Thermal and baryon density modifications to the σ-boson propagator

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Phys. Lett. B 869 139873 (2025)

December, 2025

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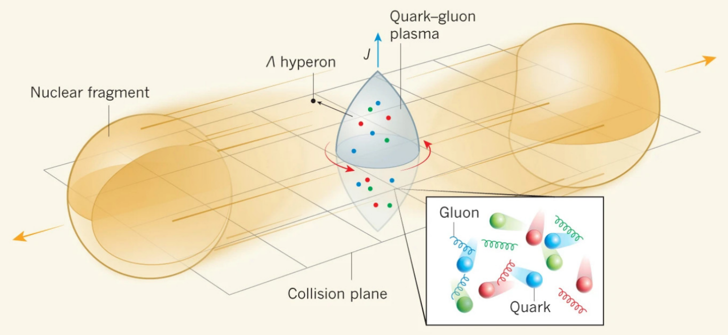
Overview

Hot, dense, swirling QCD matter in HICs

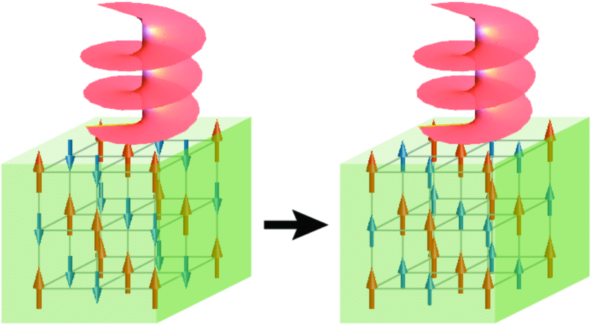


Non-central collisions have large angular momentum $L \sim 10^5 \hbar$.
Shear forces in initial condition introduce vorticity to the QGP.

Spin-orbit coupling: spin alignment, or polarization, along the direction of the vorticity - on average - parallel to J.

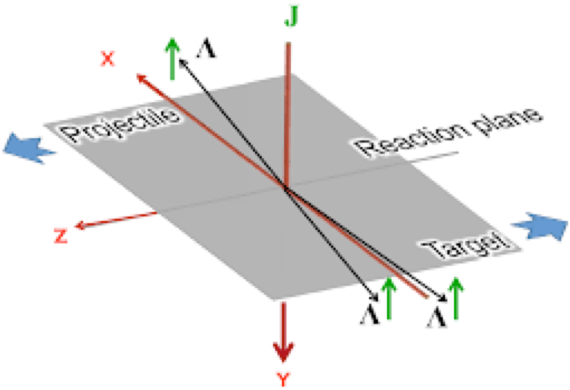
Barnett effect

- A spinning ferromagnet experiences a change of its magnetization



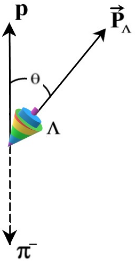
Global Λ polarization

- Λ's are produced globally polarized in a heavy-ion reaction

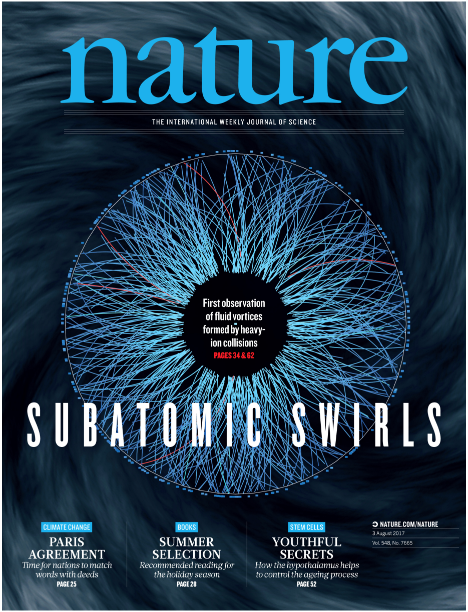


Good swirling-ness probe in HICs: Λ hyperon

- Particle Data Group:
  - $m_\Lambda = 1115.683 \pm 0.006$ MeV
  - $\tau = 2.632 \pm 0.020 \times 10^{-10}$ s (~ 7.9 cm at c)
  - $\Gamma_1(\Lambda \to p\pi^-) = (63.9 \pm 0.5)\%$
  - $\Gamma_2(\Lambda \to n\pi^0) = (35.8 \pm 0.5)\%$
- Advantages:
  - lightest hyperon with s content
  - long lifetime: good for fiducial track/reco
  - parity-violating weak decay - sort of self-analyzing
  - decay dist not-isotropic: p going off in the direction of Λ spin



Hot, dense, whirly QCD matter



STAR Collaboration, Nature 548 (2017)

$$\omega \approx (9 \pm 1) \times 10^{21}\ \mathrm{s}^{-1}$$

How whirly is this?

superfluid nanodroplets $10^{7}\ \mathrm{s}^{-1}$

turbulent flow in superfluid He-II $10^{2}\ \mathrm{s}^{-1}$

rotating, heated soap bubbles used to model climate change $10^{2}\ \mathrm{s}^{-1}$

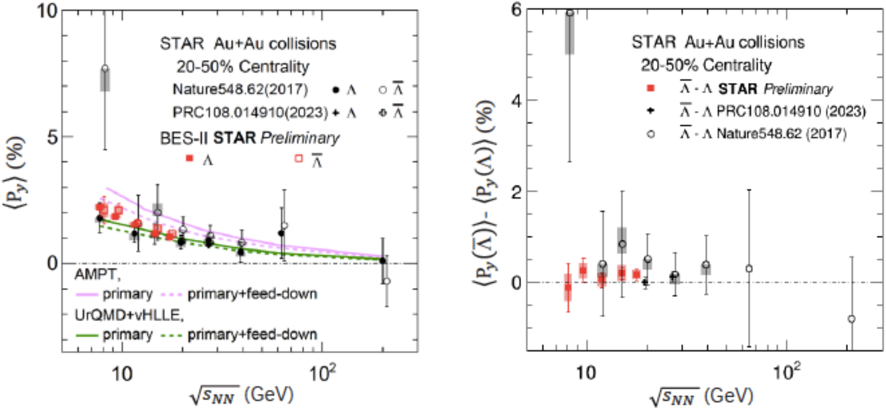
supercell tornado cores $10^{-1}\ \mathrm{s}^{-1}$

the Great Red Spot of Jupiter $10^{-4}\ \mathrm{s}^{-1}$

large-scale terrestrial atmospheric patterns $10^{-7} - 10^{-5}\ \mathrm{s}^{-1}$

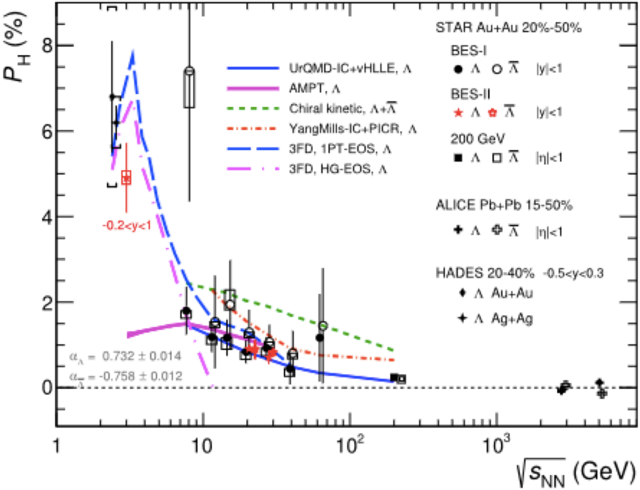
solar subsurface flow $10^{-7}\ \mathrm{s}^{-1}$

Features of Λ and $\overline{\Lambda}$ polarization excitation functions



EPJ Web of Conferences 316, 06013 (2025) SQM 2024

Features of Λ and $\overline{\Lambda}$ polarization excitation functions



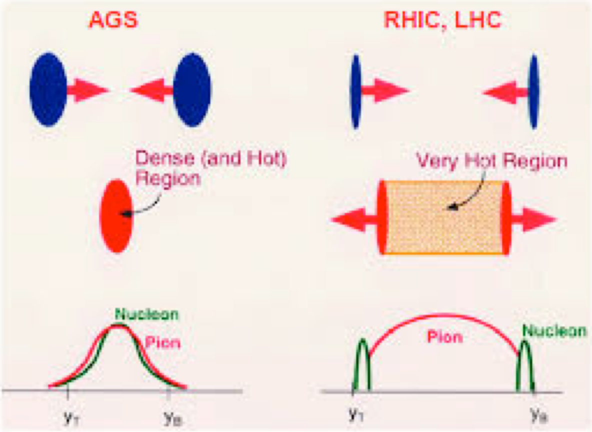
Int. J. Mod. Phys, E 33, 2430010 (2024)

Features of Λ and $\overline{\Lambda}$ polarization excitation functions

The large $\overline{\Lambda}$ polarization in the STAR 2017 Nature paper is now understood to be inflated by an outdated value of the weak decay parameter $\alpha_{\overline{\Lambda}}$, and when modern decay-parameter measurements (BES-III, 2019–2022) are used, the apparent excess of $\overline{\Lambda}$ polarization largely disappears.

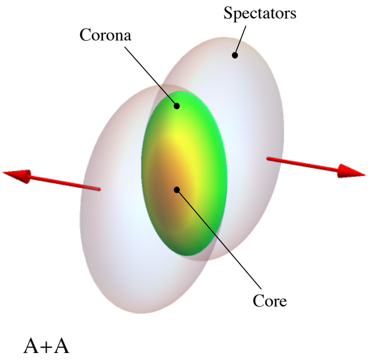
Hot vs Hot and Dense

- In a heavy-ion reaction, the interaction region has different properties depending on the energy of the colliding nuclei



Λ, $\overline{\Lambda}$ global polarization from a two-component source

Non-central heavy-ion collision of a symmetric system:
Λ, $\overline{\Lambda}$s from core via QGP processes
Λ, $\overline{\Lambda}$s from corona via n + n reactions



$$N_\Lambda = \overbrace{N_{\Lambda\,\mathrm{QGP}}}^{core} + \overbrace{N_{\Lambda\,\mathrm{REC}}}^{corona}$$

Polarization asymmetry -spin alignment asymmetry- of any baryon species produced in high-energy reactions

$$\mathcal{P} = \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

$N^\uparrow$ and $N^\downarrow$ baryons with spin aligned and opposite to a given direction.

A.A., I. Dominguez, I. Maldonado, M. E. Tejeda- Yeomans,
Particles 6, 405 (2023); , Rev. Mex. Fis. Suppl. 3, 040914 (2022); PRC 105, 034907 (2022); PLB 810, 135818 (2020)

Λ, $\bar{\Lambda}$ polarization

- We introduce the definitions

$$w = N_{\bar{\Lambda}\,\mathrm{REC}}/N_{\Lambda\,\mathrm{REC}} \quad \text{and} \quad w' = N_{\bar{\Lambda}\mathrm{QGP}}/N_{\Lambda\mathrm{QGP}}$$

(can be estimated either from data or from simple considerations)

- Define the intrinsic polarizations

$$\begin{aligned} z &= \frac{(N^{\uparrow}_{\Lambda\,\mathrm{QGP}} - N^{\downarrow}_{\Lambda\,\mathrm{QGP}})}{N_{\Lambda\,\mathrm{QGP}}} \\ \bar{z} &= \frac{(N^{\uparrow}_{\bar{\Lambda}\,\mathrm{QGP}} - N^{\downarrow}_{\bar{\Lambda}\,\mathrm{QGP}})}{N_{\bar{\Lambda}\,\mathrm{QGP}}}, \end{aligned}$$

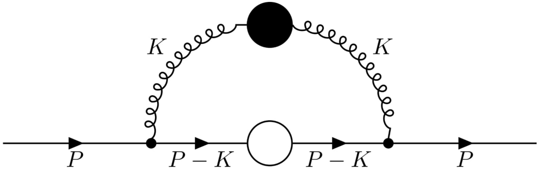
- Therefore,

$$\mathcal{P}^{\Lambda} = \frac{\mathcal{P}^{\Lambda}_{\mathrm{REC}} + z\frac{N_{\Lambda\ \mathrm{QGP}}}{N_{\Lambda\ \mathrm{REC}}}}{\left(1 + \frac{N_{\Lambda\ \mathrm{QGP}}}{N_{\Lambda\ \mathrm{REC}}}\right)}, \quad \mathcal{P}^{\bar{\Lambda}} = \frac{\mathcal{P}^{\bar{\Lambda}}_{\mathrm{REC}} + \bar{z}\left(\frac{w'}{w}\right)\frac{N_{\Lambda\ \mathrm{QGP}}}{N_{\Lambda\ \mathrm{REC}}}}{\left(1 + \left(\frac{w'}{w}\right)\frac{N_{\Lambda\ \mathrm{QGP}}}{N_{\Lambda\ \mathrm{REC}}}\right)}.$$

Relaxation time: inverse of imaginary part of self-energy

Ingredients

- Imaginary part of rotating quark self-energy Σ immersed in a thermal QCD medium
- Thermal gluon propagator in HTL approximation
- **Propagator of a quark in a rotating environment**

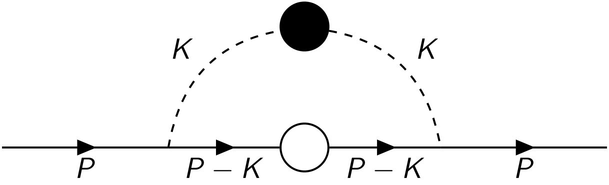


$$
\begin{aligned}
\Gamma(p_0) &= f_F(p_0)\mathrm{Tr}\left[(O^+ + O^-)\mathrm{Im}\Sigma\right] \\
&= f_F(p_0)\mathrm{Tr}\left[\mathrm{Im}\Sigma\right] \\
O^{\pm} &= \frac{1}{2}\left[1 \pm i\gamma_1\gamma_2\right]
\end{aligned}
$$

In nuclear matter, interactions modelled by σ exchange

Ingredients

- Imaginary part of rotating Λ self-energy Σ immersed in a thermal nuclear environment
- Thermal σ propagator accounting for the fact that the nucleon mass $M_N \gg T$
- **Propagator of a Λ in a rotating environment**



T- and μ_B-dependent σ propagator

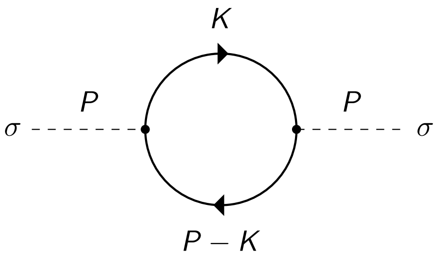
- In the **linear Relativistic Mean Field model**, the Lagrangian density that describes the coupling of nucleons and σ is

$$\mathcal{L}=\bar{\psi}\left[i\gamma^\mu\partial_\mu - M_N - g_\sigma\sigma\right]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2$$

- corona as a medium where nucleons and σs interact at finite temperature T and baryon chemical potential μ, the one-loop **effective σ propagator** Δ^* can be written, in the imaginary time formalism of finite temperature field theory, as

$$\Delta^*(i\omega, p) = \frac{1}{\omega^2 + p^2 + m_\sigma^2 + \Pi}$$

T- and μ_B-dependent σ propagator



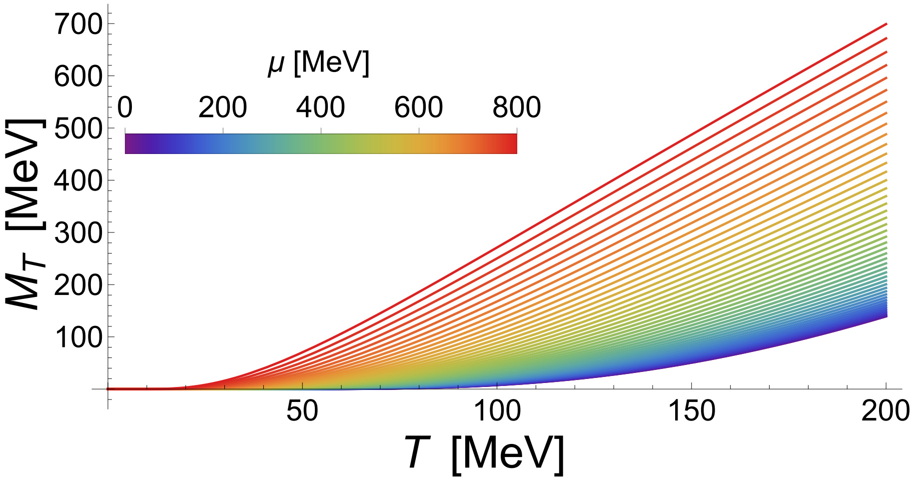
$$\Pi = -g_\sigma^2 \sum_n \int \frac{d^3k}{(2\pi)^3}\, \mathrm{Tr}\left[(M_N - \not{K})(M_N - (\not{K} - \not{P}))\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P)$$

$\tilde{\Delta}(K)$ is the Matsubara fermion propagator and we have defined

$$P \;= (i\omega, \vec{p}), \quad K \;= (i\tilde{\omega}_n + \mu, \vec{k})$$

Thermal mass

$g_\sigma = 10.5$ and $m_\sigma = 510$



The effective propagator

$$\Delta^*(p_0,p) = \frac{-1}{P^2 - M_\sigma^2 - \gamma_T p_0 F(x) - i\pi A(x)\theta(p^2 - p_0^2)}$$

$$P^2 = p_0^2 - p^2, \qquad M_\sigma^2 = m_\sigma^2 + M_T^2$$

$$\begin{aligned} A(x) &= \gamma_T \left(5x^2+1\right)\left(x - \frac{2x^2}{\sqrt{3x^2+1}}\right) \\ &+ \lambda_1\, x^2 \left(3(2x^2+1) - \frac{x(10x^2+7)}{\sqrt{3x^2+1}}\right) - \lambda_2 \end{aligned}$$

- $x \equiv p_0/p$

The effective propagator

$$
\begin{aligned}
F(x) = & \Bigg[3\left(2x^2+1\right)x\lambda_1 - \lambda_2 \\
& + \left(5x^2+1\right)\gamma_T\Bigg] x\ln\left(\frac{x+1}{x-1}\right) \\
& - \left[2\frac{\left(5x^2+1\right)}{\sqrt{3x^2+1}}\gamma_T + \frac{x\left(10x^2+7\right)}{\sqrt{3x^2+1}}\lambda_1\right] \\
& \times x^2 \ln\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right) \\
& + \left[4x\left(2x^2+1\right)\lambda_1 + 2\lambda_2 + \frac{2}{3}\left(15x^2-2\right)\gamma_T\right]
\end{aligned}
$$

Dispersion relation and damping rate

$$\begin{aligned}
\omega^2(p) &= p^2 + m_\sigma^2 + \mathrm{Re}\left[\Pi\Big|_{p_0=\omega(p)}\right] \\
&= p^2 + m_\sigma^2 + M_T^2 + \gamma_T\omega(p)F\left(\omega(p)/p\right) \\
\gamma(p) &= -\frac{1}{2\omega(p)}\mathrm{Im}\left[\Pi\Big|_{p_0=\omega(p)}\right] \\
&= \frac{\gamma_T}{2}\pi A\left(\omega(p)/p\right)\theta(p^2-\omega(p)^2)
\end{aligned}$$

At leading order

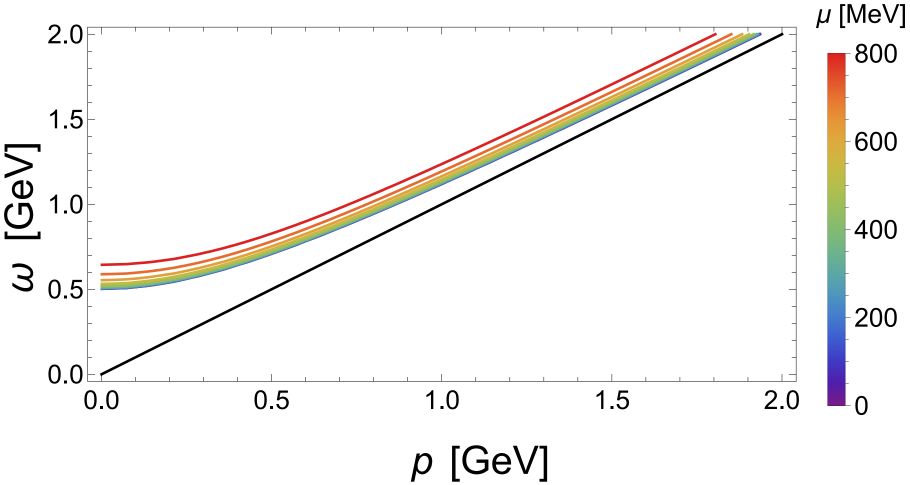
$$\omega(p) = p^2 + M_\sigma^2$$

In the limit $p \to 0$, $F(p_0/p) \to 0$, therefore,

$$\omega^2(p=0) = M_\sigma^2 = m_\sigma^2 + M_T^2$$

Dispersion relation

$T = 150$ MeV, $g_\sigma = 10.5$, $m_\sigma = 510$ MeV



Spectral density

$$
\begin{aligned}
\rho(p_0, p) &= 2\mathrm{Im}\Delta^*(q_0 + i\eta, q) \\
&= 2\pi Z(\omega(p))\left[\delta(p_0 - \omega(p)) - \delta(p_0 + \omega(p))\right] + \beta(p_0, p)
\end{aligned}
$$

- Small p

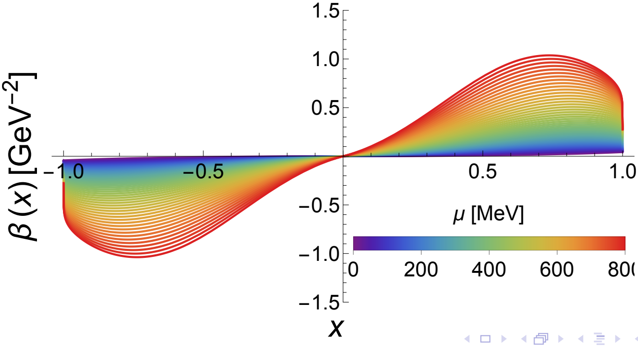
$$
Z(p) \approx \frac{1}{2\sqrt{p^2 + M_\sigma^2}}
$$

- Large p

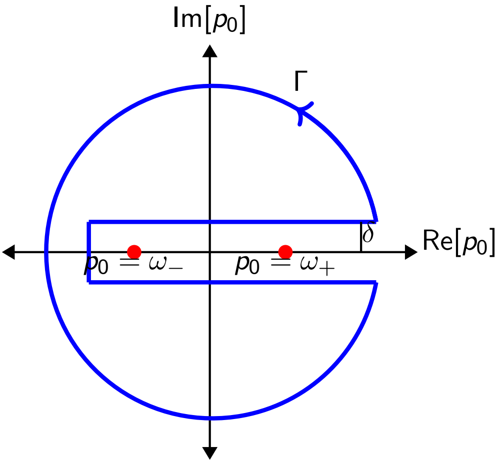
$$
Z(p) \approx \frac{12\kappa}{\lambda_1 + 2\lambda_2} \exp\left[-\frac{2p}{(\lambda_1 + 2\lambda_2)}\right]
$$

$$\beta(p_0, p) = 2\pi\gamma_T p_0 A(x)\theta\left(1 - x^2\right) \times \left[\left(P^2 - M_\sigma^2 - \gamma_T p_0 F(x)\right)^2 - \left(\gamma_T p_0 A(x)\pi\right)^2\right]^{-1}$$

T =150 MeV, $g_\sigma = 10.5$, $m_\sigma = 510$



Sum rules



Sum rules

- Use Cauchy's theorem to write

$$
\begin{aligned}
\Delta^*(p_0, p) &= \oint_\Gamma \frac{dz}{2i\pi} \frac{\Delta^*(z, p)}{z - p_0} \\
&= \int_{-\infty}^{\infty} \frac{dp_0'}{2i\pi} \frac{\Delta^*(p_0 + i\delta, p) - \Delta^*(p_0' - i\delta, p)}{p_0' - p_0} \\
&+ \oint_{\Gamma'} \frac{dz}{2i\pi} \frac{\Delta^*(z, p)}{z - p_0} \\
&= \int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} \frac{\rho(p_0', p)}{p_0' - p_0} + \oint_{\Gamma'} \frac{dz}{2i\pi} \frac{\Delta^*(z, p)}{z - p_0}
\end{aligned}
$$

Sum rules

Since $\Delta^*(z,p) \sim 1/z^2$ for $z \to \infty$, there is no contribution from Γ'. On the other hand

$$\boxed{\lim_{p_0 \to 0} \Delta^*(p_0, p) = \frac{1}{p^2 + m_\sigma^2 + M_T^2}}$$

As a consequence, thre fist sum rule is

$$\boxed{\int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} \frac{\rho(p_0', p)}{p_0'} = \frac{1}{p^2 + m_\sigma^2 + M_T^2}}$$

Sum rules

Other sum rules can be obtained considering the asymptotic expansion

$$\frac{1}{p_0' - p_0} = -\frac{1}{p_0}\sum_{n=0}^{\infty}\left(\frac{p_0'}{p_0}\right)^{2n+1}$$

which implies

$$\begin{aligned}
\Delta^*(p_0, p) &= \int_{-\infty}^{\infty}\frac{dp_0'}{2\pi}\frac{\rho(p_0', p)}{p_0' - p_0} \\
&= -\frac{1}{p_0}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_0'}{2\pi}\left(\frac{p_0'}{p_0}\right)^{2n+1}\rho(p_0', p)
\end{aligned}$$

Sum rules

For $n = 0$

$$\boxed{\int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} p_0' \; \rho(p_0', p) = 1}$$

For $n = 1$

$$\boxed{\int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} p_0'^3 \; \rho(p_0', p) = p^2 + M_\sigma^2}$$

For $n = 2$

$$\boxed{\int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} p_0'^5 \; \rho(p_0', p) \quad = \quad (p^2 + M_\sigma^2)^2}$$

Conclusions

- Temperature and baryon density modifications to the σ-meson, accounting for the large nucleon mass compared to the rest of the energy scales.
- The σ dispersion relation and the damping rate receive non-negligible thermal and baryon density contributions.
- The σ develops a thermal mass and the propagator develops an imaginary part with a piece associated to Landau damping.
- The spectral density obeys simple, HTL-like, sum rules, which shows that the adopted approximation scheme captures the leading-order behavior of the σ-meson propagator in a nucleon environment.
- We plan to use this propagator when describing the interaction of Λs and nucleons, where the latter take part of the vortical motion produced in the low-density region (the corona) in a peripheral heavy-ion collision.

THANKS

FAPESP

UNICID
Universidade
Cidade de S. Paulo

IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

IFUSP
Instituto de Física da USP

dgapa

BACKUP

Approximation for $M_N \gg T$

- Since $M_N \gg T$

$$
\begin{aligned}
\Pi = M_T^2 + (i\omega)\Bigg\{ \Bigg[& 3x^2\left(2x^2+1\right)\lambda_1 - x\lambda_2 \\
& + x\left(5x^2+1\right)\gamma_T\Bigg] \ln\left(\frac{x+1}{x-1}\right) - \Bigg[\frac{2x^2\left(5x^2+1\right)}{\sqrt{3x^2+1}}\gamma_T \\
& + \frac{x^3\left(10x^2+7\right)}{\sqrt{3x^2+1}}\lambda_1\Bigg] \ln\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right) \\
& + \Bigg[4x\left(2x^2+1\right)\lambda_1 + 2\lambda_2 + \frac{2}{3}\left(15x^2-2\right)\gamma_T\Bigg]\Bigg\}
\end{aligned}
$$

- $x \equiv i\omega/p$

where we have introduced the definitions

$$
\begin{aligned}
M_T^2 &\equiv \frac{2g_\sigma^2 M_N T}{\pi^2} K_1\left(\frac{M_N}{T}\right)\cosh\left(\frac{\mu}{T}\right), \\
\gamma_T &\equiv \frac{2g_\sigma^2 M_N}{\pi^2} K_1\left(\frac{M_N}{T}\right)\sinh\left(\frac{\mu}{T}\right), \\
\lambda_1 &\equiv \frac{2g_\sigma^2 T}{\pi^2} e^{-\frac{M_N}{T}}\cosh\left(\frac{\mu}{T}\right), \\
\lambda_2 &\equiv \frac{2g_\sigma^2 T}{\pi^2} e^{-\frac{M_N}{T}}\sinh\left(\frac{\mu}{T}\right)
\end{aligned}
$$

- M_T is the **thermal mass**, γ_T λ_1, and λ_2 are related to **Landau damping.**

Analytical properties

- Analytic continuation back to Minkowski $x \to p_0/p$ with p_0 real.
- For $-1 < x < 1$

$$\frac{x+1}{x-1} < 0, \qquad \frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1} < 0$$

- The logarithmic terms develop an imaginary part, explicitly

$$\ln\left(\frac{x+1}{x-1}\right) = \ln\left|\frac{x+1}{x-1}\right| + i\pi\theta\left(1-x^2\right)$$

$$\ln\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right) = \ln\left|\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right| + i\pi\theta\left(1-x^2\right)$$

Dispersion relation

- Large p

$$
\begin{aligned}
\omega(p) &= p\left(1+\kappa\ \exp\left[-\frac{\alpha_1\gamma_T+\alpha_2\lambda_1+\alpha_3\lambda_2}{3(\lambda_1+2\lambda_2)}p\right]\right)\\
\kappa &\equiv \exp\left[\frac{-2M\sigma^2}{p(\lambda_1+2\lambda_2)}+\frac{\alpha_1\gamma_T+\alpha_2\lambda_1+\alpha_3\lambda_2}{\lambda_1+2\lambda_2}\right]\\
\alpha_1 &\equiv \frac{2}{3}(13-18\ln(2))\approx 0.3489\\
\alpha_2 &\equiv \frac{3}{2}(2-11\ln(2))\approx 0.5631\\
\alpha_3 &\equiv \ln(2)\approx 0.6931
\end{aligned}
$$