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THE SPEED OF SOUND IN DENSE ISOSPIN-ASYMMETRIC MATTER FROM THE LINEAR SIGMA MODEL

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SIXTH WORKSHOP ON NONPERTURBATIVE ASPECTS OF QCD

Dec 3, 2025

A. Ayala, R. L. S.Farias, B. S. Lopes, L. C. P. L., Phys. Lett. B, 864, 139396 (2025)

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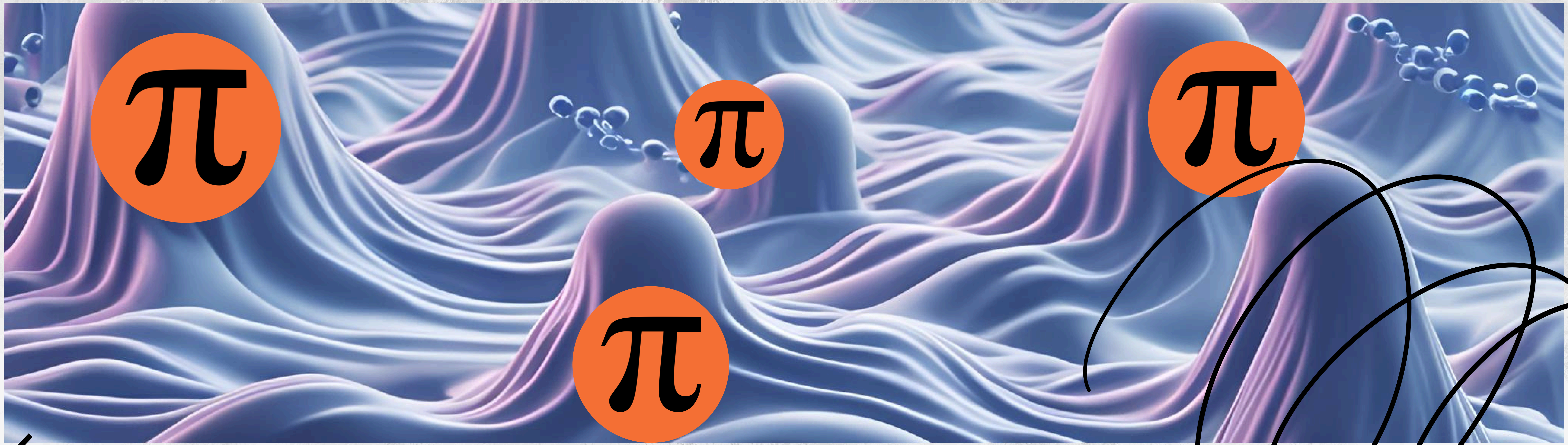
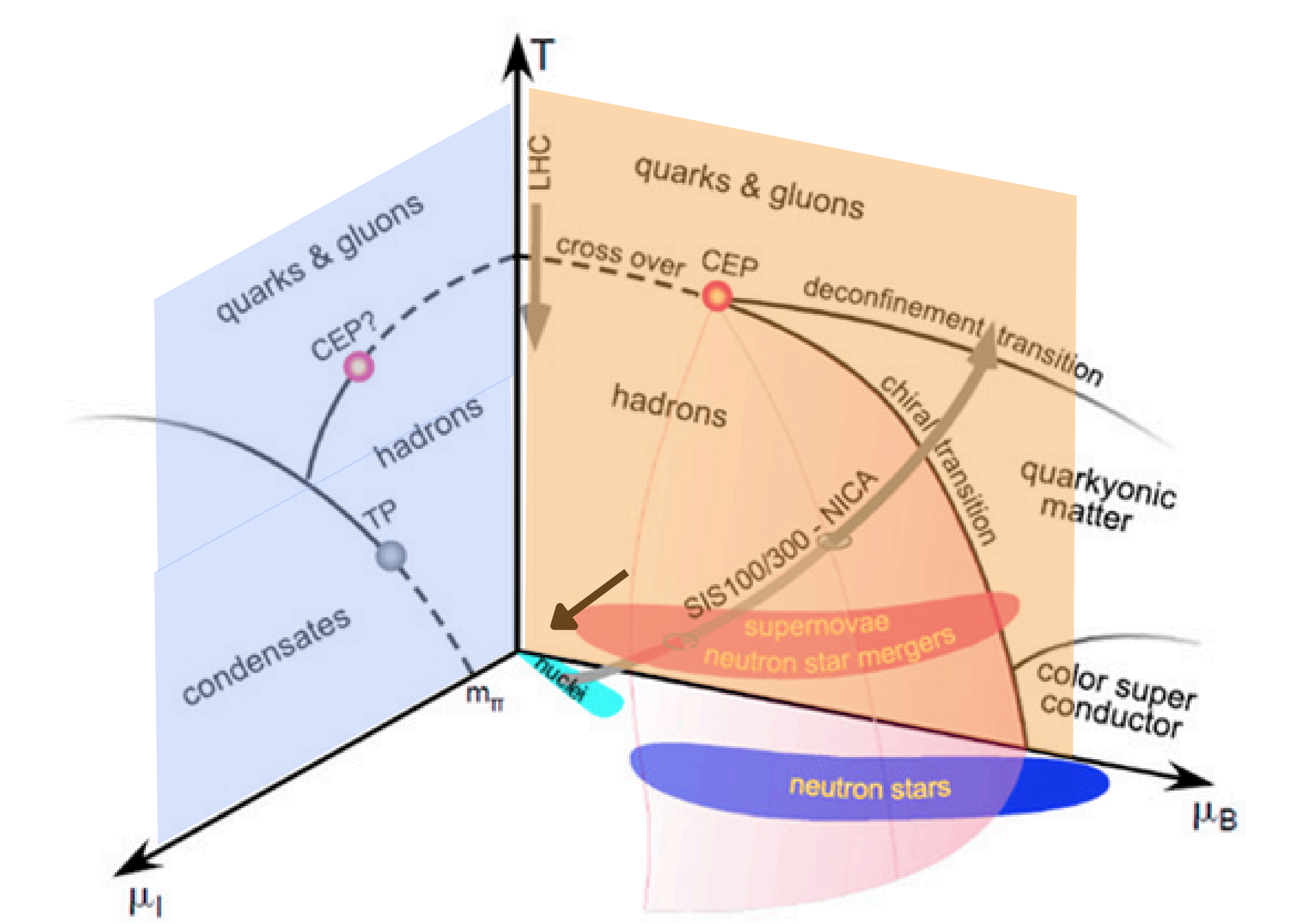
INTRODUCTION



EXTREME NUCLEAR MATTER



YOU ARE HERE

Nuclear matter, life

MOST OF THE OBSERVABLE UNIVERSE

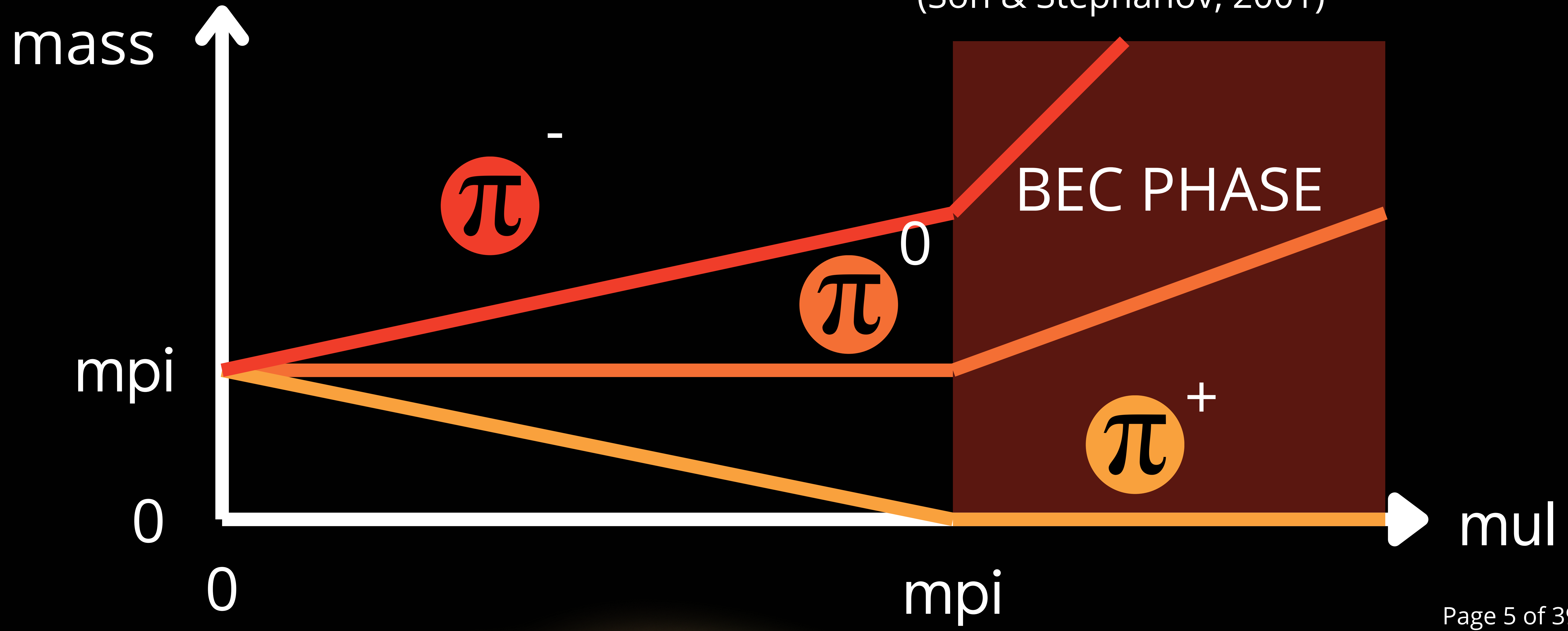
Particle Collisions, stars, early universe

OUR PROJECT

Exotic matter, pion stars

PION MASSES

(Son & Stephanov, 2001)

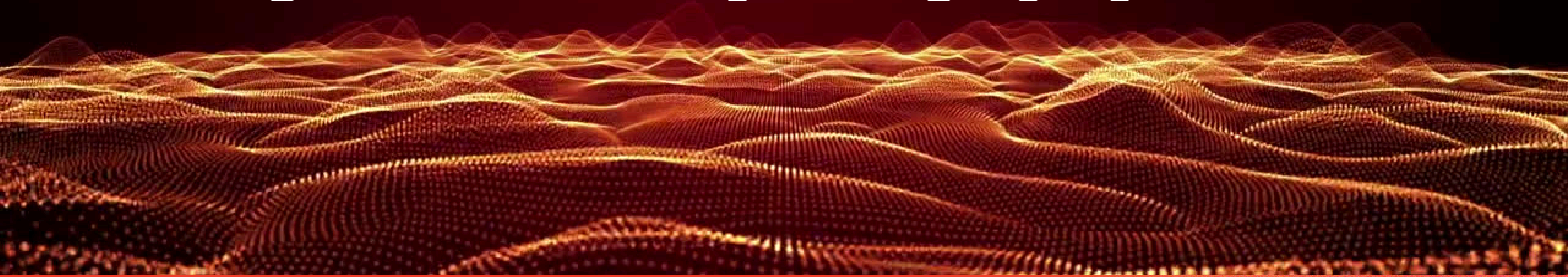


RUNNING COUPLING CONSTANT IN QCD (G_s)

Confinement

Free particles

SPEED OF SOUND



For mul ≈ mpi

Squared speed of sound linear

$$c_s^2 \approx \frac{\text{mul-mpi}}{\text{mpi}}$$

(Son & Stephanov, 2001)

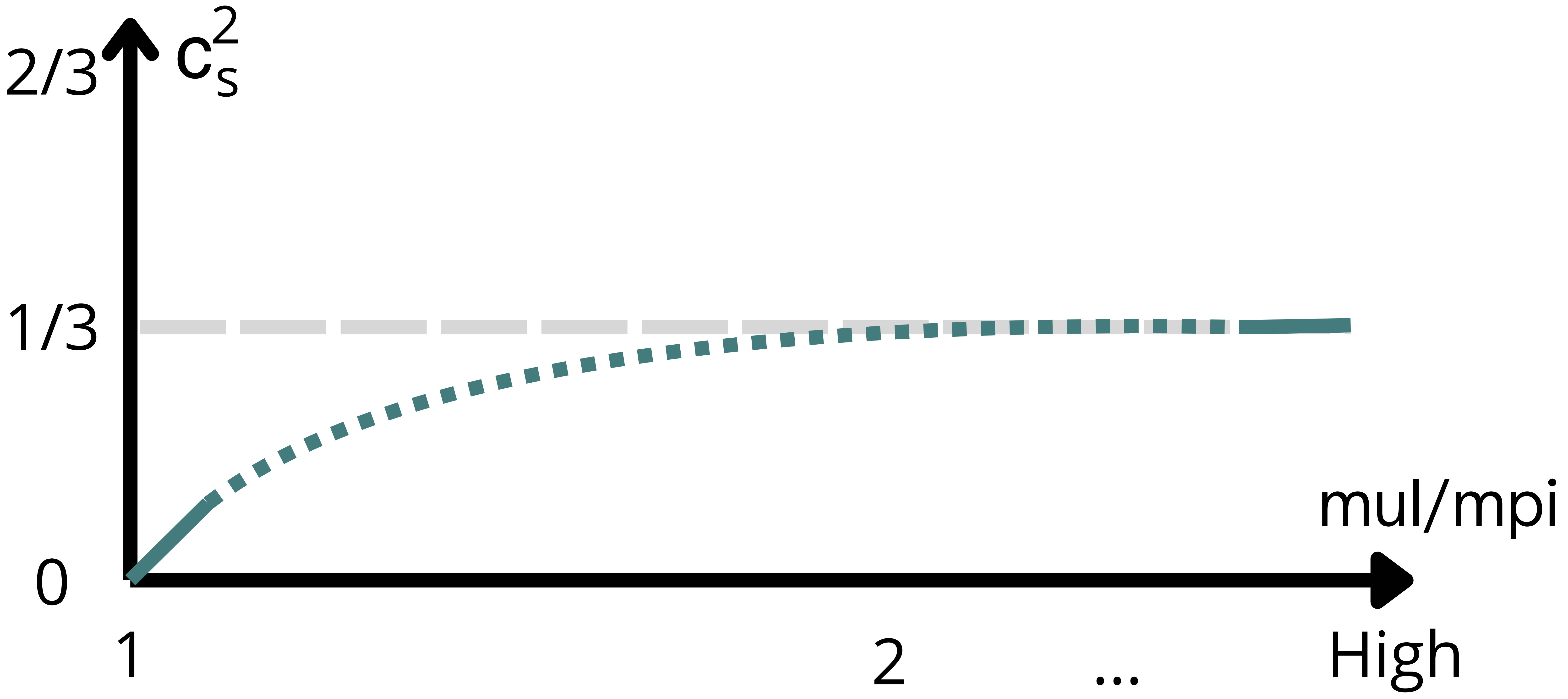
ASYMPTOTIC FREEDOM

Non-interacting relativistic matter (photon gas-like)

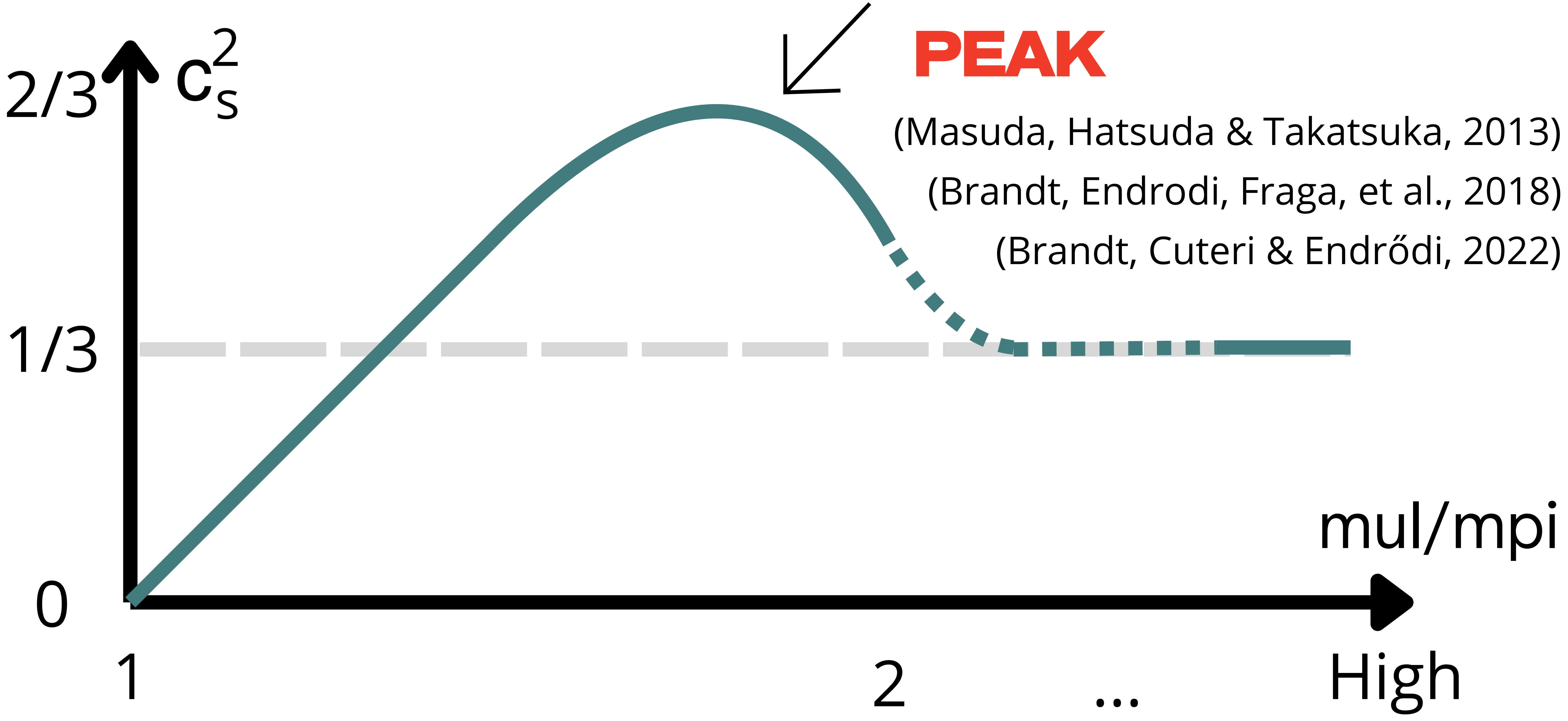
$$E = 3P$$

$$c_s^2 = dP/dE = 1/3$$

Conformal limit



EXPECTED
BEHAVIOR ↗



REAL
BEHAVIOR

State of the Art

"The speed of sound without Δ (gap energy) remains smaller than the conformal limit, while the gap corrections make exceed the con-formal limit in the intermediate density region"

K. Fukushima, S. Minato, arXiv:2411.03781v2, (2025)

pQCD

"We associate the sound velocity peak with the saturation of quark states"

R. Chiba, T. Kojo, Phys. Rev. D, **109**, 076006 (2024)

QMM

"The sound velocity is strongly affected σ, and finally it converges on 1/3"

M. Kawaguchi, D. Suenaga, Phys. Rev. D, **109**, 096034 (2024)

chPT

"The medium contributions to the coupling may be closely connected to the characterization of a non-monotonic behavior in the speed of sound"

A. Ayala, R. L. S.Farias, B. S. Lopes, L. C. P. L., EPJA, **60**, 250 (2024)

LSMq - NJL

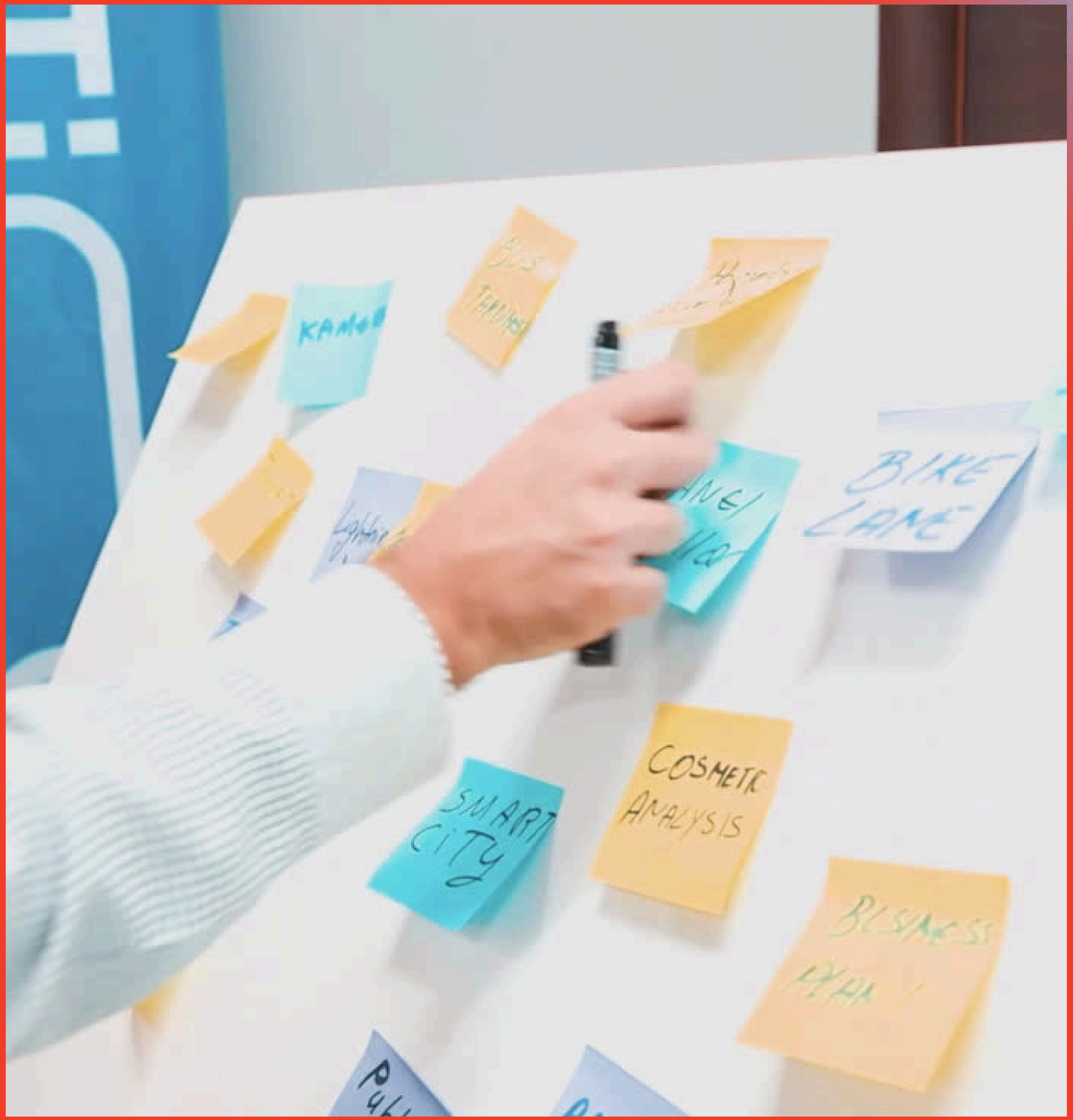
"The speed of sound peak signals for a transition from nuclear to quark matter"

K. Toru, Phys. Rev. D, **104**, 074005, (2018)

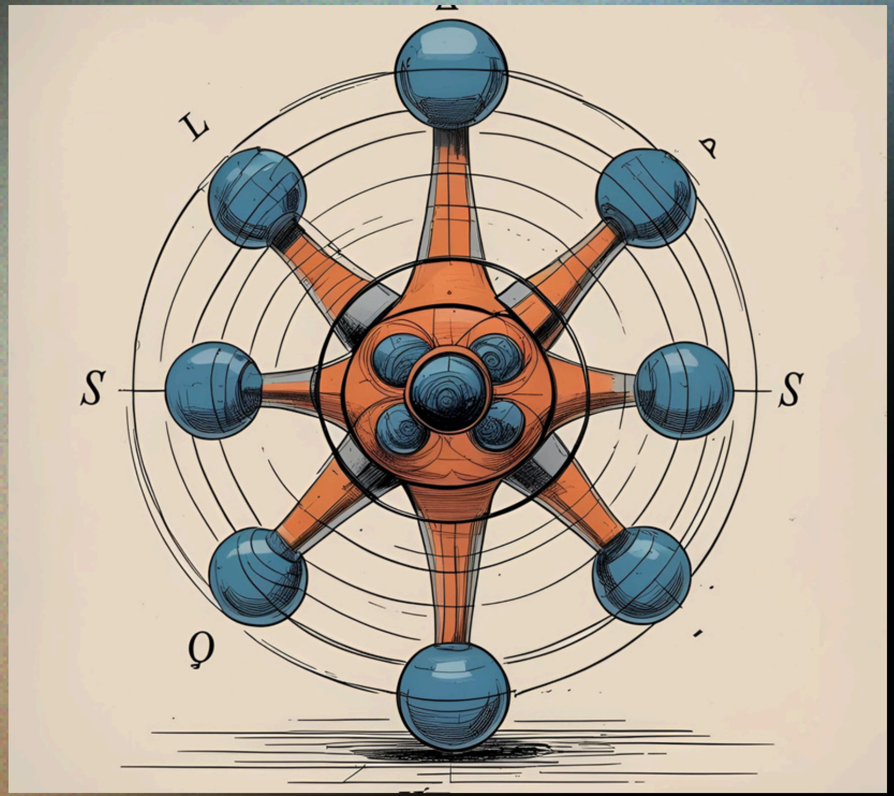
QHC18

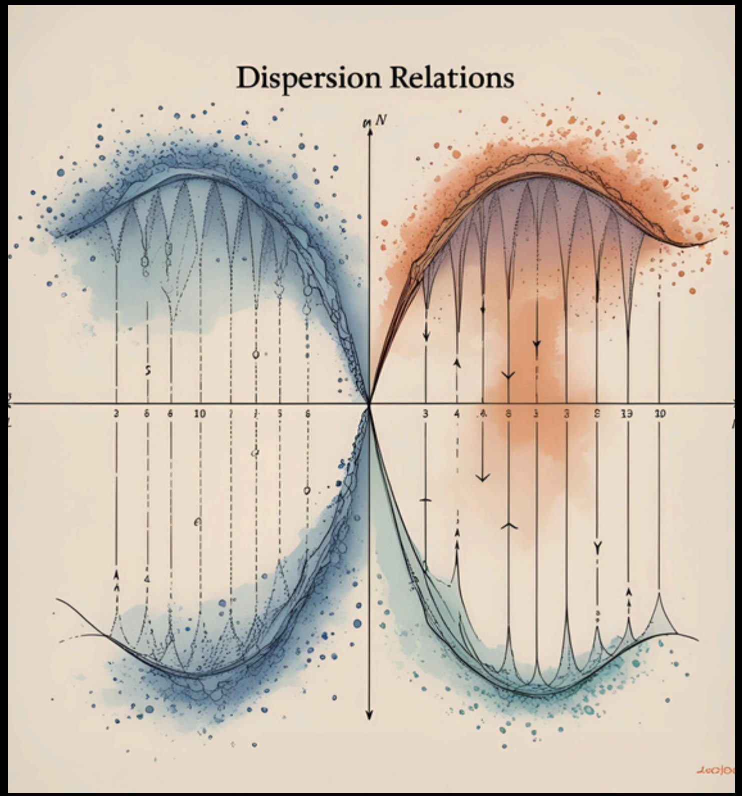
OBJECTIVE

To give an explanation for the existence of the peak in the speed of sound in the isospin imbalanced matter



THEORICAL FRAMEWORK



1. **LINEAR SIGMA MODEL WITH QUARKS**

2. **CONDENSATES**

3. **DISPERTION RELATIONS**

4. **GOLDSTONE MODES AND WARD ID.**

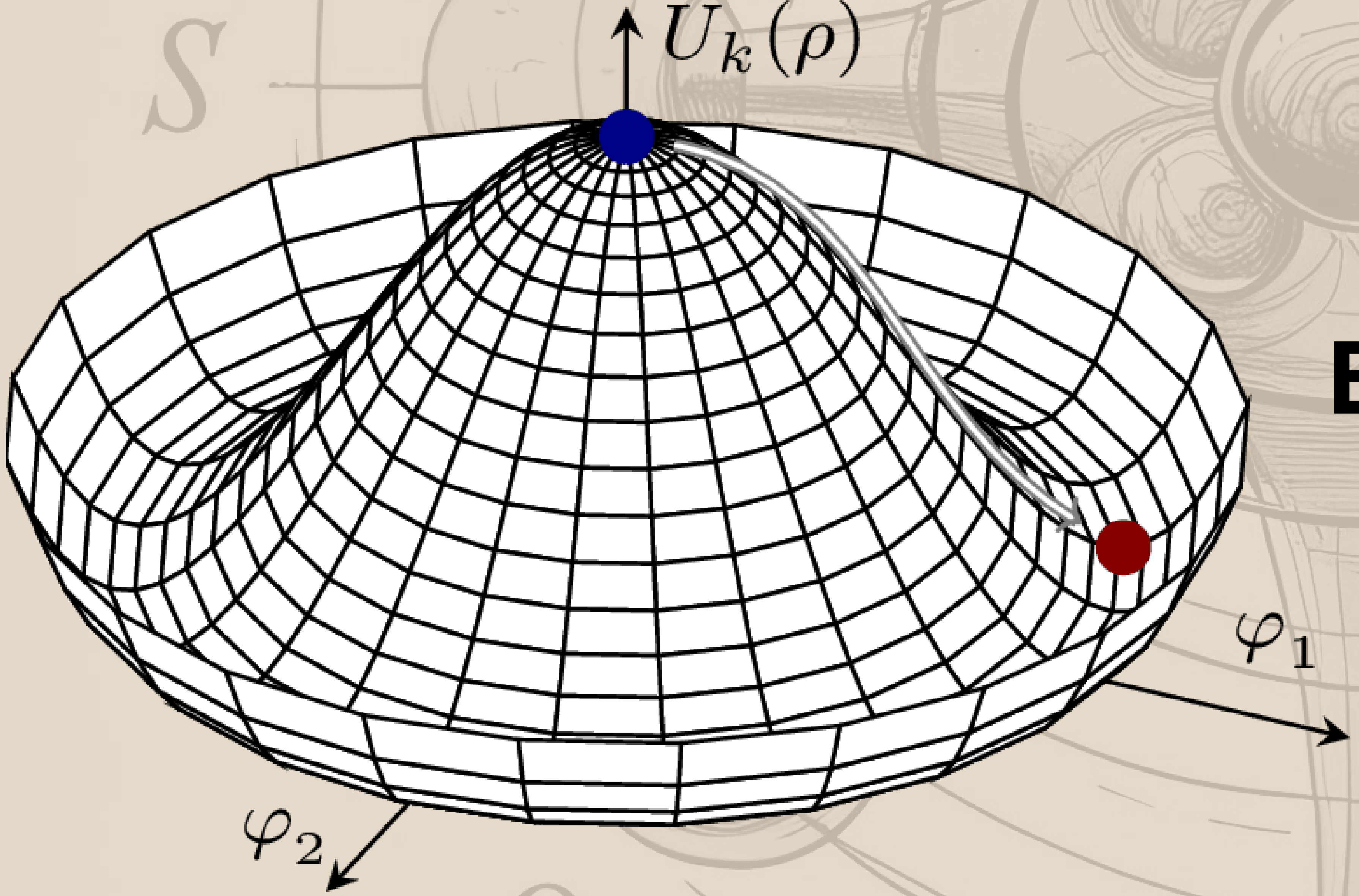
LINEAR SIGMA MODEL WITH QUARKS

$$\mathcal{L}_{MSLq} = \frac{1}{2}\left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2\right] - \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - ig\bar{\psi}\gamma^5 \vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\sigma\psi,$$

$$\psi_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R},$$

$$SU(2)_L \times SU(2)_R,$$

LINEAR SIGMA MODEL WITH QUARKS



$SU(2)_L \times SU(2)_R$, **6**

Break sym.

$SU(2)_V$, **3**

CONDENSATES

QUIRAL CONDENSATE

$$\sigma \to \sigma + v.$$

$$m_f^2 = g^2 v^2 \quad ; \quad m_{\pi^0}^2 = \lambda v^2 - a^2 \quad ; \quad m_\sigma^2 = 3\lambda v^2 - a^2,$$

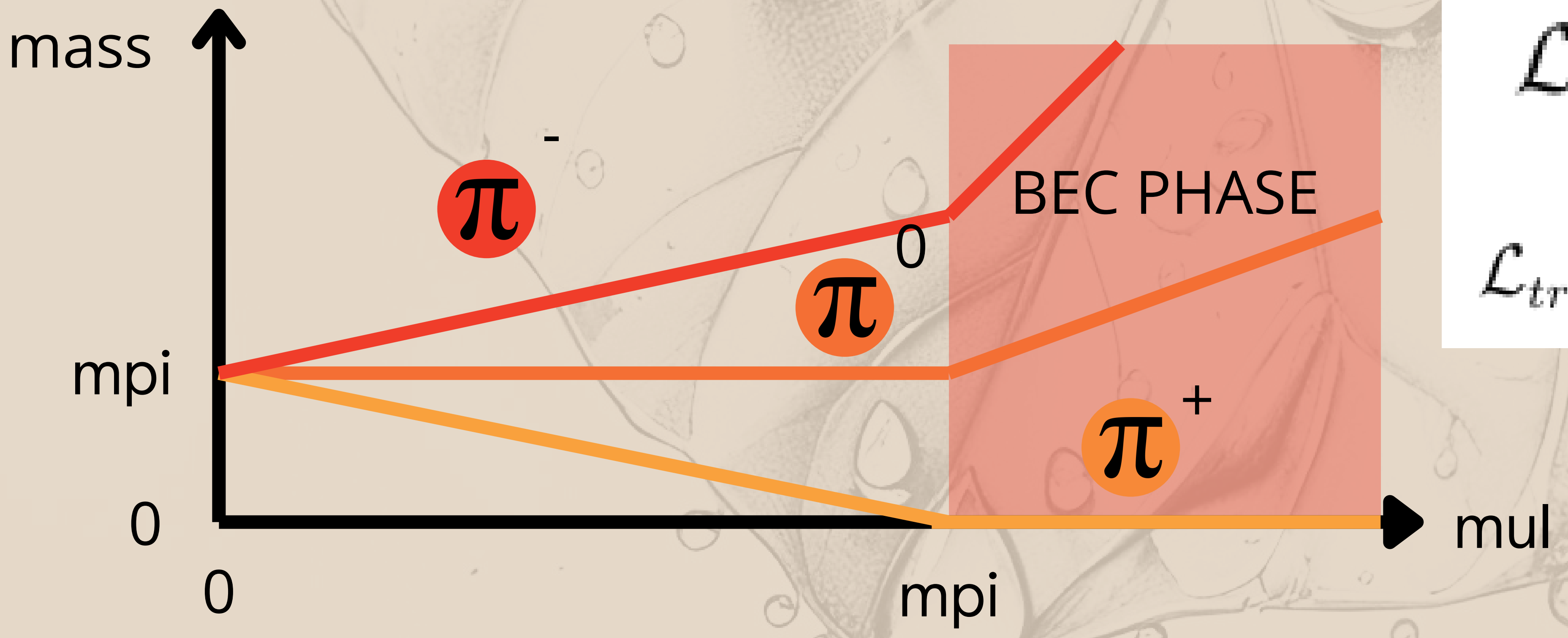
$$\mathcal{L}_{\text{explicit}} = h(\sigma + v),$$

CONDENSATES

PION CONDENSATE

$$D_\mu = \partial_\mu + i\mu_I \hat{I}_3 \delta^0_\mu \pi_\pm,$$

$$\pi_\pm \to \pi_\pm + \frac{\Delta}{\sqrt{2}} e^{\pm i\theta},$$



$$\mathcal{L}_{LSMq} = \mathcal{L}_{tree} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4,$$

$$\mathcal{L}_{tree} = \frac{a^2}{2}(v^2 + \Delta^2) - \frac{\lambda}{4}(v^2 + \Delta^2)^2 + \frac{1}{2}\mu_I^2 \Delta^2 + hv,$$

DISPERTION RELATIONS

$$\mathcal{L}_{LSMq} = \mathcal{L}_{tree} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4,$$

$$E_u = \sqrt{\left(\sqrt{k^2 + m_f^2} + \mu_I/2\right)^2 + g^2\Delta^2}$$

$$E_d = \sqrt{\left(\sqrt{k^2 + m_f^2} - \mu_I/2\right)^2 + g^2\Delta^2}.$$

QUARK DISP. RELS.

DISPERTION RELATIONS

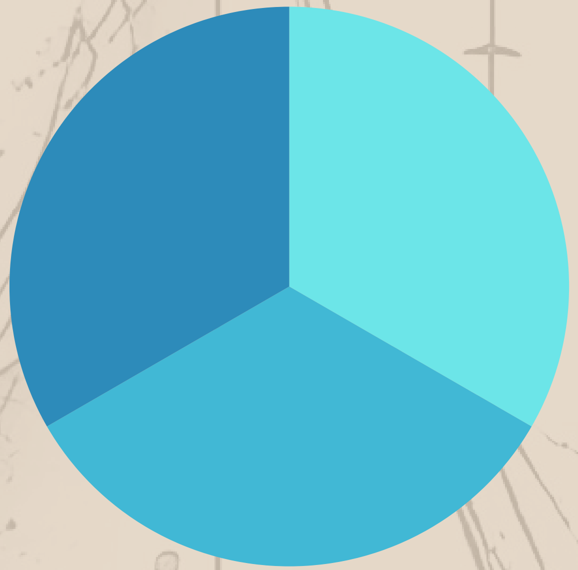
$$\mathcal{L}_M = \begin{pmatrix}\pi^- & \pi^+ & \sigma\end{pmatrix} D_M^{-1} \begin{pmatrix}\pi^+ \\ \pi^- \\ \sigma\end{pmatrix} + \frac{\pi_0^2}{2}(k_0^2 - E_{\pi^0}),$$

$$E_1 = \sqrt{\frac{a_1}{3} - 2\sqrt{-Q}\cos\left(\frac{\theta}{3}\right)},$$

$$E_2 = \sqrt{\frac{a_1}{3} - 2\sqrt{-Q}\cos\left(\frac{\theta + 2\pi}{3}\right)},$$

$$E_3 = \sqrt{\frac{a_1}{3} - 2\sqrt{-Q}\cos\left(\frac{\theta + 4\pi}{3}\right)},$$

MESON DISP. RELS.



$$E_{\pi^0} = \sqrt{\vec{k}^2 + m_\pi^2}.$$

ONE LOOP CORR.

$$V_{eff} = V_{tree} + V_f^{(1)} + V_M^{(1)},$$

$$V_M^{(1)} = -i\Omega^{-1} \ln[\mathrm{Det}[D_M^{-1}]],$$

$$V_f^{(1)} = i\Omega^{-1} \ln[\mathrm{Det}[S_{m_f}^{-1}]].$$

$$k_0^2 - E_i^2$$

$$-\frac{m^4}{2^5\pi^2}\left[\frac{1}{\epsilon} + \frac{3}{2} + \ln\left(\frac{\Lambda^2}{m^2}\right)\right]$$

ONE LOOP CORR.

$$-\frac{m^4}{2^5\pi^2}\left[\frac{1}{\epsilon}+\frac{3}{2}+\ln\left(\frac{\Lambda^2}{m^2}\right)\right]$$

$$\delta V_{CT}=\frac{\delta_a a^2}{2}\left(v^2+\Delta^2\right)$$

$$-\frac{\delta_\lambda \lambda}{4}\left(v^2+\Delta^2\right)^2+\frac{\delta}{2}\mu_I^2\Delta^2.$$

$$\delta_a=\frac{3\lambda}{8\pi^2\epsilon}+\text{términos finitos}$$

$$\delta_\lambda=\frac{3\lambda}{4\pi^2\epsilon}-\frac{g^4\text{Nc}}{2\pi^2\lambda\epsilon}+\text{términos finitos}$$

$$\delta=\frac{g^2\text{Nc}}{4\pi^2}+\text{términos finitos}$$

$$\left.\frac{\partial V_{eff}}{\partial v}\right|_{\mu_I\to m_\pi}=0,\quad \left.\frac{\partial^2 V_{eff}}{\partial v^2}\right|_{\mu_I\to m_\pi}=m_\sigma^2,\quad \left.\frac{\partial^2 V_{eff}}{\partial \Delta^2}\right|_{\mu_I\to m_\pi}=0.$$

WARD IDENTITIES PCAC

$$\partial_z^\mu T[\vec{A}_\mu(z)\sigma(x)\pi^0(y)]$$

$$\partial_\mu A^{i\mu}(x) = h\pi^i(x).$$

$$-iv\Gamma_{\sigma\pi\pi}(p;0,-p) = \Delta_\sigma^{-1}(p) - \Delta_\pi^{-1}(p).$$

$$-2\lambda v^2 = \Delta_\sigma^{-1}(p) - \Delta_\pi^{-1}(p).$$

→

$$\delta_\lambda = \delta_a.$$

$$4g^4 N_c = 3\lambda^2,$$

$$m_{0\sigma}^2 = 4\,m_{0f}^2 + m_{0\pi}^2.$$

$$a = \sqrt{\frac{1}{2}\left(m_{0\sigma}^2 - 3m_{0\pi}^2\right)}, \quad \lambda = \frac{m_{0\sigma}^2 - m_{0\pi}^2}{2f_\pi^2}, \quad g = \frac{m_{0f}}{f_\pi}.$$

GOLDSTONE MODES

$$\mathrm{Det}[D'^{-1}_M(k=0)] = m_\sigma^2\left(m_\pi^2 - \mu_I^2\right)\left(m_\pi^2 - \mu_I^2 + 2\lambda\Delta^2\frac{m_\pi^2}{m_\sigma^2}\right).$$

C **NC**

NON CLASICAL

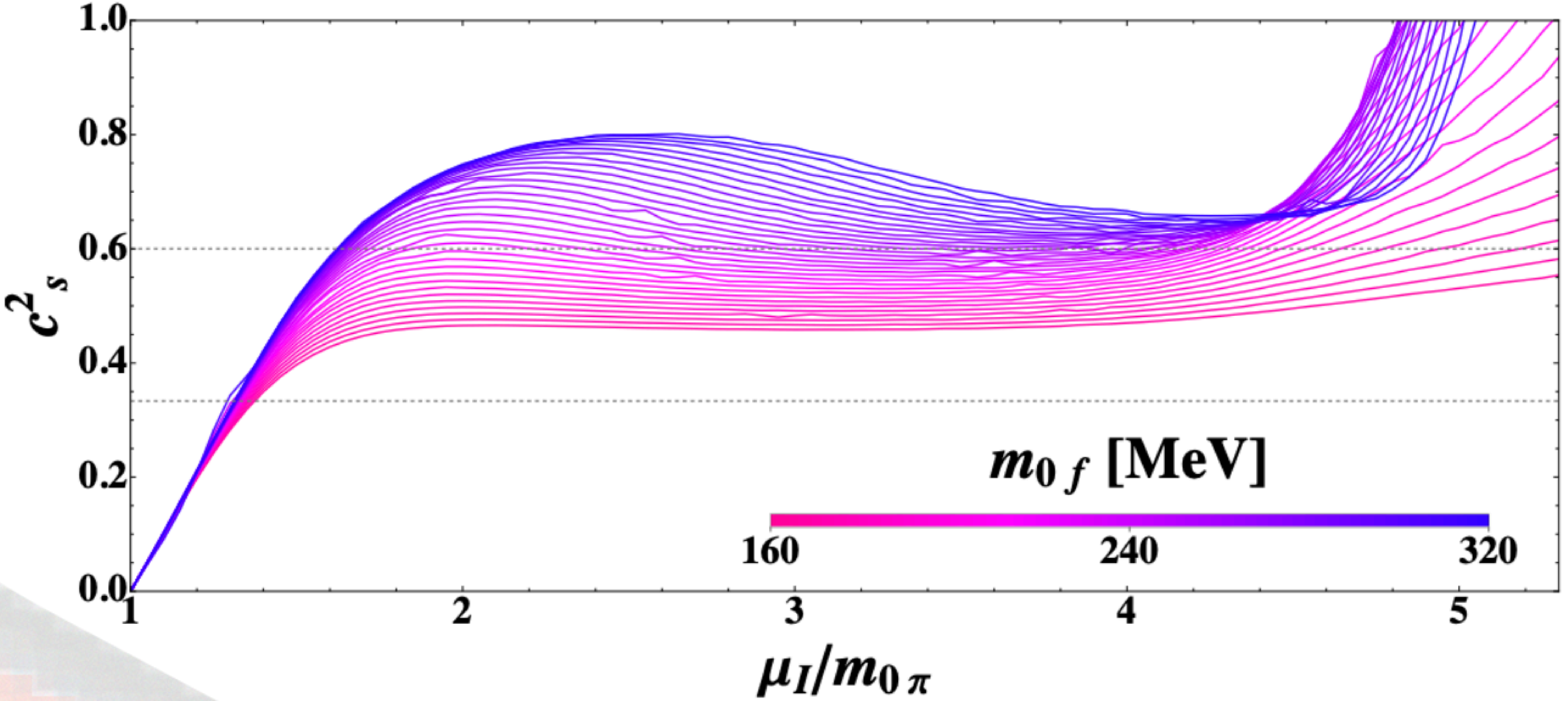
$$\Delta_1 = \sqrt{\frac{4a^2 + \mu_I^2 - 6\lambda v^2 + \sqrt{4a^4 - 4a^2\mu_I^2 + \mu_I^4 + 24\lambda\mu_I^2 v^2}}{6\lambda}},$$

CLASICAL

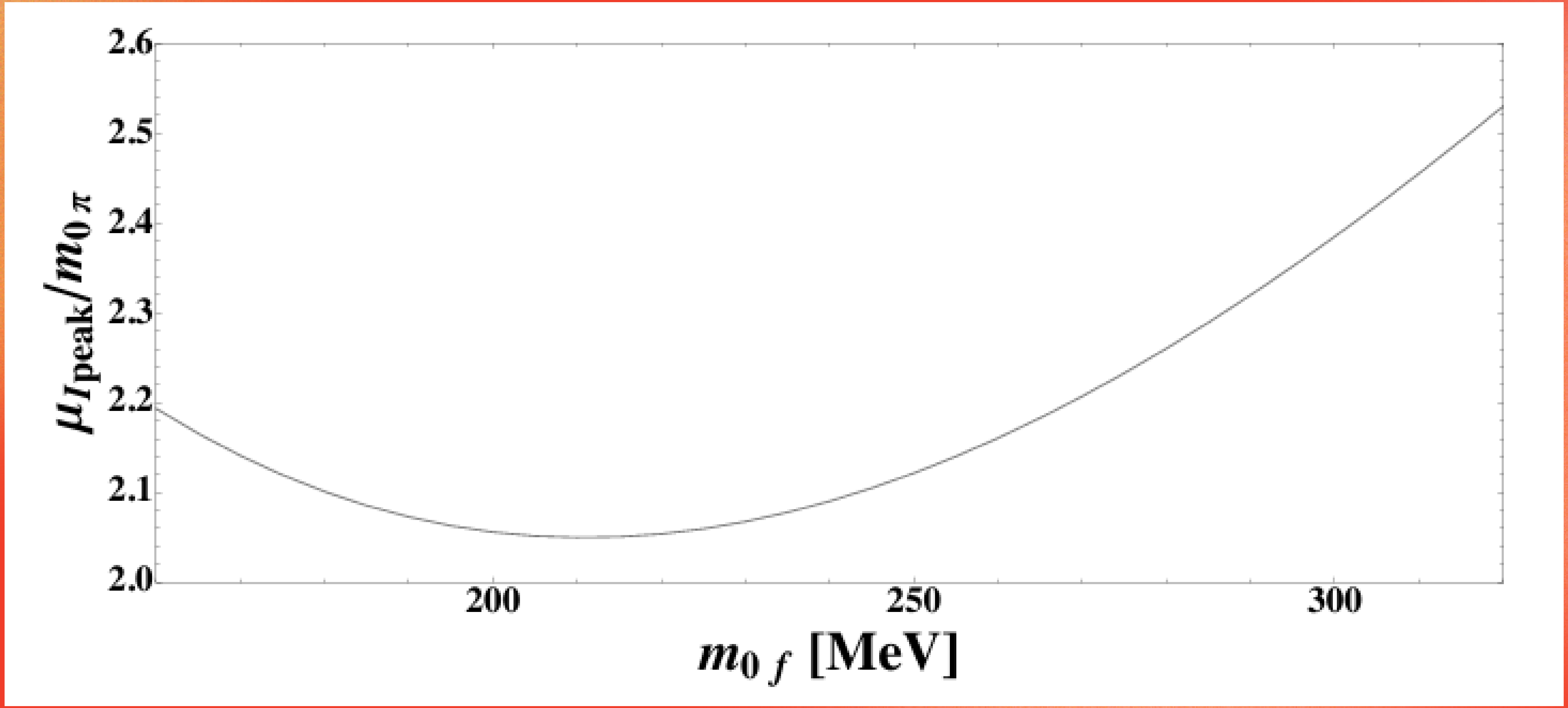
$$\Delta_2 = \sqrt{\frac{\mu_I^2 - (\lambda v^2 - a^2)}{\lambda}},$$

RESULTS

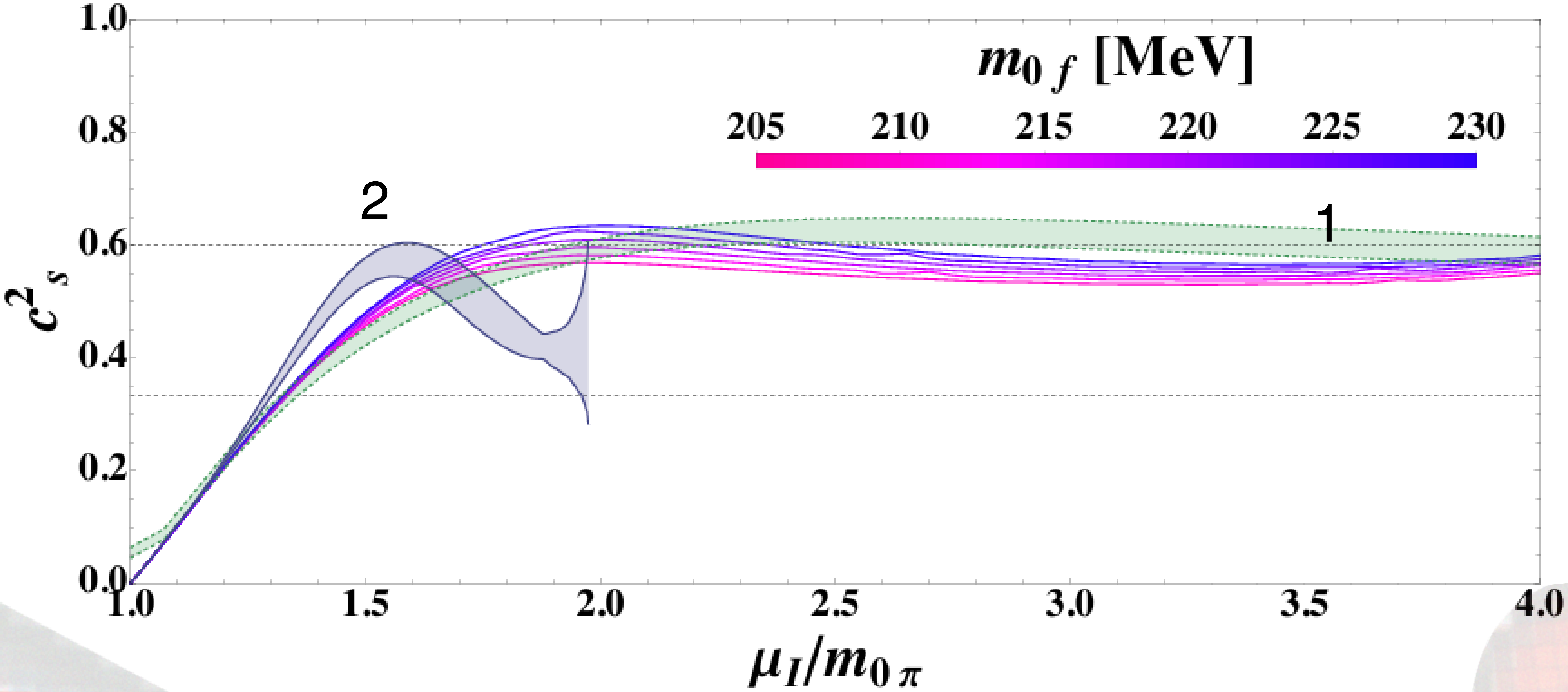
SPEED OF SOUND PEAK



PEAK POSITION



PEAK COMPARED



1 (ABBOTT, DETMOLD, ROMERO-LÓPEZ, ET AL. 2023)
2 (BRANDT, CUTERI & ENDRODI, 2023)

CONCLUSIONS

CONCLUSIONS

THE LSMQ DESCRIBES THE PEAK IN THE SPEED OF SOUND

AGREEMENT BETWEEN LSMQ AND LQCD

RELEVANCE OF GOLDSTONE MODES

ORIGIN OF THE PEAK IN THE SPEED OF SOUND

THANK YOU

Luis Carlos Parra Lara

Posgrado en Ciencias Fisica

IS MUI MESSURABLE?

Experimento	$\sqrt{s_{NN}}$	μ_B [MeV]	μ_{I_3} [MeV]
LHC (Pb+Pb)	2.76–5.36 TeV	1−10	10-30
RHIC (Au+Au)	7.7–200 GeV	20−400	30−80
NICA (Au+Au)	4–11 GeV	400−700	80−150
FAIR (SIS100)	2–8 GeV	500−800	150−300

Notas:

- Todos los sistemas presentan desbalance por usar núcleos pesados (Pb/Au) con exceso de neutrones
- μ_{I_3} estimado usando relaciones de QCD en equilibrio químico
- Valores dependen de modelos teóricos (e.g. HRG, NJL) → **no conclusive**

Referencias:

- J. Randrup y J. Cleymans, *Phys. Rev. C* **74**, 047901 (2006).
- ALICE Collaboration, *Nature Phys* **13**, 535–539 (2017).
- V. Kekelidze et al., *Nucl. Phys. A* **956**, 846 (2016).
- X. Xia et al., *Phys. Rev. C* **101**, 054903 (2020) [Isospin effects]

DOES MUI BREAK CP?

$$-\frac{\lambda\Delta}{\sqrt{2}}(\sigma^2+\pi^2)(e^{-i\theta}\pi_+ + e^{i\theta}\pi_-)$$

MOST PROBLEMATIC TERM

$$\Delta = \langle \pi^+(x) \rangle,$$

$$\pi_\pm \to \pi_\pm + \frac{\Delta}{\sqrt{2}} e^{\pm i\theta},$$