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Nucleares
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Signs of criticality in Heavy-ion collisions via two-pion correlations

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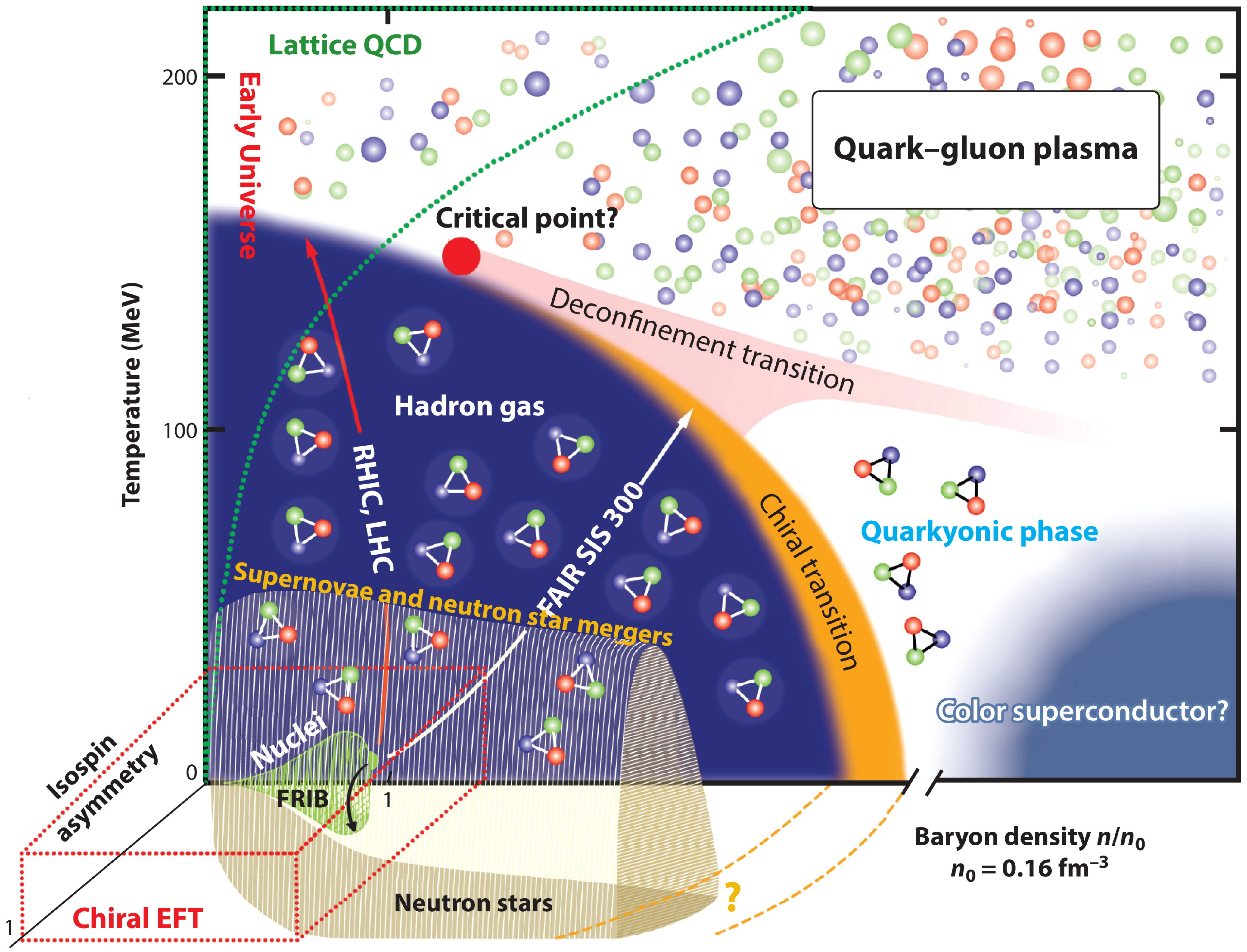
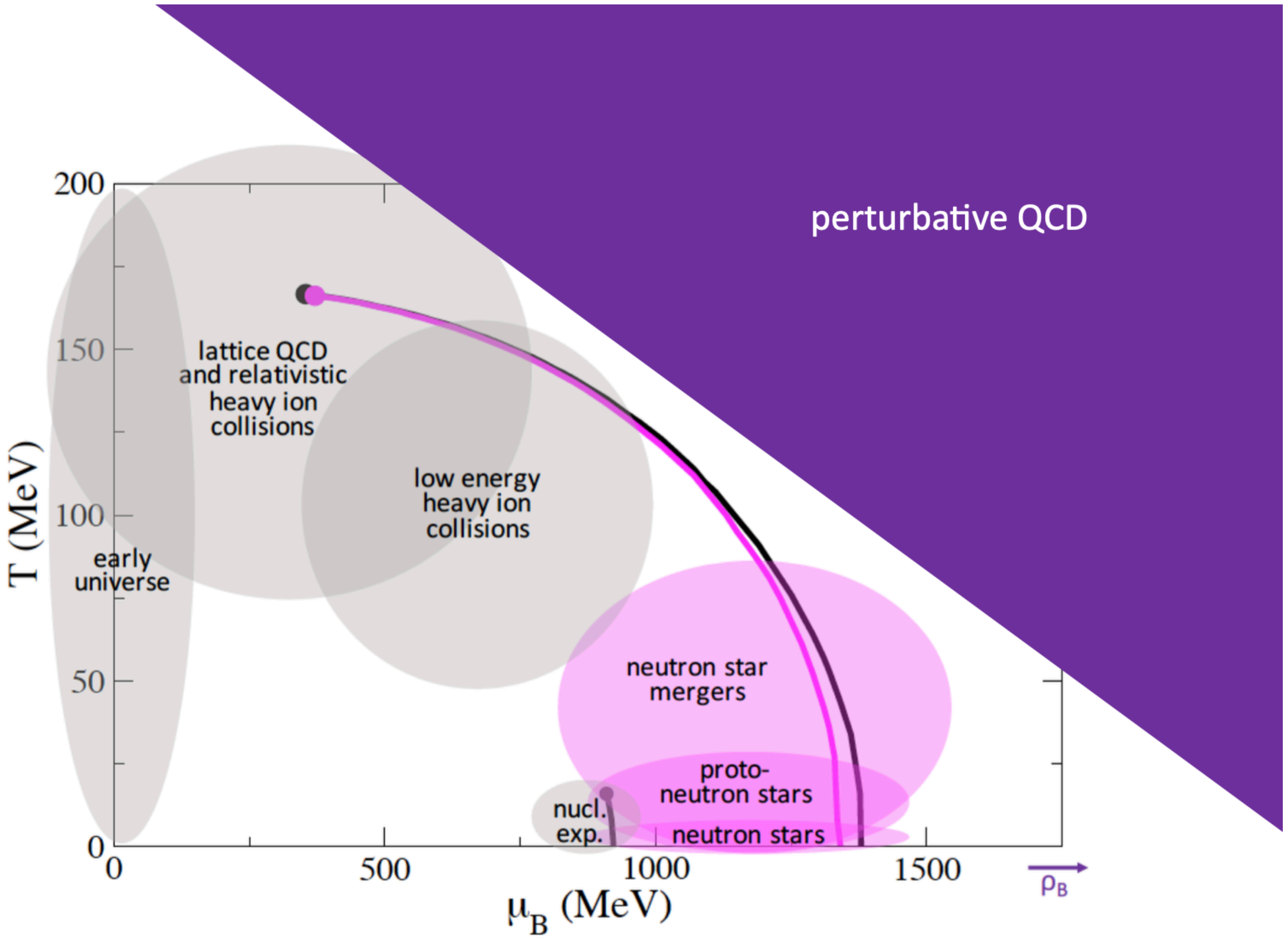
Outline of this talk

- QCD phase diagram at finite baryonic density and its critical exponents
- LQCD-based finite density EoS
- Femtoscopic measurements and Lévy sources
- Preliminary results
- Summary

QCD phase diagram



Drischler C, et al. 2021
Annu. Rev. Nucl. Part. Sci. 71:403–32



V. Dexheimer

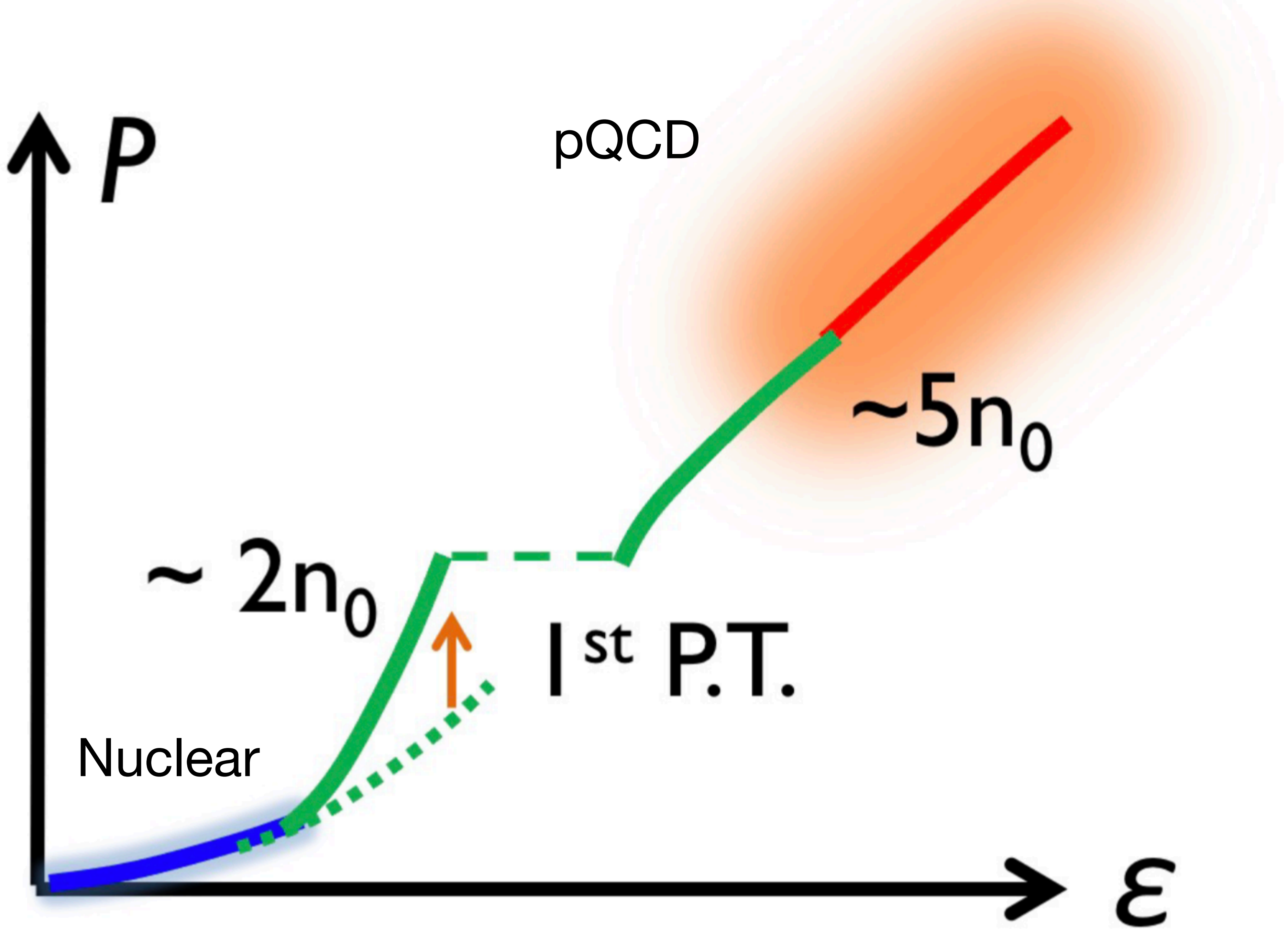
Critical exponents of QCD

And its universality class

- In the vicinity of the CEP, where a second order phase transition occurs, critical phenomena occur

- To characterize this critical behavior, critical exponents are introduced

- In QCD, there exist 6 critical exponents, 2 of which are independent

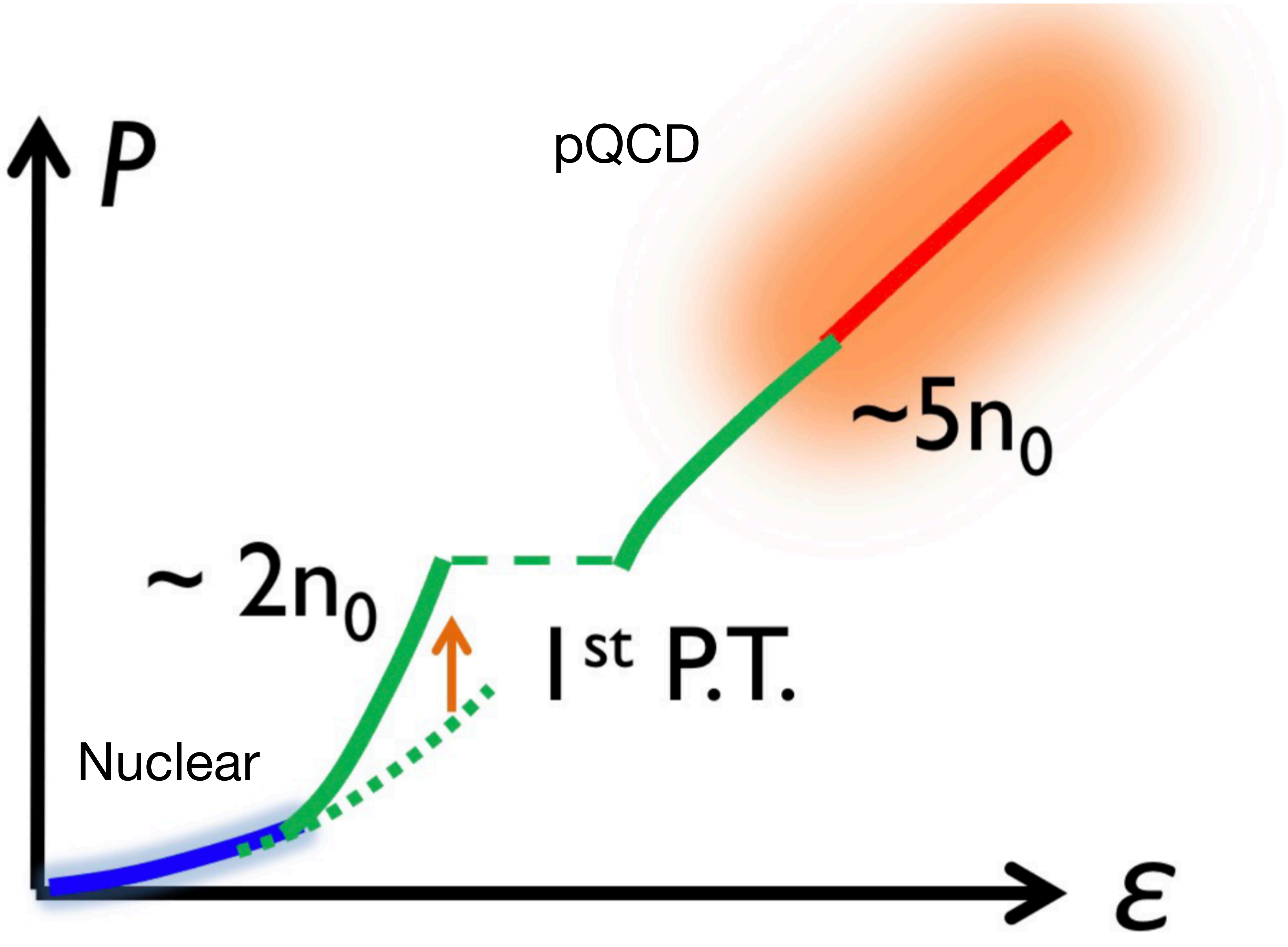


arXiv:2011.10940

Critical exponents of QCD

And its universality class

- In Nucl. Phys. B **399**, 395 (1993), a two-flavour model with massless quarks, the critical exponents were computed and it was shown that the universality class of this QCD is the same as that of the 3d-Ising model
- Due to the nature of HIC, the true universality class is the same as in the random 3d-Ising model [PRB **52**, 6659 (1995)] :
- $\eta = 0.5 \pm 0.05, \nu = 1.1 \pm 0.2,$ $\alpha = -1.3 \pm 0.6, \beta = 0.6 \pm 0.1,$ $\gamma = 2.2 \pm 0.4, \delta = 4.7 \pm 0.3$

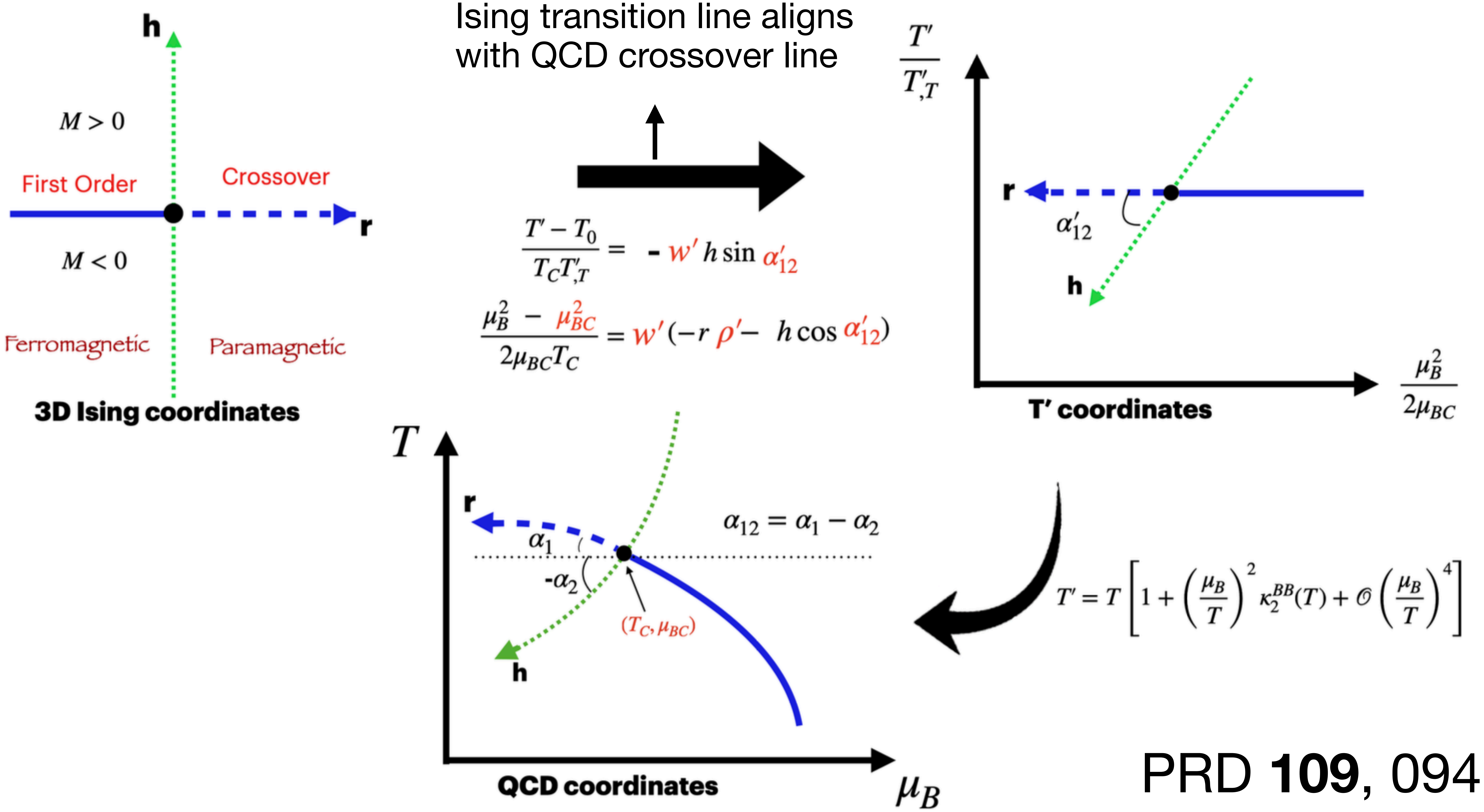


arXiv:2011.10940

Finite density QCD equation of state

Lattice-based T expansion and critical point

Ising transition line aligns with QCD crossover line



$$\frac{T' - T_0}{T_C T'_{,T}} = - w' h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC} T_C} = w'(-r \rho' - h \cos \alpha'_{12})$$

$$\alpha_{12} = \alpha_1 - \alpha_2$$

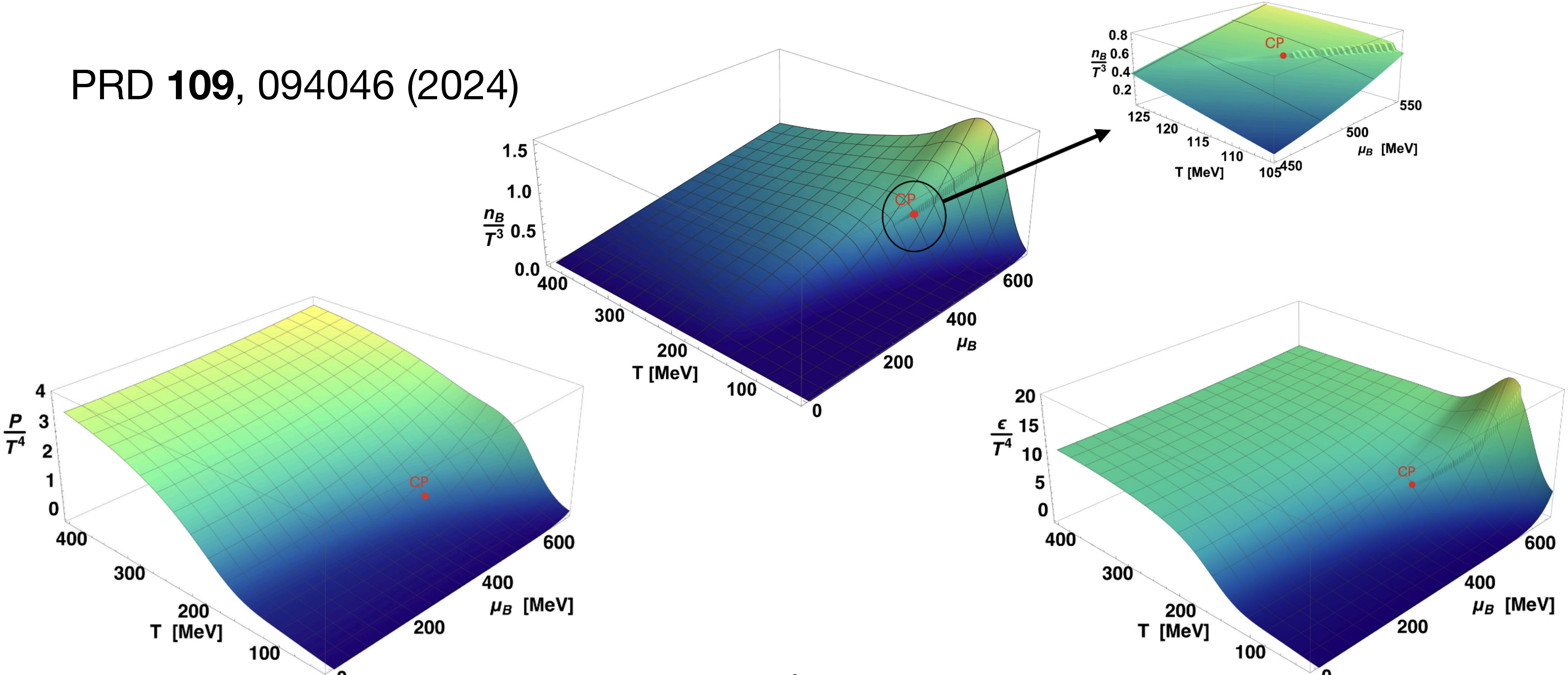
$$T' = T\left[1 + \left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4\right]$$

PRD **109**, 094046 (2024)

Finite density QCD equation of state

Lattice-based T expansion and critical point

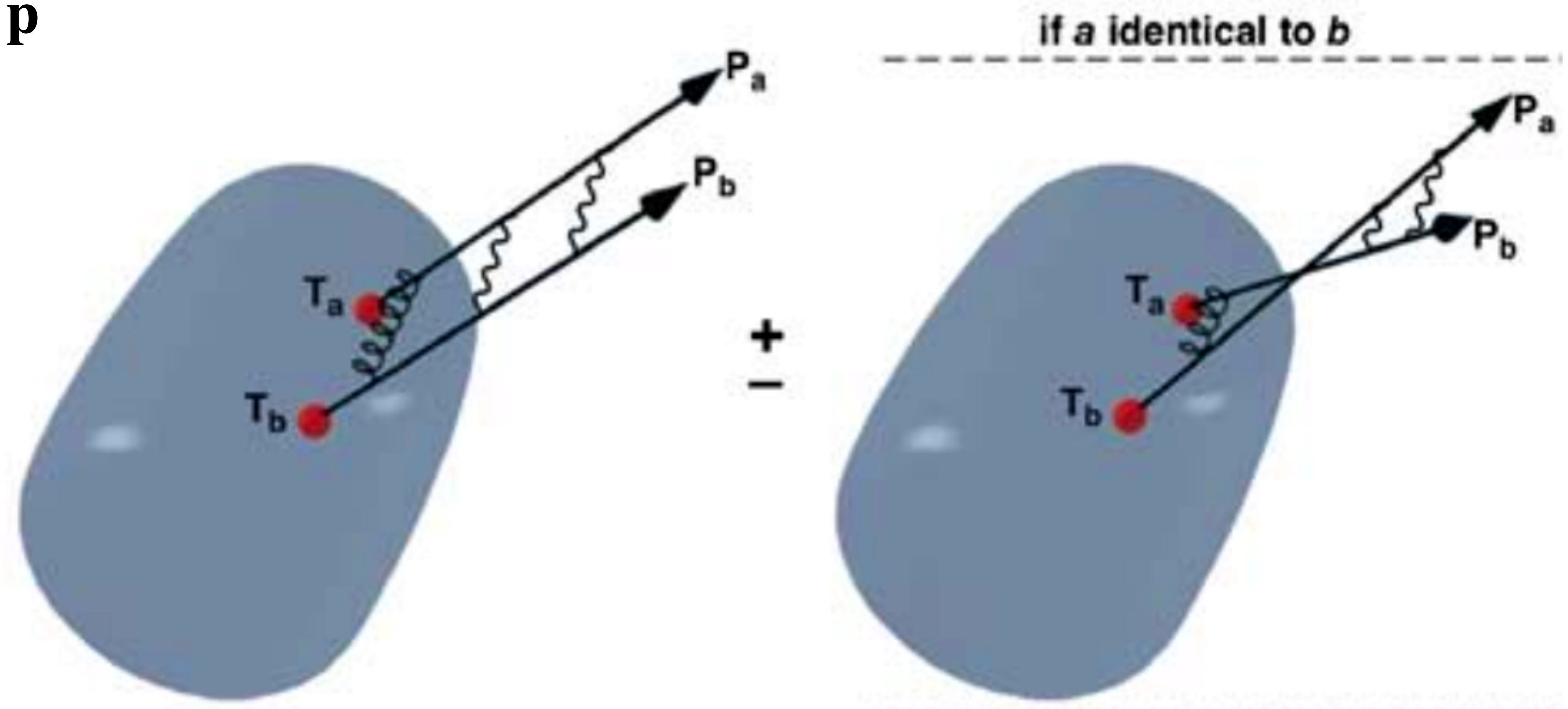
PRD **109**, 094046 (2024)



Two-identical particle correlations

Or HBT femtoscopy

- After a collision, the one-particle momentum distribution can be found as
- $\mathscr{P}_1(\mathbf{p}) \equiv \dfrac{\mathrm{d}^3N}{\mathrm{d}p^3} \longrightarrow$ Probability of emisión of a particle with momentum $\mathbf{p}$
- In a similar way, the two-particle momentum distribution
- $\mathscr{P}_2(\mathbf{p}_1, \mathbf{p}_2) \equiv \dfrac{\mathrm{d}^6N}{\mathrm{d}^3p_1\mathrm{d}^3p_2} \longrightarrow$ Probability simultaneous emission of particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$
- When the emission process of the particles are independent of each other, $\mathscr{P}_2$ can be factorized as the product of $\mathscr{P}_1$'s
- $\mathscr{P}_2(\mathbf{p}_1, \mathbf{p}_2) = \mathscr{P}_1(\mathbf{p}_1)\mathscr{P}_1(\mathbf{p}_2)$

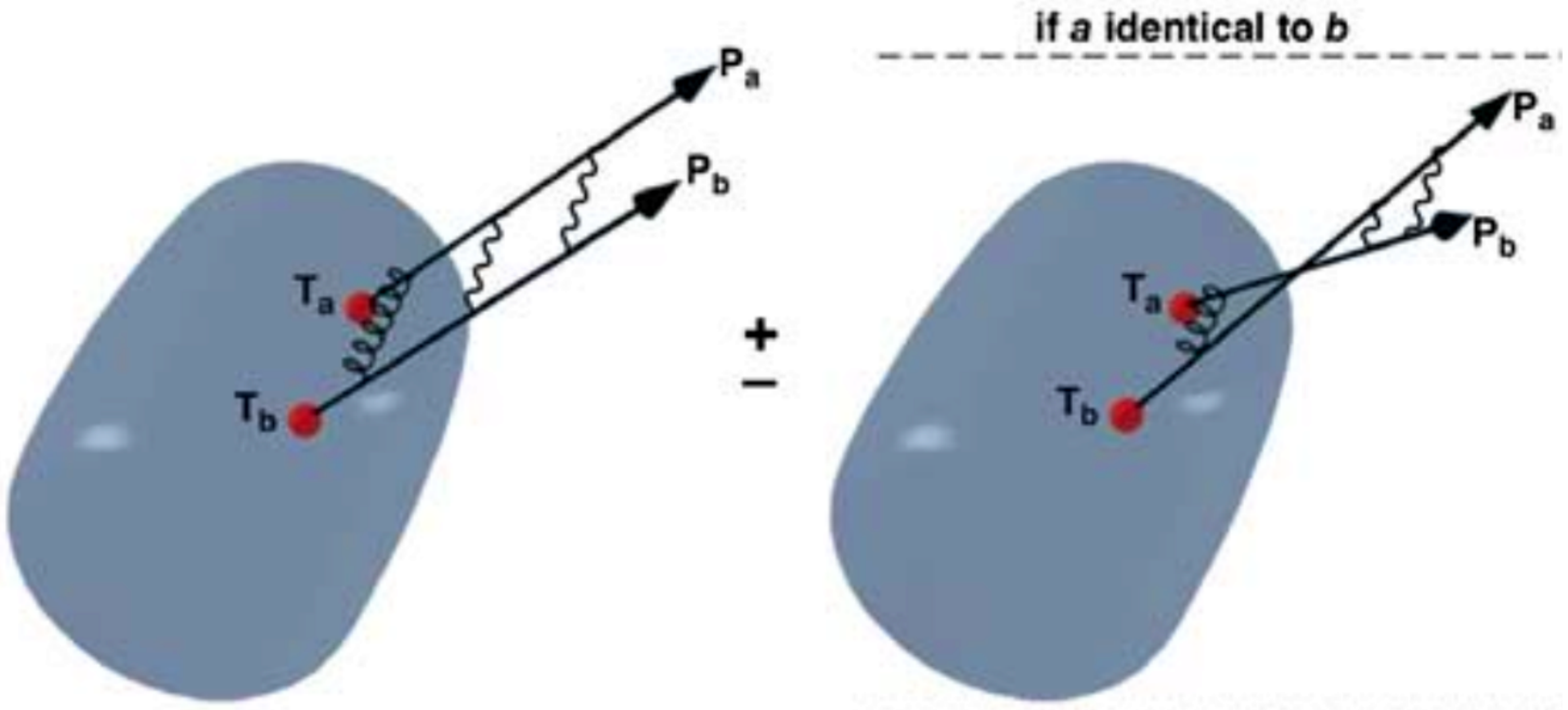


Annu. Rev. Nucl. Part. Sci. **55**, 357 (2005)

Two-identical particle correlations

Or HBT femtoscopy

- If this factorization is not valid, then it is because the emission processes are not independent of each other, but correlated due to:
- Conservation laws
- Decays
- Quantum nature of the particles
- Long etcetera …



Annu. Rev. Nucl. Part. Sci. **55**, 357 (2005)

Two-particle correlation functions

Femtoscopy

- From the theoretical point of view, this function is defined as
- $$C_2(\mathbf{p}_1, \mathbf{p}_2) = \frac{\mathscr{P}_2(\mathbf{p}_1, \mathbf{p}_2)}{\mathscr{P}_1(\mathbf{p}_1) \cdot \mathscr{P}_1(\mathbf{p}_2)}$$

Two-particle momentum distribution

One-particle momentum distribution

- Correlation functions are usually described as functions of the **pair relative momentum**, $q = p_1 - p_2$, and the **pair average momentum**, $K = \frac{p_1 + p_2}{2}$. On-shell conditions imply that C_2 is only a function of $\mathbf{q}$ and $\mathbf{K}$

Two-particle correlation functions

Femtoscopy

- Two-particle correlation functions can also be related to the particle emission source in phase-space $S(x, p)$. Assuming that the emission is not initially correlated, that particles are bosons and that they not interact in their final state,
- If the Fourier transformation of the source is $\tilde{S}(q, p) = \int d^4x\, e^{iq\cdot x} S(x, p)$,

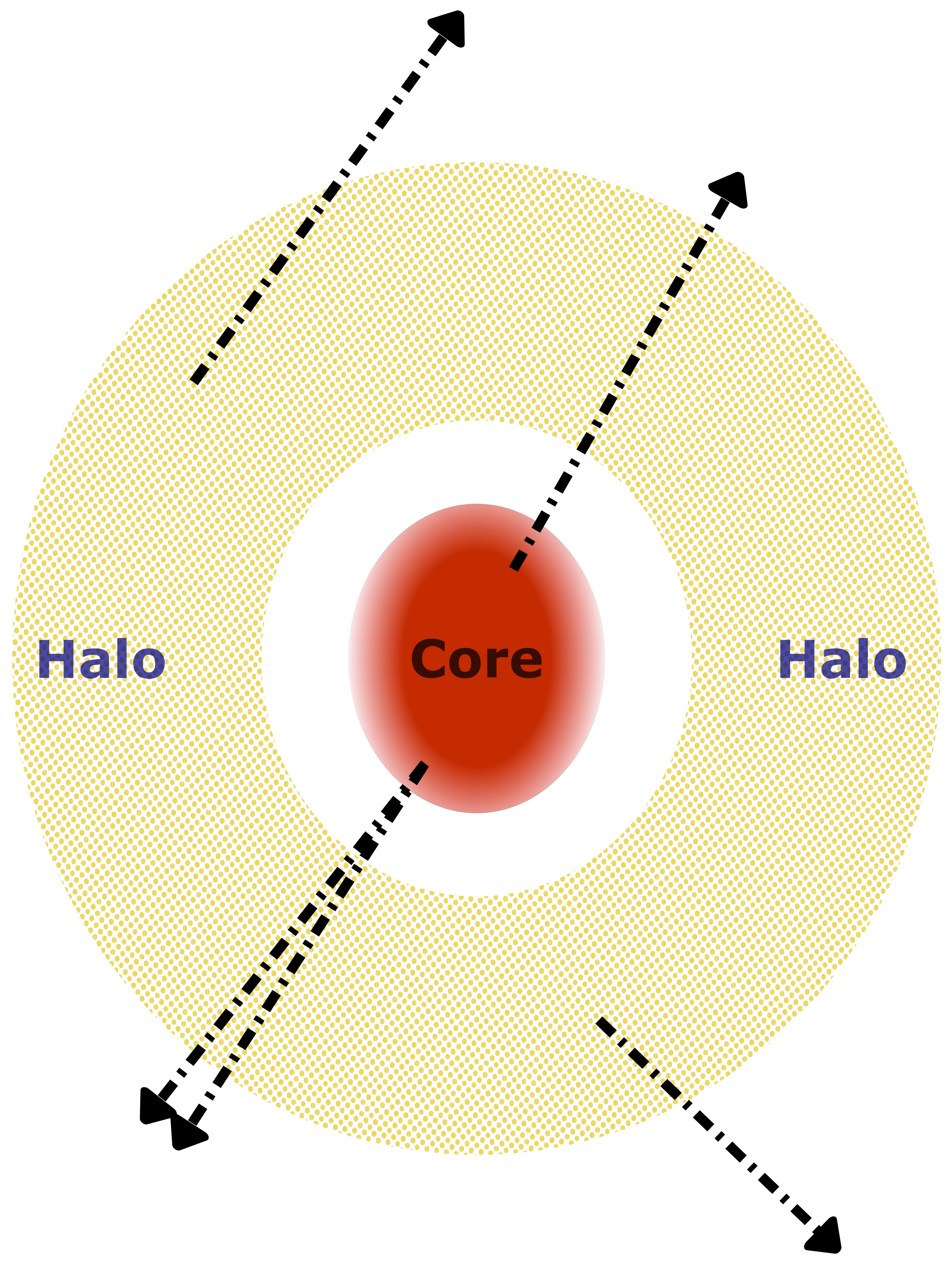
  then
- $$C_2(q, K) = 1 + \frac{\left|\tilde{S}(q, K)\right|^2}{\left|\tilde{S}(0,K)\right|^2}$$

Correlation functions

Core - halo model [PRD 47, 3860 (1993), Z. Phys. C 71, 491 (1996)]

- The correlation function for $q_{\mathrm{inv}} \to 0$, reaches a value of $1 + \lambda$, with $0 \leq \lambda \leq 1$, this is due to the resonances that decay and form a halo around the interaction region

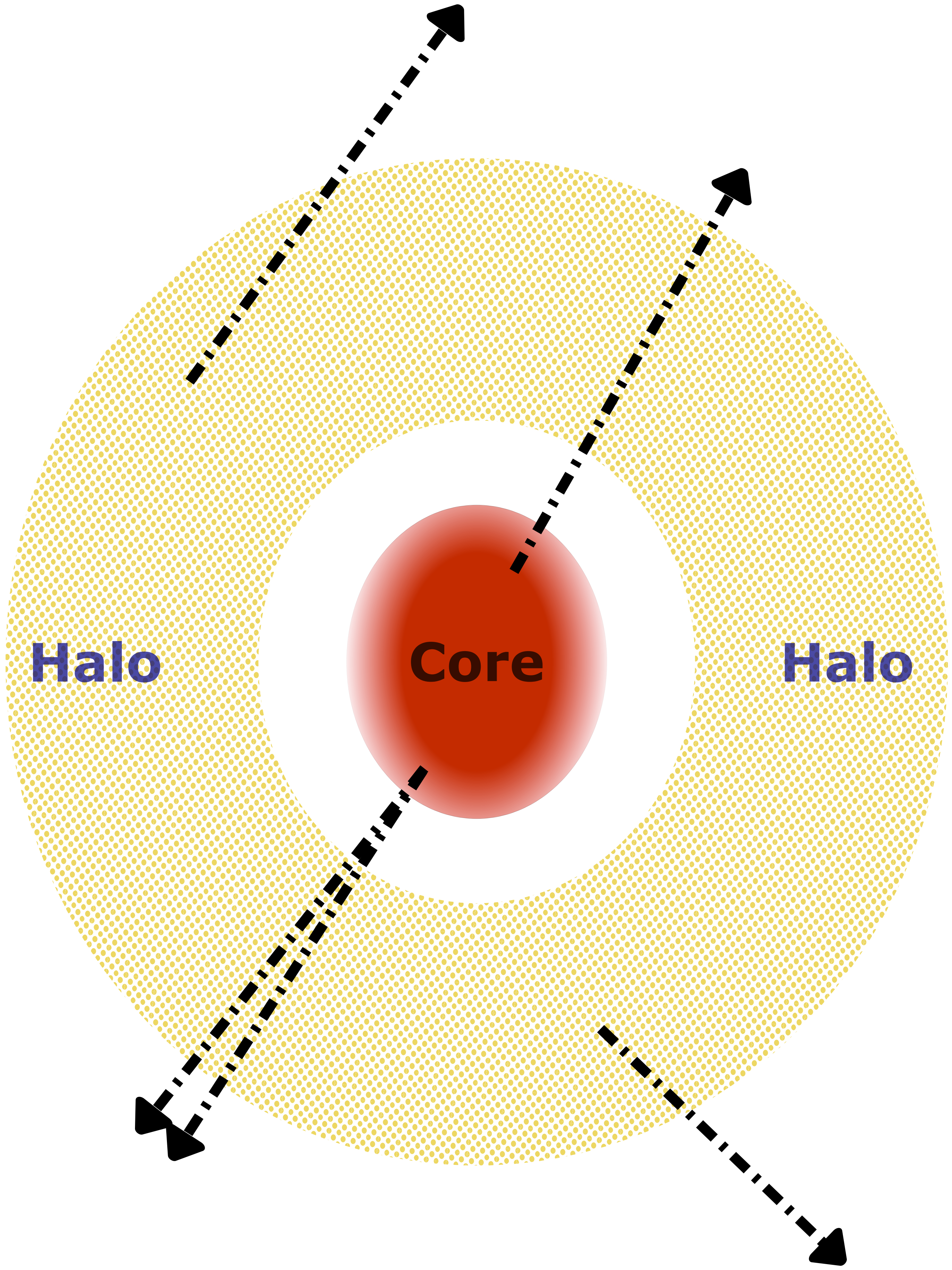
- A detector with finite resolution will not be able to solve the halo if its characteristic size is larger than the momentum resolution



Correlation functions

Core - halo model [PRD 47, 3860 (1993), Z. Phys. C 71, 491 (1996)]

- Let us assume that the detector has a momentum resolution Δ_q and that the source is made of two components, such that
- $S = S_{\text{core}} + S_{\text{halo}}$
- Where the halo is composed of particles produced through the decays of resonance
- The characteristic scale of the halo is $R_{\text{halo}} \gtrsim \dfrac{1}{\Delta_q}$, then
- $R_{\text{core}} < \dfrac{1}{\Delta_q} \lesssim R_{\text{halo}}$



Correlation functions

Core - halo model [PRD 47, 3860 (1993), Z. Phys. C 71, 491 (1996)]

- The core and halo distributions can be written as
- $\mathscr{P}_{1,\text{core}}(K) = \int d^4x\, S_{\text{core}}(x, K) = \tilde{S}_{\text{core}}(0,K) \equiv N_{\text{core}}$
- $\mathscr{P}_{1,\text{halo}}(K) = \int d^4x\, S_{\text{halo}}(x, K) = \tilde{S}_{\text{halo}}(0,K) \equiv N_{\text{halo}}$
- Since, for the halo, the region with $q < \Delta_q$ cannot be solved, then $\tilde{S}_{\text{halo}}(q, K) \approx 0$, and $\tilde{S}(q, K) \simeq \tilde{S}_{\text{core}}(q, K)$. Hence
- $C_2(q, K) = 1 + \left(\frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}} \right)^2 \frac{\left| \tilde{S}_{\text{core}}(q, K) \right|^2}{\left| \tilde{S}_{\text{core}}(0,K) \right|^2}$, with $\lambda = \left(\frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}} \right)^2$

Correlation functions

And its parameterizations

- Once the correlation function is obtained, it can be fitted to extract the source characteristics
- As an example, assume a 1D source that can be factorized into a space-time distribution and a momentum distribution $S(x,p) = f(x) \cdot g(p)$, with
- $\displaystyle\int dx\, f(x) = 1$ and $\displaystyle\int dp\, g(p) = N$
- Then
- $\boxed{C_2(q,K) = 1 + \left|\tilde{f}(q)\right|^2 \approx 2 - q^2\left(\langle x^2\rangle - \langle x\rangle^2\right) + \ldots \approx 1 + \exp(-q^2R^2)}$
- Where $R^2 = \langle x^2\rangle - \langle x\rangle^2$ and $\tilde{f}(q) = \displaystyle\int dx\ \exp(iqx)\, f(x)$ is known as the characteristic function

Correlation functions

And its parameterizations

- Under which conditions are these Gaussian assumptions valid?
- The emission is a superposition of several independent proceses whose emission coordinate is separated by δx_i
- If the variance that characterize this separation is finite, then by means of the central limit theorem, the probability distribution will tend to a Gaussian
- Since the Fourier transformation of a Gaussian is also a Gaussian, the correlation function will also be a Gaussian

Correlation functions

And its parameterizations

- In the neighborhood of a CEP, where the correlation length diverges, and the distributions that characterize physical quantities behave as power-laws
- These kind of distributions have non-finite variance (and even non-finite mean) and hence have a non-analytic behavior for certain values
- Then the probability distribution will be different from a Gaussian
- These special case distributions are called Lévy distributions

Correlation functions

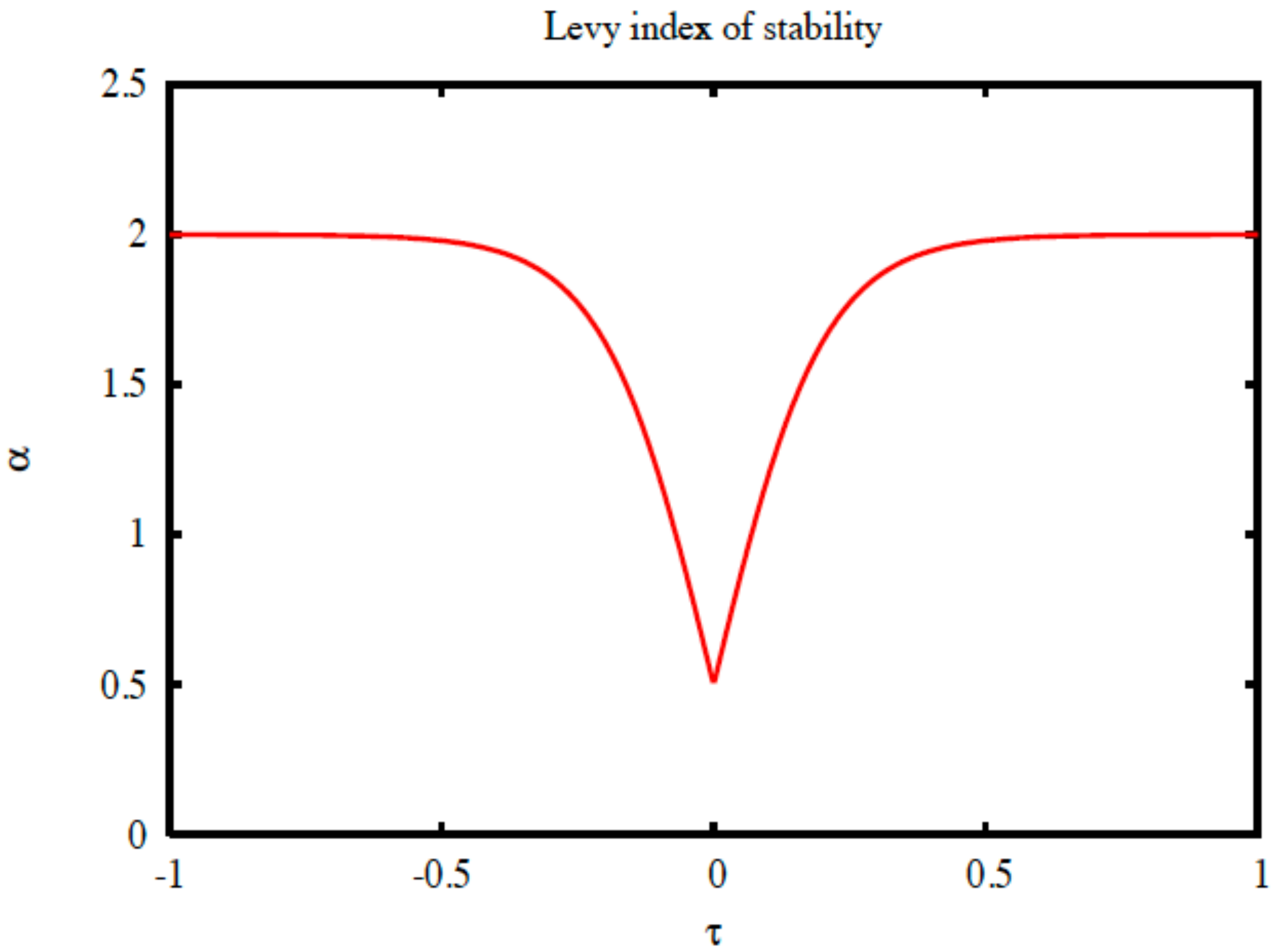
And its parameterizations

- The Lévy characteristic function is
- $\tilde{f}(q) = \exp\left(-\gamma^{\alpha}|q|^{\alpha} + i\beta\gamma^{\alpha}\,\text{sign}(q)\,\tan\left(\frac{\alpha\pi}{2}\right) + iq\delta\right)$, with $0 \leq \alpha \leq 2$, $-1 \leq \beta \leq 1, \gamma > 0$ and $-\infty < \delta < \infty$
- A special case happens when $\beta = 0$, $\gamma = R/2^{1/\alpha}$ and $\delta = x_0$, then
- $\tilde{f}(q) = \exp\left(iqx_0 + |qR|^{\alpha}\right) \approx 1 + iqx_0 - \frac{1}{2}|qR|^{\alpha}$
- Therefore, $\boxed{C_2(q;\alpha) = 1 + \exp\left(-|qR|^{\alpha}\right)}$

Correlation functions

Relationship between the Lévy index and QCD critical exponents

- For QCD, the correlation function of the order parameter decays as a power-law in the vicinity of the critical point $\rho \propto r^{-(d-2+\eta)}$
- For Lévy-type sources, the correlation between initial and final positions decays also as a power-law $\rho \propto r^{-(1+\alpha_{Levy})}$
- Para $d = 3, \eta = \alpha_{\text{Lévy}}$
- In this model, $\eta = 0.5 \pm 0.05$

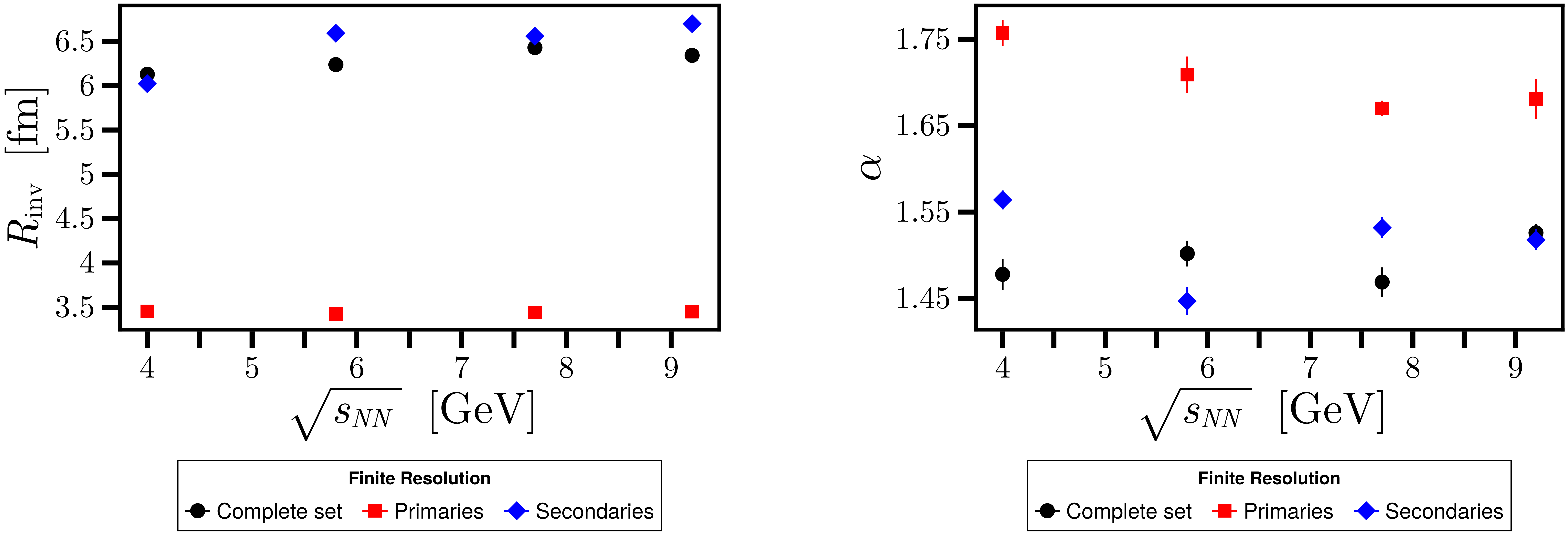


$\tau = (T - T_c)/T_c$

AIP Conf. Proc. **828**, 525 (2006)

Two-pion correlation functions

With finite resolution at different energies



EPJ A **60**, 135 (2024)

Summary

And future work

- Two-pion correlation functions are a prime tool to study the interaction region of heavy-ion collisions
- Radii grow as the energy increases, and the Lévy index of stability decreases
- At low collision energies, the core has a large contribution from secondary pions
- This type of studies including EoS effects could help us determine if it can be used to signal critical phenomena
- Stay tuned for new results!

Thank you!

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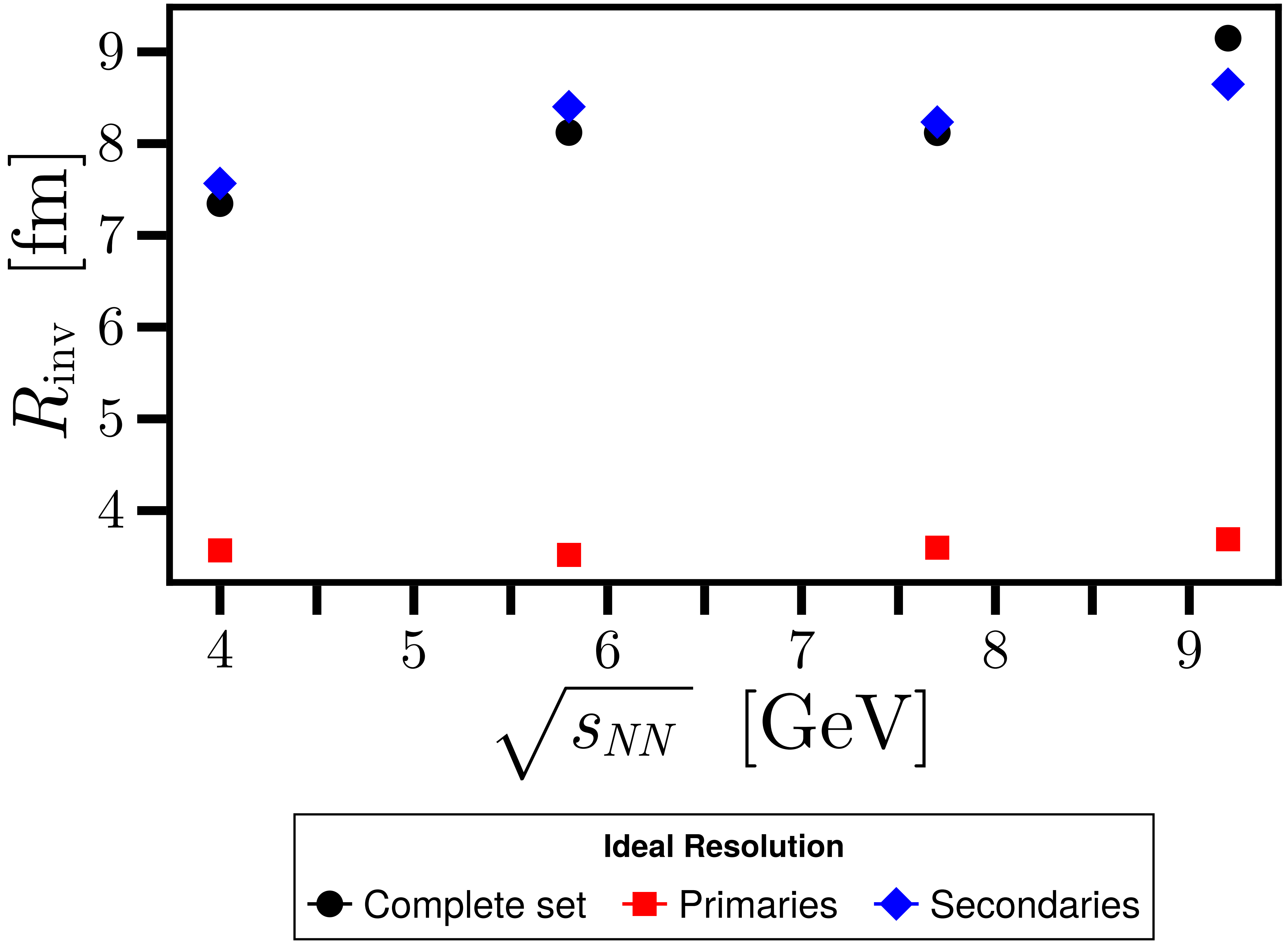
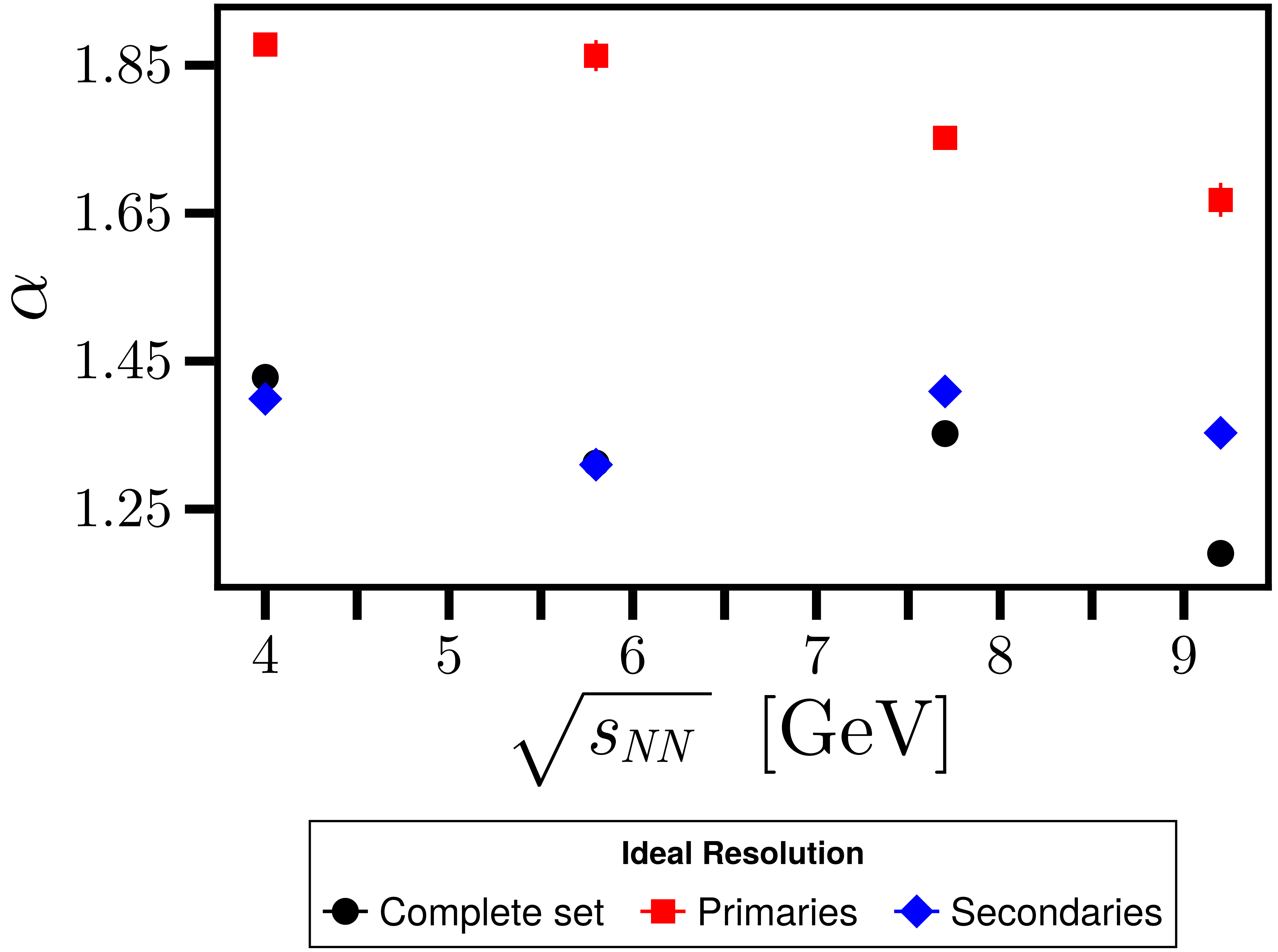
Backup

Two-pion correlation functions

With ideal resolution at different energies





EPJ A **60**, 135 (2024)

Two-pion correlation functions

And core - halo model

- In the core - halo model
- $\lambda = \left(\frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}} \right)^2$
- This means that between 82 and 77 % of the pions come from the core
- Between 6 y 13 % of the pions are produced from primary processes, hence the core has a large fraction of secondary pions
- Since $\Delta_q \cdot R_{\text{inv}} \sim 1$, then $R_{\text{inv}} \lesssim 20$ fm

$\sqrt{s_{NN}}$	λ_{all}	λ_{primary}	$\lambda_{\text{secondary}}$
4.0	0.677 ± 0.003	0.907 ± 0.002	0.651 ± 0.004
5.8	0.632 ± 0.004	0.905 ± 0.003	0.647 ± 0.005
7.7	0.625 ± 0.004	0.9 ± 0.003	0.608 ± 0.003
9.2	0.595 ± 0.007	0.887 ± 0.005	0.602 ± 0.003