Meson Observables from AdS/QCD via the Segrè Formula

Alfredo Vega

6th Workshop on Nonperturbative Aspects of QCD.
UTFSM. Valparaíso, Chile

In collaboration with Miguel Ángel Martín and Saulo Diles

December 2, 2025

Outline

General Ideas about AdS/QCD Models

Masses and Decay Constants in AdS/QCD Models

Segrè Formula and AdS/QCD models

Final Comments

General Ideas about AdS/QCD Models

⋆ Basic ideas about AdS/QCD models.

- Gauge/Gravity duality can be used to study hadron properties
- There are two main ways to extend AdS/CFT through to QCD:
  - Top-Down Approach.
    Starting from string theory, taking a low-energy limit, and compactifying extra dimensions, one tries to obtain a QCD-like theory at the boundary.
  - Bottom-Up Approach.
    Hadron properties are used to construct a model in a higher-dimensional gravitational background.
- A dictionary exists that relates quantities on both sides of the holographic correspondence.
- In the bottom-up approach, we consider

$$S = \int d^{d+1}x \sqrt{g}\, e^{-\Phi(z)} \left(\mathcal{L}_{Part} + \mathcal{L}_{Int} \right)$$

Masses and Decay Constants in AdS/QCD Models

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Part} + \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Int}$$

Example: Scalar hadrons case [1]

$$\mathcal{L} = g^{MN}\,\partial_M\psi\,(x,z)\,\partial_N\psi\,(x,z) + m_5^2\psi^2\,(x,z)\,,$$

with $M, N = 0, 1, 2, 3, z$.

- z corresponds to the holographic coordinate.
- $\phi\,(z)$ is a dilaton field introduced to discretize the spectrum (you can use a hard cut-off also).
- m_5 is the bulk mass. It is related to the dimension of operators that create hadrons. For scalars $m_5^2 R^2 = \Delta\,(\Delta - 4)$

Δ_0	(nQ)(mG)
3	(2Q)
4	(2G)

[1] e.g., see A. V and I. Schmidt Phys.Rev.D 78 (2008) 017703

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Part} + \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Int}$$

Example: Scalar hadrons case

$$\mathcal{L} = g^{MN}\,\partial_M\Psi\left(x,z\right)\,\partial_N\Psi\left(x,z\right) + m_5^2\Psi^2\left(x,z\right),$$

A usual choice for hadrons in vacuum at zero temperature is

$$d^2s = e^{2A(z)}\left(z\right)\eta_{MN}dx^M dx^N,$$

where $e^{2A(z)}$ is a warp factor and $\eta_{MN} = diag(-1,1,1,1,1)$.
From this action we obtain an equation of motion in 5 dimensions for scalars, and we use the transformation $\Psi\left(X,z\right) = e^{-iPX}f\left(z\right)$, where P and X correspond to momentum and position on the 4D boundary.

$$-f''\left(z\right) + B'(z)f'\left(z\right) + e^{2A(z)}m_5^2 f\left(z\right) = M^2 f\left(z\right),$$

where $B(z) = \phi(z) - 3A(z)$ and $P^2 = M^2$, i.e., M is the mass of the hadron studied in these kinds of models.

Considering the transformation

$$f(z) = e^{\frac{1}{2}B(z)}\psi(z),$$

we obtain a Schrödinger like equation

$$\left(-\partial_z^2 + V(z)\right)\psi(z) = M^2\psi(z).$$

In terms of AdS metric ($A(z) = Ln(1/z)$), $\phi(z)$ and m_5, potential is

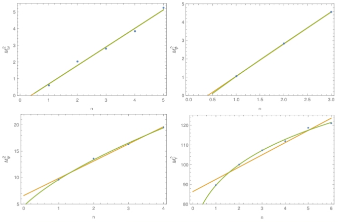
$$V(z) = \frac{15}{4z^2} + \frac{m_5^2 R^2}{z^2} - \frac{3}{2z}\phi'(z) + \frac{1}{4}\phi'^2(z) - \frac{1}{2}\phi''(z).$$

In vector the sector, the procedure is similar, but potential for vector mesons is

$$V(z) = \frac{3}{4z^2} + \frac{1}{2z}\phi'(z) + \frac{1}{4}\phi'^2(z) - \frac{1}{2}\phi''(z).$$

- $\phi(z) = cte$ with a hard cutoff at z_0. (Hard Wall model)
- $\phi(z) = cz^2$ (Traditional Soft Wall Model)
- $\phi(z) = (\kappa z)^{2-\alpha}$ (Non quadratic dilaton) [2]

	Linear Regge Trajectory: $M^2 = a(n+b)$			Non Linear Regge Trajectory ($M^2 = a(n+b)^\nu$)			
Meson	a	b	R^2	a	b	ν	R^2
ω	1.1074	−0.3781	0.9978	1.1078	−0.3784	0.9998	0.9978
ϕ	1.7595	−0.4048	0.9999	1.8545	−0.4524	0.9617	1.000
ψ	3.2607	2.0259	0.9997	7.6516	0.4460	0.6249	0.9999
Υ	6.2015	13.9182	0.9996	85.3116	0.2849	0.1917	0.9999



[2] M. A. Martín and A. V, Phys.Rev.D 102 (2020) 4, 046007

Let us consider the holographic calculation of decay constants.
If we have AdS modes that satisfy

$$-\psi''(z) + B'(z)\psi'(z) + e^{2A(z)} m_5^2 \psi(z) = M^2 \psi(z),$$

where $B(z) = \phi(z) - A(z)$ for vector mesons. The bulk-to-boundary propagator can be decomposed in terms of the bulk normalizable modes $\psi_n(z, q)$ as poles and residues as

$$\Pi_V\left(-q^2\right) = \sum_{n=1}^{\infty} \frac{f_n^2}{p^2 - m_n^2 + i\epsilon}.$$

corresponding to the vector meson mass spectrum, and the holographic decay constants f_n, defined as

$$f_n^2 = \frac{1}{g_5^2 M_n^2} \lim_{\varepsilon \to 0} e^{-2B(\varepsilon)} \left|\psi_n(\varepsilon, q)\right|^2.$$

Masses and Decay Constants in AdS/QCD Models

Experimental masses and the corresponding decay constants for the charmonium S-wave resonances.

	Charmonium data	
State	Mass (MeV)	Decay constant (MeV)
$1S$	3096.916 ± 0.011	416 ± 5.3
$2S$	3686.109 ± 0.012	296.1 ± 2.5
$3S$	4039 ± 1	187.1 ± 7.6
$4S$	4421 ± 4	160.8 ± 9.7

Previously we present a dilaton that reproduce well the mass spectrum, but unfortunately, they are not able to reproduce the behavior of the decay constant. A dilaton that reproduces the decay-constant behavior is

$$\phi(z) = k^2 z^2 + Tanh\left(\frac{1}{Mz} - \frac{k}{\sqrt{\Gamma}}\right),$$

where k, M and Γ are parameters [3].

[3] N.R.F.Braga et al. PhysicsLettersB774(2017)476–481

We consider [4],

$$\phi(z) = (\kappa z)^{2-\alpha} + Mz + Tanh\left(\frac{1}{Mz} - \frac{\kappa}{\sqrt{\Gamma}}\right),$$

Charmonium States $I^G(J^{PC}) = 0^+(1^{--})$							
Parameters:			$\kappa = 1.8$ GeV, $M = 1.7$ GeV, $\sqrt{\Gamma} = 0.53$ GeV and $\alpha = 0.54$				
n	*State*	M_{Exp} *(MeV)*	M_{Th} *(MeV)*	$\%M$	f_{Exp} *(MeV)*	f_{Th} *(MeV)*	$\%f$
1	J/ψ	3096.916 ± 0.011	3140.1	1.42	416.16 ± 5.25	412.4	1.4
2	$\psi(2S)$	3686.109 ± 0.012	3656.9	0.9	296.08 ± 2.51	272.7	8.0
3	$\psi(4040)$	4039 ± 1	4055.7	0.4	187.13 ± 7.61	201.8	7.8
4	$\psi(4415)$	4421 ± 4	4376	0.9	160.78 ± 9.70	164.1	2.0
Nonlinear Regge Trajectory:					$M_n^2 = 8.097(0.39 + n)^{0.58}$ GeV^2 with $R^2 = 0.999$		

[4] M. A. Martín, S. Diles and A. V, PRD 103, 086008 (2021)

We consider [5],

$$\phi(z) = (\kappa z)^{2-\alpha} + Mz + Tanh\left(\frac{1}{Mz} - \frac{\kappa}{\sqrt{\Gamma}}\right),$$

Bottomonium States $I^G(J^{PC}) = 0^+(1^{--})$								
Parameters:			$\kappa = 9.9$ GeV, $M = 2.74$ GeV, $\sqrt{\Gamma} = 1.92$ GeV and $\alpha = 0.863$					
n	*State*	M_{Exp} *(MeV)*	M_{Th} *(MeV)*	$\%M$	f_{Exp} *(MeV)*	f_{Th} *(MeV)*	$\%f$	
1	$\Upsilon(1S)$	9460.3 ± 0.26	9506.5	0.5	714.99 ± 2.40	718.8	0.5	
2	$\Upsilon(2S)$	10023.26 ± 0.32	9892.9	1.0	497.37 ± 2.23	575.7	16	
3	$\Upsilon(3S)$	10355.2 ± 0.5	10227.2	1.2	430.11 ± 1.94	413.0	4.0	
4	$\Upsilon(4S)$	10579.4 ± 1.2	10497.5	0.8	340.65 ± 9.08	324.3	4.8	
5	$\Upsilon(10860)$	$10889.9^{+3.2}_{-2.6}$	10721.5	1.5	–	–	–	
6	$\Upsilon(11020)$	$10992.9^{+10.0}_{-3.1}$	10912.7	0.7	–	–	–	
Nonlinear Regge Trajectory:			$M_n^2 = 7.376(1.31 + n)^{0.24}$ GeV^2 with $R^2 = 0.999$					

[5] M. A. Martín, S. Diles and A. V, PRD 103, 086008 (2021)

Segrè Formula and AdS/QCD models [6]

[6] S. Diles, M. A. Martín and A. V, Phys. Rev. D 112, 056010 (2025).

In the context of non-relativistic quantum mechanical modeling of heavy quarkonium, we identify a relationship, referred to as the Segrè formula, between the radial wave function at the origin for a radial excitation of the s-wave state $\Psi_n(0)$ and the dependence of the binding energy on the excitation level E_n. This relationship establishes that [7]:

$$|\Psi_n(0)|^2 = \frac{(2\mu)^{\frac{3}{2}}}{4\pi^2} E_n^{\frac{1}{2}} \frac{d}{dn} E_n.$$

Considering

$$f_n^2 = \frac{12\,\pi\, Q_q}{M_n}\, |\Psi_n(0)|^2 \text{ and } M_n = 2m_q + E_n.$$

These expressions involve quantities that can be calculated in AdS if you have a good dilaton. In our case, we consider the Braga non-quadratic dilaton

$$\phi(z) = (\kappa z)^{2-\alpha} + Mz + Tanh\left(\frac{1}{Mz} - \frac{\kappa}{\sqrt{\Gamma}}\right),$$

⋆ Calculation of constituent quark masses

$$\frac{|\Psi_{n_1}(0)|^2}{|\Psi_{n_2}(0)|^2} = \frac{M_{n_1} f_{n_1}^2}{M_{n_2} f_{n_2}^2} = \frac{\left(M_{n_1} - 2\, m_q\right)^{1/2}}{\left(M_{n_2} - 2\, m_q\right)^{1/2}} \frac{\frac{d\, M_n}{d\, n}\Big|_{n=n_1}}{\frac{d\, M_n}{d\, n}\Big|_{n=n_2}}.$$

$$m_c = 1.97 \pm 0.28 \text{ GeV and } m_b = 5.21 \pm 0.36 \text{ GeV}.$$

⋆ Three-gluons decay and the fine structure constant of the strong interaction

$$\Gamma_{V_n \to \mathrm{ggg}} = \frac{40(\pi^2-9)}{81 m_q^2}\, \alpha_{\mathrm{s}}^3\, |\Psi_n(0)|^2 = \frac{10(\pi^2-9)\, f_1^2\, M_1}{243\, \pi\, m_q^2\, Q_q} \alpha_s(q)^3.$$

$$\alpha_s(c) = 0.293 \pm 0.008 \text{ and } \alpha_s(b) = 0.192 \pm 0.009.$$

⋆ The first-order corrections on α_s

$$\Gamma_{V_1 \to ggg} = \frac{10(\pi^2 - 9)\, f_1^2\, M_1}{243\, \pi\, m_q^2\, Q_q} \alpha_s^3 \left[1 + \frac{\alpha_s}{\pi}\left(\frac{16}{3} - \lambda_q\right)\right],$$

where λ_q encodes a characteristic energy associated with each meson family and depends only on the number of lighter quark flavors. For charmonium $\lambda_c = 3.7$, and for bottomonium $\lambda_b = 4.9$. Next, we obtain the first-order α_s for the charm and bottom quark families as:

$$\alpha_s^1(c) = 0.280 \pm 0.007 \text{ and } \alpha_s^1(b) = 0.191 \pm 0.009.$$

TABLE I. Summary of the results for the three-gluon, three-photon, and $g\gamma\gamma$ decays for the heavy quarkonium vector (1^3S_n) trajectories. Experimental results are read from PDG [33].

Quarkonium	State	$\Gamma_{V\to ggg}$		$\Gamma_{V\to\gamma\gamma\gamma}$		$\Gamma_{V\to\gamma gg}$	
$c\bar{c}$	$n^{2S+1}L_J$	Experimental KeV	Theoretical KeV	Experimental eV	Theoretical eV	Experimental KeV	Theoretical KeV
	1^3S_1	59.3 ± 1.43[a]	59.3 ± 1.43[a]	1.07 ± 0.20	0.96 ± 0.09	8.15 ± 1.31	3.79 ± 0.36
	2^3S_1	30.3 ± 4.9	30.22 ± 5.88	⋯	0.49 ± 0.09	2.94 ± 0.84	1.93 ± 0.35
	3^3S_1	⋯	18.36 ± 3.57	⋯	0.30 ± 0.05	⋯	1.17 ± 0.21
	4^3S_1	⋯	13.09 ± 1.59	⋯	0.21 ± 0.02	⋯	0.83 ± 0.08
$b\bar{b}$	$n^{2S+1}L_J$	Experimental KeV	Theoretical KeV	Experimental eV	Theoretical eV	Experimental KeV	Theoretical KeV
	1^3S_1	44.13 ± 1.09[a]	44.13 ± 1.09[a]	⋯	0.0627 ± 0.0095	1.18 ± 0.32	1.15 ± 0.19
	2^3S_1	18.80 ± 1.59	29.46 ± 11.12	⋯	0.0419 ± 0.0147	0.60 ± 0.10	0.77 ± 0.28
	3^3S_1	7.25 ± 0.84	6.21 ± 1.33	⋯	0.0088 ± 0.0014	0.20 ± 0.04	0.16 ± 0.03
	4^3S_1	⋯	4.14 ± 0.91	⋯	0.0059 ± 0.0010	⋯	0.11 ± 0.02

Summary of data

[a]The asterisk (*) indicates that we used this data for fitting the corresponding decay width.

Final Comments

Final Comments

- Constituent Quark models give an opportunity to access a more hadronic phenomenology with AdS/QCD models. In the cases considered, the key was the use of Segrè Formula as a complement to AdS/QCD results.
- With the help of the Segrè Formula, it was possible to calculate new properties and to do some predictions (constituent quark masses, three-photon decay widths, α_s from the three-gluon channel, three-gluon and mixed γgg decay widths for ground and excited heavy-quarkonium states.
- Results are not highly accurate, but they are consistent and of the correct order of magnitude, and shows that holographic phenomenology now extends to decay widths, strong-coupling extraction, and quark-mass determination.
- Non-relativistic quantum mechanics provides a useful bridge for deriving new holographic observables.


That's all Folks!