Non-Abelian Casimir Effect in the Curci-Ferrari Model

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Outline

The Curci-Ferrari Model

Yang-Mills in Landau gauge $\partial_\mu A^a_\mu = 0$ with phenomenological mass:

$$S_{\mathsf{CF}} = \int \mathrm{d}^4 x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a D_\mu c^a + i h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 A^a_\mu A^a_\mu\right]$$

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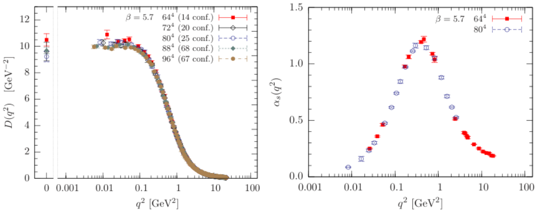
CF perturbatively models the effect of Gribov copies in YM as seen on the lattice (cfr. Peláez, Reinosa, Serreau, Tissier, Wschebor, *A window on infrared QCD with small expansion parameters*, 2021):

- gluon propagator with non-zero value for vanishing momenta: screening mass
- IR safe: renormalization scheme without Landau pole



Bogolubsky, Ilgenfritz, Muller-Preussker, Sternbeck, 2009

Quadratic approximation:

$$S'_{\mathsf{CF}} = \int \mathrm{d}^4x \left[\frac{1}{4}\mathcal{F}^a_{\mu\nu}\mathcal{F}^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \frac{1}{2}m^2 A^a_\mu A^a_\mu\right],$$

with $\mathcal{F}^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$.

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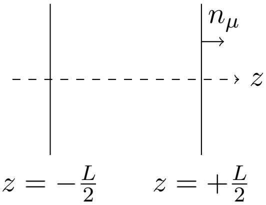

PMC boundary conditions: $\left. \mathcal{F}^a_{\mu\nu} n_\nu \right|_{z=\pm\frac{L}{2}} = 0$

PEC boundary conditions: $\left. \tilde{\mathcal{F}}^a_{\mu\nu} n_\nu \right|_{z=\pm\frac{L}{2}} = 0$

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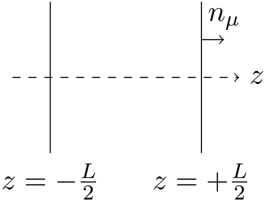

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$$S_{\mathsf{BC}} = \int \mathrm{d}^4x \left[\Big(b_i^{-,a}(\overleftarrow{x})\delta(z+L/2) + b_i^{+,a}(\overleftarrow{x})\delta(z-L/2)\Big)\mathcal{F}^a_{i\nu} n_\nu\right]$$

$$\equiv \int \mathrm{d}^4x \,[b_i^{\gamma,a}(\overleftarrow{x})\delta(z-z^\gamma)\mathcal{F}^a_{i\nu} n_\nu]$$

Casimir Energy

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Then shift $A^a_\mu(k) \to A^a_\mu(k) - v^b_\rho(k) \left(K^{-1}\right)^{ab}_{\mu\nu}$ to get

$$S = \frac{1}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[A^a_\mu(k) K^{ab}_{\mu\nu} A^b_\nu(-k) - v^a_\mu(k) \left(K^{-1}\right)^{ab}_{\mu\nu} v^b_\nu(-k)\right].$$

Rewrite the effective boundary action

$$S_{\mathsf{bnd}} = -\frac{1}{2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\left[v^a_\mu(k)\left(K^{-1}\right)^{ab}_{\mu\nu} v^b_\nu(-k)\right] = \frac{1}{2}\int \frac{\mathrm{d}^3 \overleftarrow{k}}{(2\pi)^3}\, b^{\gamma,a}_i(\overleftarrow{k})\,\mathbb{K}^{\gamma\lambda}_{ij}(\overleftarrow{k})\, b^{\lambda,a}_j(-\overleftarrow{k}),$$

then

$$Z = (\mathrm{Det}\,\mathbb{K})^{-1/2}.$$

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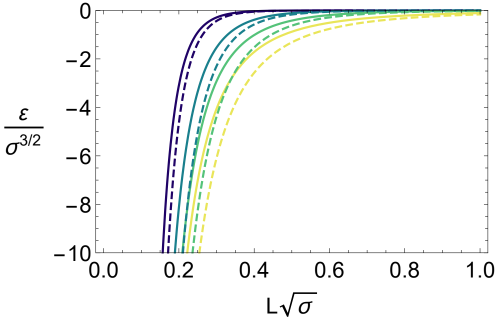
Using $\log \mathrm{Det}\, A = \mathrm{Tr}\log A$, we find

$$\mathcal{E} = \frac{1}{2}\int \frac{d^3\overleftarrow{k}}{(2\pi)^3}\log\left(\det \mathbb{K}(\overleftarrow{k})\right).$$

At last we get

$$\mathcal{E}_{\text{PEC}} = \frac{(N^2-1)}{2\pi^2}\int_0^\infty dk\, k^2 \log\left[1 - e^{-2L\sqrt{k^2+m^2}}\right]$$
$$= -2(N^2-1)\frac{m^2}{8\pi^2 L}\sum_{n=1}^{\infty}\frac{1}{n^2}K_2(2nmL),$$

$$\mathcal{E}_{\text{PMC}} = \frac{3}{2}\mathcal{E}_{\text{PEC}}.$$



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Discontinuity in massless limit

As one would expect, quadratic CF reduces to copies of Maxwell in massless limit:

$$\lim_{m\to 0} \mathcal{E}_{\mathsf{PEC}} = (N^2 - 1)\left(-\frac{\pi^2}{720L^3}\right) \equiv \mathcal{E}_{\mathsf{Maxwell}},$$

but not for PMC:

$$\lim_{m\to 0} \mathcal{E}_{\mathsf{PMC}} = \frac{3}{2}\mathcal{E}_{\mathsf{Maxwell}}$$

$\to$ *van Dam–Veltman–Zakharov discontinuity* for PMC.

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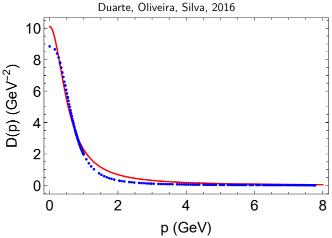
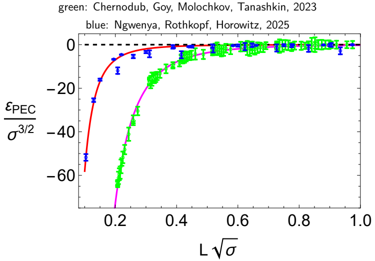
Origin: auxiliary gauge freedom in auxiliary field

$$\begin{aligned} b \to b + \partial\phi \implies \ & \mathsf{PEC:} \quad bn\tilde{\mathcal{F}} \to bn\tilde{\mathcal{F}} - \phi n \overbrace{\partial\tilde{\mathcal{F}}}^{0} \\ \implies \ & \mathsf{PMC:} \quad bn\mathcal{F} \to bn\mathcal{F} - \phi n \underbrace{\partial\mathcal{F}}_{m^2 A} \end{aligned}$$

Comparison with PEC lattice data in (3+1)D

2 parameters to fit: mass m, global multiplicative constant C.



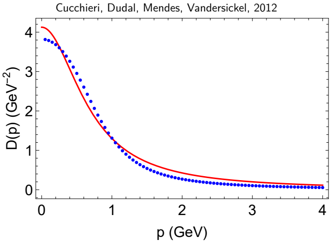


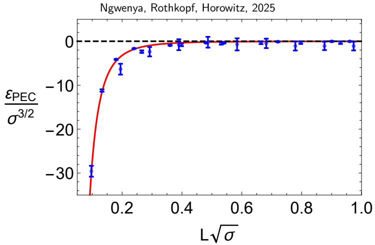
$SU(3)$

$m = 0.54$ GeV

$C^{\text{green}} = 5.63$

$C^{\text{blue}} = 0.54$





$SU(2)$

$m = 0.68$ GeV

$C = 0.69$

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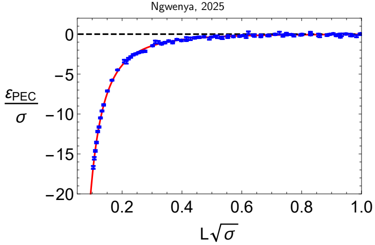
Comparison with PEC lattice data in (2+1)D

2 parameters to fit: mass m, global multiplicative constant C.

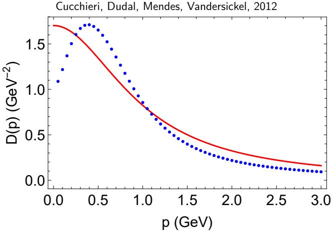
Not available

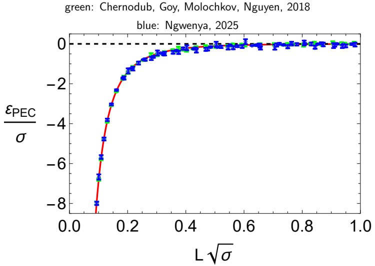


$SU(3)$

$m = 0.76$ GeV

$C = 1.01$





$SU(2)$

$m = 0.97$ GeV

$C = 1.07$

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David Dudal, Philipe De Fabritiis, Sebbe Stouten,
Non-Abelian Casimir energy in the Curci-Ferrari model through a functional approach,
arXiv 2509.07256,
JHEP

Summary

- CF action + boundary conditions $\rightarrow$ effective boundary theory $\rightarrow$ Casimir energy
- PEC $\neq$ PMC
- Analytic result fits PEC lattice data well

Outlook

- Wanted! Lattice data for PMC Casimir energy:
  confirm or debunk PEC $\neq$ PMC Casimir energy
  (Karabali, Maj, Nair, *On Casimir effect in Yang-Mills theories in three and four dimensions*, 2025)
- Compute non-abelian Casimir energy via Gribov-Zwanziger action: role of new gauge freedom?

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