LSZ Reduction Formula for massive charged vector bosons in an constant magnetic field

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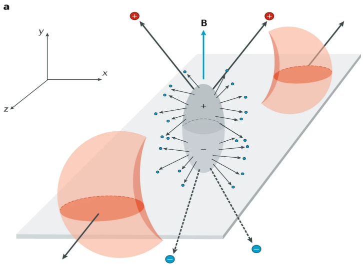
Intense Magnetic Fields

- Presence of intense magnetic fields in high energy physics.
  Magnetic field intensity in magnetars between $10^{-2}m_\pi^2$ on the surface[1] and up to $10^0 m_\pi^2$ in its core[2]. In heavy ion colisions $\sim 1 - 10m_\pi^2$.[3]

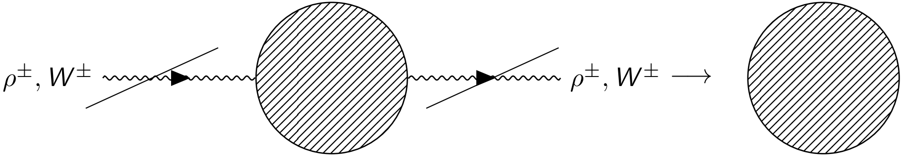


E. Kharzeev, & J. Liao, (2021) **3**, Nat. Rev. Phys., 55-63.

1 R. Turolla, S. Zane, & A. L. Watts, (2015), Rep. Prog. Phys. **78**, 116901.
2 G. Chanmugam, (1992), Annu. Rev. Astron. Astrophys. , **30**, 143-184.
3 Y. Zhong, et. al, (2014), Adv. High Energy Phys., **193039**.

Massive Charged Vector Bosons as External States

There are certain physical scenarios in which massive charged vector bosons exist as external states under the presence of a strong magnetic field such as heavy ion collisions and in compact objects such as neutron stars and magnetars[4].



Amputation process of the external states in the vacuum.

To extract the self-energy information from this type of diagrams it is important to apply the LSZ reduction formula and cancel out the external states.

[4] E.E. Kolomeitsev, K.A. Maslov, D.N. Voskresensky, (2018), Nucl. Phys. A 970, 291-315

LSZ Reduction Formula

The Lehmann-Symanzik-Zimmermann (LSZ) Reduction Formula is a non-perturbative operation which relates the scattering amplitude of a process to the Green's functions of the theory.
The simplest spin 0 case of the reduction formula for $n(m)$ in-going (out-going) particles is

$$\begin{aligned}\langle f|\, S\, |i\rangle &= \langle p_1', ...p_m'|\, S\, |p_1, ..., p_n\rangle \\ &= \prod_{i,f}^{n,m} \left(p_i^2 - m^2\right)\left(p_f^2 - m^2\right) \int d^4x_i\ e^{-ip_i\cdot x_i} \int d^4x_f'\ e^{ip_f\cdot x_f'} \\ &\qquad\qquad \times \langle 0|\, \mathcal{T}\left\{\phi(x_1)...\phi(x_n)\phi(x_1')...\phi(x_m')\right\}|0\rangle\,.\end{aligned}$$

LSZ Reduction Formula (Vacuum): Charged Vector Boson

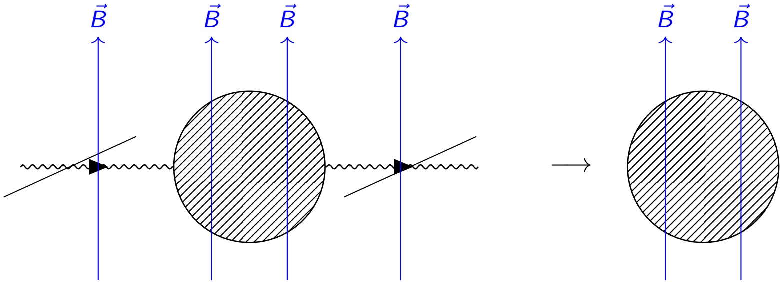
The probability amplitude can be written as

$$
\begin{aligned}
\langle f|\, S\, |i\rangle &= \int d^4x_f \int d^4x_i \; e^{ip_f\cdot x_f} e^{-ip_i\cdot x_i} \varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i) \\
&\qquad \times \left(\partial^\alpha_{x_f}\partial_{\alpha,x_f} + m_W^2\right)\left(\partial^\beta_{x_i}\partial_{\beta,x_i} + m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle \\
&= \int d^4x_f \int d^4x_i \; e^{ip_f\cdot x_f} e^{-ip_i\cdot x_i} \varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i) \\
&\qquad \times \left(p_f^2 - m_W^2\right)\left(p_i^2 - m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle\,.
\end{aligned}
$$

This is the result in the case of $B = 0$.

LSZ Reduction Formula in a Magnetic Field

The advantage of using Ritus Eigenfunctions is that the structure, of the equations in the vacuum case are preserved.
In the magnetic case



Amputation process of the external states in the presence of a magnetic field.

LSZ Reduction Formula (Magnetic): Charged Vector Boson

The probability amplitude can be written as

$$
\begin{aligned}
\langle f|\, S\,|i\rangle_B = & \int d^4x_f \, \left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\hat{s}_3}(x_f)\right)^{\mu}{}_{\alpha}\, \bar{\varepsilon}^{-*}_{\mu}\left(\vec{\bar{p}}_f,\lambda_f\right) \sum_{s_3'} \left(D^{\sigma}_{x_f} D_{x_f,\sigma} + m_W^2 - 2eBs_3'\right) \Delta^{\alpha}{}_{\beta}(s_3') \\
& \times \int d^4x_i \, \left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\hat{s}_3''}(x_i)\right)^{\nu}{}_{\rho}\, \bar{\varepsilon}^{-}_{\nu}\left(\vec{\bar{p}}_i,\lambda_i\right) \sum_{s_3'''} \left(D^{\gamma *}_{x_i} D^{*}_{x_i,\gamma} + m_W^2 - 2eBs_3'''\right) \Delta^{\rho}{}_{\delta}(s_3''') \\
& \times \frac{1}{2\pi} \langle\Omega|\, \mathcal{T}\left\{W^{\beta-}(x_f)W^{\delta+}(x_i)\right\}|\Omega\rangle \\
& = \frac{1}{2\pi}\int d^4x_f \, \left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\hat{s}_3}(x_f)\right)^{\mu}{}_{\alpha}\, \bar{\varepsilon}^{-*}_{\mu}\left(\vec{\bar{p}}_f,\lambda_f\right)\left(\bar{\mathcal{P}}^{\sigma}_{f}\bar{\mathcal{P}}^{*}_{f,\sigma} - m_W^2\right) \\
\times \int d^4x_i \, & \left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\hat{s}_3'}(x_i)\right)^{\nu}{}_{\beta}\, \bar{\varepsilon}^{-}_{\nu}\left(\vec{\bar{p}}_i,\lambda_i\right)\left(\bar{\mathcal{P}}^{\rho}_{i}\bar{\mathcal{P}}^{*}_{\rho,i} - m_W^2\right)\langle\Omega|\, \mathcal{T}\left\{W^{\alpha-}(x_f)W^{\beta+}(x_i)\right\}|\Omega\rangle\,,
\end{aligned}
$$

This is the result for a constant magnetic field in the $\hat{z}$ direction $B_z \neq 0$.

LSZ Formula: Comparison

$$\langle f|\,S\,|i\rangle = \int d^4x_f \int d^4x_i\; e^{ip_f\cdot x_f} e^{-ip_i\cdot x_i} \varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i)$$
$$\times \left(\partial^\alpha_{x_f}\partial_{\alpha,x_f} + m_W^2\right)\left(\partial^\beta_{x_i}\partial_{\beta,x_i} + m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle$$

$$\langle f|\,S\,|i\rangle_B = \int d^4x_f \left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\hat{s}_3}(x_f)\right)^\mu{}_\alpha \bar{\varepsilon}^{-*}_\mu(\vec{\bar{p}}_f,\lambda_f)\sum_{s_3'}\left(D^\sigma_{x_f}D_{x_f,\sigma} + m_W^2 - 2eBs_3'\right)\Delta^\alpha{}_\beta(s_3')$$
$$\times \int d^4x_i \left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\hat{s}_3''}(x_i)\right)^\nu{}_\rho \bar{\varepsilon}^{-}_\nu(\vec{\bar{p}}_i,\lambda_i)\sum_{s_3'''}\left(D^{\gamma*}_{x_i}D^*_{x_i,\gamma} + m_W^2 - 2eBs_3'''\right)\Delta^\rho{}_\delta(s_3''')$$
$$\times \frac{1}{2\pi}\langle\Omega|\,\mathcal{T}\left\{W^{\beta-}(x_f)W^{\delta+}(x_i)\right\}|\Omega\rangle$$

LSZ Formula: Comparison

$$\langle f|\, S\, |i\rangle = \int d^4x_f \int d^4x_i\; e^{ip_f\cdot x_f} e^{-ip_i\cdot x_i} \varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i)$$
$$\times \left(p_f^2 - m_W^2\right)\left(p_i^2 - m_W^2\right)\langle\Omega|\, \mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle$$

$$\langle f|\, S\, |i\rangle_B = \frac{1}{2\pi}\int d^4x_f \left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\hat{s}_3}(x_f)\right)^{\mu}{}_{\alpha}\, \bar{\varepsilon}^{-*}_{\mu}\left(\vec{\bar{p}}_f,\lambda_f\right)\left(\bar{\mathcal{P}}^{\sigma}_f\bar{\mathcal{P}}^*_{f,\sigma} - m_W^2\right)$$
$$\times \int d^4x_i \left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\hat{s}'_3}(x_i)\right)^{\nu}{}_{\beta}\, \bar{\varepsilon}^{-}_{\nu}\left(\vec{\bar{p}}_i,\lambda_i\right)\left(\bar{\mathcal{P}}^{\rho}_i\bar{\mathcal{P}}^*_{\rho,i} - m_W^2\right)\langle\Omega|\, \mathcal{T}\left\{W^{\alpha-}(x_f)W^{\beta+}(x_i)\right\}|\Omega\rangle$$

See you in the poster session!

LSZ Reduction Formula for Massive Charged Vector Bosons in a Constant Magnetic Field

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Abstract

Using the Ritus Eigenfunction representation of the massive charged vector boson quantum operator in the unitary gauge ($\xi \to \infty$) under the influence of a constant magnetic field, a novel derivation of the Lehmann-Symanzik-Zimmermann (LSZ) Reduction Formula is presented. To make explicit the usefulness of the representation used in this work, the vacuum case is also presented hoping to make clear the conservation of the physical structure.

Introduction and Motivation

Intense magnetic fields appear in many physical scenarios in which high energy physics are present. Certain compact objects, such as Neutron Stars, present magnetic fields going from $10^{12}\,G - 10^{16}\,G$ on their surface [1] and up to $10^{18}\,G$ in their innermost core [2] which are one of the most intense in the universe. It has been proposed that charged condensed ρ mesons could appear in the core of neutron stars considering in-medium and magnetic effects [3]. Then, the condensed ρ mesons will be affected by the external magnetic field as external states. In this sense, a Reduction Formula that considers the magnetic effects on the external states is required when calculating, for example, self-energies of these massive charged vector bosons. However, this procedure has not been taken into account in some papers [4, 5].

In the case of $n(m)$ in-going (out-going) scalar particles in a process scattering process, the scattering amplitude, using the LSZ Reduction Formula, is

$$\begin{aligned}\langle f|\,S\,|i\rangle &= \langle p_1',...p_m'|\,S\,|p_1,...,p_n\rangle \\ &= \prod_{i,f}^{n,m}\left(p_i^2-m^2\right)\left(p_f^2-m^2\right) \\ &\times \int d^4x_i\, e^{-ip_i\cdot x_i}\int d^4x_f'\, e^{ip_f\cdot x_f'} \\ &\times \langle 0|\,\mathcal{T}\left\{\phi(x_1)...\phi(x_n)\phi(x_1')...\phi(x_m')\right\}|0\rangle\,,\end{aligned}$$

which is a non-perturbative process as it relates the scattering amplitude with all the possible processes of propagation. Diagrammatically, the Reduction Formula amputates the external state legs of the process

Figure 1. Amputation process of a Feynman diagram.

The derivation in the case of a charged massive spin-1 boson is presented in both cases of $B=0$ and $B\neq 0$.

LSZ Reduction Formula: Vacuum Case

The standard way of deriving the LSZ Reduction Formula for a massive vector boson is reported in the literature [6]. However, to make explicit the usefulness of the Ritus representation some key parts are performed here. The scattering amplitude is

$$\begin{aligned}\langle f|\,S\,|i\rangle &= 2\sqrt{E_{p_i}E_{p_f}}\,\langle\Omega|\,a_{p_f}(\infty)a^\dagger_{p_i}(-\infty)\,|\Omega\rangle \\ &= 2\sqrt{E_{p_i}E_{p_f}}\,\langle\Omega|\,\mathcal{T}\Big\{\left[a_{p_f}(\infty)-a_{p_f}(-\infty)\right] \\ &\qquad\times\left[a^\dagger_{p_i}(\infty)-a^\dagger_{p_i}(-\infty)\right]\Big\}\,|\Omega\rangle\,,\end{aligned}$$

and the Fourier expansion of the quantum field

$$\begin{aligned}\hat{W}^{\mu-}(x) &= \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}} \\ &\times\sum_{\lambda=1}^{3}\left[\hat{a}_{p,\lambda}\varepsilon^\mu(\vec{p},\lambda)e^{-ip\cdot x}+\hat{b}^\dagger_{p,\lambda}\varepsilon^{\mu*}(\vec{p},\lambda)e^{ip\cdot x}\right].\end{aligned}$$

The Fourier analysis over the quantum field

$$\begin{aligned}&\int d^3x\, e^{-i\vec{p}\cdot\vec{x}}\,(i\partial_t+E_p)\,\hat{W}^{\mu-}(x) \\ &\qquad= \sqrt{2E_p}\sum_{\lambda=1}^{3}\left[\hat{a}_{p,\lambda}\varepsilon^\mu(\vec{p},\lambda)e^{-iE_pt}\right],\end{aligned}$$

is performed so that the creation operator

$$\hat{a}_{p,\lambda} = \frac{-1}{\sqrt{2E_p}}\int d^3x\, e^{ip\cdot x}\varepsilon^*_\mu(\vec{p},\lambda)\,(i\partial_t+E_p)\,\hat{W}^{\mu-}(x)$$

is found. The difference

$$\begin{aligned}\sqrt{2E_p}\left[a^\lambda_p(\infty)-a^\lambda_p(-\infty)\right] &= -i\int_{-\infty}^{\infty} d^4x\, e^{ip\cdot x}\varepsilon^*_\mu(\vec{p},\lambda) \\ &\times\left[\partial_t^2+E_p^2\right]\hat{W}^{\mu-}(x),\end{aligned}$$

is computed for the given scattering amplitude, and the Fourier analysis

$$\begin{aligned}&\int d^4x\, e^{ip\cdot x}\varepsilon^*_\mu(\vec{p},\lambda)\left(\partial^\alpha\partial_\alpha+m_W^2\right)W^{\mu-}(x) \\ &\quad= \int d^4x\, e^{ip\cdot x}\varepsilon^*_\mu(\vec{p},\lambda)\left(\partial_t^2+E^2\right)W^{\mu-}(x) \\ &\quad= i\sqrt{2E_p}\left[a^\lambda_p(\infty)-a^\lambda_p(-\infty)\right]\end{aligned}$$

is performed. An analogous procedure is done for the creation operator. The scattering amplitude is then

$$\begin{aligned}\langle f|\,S\,|i\rangle &= \int d^4x_f\int d^4x_i\, e^{ip_f\cdot x_f}e^{-ip_i\cdot x_i}\varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i) \\ &\times\left(\partial^\alpha_{x_f}\partial_{\alpha,x_f}+m_W^2\right)\left(\partial^\beta_{x_i}\partial_{\beta,x_i}+m_W^2\right)\mathcal{M}^{\mu\nu}(x_f,x_i) \\ &= \int d^4x_f\int d^4x_i\, e^{ip_f\cdot x_f}e^{-ip_i\cdot x_i}\varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i) \\ &\times\left(p_f^2-m_W^2\right)\left(p_i^2-m_W^2\right)\mathcal{M}^{\mu\nu}(x_f,x_i).\end{aligned}$$

where $\mathcal{M}^{\mu\nu}(x_f,x_i)\equiv\langle\Omega|\,\mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle$ is the matrix element. This is the LSZ Reduction Formula for charged vector bosons in a vacuum.

LSZ Reduction Formula: Magnetic Case

In the magnetic case, using the Ritus Eigenfunction representation, the steps are analogous except certain complications that will be highlighted.

The scattering amplitude is

$$\begin{aligned}\langle f|\,S\,|i\rangle_B &= 2\sqrt{E_{\bar{p}_i}E_{\bar{p}_f}}\,\langle\Omega|\,a_{\bar{p}_f}(\infty)a^\dagger_{\bar{p}_i}(-\infty)\,|\Omega\rangle \\ &= 2\sqrt{E_{\bar{p}_i}E_{\bar{p}_f}}\,\langle\Omega|\,\mathcal{T}\Big\{\left[a_{\bar{p}_f}(\infty)-a_{\bar{p}_f}(-\infty)\right] \\ &\qquad\times\left[a^\dagger_{\bar{p}_i}(\infty)-a^\dagger_{\bar{p}_i}(-\infty)\right]\Big\}\,|\Omega\rangle\,,\end{aligned}$$

and the Fourier expansion on the quantum field is

$$\begin{aligned}\hat{W}^{\mu-}_B(x) &= \sum\!\!\!\!\!\!\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E}} \\ &\times\sum_{\lambda=1}^{3}\left[\hat{a}^\lambda_{\bar{p}}\mathbb{E}^{+\mu}_{n,\,\alpha}(\rho_+)\varepsilon^{\alpha-}_\lambda(\bar{p})+\hat{b}^{\lambda\dagger}_{\bar{p}}\mathbb{E}^{-\mu}_{n,\,\alpha}(\rho_-)\left(\varepsilon^{\alpha+}_\lambda(\bar{p})\right)^*\right].\end{aligned}$$

The Fourier analysis over the quantum field is

$$\begin{aligned}&\int d^3x\sum_{s_3'}X^+_{\ell'-1+s_3'}(\rho_+')\Delta^\alpha{}_\mu(s_3')e^{i(p^{2'}x_2+p^{3'}x_3)} \\ &\qquad\times(i\partial_t+E_{\bar{p}'})\,\hat{W}^{\mu-}_B(x) = -\sqrt{4\pi E_{\bar{p}'}}\,\hat{a}_{\bar{p}',\lambda'}e^{-iE_{\bar{p}'}x_0}.\end{aligned}$$

With this, the creation operator becomes

$$\hat{a}_{\bar{p},\lambda} = \frac{-1}{\sqrt{4\pi E_{\bar{p}}}}\int d^3x\,\left(\mathbb{E}^{+\dagger}_n(\rho_+)\right)^\alpha{}_\mu\,\bar{\varepsilon}^{-*}_\alpha(\vec{p},\lambda)\,(i\partial_t+E_{\bar{p}})\,\hat{W}^{\mu-}_B(x)$$

and the difference

$$\begin{aligned}&\sqrt{4\pi E_{\bar{p}}}\left[a_{\bar{p},\lambda}(\infty)-a_{\bar{p},\lambda}(-\infty)\right] \\ &\quad= -i\int_{-\infty}^{\infty}d^4x\,\left(\mathbb{E}^{+\dagger}_n(\rho_+)\right)^\alpha{}_\mu\,\bar{\varepsilon}^{-*}_\alpha(\vec{p},\lambda)\left[\partial_t^2+E_{\bar{p}}^2\right]\hat{W}^{\mu-}_B(x),\end{aligned}$$

is computed. The Fourier analysis

$$\begin{aligned}&\int d^4x\,\left(\mathbb{E}^{+\dagger}_n(\rho_+)\right)^\mu{}_\alpha\,\bar{\varepsilon}^{-*}_\mu(\vec{p},\lambda) \\ &\quad\times\sum_{s_3'}\left(D^\sigma D_\sigma+m_W^2-2eBs_3'\right)\Delta^\alpha{}_\beta(s_3')W^{\beta-}_B(x) \\ &\quad= \int d^4x\,\left(\mathbb{E}^{+\dagger}_n(x)\right)^\mu{}_\alpha\,\bar{\varepsilon}^{-*}_\mu(\vec{p},\lambda)\left[\partial_t^2+E_{\bar{p}}^2\right]W^{\alpha-}_B(x) \\ &\quad= i\sqrt{4\pi E_{\bar{p}}}\left[\hat{a}_{\bar{p},\lambda}(\infty)-\hat{a}_{\bar{p},\lambda}(-\infty)\right]\end{aligned}$$

is performed over the Magnetic Proca Equation. Likewise, an analogous procedure is done for the annihilation operator. The scattering amplitude is

$$\begin{aligned}\langle f|S|i\rangle_B &= \frac{1}{2\pi}\int d^4x_f\left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\bar{s}_3}(x_f)\right)^\mu{}_\alpha\,\bar{\varepsilon}^{-*}_\mu\left(\vec{p}_f,\lambda_f\right) \\ &\times\int d^4x_i\left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\bar{s}_3''}(x_i)\right)^\nu{}_\rho\,\bar{\varepsilon}^{-}_\nu\left(\vec{p}_i,\lambda_i\right) \\ &\times\sum_{s_3'}\left(D^\sigma_{x_f}D_{x_f,\sigma}+m_W^2-2eBs_3'\right)\Delta^\alpha{}_\beta(s_3') \\ &\times\sum_{s_3''}\left(D^{\gamma*}_{x_i}D^*_{x_i,\gamma}+m_W^2-2eBs_3'''\right)\Delta^\rho{}_\delta(s_3''')\mathcal{M}^{\beta\delta}_B(x_f,x_i) \\ &= \frac{1}{2\pi}\int d^4x_f\left(\mathbb{E}^{+\dagger}_{\bar{p}_f,\ell_f-1+\bar{s}_3}(x_f)\right)^\mu{}_\alpha\,\bar{\varepsilon}^{-*}_\mu\left(\vec{p}_f,\lambda_f\right) \\ &\times\int d^4x_i\left(\mathbb{E}^{+}_{\bar{p}_i,\ell_i-1+\bar{s}_3'}(x_i)\right)^\nu{}_\beta\,\bar{\varepsilon}^{-}_\nu\left(\vec{p}_i,\lambda_i\right) \\ &\times\left(\bar{\mathcal{P}}^\sigma_f\bar{\mathcal{P}}^*_{f,\sigma}-m_W^2\right)\left(\bar{\mathcal{P}}^\rho_i\bar{\mathcal{P}}^*_{\rho,i}-m_W^2\right)\mathcal{M}^{\alpha\beta}_B(x_f,x_i).\end{aligned}$$

where $\mathcal{M}^{\alpha\beta}_B(x_f,x_i)\equiv\langle\Omega|\,\mathcal{T}\left\{W^{\alpha-}_B(x_f)W^{\beta+}_B(x_i)\right\}|\Omega\rangle$ is the magnetic matrix element. This is the LSZ Reduction Formula for a charged vector boson in the presence of a constant magnetic field in the $\hat{z}$ direction.

Conclusions

- An analogous derivation of the LSZ Reduction Formula for the case of an external constant magnetic field is presented. The resulting formula, and the derivation in between, maintains the same structure as in the $B=0$ case.

Future Work

- Implementing this result in the physical scenario of a magnetic field in the β decay process $W^+\to e^+\bar{\nu}_e$ could be of interest as the decay rate Γ could be affected by the presence of the magnetic field.
- Extending this calculation for a massive charged spin $\frac{1}{2}$ particle would be interesting.

Acknowledgments

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References

[1] R. Turolla et. al.
Magnetars: the physics behind observations. a review.
Rep. Prog. Phys., 78, 2015.

[2] D. Lai.
Equation of state in metals and cold stars: Evaluation of statistical models.
ApJ, 1991.

[3] Ritam Mallick, Stefan Schramm, Veronica Dexheimer, and Abhijit Bhattacharyya.
On the possibility of rho-meson condensation in neutron stars.
Monthly Notices of the Royal Astronomical Society, 449(2):1347–1351, May 2015.

[4] V. V. Skalozub.
Effective mass of W boson in a magnetic field.
Phys. Atom. Nuclei, 77(7):901–907, July 2014.

[5] Hao Liu, Lang Yu, and Mei Huang.
Charged and neutral vector ρ mesons in a magnetic field.
Phys. Rev. D, 91(1):014017, January 2015.

[6] Claude Itzykson and Jean-Bernard Zuber.
Quantum Field Theory.
International Series in Pure and Applied Physics. McGraw-Hill, New York [usw.], Hamburg [usw.], 1980.

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LSZ Formula in a Magnetic Field

The dispersion amplitude in the presence of a magnetic field should involve the magnetic information

$$\begin{aligned}\langle f|\, S\,|i\rangle_B &= 2\sqrt{E_{\bar{p}_i} E_{\bar{p}_f}}\,\langle\Omega|\, a_{\bar{p}_f}(\infty) a^{\dagger}_{\bar{p}_i}(-\infty)\,|\Omega\rangle \\ &= 2\sqrt{E_{\bar{p}_i} E_{\bar{p}_f}}\,\langle\Omega|\,\mathcal{T}\left\{\,[a_{\bar{p}_f}(\infty) - a_{\bar{p}_f}(-\infty)]\left[a^{\dagger}_{\bar{p}_i}(\infty) - a^{\dagger}_{\bar{p}_i}(-\infty)\right]\,\right\}|\Omega\rangle\,,\end{aligned}$$

and the Fourier expansion on the quantum field is

$$\hat{W}^{\mu-}(x) = \sum\!\!\!\!\!\!\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E}}\sum_{\lambda=1}^{3}\left[\hat{a}^{\lambda}_{\bar{p}}\mathbb{E}^{+\mu}_{n,\;\alpha}(\rho_+)\varepsilon^{\alpha-}_{\lambda}(\bar{p}) + \hat{b}^{\lambda\dagger}_{\bar{p}}\mathbb{E}^{-\mu}_{n,\;\alpha}(\rho_-)\left(\varepsilon^{\alpha+}_{\lambda}(\bar{p})\right)^*\right].$$

Here, the spin projection matrix

$$\Delta^{\mu}{}_{\nu}(s_3) \equiv= \begin{pmatrix} 1-s_3^2 & 0 & 0 & 0 \\ 0 & \frac{s_3^2}{2} & \frac{-is_3}{2} & 0 \\ 0 & \frac{is_3}{2} & \frac{s_3^2}{2} & 0 \\ 0 & 0 & 0 & 1-s_3^2 \end{pmatrix}.$$

LSZ Formula in a Magnetic Field: $\hat{a}_{\bar{p},\lambda}$ and $\hat{a}^{\dagger}_{\bar{p},\lambda}$

The Fourier analysis over the quantum field is

$$\int d^3x \sum_{s_3'} X^{+}_{\ell' -1+\tilde{s}_3'}(\rho_+')\Delta^{\alpha}{}_{\mu}(s_3')e^{i\left(p^{2\prime} x_2 + p^{3\prime} x_3\right)}\left(i\partial_t + E_{\bar{p}'}\right)\hat{W}^{\mu -}(x) = -\sqrt{4\pi E_{\bar{p}'}}\,\hat{a}_{\bar{p}',\lambda'}\,e^{-iE_{\bar{p}'}x_0}$$

With this, the creation and annihilation operators become

$$\hat{a}_{\bar{p},\lambda} = \frac{-1}{\sqrt{4\pi E_{\bar{p}}}}\int d^3x\ \left(\mathbb{E}^{+\dagger}_{n}(\rho_+)\right)^{\alpha}{}_{\mu}\,\bar{\varepsilon}^{-*}_{\alpha}\left(\vec{\bar{p}},\lambda\right)\left(i\partial_t + E_{\bar{p}}\right)\hat{W}^{\mu -}(x),$$

$$\hat{a}^{\dagger}_{\bar{p},\lambda} = \frac{-1}{\sqrt{4\pi E_{\bar{p}}}}\int d^3x\ \mathbb{E}^{+\alpha}_{n,\ \mu}(\rho_+)\bar{\varepsilon}^{-}_{\alpha}\left(\vec{\bar{p}},\lambda\right)\left(i\partial_t - E_{\bar{p}}\right)\hat{W}^{\mu +}(x).$$

LSZ Formula in a Magnetic Field: Differences

The differences are then

$$\sqrt{4\pi E_p}\left[a_{\bar{p},\lambda}(\infty) - a_{\bar{p},\lambda}(-\infty)\right] = -i\int_{-\infty}^{\infty} d^4x \, \left(\mathbb{E}_n^{+\dagger}(\rho_+)\right)^{\alpha}{}_{\mu} \bar{\varepsilon}_{\alpha}^{-*}(\vec{\bar{p}},\lambda)\left[\partial_t^2 + E_{\bar{p}}^2\right]\hat{W}^{\mu-}(x),$$

$$\sqrt{4\pi E_p}\left[a^{\dagger}_{\bar{p},\lambda}(\infty) - a^{\dagger}_{\bar{p},\lambda}(-\infty)\right] = i\int_{-\infty}^{\infty} d^4x \, \mathbb{E}_{n,\ \mu}^{+\alpha}(\rho_+)\bar{\varepsilon}_{\alpha}^{-}(\vec{\bar{p}},\lambda)\left[\partial_t^2 + E_{\bar{p}}^2\right]\hat{W}^{\mu+}(x),$$

LSZ Formula in a Magnetic Field: Proca Fourier Analysis

On the other hand, the Magnetic Proca Equation Fourier Analysis gives

$$\begin{aligned}
\int d^4x \ \left(\mathbb{E}_n^{+\dagger}(\rho_+)\right)^{\mu}{}_{\alpha}\, \bar{\varepsilon}_{\mu}^{-*}\left(\vec{\bar{p}},\lambda\right) \sum_{s_3'} \left(D^{\sigma} D_{\sigma} + m_W^2 - 2eBs_3'\right) \Delta^{\alpha}{}_{\beta}(s_3') W^{\beta -}(x) \\
= \int d^4x \ \left(\mathbb{E}_n^{+\dagger}(x)\right)^{\mu}{}_{\alpha}\, \bar{\varepsilon}_{\mu}^{-*}\left(\vec{\bar{p}},\lambda\right) \left[\partial_t^2 + E_{\bar{p}}^2\right] W^{\alpha -}(x) \\
= i\sqrt{4\pi E_{\bar{p}}}\left[\hat{a}_{\bar{p},\lambda}(\infty) - \hat{a}_{\bar{p},\lambda}(-\infty)\right]
\end{aligned}$$