > |a_l| \approx (|m_\tau|/|m_\mu|)|m_e$$

- Diagonalization matrix U1L almost identity
- Again charged lepton will provide deviation from cobimaximal in the PMNS matrix $\Longrightarrow$ 5 free parameters

$$\gamma_{12},\ \gamma_{13} \text{ and } |a_l|$$

$$\eta_\mu \text{ and } \eta_\tau$$

The limit gives t

$$|a_l| = (|m_\tau|/|m_\mu|)|m_e|$$

$$|\bar{m}_e| = |m_e|/|m_\mu| \sim \mathcal{O}\left(10^{-3}\right)$$

$$\begin{aligned}
\sin\theta_{13} &\approx \sin\gamma_{13}\left[1 - \frac{|\bar{m}_e|}{\sqrt{2}}\cot\gamma_{13}\sin\eta_\tau\right]; \\
\sin\theta_{12} &\approx \sin\gamma_{12}\left[1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau\right]; \\
\sin\theta_{23} &\approx \frac{1}{\sqrt{2}}\left[1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau\right]; \\
\sin\delta_{CP} &\approx -1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau .
\end{aligned}$$

3 free parameters for PMNS

η_μ negligible

FIT FOR LEPTONS

- We fit with the 5 effective parameters, $\chi^2 = 0.497$:

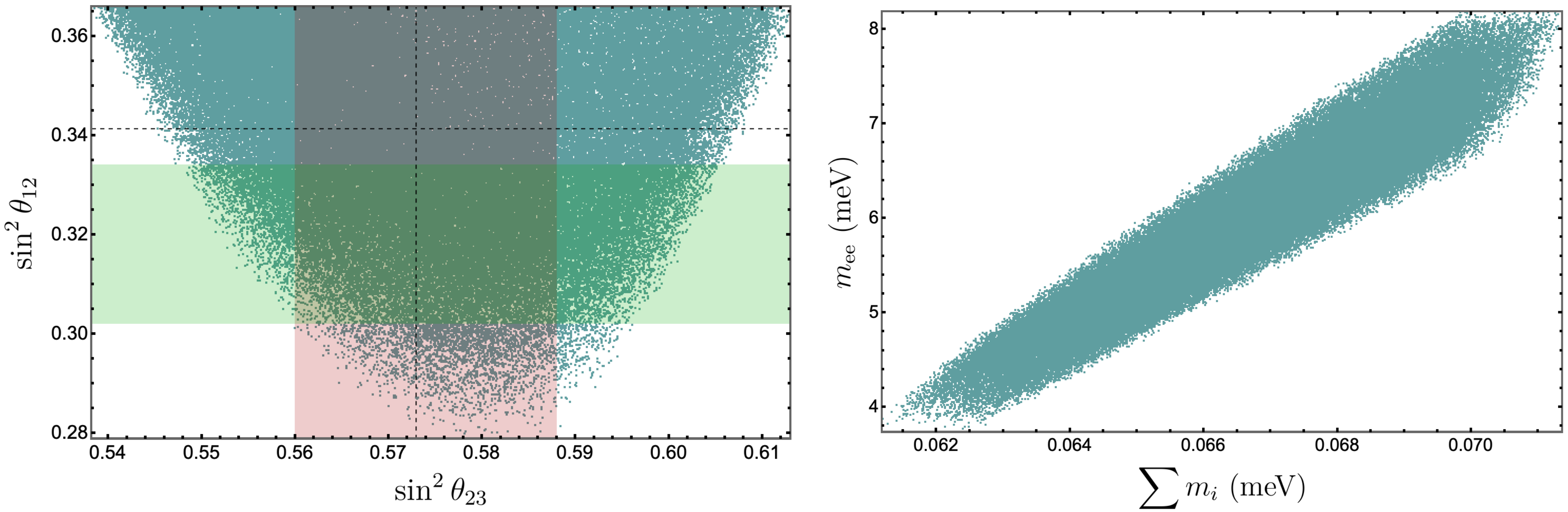
$$\chi^2 = \frac{\left(\Delta m_{21}^{2\ \mathrm{exp}} - \Delta m_{21}^{2\ \mathrm{th}}\right)^2}{\sigma^2_{\Delta m^2_{21}}} + \frac{\left(\Delta m_{31}^{2\ \mathrm{exp}} - \Delta m_{31}^{2\ \mathrm{th}}\right)^2}{\sigma^2_{\Delta m^2_{31}}} + \frac{\left(\sin^2\theta_{12}^{(l)\,\mathrm{exp}} - \sin^2\theta_{12}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta^{(l)}_{12}}}$$
$$+ \frac{\left(\sin^2\theta_{23}^{(l)\,\mathrm{exp}} - \sin^2\theta_{23}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta^{(l)}_{23}}} + \frac{\left(\sin^2\theta_{13}^{(l)\,\mathrm{exp}} - \sin^2\theta_{13}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta^{(l)}_{13}}} + \frac{\left(\delta_{\mathrm{CP}}^{\mathrm{exp}} - \delta_{\mathrm{CP}}^{\mathrm{th}}\right)^2}{\sigma^2_{\delta_{\mathrm{CP}}}} ,$$

- And obtain m_i and m_{ee}:

$$\sum m_i \qquad\qquad m_{ee} = \left|\sum_i \mathbf{U}_{ei}^2 m_{\nu i}\right|$$

Observable	range	Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$\sin^2\theta^{(l)}_{12}/10^{-1}$	$\sin^2\theta^{(l)}_{13}/10^{-2}$	$\sin^2\theta^{(l)}_{23}/10^{-1}$	$\delta^{(l)}_{\mathrm{CP}}(^\circ)$
Experimental	1σ	$7.50^{+0.22}_{-0.20}$	$2.55^{+0.02}_{-0.03}$	3.18 ± 0.16	$2.200^{+0.069}_{-0.062}$	5.74 ± 0.14	194^{+24}_{-22}
Value [97]	3σ	$6.94 - 8.14$	$2.47 - 2.63$	$2.71 - 3.69$	$2.000 - 2.405$	$4.34 - 6.10$	$128 - 359$
Experimental	1σ	7.49 ± 0.19	$2.513^{+0.021}_{-0.019}$	$3.08^{+0.12}_{-0.11}$	$2.215^{+0.058}_{-0.056}$	$4.7^{+0.17}_{-0.13}$	212^{+26}_{-41}
Value [98]	3σ	$6.92 - 8.05$	$2.451 - 2.578$	$2.75 - 3.45$	$2.03 - 2.388$	$4.35 - 5.85$	$124 - 364$
Fit	$1\sigma - 3\sigma$	7.69	2.54	3.41	2.24	5.73	219.7

Table III: Model predictions for the scenario of normal order (NO) neutrino mass.



(a) (b)

Figure 2: Correlation plot between mixing angle, effective Majorana mass, and sum lightest mass neutrino.

NO shown

IO disfavored

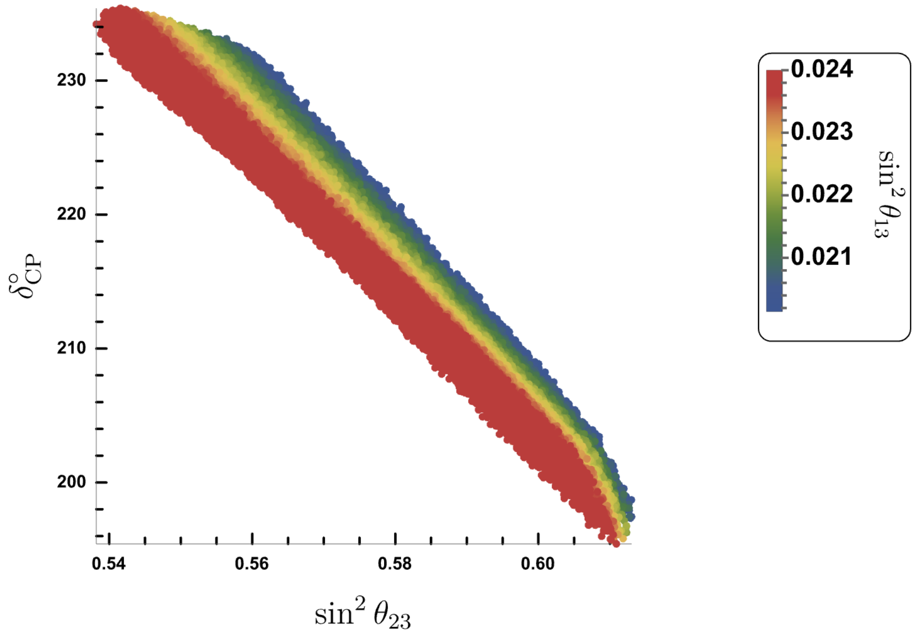


Figure 3: Correlation plot between mixing angles and CP violation phase, for different values of $\sin^2\theta_{13}$.

SCALAR SECTOR AND DARK MATTER ANALYSIS

- Implement model in SARAH, neglect off-diagonal terms and masses of first and second generation
- Then implement SARAH-SPheno
- Scalar potential analyzed with EVADE
- Exclusion limits with Higgs/Tools, Higgs/Predictions, Higgs/Bounds
- To generate input for Higgs/Toos we use CalcHEP and Micromegas
- Several dark matter candidates:
  lightest right-handed neutrino N_R
  one component of φ_1 and φ_2 ,
  we denote them ϕ_1 and ϕ_2
- We calculate relic density

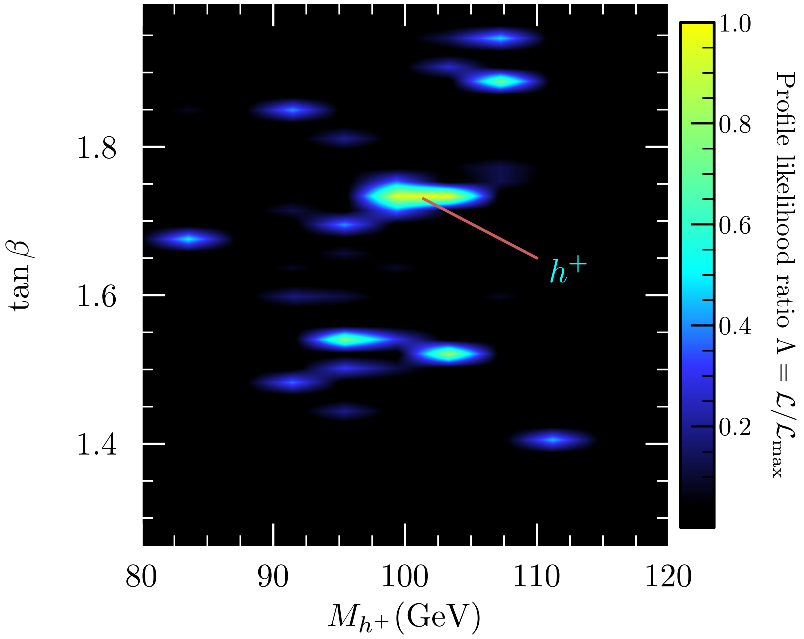
- Hard cuts, anything that doesn't pass this
- Then couplings and decays using Higgs/Tools, Higgs/Predictions
- Likelihood function Higgs

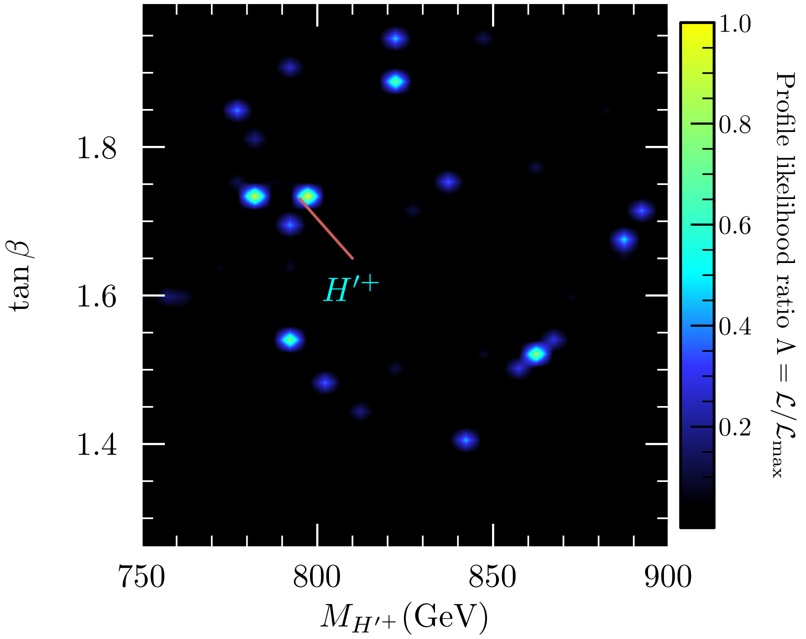
$$\log \mathcal{L}_{\text{scalar}} = \log \mathcal{L}_{\text{Higgs}} + \log \mathcal{L}_{H_0 \to \gamma\gamma}$$

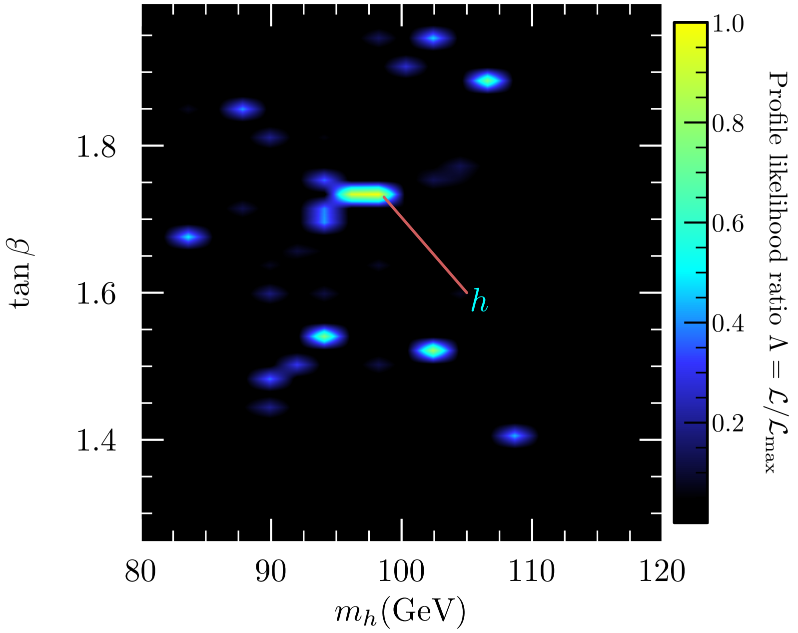
- Likelihood function DM

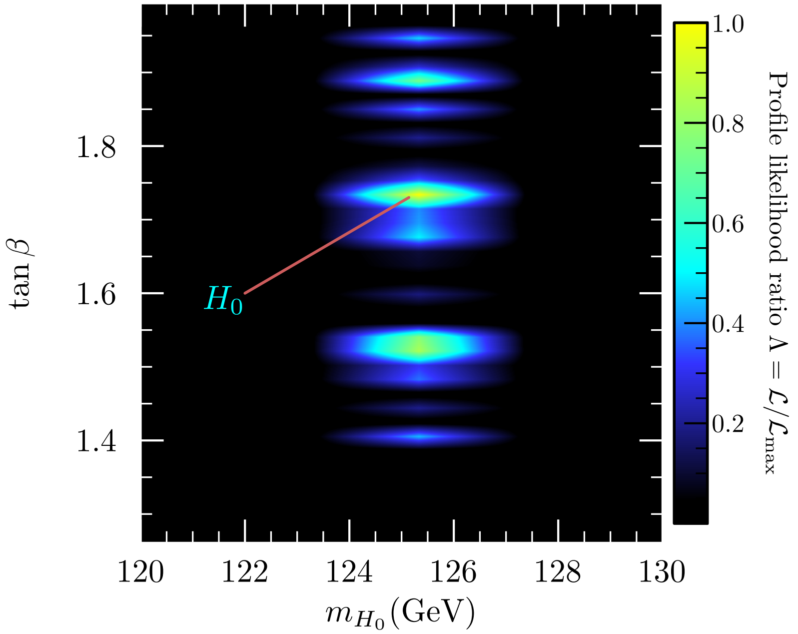
$$\log \mathcal{L} = \log \mathcal{L}_{\text{scalar}} + \log \mathcal{L}_{\text{DD}} + \log \mathcal{L}_{\Omega h^2}$$

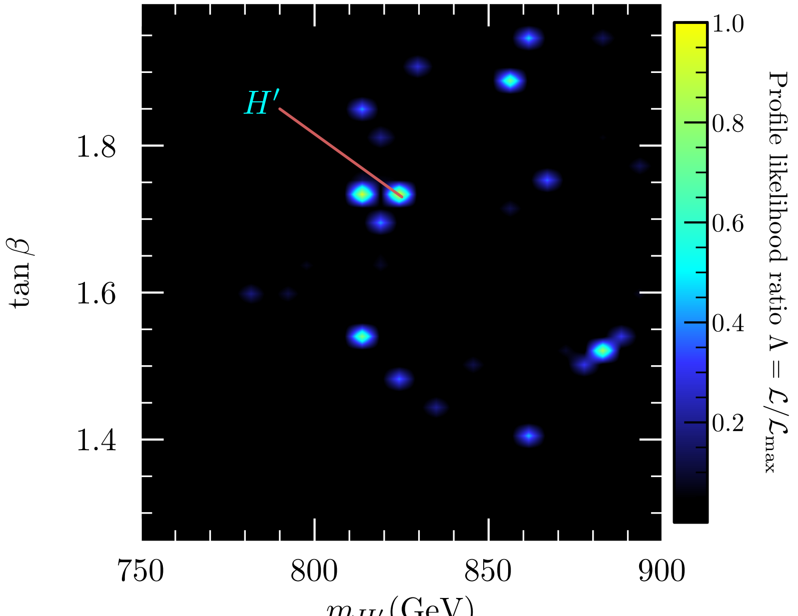
HIGGS SECTOR RESULTS











Interesting BFP:

$$(m_h, m_{H_0}, m_{H'}) = (98.7, 125.1, 825)\ \text{GeV}$$

$$(m_A, m_{A'}, m_{h^+}, m_{H'^+}) = (348.3, 795.6, 101.4, 795)\ \text{GeV and } \tan\beta = 1.73.$$

BFP signals out a neutral scalar at ~ 98 GeV
and a pseudoscalars at ~ 348 and 795 GeV

CMS new scalar at ~ 95 GeV ?
pseudoscalar ~ 365 GeV ?   pbbly toponium

DM RESULTS

Direct detection

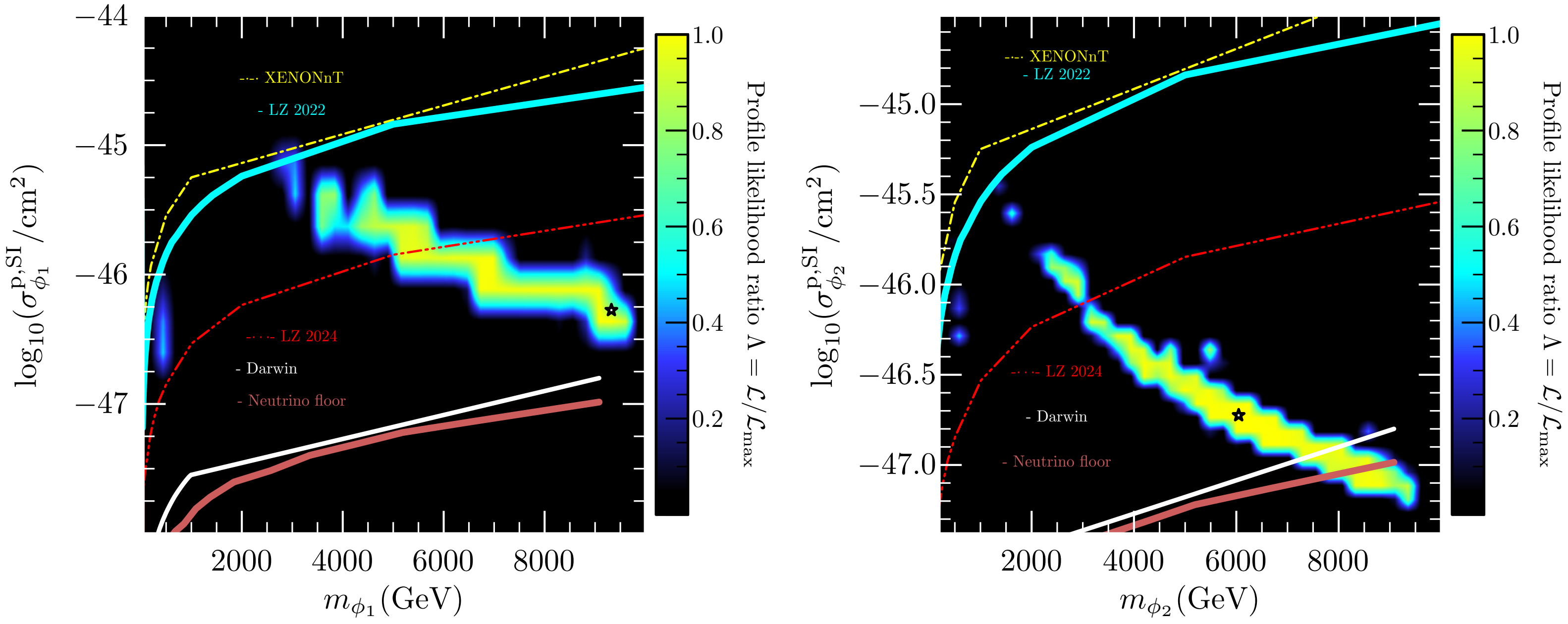
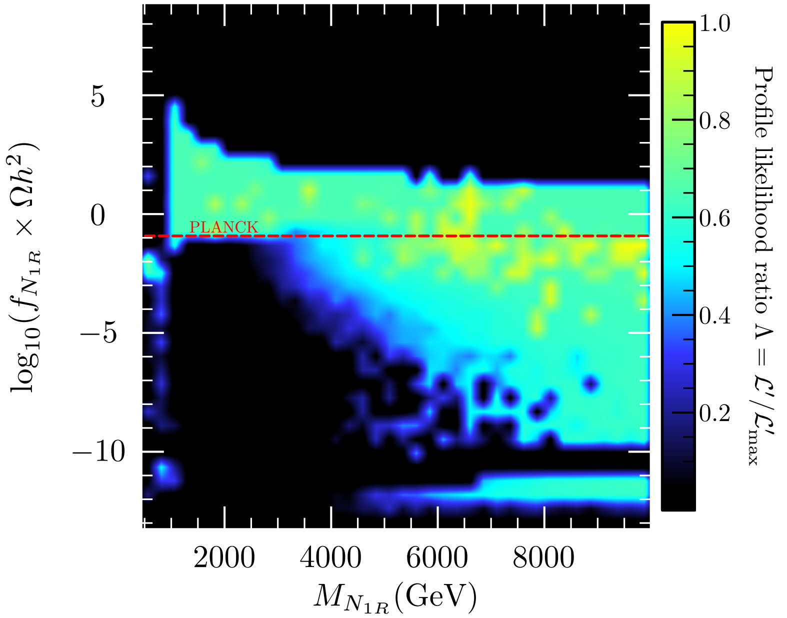
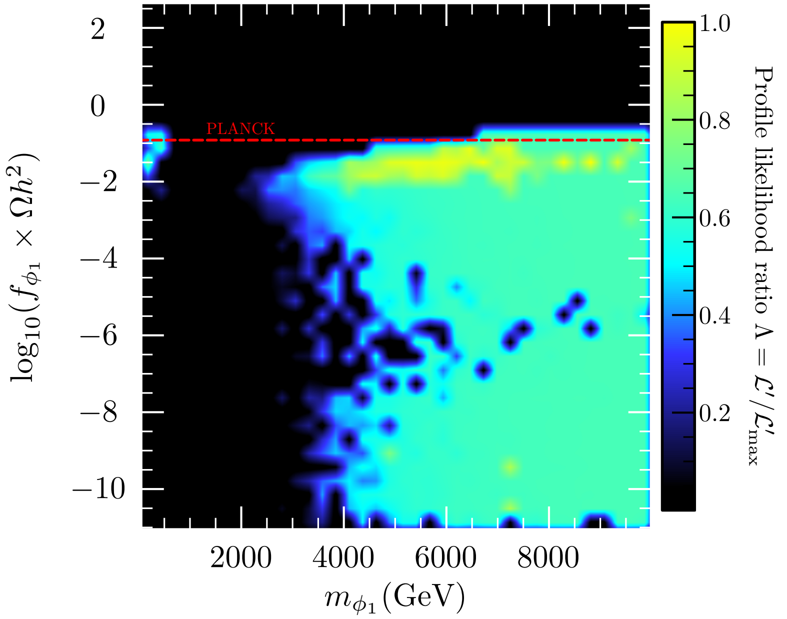
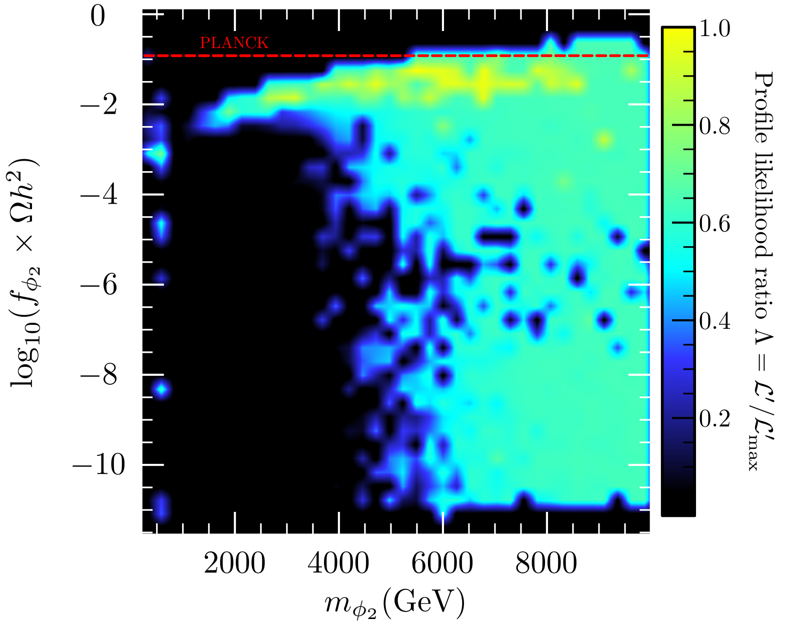


Figure 8: DM-proton spin-independent elastic scattering cross section as a function of the masses of the scalar DM candidates. Brightest areas are most consistent with all imposed constraints, dark areas are excluded by the analysis. The best fit point (BFP) is marked with a small star. For comparison, exclusion limits of the XENONnT, LZ 2022 and LZ 2024 experiments are shown, alongside with the projection of the DARWIN experiment and the neutrino floor.





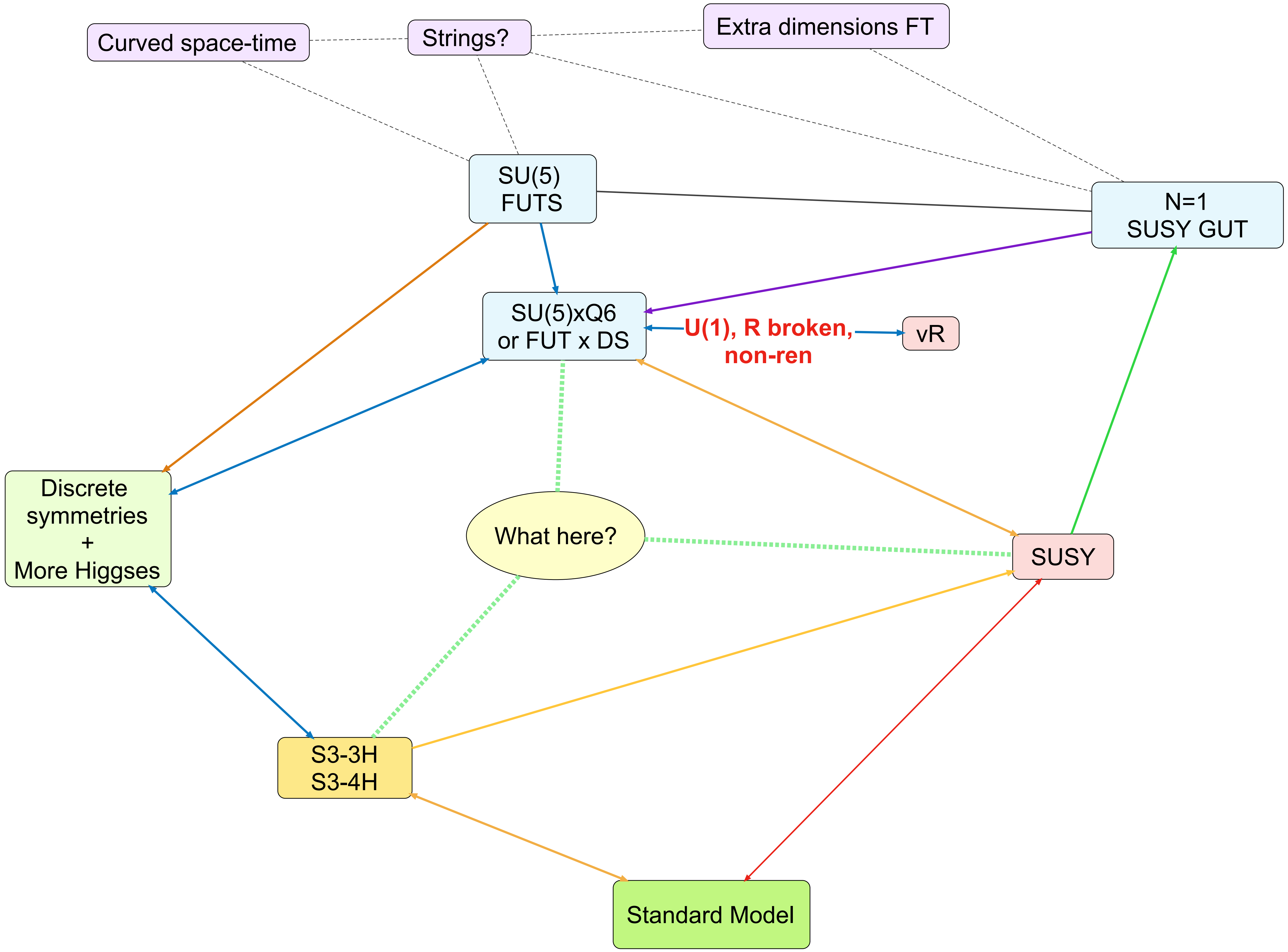


DM relic abundance weighed by respective DM fractions of N1R, ϕ_1 and ϕ_2

$$\log \mathcal{L}' = \log \mathcal{L} - \log \mathcal{L}_{\Omega h^2}$$

RECAP Q6

- 2+1 in quarks and leptons works well
- 2-loop generated neutrino masses
    - again cobimaximal pattern “corrected” by lepton sector
- Number of effective parameters 4 in quark sector and 5 in neutrino sector
- Higgs sector with interesting new possibilities
  and passes all known constraints
- Viable DM sector with three candidates
- To do: more phenomenology…



Curved space-time

Strings?

Extra dimensions FT

SU(5)
FUTS

N=1
SUSY GUT

SU(5)xQ6
or FUT x DS

U(1), R broken,
non-ren

vR

Discrete
symmetries
+
More Higgses

What here?

SUSY

S3-3H
S3-4H

Standard Model

OUTLOOK AND CONCLUSIONS

- Among the different ways to go BSM finiteness proves to be a good guiding principle.
  Reduces greatly the number of free parameters, RG flow of the third family in the right direction
- Needs extended Higgs sector and discrete flavor symmetries
- At low energies S3, S4, Q4, Q6 theories with extended Higgs sector explain well CKM and have predictions for neutrino sector
  - Provide baryogenesis through leptogenesis
  - Good DM candidates

- Maybe is possible to connect both approaches
- How do astroparticle physics and cosmology help?

Thank you!