Critical properties for RFT at One loop improved Wilsonian regulator and ε – expansion

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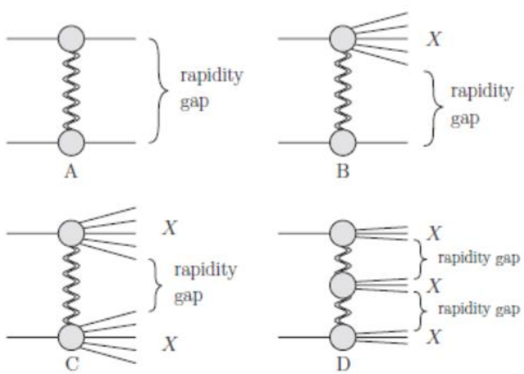
Outline

- Motivation
- N-Pomerons fields and Reggeons interactions
- One loop and interaction of Reggeons
- Numerical results for 2 Reggeons Interaction
- Summary and Outlook

JHEP 1603 (2016) 201
PRD 95 (2017) 014013
JHEP 05 (2024) 032
Two Reggeons: arXiv to appear

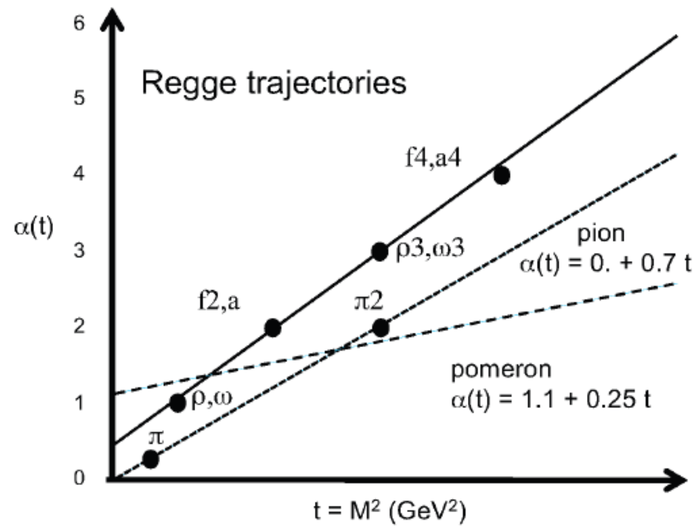
Motivation

- For Diffractive process, in the Regge Limit $(s > t)$, t-channel exchange dominate and then becomes an effective 2+1dimensional: transversal space and rapidity (Lipatov effective action/CGC/Dipole approximation)
- At very small transverse distances: pQCD and BFKL Pomeron (1958)
- At very large transverse distances before QCD era, there was the Reggeon Field Theory description introduced by Gribov
- The Pomeron is usually related to gluonic Exchange: state of two gluon and C=1



Reggeon Field Theory before QCD

- V. N. Gribov introduce in the 60´s: RFT
- Scattering amplitude at high energies for hadrons is according Regge Theory: the exchange of "quasi particles" characterized by its Regge trajectories : $\alpha_i\,(t)$

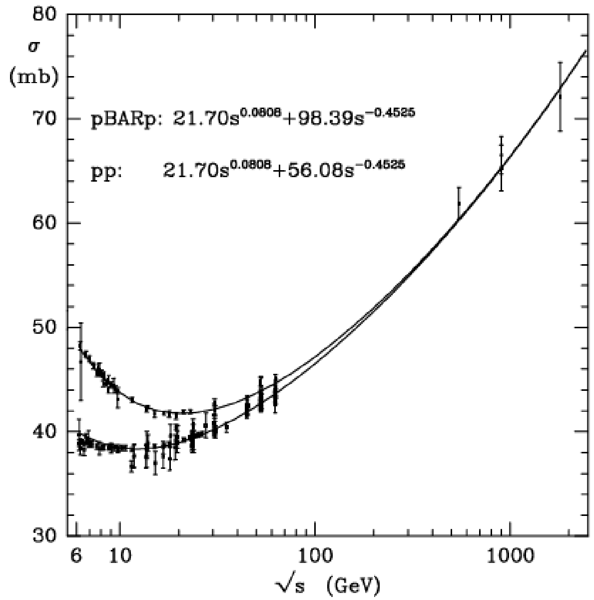


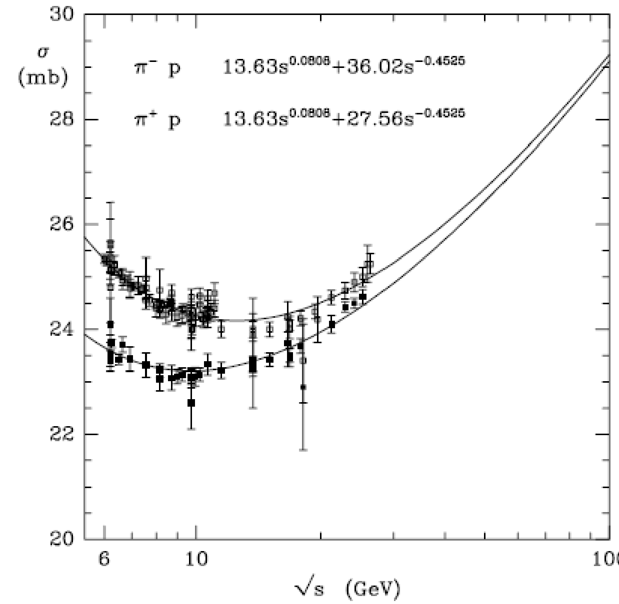
- the total Cross section, is given by: $\sigma_T = A_i s^{\alpha_i(0)-1}$
- Leading Pole: is Called Pomeron with vacuum quantum numbers

$$\alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t$$

$\mu = \alpha_0 - 1$ is the Pomeron intercept and α' is the Pomeron slope

A. Donnachie and Landshoff : Phys. Lett. B 727 (2013) 500 arXiv 1309.1292





- cross section grows with energy

  Unexpected behavior: Soft Pomeron exchange

$$\alpha_P(t) \;=\; 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$$

BFKL or Hard Pomeron pQCD

Balinsky, Fadin, Kuraev, Lipatov (1977)

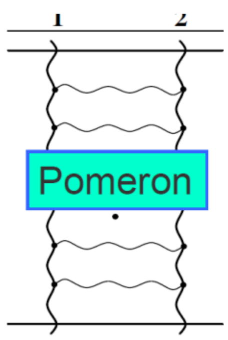
The BFKL Pomeron which has been studied up to NLO in perturbation theory is a composite states of Reggeized gluons.

Resumation of the Ladder at leading log approximation (multi Regge Kinemmatics MRK) Lipatov, Bartels

- **The intercept of the Pomeron is related with the eigenvalues of the BFKL Kernel**

$\boldsymbol{\omega_0(\gamma) = \alpha_s N_c/\pi\ [2\,\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)]}$ $\quad \Psi(\gamma) = \frac{d}{d\gamma} Ln\,\Gamma(\gamma)$ Digamma function

$$\alpha_P(0) \approx 1 + \frac{\alpha_s N_c}{\pi} 4 \ln 2 \approx 1 + 0.5295 \quad Hard\ BFKL/QCD$$



$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$

$\alpha_{P,k}(0)$

we can connect Hard –Soft Pomeron regions of different sizes and different sorts of Pomeron using

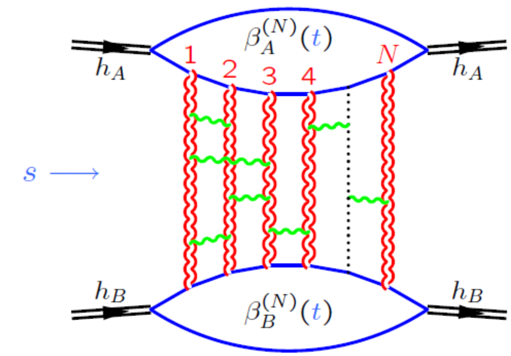
Renormalization Group approach

- *Exact/Functional Renormation Group*
- *One loop improved approximation*

How we can study another states with 3, 4 gluons ?

1973 Lukaszuk and Nicolescu proposal the Odderon as the odd partner of the Pomeron (C =-1) 3 gluon

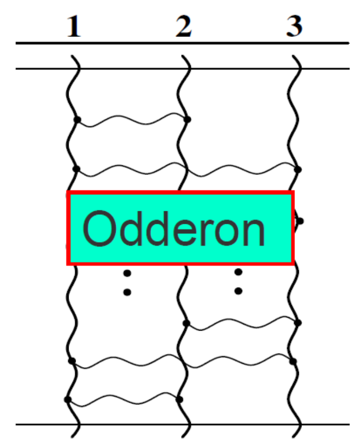
1980 J. Bartels; J. Kwiecinski and M. Praszalowicz:



Multi-Reggeons equation BKP

What we can study:

- Solutions
- Intercept and the Slope



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

- t-dependence of elastic cross section shows difference between pp and $p\bar{p}$ (evidence for existence of Odderon)

***Phys. Lett.B* 778 (2018) 414-418**

Did TOTEM experiment discover the Odderon?

Evgenij Martynov[a], Basarab Nicolescu[b]

[a]*Bogolyubov Institute for Theoretical Physics, Metrologichna 14b, Kiev, 03680 Ukraine*
[b]*Faculty of European Studies, Babes-Bolyai University, Emmanuel de Martonne Street 1, 400090 Cluj-Napoca, Romania*

Experimetal data from Tevatron and $D\emptyset$

T. Csörgö, R. Pasechnik and A. Ster *Eur. Phys. J. C* 79 (2019) 1, 62

M. Broilo E.G.S. Luna M.J. Menon ***arXiv:1803.06560***

Csorgo et al. EPJC 81 (2021) 2

- There are evidence for the non-perturbative Odderon
- Intercept and the Slope

N-Pomerons

- The BFKL Pomeron spectrum then becomes discrete, even at relatively short distances. This corresponds to the appearance of bound state Regge poles (Pomeron states)

- For these discrete Pomeron states the eigenfunctions have been studied
  - J. Bartels, C. Contreras and G. P. Vacca, Phys. Rev. D 95 (2017 [arXiv:1608.08836 [hep-th]].
- What remains is the 'unitarization' of this set of Pomeron states: this requires, in particular, the introduction of the triple Pomeron vertex.

- Kowalsky and others have used the discrete Pomerons to fit the small-x and low-Q2 HERA data, which allows to fix the infrared boundary values. All these approaches only use the contribution of the discrete BFKL Pomeron, and so far no attempt has been made to introduce the triple Pomeron vertex.

  H. Kowalski, L. N. Lipatov, D.A. Ross and O. Schulz, Eur. Phys. J. C 77 (2017) no.11, 777 [arXiv:1707.01460 [hep-ph]].

- How we can study this interaction of N-Pomerons /Reggeons with interactions?

The Reggeons Field Theory

Let us define here the theory which we want to investigate. We shall consider a set of fields and their conjugate ψ_i , $\psi_i^\dagger$, for $\mathrm{i} = 1..\mathrm{N}$, whose evolution and interactions are defined (Gribov - action)

$$\Gamma\left[\psi_i, {\psi_i}^\dagger\right] = \int d^D x d\tau \left[\left(\frac{1}{2}{\psi_i}^\dagger \partial_\tau \psi_i + \alpha_i' \psi_i^\dagger \nabla^2 \psi_i\right) + \mu_i \psi_i^\dagger \psi_i + V[\psi_i, \psi_i^\dagger; \lambda_n]\right]$$

$$V(\psi, \psi^\dagger) = \sum_{jkl} \frac{i}{2} \alpha' \lambda_{j,kl} \left(\psi_j^\dagger \psi_k \psi_l + \psi_j \psi_k^\dagger \psi_l^\dagger\right) \quad \lambda_{i,jk} = \lambda_{i,kj}$$

The interaction N=2 discrete fields is described by the cubic potential V_k:

$$(\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6) = (\lambda_{1,11},\ \lambda_{1,22},\ \lambda_{2,11},\ \lambda_{2,22},\ \lambda_{1,21} = \lambda_{1,12},\ \lambda_{2,21} = \lambda_{2,12})$$

$$\begin{aligned} V_k[\psi_1, \psi_1^\dagger, \psi_2, \psi_2^\dagger] \ = \ & i\lambda_1 \psi_1^\dagger(\psi_1^\dagger + \psi_1)\psi_1 + i\lambda_2 \psi_2^\dagger(\psi_2^\dagger + \psi_2)\psi_2 + i\lambda_3 \psi_1^\dagger(\psi_2^\dagger + \psi_2)\psi_1 + \\ & i\lambda_4 \psi_2^\dagger(\psi_1^\dagger + \psi_1)\psi_2 + i\lambda_5(\psi_2^{\dagger 2}\psi_1 + \psi_2^2 \psi_1^\dagger) + i\lambda_6(\psi_1^{\dagger 2}\psi_2 i + \psi_1^2 \psi_2^\dagger), \end{aligned}$$

Renormalization Group approach

We need to study the evolution with the k-scale:

- *Reggeon Intercept:* $\mu_{i,k}$
- *Reggeon slope:* $\alpha'_{i.k}$ $\qquad r = \frac{\alpha_2'}{\alpha_1'}$
- *Coupling constants* $\lambda_{i,k}$
- *Scaling behavior for the cross section to high energy*

Exact/Functional Renormalization Group

Using FRG and Wetterich Equation 93

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

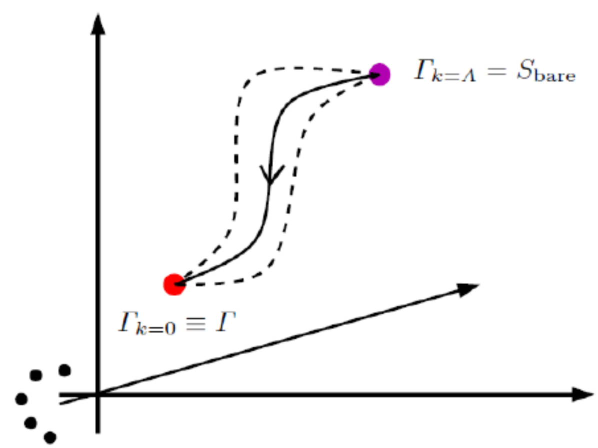
Action $\Gamma_k(\psi_i, \psi_i^\dagger\ ,\ \lambda_i)$

Regulator $R_k(q)$

Initial Condition $\Gamma_{k=\Lambda}(\phi, g_i)$

Interaction $V = \sum \lambda_{i;jk}\,(\psi_i^\dagger \psi_j \psi_k + cc)$

$$R_1 = Z_1\alpha_1'(k^2 - q^2)\Theta(k^2 - q^2)$$
$$R_2 = Z_2\alpha_2'(k^2 - q^2)\Theta(k^2 - q^2) = rZ_2\alpha_1'(k^2 - q^2)\Theta(k^2 - q^2),$$



Calculation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\, \delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

$$Tr[\ \dots\ ] \equiv \int\int \frac{dq^D}{(2\pi)^D} \frac{d\omega}{(2\pi)} Tr[\ \dots\ ].$$

$\Gamma_k(\phi, \lambda_i, \mu_i) = \sum_i g_i(k) O_i\,(\phi)$
$\partial_t\, \Gamma_k \rightarrow \sum_i \partial_t g_i(k) * O_i\,(\phi) \rightarrow \beta_i(k)$ asociadas al $\{O_i\}$ operator basis.

Beta Funtions: $\beta_i(k) = \partial_t g_i(k)$

Fixed Points Conditions $\partial_t \Gamma_k^* \cong 0 \qquad y \quad t = \ln(\frac{k}{\Lambda})$

Flow equation for the Reggeon theory $\beta_i = \frac{\partial g_i}{\partial t}$

Beta Funtions: $g_i(k) = (\,\mu_i, m, \lambda_{1:11}\,, \lambda_{2:22}, \lambda_{1:12}, \lambda_{1:22,}\ \lambda_{2:21,}\ \lambda_{2:11})$

Dimensionaless variable

Cancino and Contreras, Universe **10 (2024) 3, 103**

$$\tilde{\psi}_1 = \sqrt{Z_1} k^{-D/2} \psi_1 \,,\ \tilde{\psi}_1 = \sqrt{Z_2} k^{-D/2} \psi_2 \,,\ \tilde{V} = \frac{V}{\alpha'_1 k^{D+2}}$$

$$\tilde{m} = \frac{m}{\sqrt{Z_1 Z_2 \alpha'_1 k^2}} \,,\ \tilde{\mu}_1 = \frac{\mu_1}{Z_1 \alpha'_1 k^2} \,,\ \tilde{\mu}_2 = \frac{\mu_2}{Z_2 \alpha'_1 k^2}$$

$$\tilde{\lambda}_1 = \frac{\lambda_1 k^{D/2}}{Z_1^{3/2} \alpha'_1 k^2} \,,\ \tilde{\lambda}_2 = \frac{\lambda_2 k^{D/2}}{Z_2^{3/2} \alpha'_1 k^2} \,,\ \tilde{\lambda}_{3,6} = \frac{\lambda_{3,6} k^{D/2}}{Z_1 \sqrt{Z_2} \alpha'_1 k^2} \,,\ \tilde{\lambda}_{4,5} = \frac{\lambda_{4,5} k^{D/2}}{Z_2 \sqrt{Z_1} \alpha'_1 k^2}$$

$$\eta_{\mathrm{i}} = -\frac{1}{\mathrm{Z_i}} \partial_t Z_i \qquad\qquad \xi_i = -\frac{1}{\alpha'_i}\, \partial_t \alpha'_i$$

$$r_2 = \alpha'_2 / \alpha'_1 \qquad\qquad \dot{r} = r\,(\xi_1 - \ \xi_2)$$

See Talk from Cancino on the Fixed point Position in this 10 dimensional space parameter and about the diagonalization process

It is possible to obtain a similar result that verifies this analysis.?

Bartels, Contreras and Vacca to appear in arXiv
M. Braun et al arXiv : 2311.13870

One loop improved perturbation theory using IR regulator

One loop calculation with non Reggeon intercept. **M. Braun et al arXiv : 2311.13870**

- At a low approximation the FRG/Wetterich reduces to one loop results.

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right] = \frac{1}{2}\partial_t \mathrm{Tr\ Log}[\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k]$$

- Perturbation theory can immediately be re-derived from the flow equation. For instance, imposing the loop expansion on Γ_k, it becomes obvious that, to one-loop order

- The last formula corresponds to the standard effective action of a loop, as it should be. In the perturbative limit, it is the expansion of the loop for the effective potential, with a modified infrared propagator.

$$G[E,\vec{q};k] = \frac{1}{E - \alpha`(q^2 + R_k(q)) + \mu_0}$$

$$q^2 \to q^2 + R_k(q^2), \text{ and } R_k(q) = (k^2 - q^2)\theta(k^2 - q^2)$$

- The results obtained in the one loop approximation describe relative good the nonperturbative result from FRG in the IR region

- Obtain information in this approximation can help to obtained the physical fixed point in this N dimensional functional space of parameters

$$g_i(k) = (\mu_i, \lambda_{1:11}, \lambda_{2:22}, \lambda_{1:12}, \lambda_{1:22}, \lambda_{2:21}, \lambda_{2:11})$$

D, Litim arxiv 0103195
D. Litim. J. Pawlosky arXiv: 0111191
M. Braun et al arXiv :2311.13870
Blaizot, Pawlowski, Reinosa: arXiv 1009.6048

- To obtain the beta functions, we need to renormalize the 2 point function, the 3 Reggeon vertex, and for the anomalous dimension the Wave function.

$$\Gamma\left[\psi_i,\psi_i{}^\dagger\right]=\int d^Dxd\tau\left[\left(\frac{1}{2}\psi_i{}^\dagger\partial_\tau\psi_i+\alpha_i'\psi_i^\dagger\nabla^2\psi_i\right)+\mu_i\psi_i^\dagger\psi_i+V[\psi_i,\psi_i^\dagger;\lambda_n]\right]$$

$$\lambda_{i,jk}=\lambda_{i,kj}$$

- $r_1=\sqrt{\alpha'_1/\alpha'_2}$; $r_2=\sqrt{\alpha'_2/\alpha'_1}$ $\qquad \dot{r}=r\left(\xi_1-\xi_2\right)$

To calculate the Pomeron corrections to the n-point Green function, we consider the following rules:

(a) define the IR regulated free Pomeron propagator of momentum q and energy E as

$$G(E,q^2;k)=\frac{i}{E-\alpha_0'(q^2+R_k(q))+m_0+i\epsilon}, \tag{A.1}$$

where E is integrated along the real axis;

(b) for each loop $\frac{dEd^Dq}{(2\pi)^{D+1}}$;

(c) if the triple coupling λ_0 would be real, each vertex comes with $(-i\lambda_0)$. Alternatively, since we choose an imaginary coupling we put λ_0.

One loop improved perturbation theory using IR regulator

- The Reggeon propagator in the RFT is modified, considering the IR Cut-off

$$G(E,q^2;k) = \frac{i}{E - \alpha_0'(q^2 + R_k(q)) + m_0 + i\epsilon};$$

where $R_{\mathrm{k}}(q)$ is the IR cutoff regulator: Litim regulator $R_{\mathrm{k}}(q) = (k^2 - q^2)\theta[k^2 - q^2]$

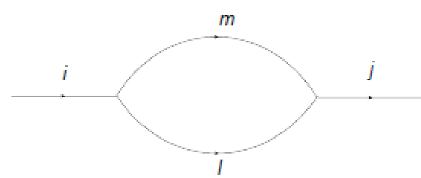


Figure 1: One loop 2-point function

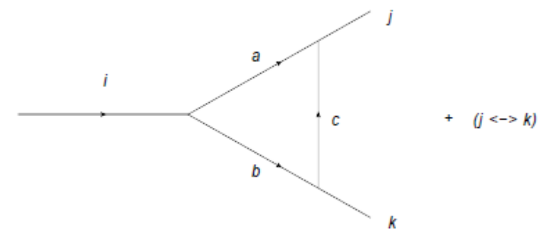


Figure 2: One loop 3-point function

Renormalization condition a la Abarbaned and Bronzan **Phy Rev D 9 (1974) 2397**

For one Pomeron we will extender to 2 Reggeons.

Self energy and wave function renormalization: Z

2 point function at one loop approximation

$$i\Gamma_U^{(1,1)}(W,q^2;k) = \begin{pmatrix} E-\alpha'_{0;1}q^2+m_{0;1} & 0 \\ 0 & E-\alpha'_{0;2}q^2+m_{0;2} \end{pmatrix} - i\Gamma_{AB;k}^{(1,1)}(E,q^2) \qquad \Gamma_{AB,k}^{(1,1)} = \begin{pmatrix} \Gamma_{AB;11}^{(1,1)} & \Gamma_{AB;12}^{(1,1)} \\ \Gamma_{AB;21}^{(1,1)} & \Gamma_{AB;22}^{(1,1)} \end{pmatrix}$$

$$\Gamma_{ren;first\,step}^{(1,1)}(E,q^2;k) = Z_k^T\Gamma_U^{(1,1)}(E,q^2;k)Z_k \qquad Z_k = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

Z is a matrix and fixed by the renormalization condition

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\partial}{\partial E} i\Gamma_{ren;first\,step}^{(1,1)}(0,0;k) = Z_k^T\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + S_k\right)Z_k \quad S_k = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \frac{\partial}{\partial E}(-i\Gamma_{AB,k}^{(1,1})|_{(0,0)},$$

Then we obtain after the Z

$$Z^T i\Gamma_U^{(1,1)} Z = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} + Z^T\left(M - q^2 A\right)Z + \ldots$$

$$\bar{Z}^T (1+S) \bar{Z} = \begin{pmatrix} 1+e_1 & 0 \\ 0 & 1+e_2 \end{pmatrix} = \begin{pmatrix} l_1 & 0 \\ 0 & l_2 \end{pmatrix} = L$$

$$Z = \bar{Z}\sqrt{L}^{-1}, \; Z^T = \sqrt{L}^{-1}\bar{Z}^T$$

$$e_{1,2} = \tfrac{s_{11}+s_{22}}{2} \pm R_s, \text{ where } R_s = \sqrt{(\tfrac{s_{11}-s_{22}}{2})^2 + s_{12}^2}.$$

$$\bar{Z} = (E_1 E_2) = \begin{pmatrix} \sqrt{1-\delta} & -\sqrt{\delta} \\ \sqrt{\delta} & \sqrt{1-\delta} \end{pmatrix}.$$

$$\delta = \frac{1}{2}\left(1 - \frac{s_{11}-s_{22}}{2R_s}\right), \; 1-\delta = \frac{1}{2}\left(1 + \frac{s_{11}-s_{22}}{2R_s}\right)$$

Self energy and wave function renormalization Z

$$Z^T i\Gamma_U^{(1,1)} Z = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} + Z^T \left(M - q^2 A\right) Z + \dots$$

Then we obtain after the Z

Diagonalization of the Slope Matrix Z_α *and* intercept Z_m

$$A = \left(\begin{pmatrix} \alpha'_{0,1} & 0 \\ 0 & \alpha'_{0,2} \end{pmatrix} + a \right) = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad a_k = -\frac{\partial}{\partial q^2}(-i\Gamma^{(1,1)}_{AB,k}|_{(0,0)}) = \begin{pmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{pmatrix}$$

Diagonalization of the Slope Matrix Z_α

$$\begin{pmatrix} \alpha'_1 & 0 \\ 0 & \alpha'_2 \end{pmatrix} = Z_\alpha^T \left(Z^T A Z\right) Z_\alpha \quad = Z_\alpha^T Z^T \left(\begin{pmatrix} \alpha'_{0,1} & 0 \\ 0 & \alpha'_{0,2} \end{pmatrix} + a \right) Z Z_\alpha .$$

We define the renormalized matrix $Z_1 = Z\, Z_\alpha$

$$\alpha'_1 = \alpha'_{0;1}(1 - s_{11}) + \alpha_{11} \ \text{ and } \ \alpha'_2 = \alpha'_{021}(1 - s_{22}) + \alpha_{22}$$

$$\frac{\partial}{\partial t}\alpha'_1 = -\alpha'_{0;1}\,\dot{s}_{11} + \frac{\partial}{\partial t}\alpha_{11}; \ \frac{\partial}{\partial t}\alpha'_2 = -\alpha'_{0;2}\,\dot{s}_{22} + \frac{\partial}{\partial t}\alpha_{22}.$$

$$\alpha'_{1,2} = \frac{\tilde{A}_{11} + \tilde{A}_{22}}{2} \pm R_a$$

$$Z_\alpha = \begin{pmatrix} \sqrt{1-\delta_\alpha} & -\sqrt{\delta_\alpha} \\ \sqrt{\delta_\alpha} & \sqrt{1-\delta_\alpha} \end{pmatrix},$$

where $R_a = \sqrt{(\frac{\tilde{A}_{11}-\tilde{A}_{22}}{2})^2 + \tilde{A}_{12}^2}$ and $\delta_a = \frac{1}{2}\left(1 - \frac{\tilde{A}_{11}-\tilde{A}_{22}}{2R_A}\right)$.

$$\begin{pmatrix} \alpha'_1 & 0 \\ 0 & \alpha'_2 \end{pmatrix} = Z_\alpha^T \left(Z^T A Z\right) Z_\alpha$$

For the coupling constants we now have:

$$\lambda_{i;jk} \to \frac{1}{2}\Big((Z_1^T)_{ii'}\lambda_{i';j'k'}(Z_1)_{j'j}(Z_1)_{k'k} + (Z_1)_{ii'}\lambda_{i';j'k'}(Z_1^T)_{j'j}(Z_1^T)_{k'k}\Big).$$

Diagonalization of the intercept (m_1, m_2): Z_m

$$Z^T i\Gamma_U^{(1,1)} Z = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} + Z^T\left(M - q^2 A\right) Z + \ldots$$

$$M = \left(\begin{pmatrix} m_{0,1} & 0 \\ 0 & m_{0,2} \end{pmatrix} + m\right) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \qquad m_k = -i\Gamma^{(1,1)}_{AB,k}|_{(0,0)} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

The eigenvalues of the matrix = intercept are:

$$m_1 = m_{0;1}(1 - s_{11}) + m_{11} \ , \ m_2 = m_{0;2}(1 - s_{22}) + m_{22}$$

$$\frac{\partial}{\partial t} m_1 = -m_{0;1}\, \dot{s}_{11} + \frac{\partial}{\partial t} m_{11} \ , \ \frac{\partial}{\partial t} m_2 = -m_{0;2}\, \dot{s}_{22} + \frac{\partial}{\partial t} m_{22}.$$

$$\tilde{m}_i = \frac{m_i}{\sqrt{\alpha_1' \alpha_2' k^2}},$$

$$\beta_{\tilde{m}_1} = \frac{\partial}{\partial t}\tilde{m}_1 = \left(-2 - \frac{1}{2}\tilde{\zeta}_1 - \frac{1}{2}\tilde{\zeta}_2\right)\tilde{m}_1 - \tilde{m}_1 s_{11}' + \frac{1}{\sqrt{\alpha_1'\alpha_2' k^2}}\frac{\partial}{\partial t} m_{11}$$
$$\beta_{\tilde{m}_2} = \frac{\partial}{\partial t}\tilde{m}_2 = \left(-2 - \frac{1}{2}\tilde{\zeta}_1 - \frac{1}{2}\tilde{\zeta}_2\right)\tilde{m}_2 - \tilde{m}_2 s_{22}' + \frac{1}{\sqrt{\alpha_1'\alpha_2' k^2}}\frac{\partial}{\partial t} m_{22}$$

We start to calculated the beta function for the masses, then the E-derivative of $\Gamma_{AB}^{1,1}$ give the matrix elements s_{ij}

$$s_{ij} = \tilde{\lambda}_{i,11}\tilde{\lambda}_{j,11}cs(11) + \tilde{\lambda}_{i,22}\tilde{\lambda}_{j,22}cs(22) + \tilde{\lambda}_{i,12}\tilde{\lambda}_{j,12}cs(12),$$

$$cs(11) = -\frac{N_D}{2}\frac{1}{4(r_1-\tilde{m}_{0;1})^2}\,[\frac{1}{D}+\frac{1}{2}B(1,\frac{4-D}{2})_2F_1(2,1,\frac{6-D}{2};-\frac{\tilde{m}_{0;1}}{r_1-\tilde{m}_{0;1}})]$$

$$cs(22) = -\frac{N_D}{2}\frac{1}{4(r_2-\tilde{m}_{0;2})^2}\,[\frac{1}{D}+\frac{1}{2}B(1,\frac{4-D}{2})_2F_1(2,1,\frac{6-D}{2};-\frac{\tilde{m}_{0;2}}{r_2-\tilde{m}_{0;2}})]$$

$$cs(12) = -\frac{N_D}{2}\frac{1}{(r_1+r_2-\tilde{m}_{0;1}-\tilde{m}_{0;2})^2} \cdot[\frac{2}{D}+B(1,\frac{4-D}{2})_2F_1(2,1,\frac{6-D}{2};-\frac{\tilde{m}_{0;1}+\tilde{m}_{02}}{r_1+r_2-\tilde{m}_{0;1}-\tilde{m}_{0;2}})]. \qquad (3.25)$$

Next we take the derivative with respect to $t = k\frac{\partial}{\partial k}$. We find

$$s'_{ij} = \tilde{\lambda}_{i,11}\tilde{\lambda}_{j,11}csd(11) + \tilde{\lambda}_{i,22}\tilde{\lambda}_{j,22}csd(22) + \tilde{\lambda}_{i,12}\tilde{\lambda}_{j,12}csd(12)$$

$$csd(11) = 4\frac{N_D}{D}\frac{r_1}{8(r_1-\tilde{m}_{0;1})^3}\;,\; csd(22) = 4\frac{N_D}{D}\frac{r_2}{8(r_2-\tilde{m}_{0;2})^3}$$

$$csd(12) = 4\frac{N_D}{D}\frac{r_1+r_2}{(r_1+r_2-\tilde{m}_{01}-\tilde{m}_{0;2})^3}.$$

The t-derivative of the mass matrix

$$\frac{1}{\sqrt{\alpha'_1\alpha'_2}k^2}\frac{\partial}{\partial t}M_{ij} = \tilde{\lambda}_{i,11}\tilde{\lambda}_{j,11}cmd(11) + \tilde{\lambda}_{i,22}\tilde{\lambda}_{j,22}cmd(22) + \tilde{\lambda}_{i,12}\tilde{\lambda}_{j,12}cmd(12)$$

$$cmd(11) = 2\frac{N_D}{D}\frac{r_1}{4(r_1-\tilde{m}_{0;1})^2}\ ,\ cmd(22) = 2\frac{N_D}{D}\frac{r_2}{4(r_2-\tilde{m}_{0;2})^2}$$

$$cmd(12) = 2\frac{N_D}{D}\frac{r_1+r_2}{(r_1+r_2-\tilde{m}_{0;1}-\tilde{m}_{0;2})^2}.$$

Next we take the derivative with respect t for the slope

$$\frac{\partial}{\partial t}A_{ij} = \tilde{\lambda}_{i,11}\tilde{\lambda}_{j,11}\ \alpha'_{0;1}cad(11) + \tilde{\lambda}_{i,22}\tilde{\lambda}_{j,22}\ \alpha'_{0;2}cad(22) + \tilde{\lambda}_{i,12}\tilde{\lambda}_{j,12}\ \sqrt{\alpha'_{0;1}\alpha'_{0;2}}cad(12)$$

$$cad(11) = 2\frac{N_D}{D}\frac{r_1}{8(r_1-\tilde{m}_{0;1})^3}\ ,\ cad(22) = 2\frac{N_D}{D}\frac{r_2}{8(r_2-\tilde{m}_{0;2})^3}$$

$$cad(12) = 4\frac{N_D}{D}\frac{1}{(r_1+r_2-\tilde{m}_{0;1}-\tilde{m}_{0;2})^3}.$$

Finally we take the t-derivative of the tensor $\Gamma^{1,2}_{AB:ijk}$

$$
\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'}k^2}\,\partial_t\Gamma^{(1,2)}_{AB;i,jk} =
$$
$$
\Big(\tilde\lambda_{i;11}\tilde\lambda_{j;11}\tilde\lambda_{1;1k}\ cld(11;11)+\tilde\lambda_{i;22}\tilde\lambda_{j;22}\tilde\lambda_{2;2k}\ cld(22;22)
$$
$$
+\tilde\lambda_{i;12}\tilde\lambda_{j;12}\tilde\lambda_{2;2k}\ cld(12;12)+\tilde\lambda_{i;21}\tilde\lambda_{j;21}\tilde\lambda_{1;1k}\ cld(21;21)
$$
$$
+\Big[\tilde\lambda_{i;11}\tilde\lambda_{j;12}\tilde\lambda_{1;2k}+\tilde\lambda_{i;12}\tilde\lambda_{j;11}\tilde\lambda_{2;1k}\Big]\ cld(11;12)
$$
$$
+\Big[\tilde\lambda_{i;12}\tilde\lambda_{j;22}\tilde\lambda_{1;2k}+\tilde\lambda_{i;22}\tilde\lambda_{j;12}\tilde\lambda_{2;1k}\Big]\ cld(21,22)\Big)+(j\leftrightarrow k)
$$

$$
cld(ab;ac)=2\frac{N_D}{D}
$$
$$
\cdot\left(\frac{r_a+r_b}{(r_a+r_b-\tilde m_a-\tilde m_b)^2}\frac{1}{r_a+r_c-\tilde m_a-\tilde m_c}+\frac{1}{r_a+r_b-\tilde m_a-\tilde m_b}\frac{r_a+r_c}{(r_a+r_c-\tilde m_a-\tilde m_c)^2}\right).
$$

Betas Functions for the Reggeon parameter at the one loop in $D = 4 - \epsilon$:

$$\alpha_1, \alpha_2; \ \ \mu_1, \mu_2; \ \ \lambda_{ijk}$$

$$r_1 = \sqrt{\frac{\alpha'_1}{\alpha'_2}} = \sqrt{\frac{\alpha'_{0;1}}{\alpha'_{0;2}}} + O(\lambda^2); \quad r_2 = \sqrt{\frac{\alpha'_2}{\alpha'_1}} = \sqrt{\frac{\alpha'_{0;2}}{\alpha'_{0;1}}} + O(\lambda^2)$$

$$\begin{aligned}
\tilde{\zeta}_1 = \frac{1}{\alpha'_1}\zeta_1 &= \frac{1}{\alpha'_1}\frac{\partial}{\partial t}\alpha'_1 = \frac{1}{\alpha'_1}\left(-\alpha'_{0;1}\, s'_{11} + \frac{\partial}{\partial t}\alpha'_{11}\right) \\
&= -s'_{11} + \frac{1}{\alpha'_1}\frac{\partial}{\partial t}\alpha'_{11} \\
\tilde{\zeta}_2 = \frac{1}{\alpha'_2}\zeta_2 &= \frac{1}{\alpha'_2}\frac{\partial}{\partial t}\alpha'_2 = \frac{1}{\alpha'_2}\left(-\alpha'_{0;2}\, s'_{22} + \frac{\partial}{\partial t}\alpha'_{22}\right) \\
&= -s'_{22} + \frac{1}{\alpha'_2}\frac{\partial}{\partial t}\alpha'_{22}.
\end{aligned}$$

$$\begin{aligned}
\beta_{r_1} &= \frac{\partial}{\partial_t}r_1 = \frac{1}{2}r_1\left(\tilde{\zeta}_1 - \tilde{\zeta}_2\right) \\
\beta_{r_2} &= \frac{\partial}{\partial_t}r_2 = \frac{1}{2}r_2\left(\tilde{\zeta}_2 - \tilde{\zeta}_1\right).
\end{aligned}$$

$$\beta_{r_1} = \frac{\partial}{\partial_t}r_1 = \frac{1}{2}r_1\left(\tilde{\zeta}_1 - \tilde{\zeta}_2\right)$$

$$s'_{ij} = \tilde{\lambda}_{i,11}\tilde{\lambda}_{j,11}csd(11) + \tilde{\lambda}_{i,22}\tilde{\lambda}_{j,22}csd(22) + \tilde{\lambda}_{i,12}\tilde{\lambda}_{j,12}csd(12)$$

Betas Functions at the one loop in $D = 4 - \epsilon$

$$\tilde\lambda_{i,jk} = \frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'}k^2}\Big((Z_1^T)_{ii'}\lambda_{i',j'k'}(Z_1)_{j'j}(Z_1)_{k'k} + (Z_1)_{ii'}\lambda_{i',j'k'}(Z_1^T)_{j'j}(Z_1^T)_{k'k}\Big)$$

$$\begin{aligned}
\beta_{i,jk} &= \frac{\partial}{\partial t}\tilde\lambda_{i,jk} \\
&= \left(\frac{D-4}{2} - \frac{1}{2}\tilde\zeta_1 - \frac{1}{2}\tilde\zeta_2\right)\tilde\lambda_{i,jk} \\
&\quad + \frac{1}{2}\Big[\left(Z^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{i,i'}\tilde\lambda_{i',jk} + \tilde\lambda_{i,j'k}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)_{j'j} + \tilde\lambda_{i,jk'}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)_{k'k} \\
&\quad + \left(Z^{-1}\frac{\partial}{\partial t}Z_1\right)_{i,i'}\tilde\lambda_{i',jk} + \tilde\lambda_{i,j'k}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{j'j} + \tilde\lambda_{i,jk'}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{k'k}\Big] \\
&\quad + \frac{1}{2}\Big[Z^T_{1\,ii'}\left(\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'}k^2}\frac{\partial}{\partial t}\Gamma^{(1,2)}_{AB;i',j'k'}\right)Z_{1j'j}Z_{1k'k} \\
&\qquad + Z_{1ii'}\left(\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'}k^2}\frac{\partial}{\partial t}\Gamma^{(1,2)}_{AB;i',j'k'}\right)Z^T_{1\,j'j}Z^T_{1\,k'k}\Big]. \qquad (
\end{aligned}$$

Anomalous dimensions: $\qquad (Z_1^{-1}\frac{\partial}{\partial t}Z_1)$

$$\begin{aligned}
Z_1^{-1}\frac{\partial}{\partial t}Z_1 &= -\frac{1}{2}Z_1^T(\frac{\partial}{\partial t}S)Z_1 + \frac{D_s}{4R_s^2}\frac{s_{11}+s_{22}}{2}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \\
&\quad + \frac{1}{r_1 - r_2}\left(\frac{1}{\sqrt{\alpha'_{0;1}\alpha'_{0;2}}}\frac{\partial}{\partial t}\alpha_{12} - \frac{\dot s_{12}}{2}(r_1+r_2)\right)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}.
\end{aligned}$$

- **Flow equation** $for\ \alpha_1, \alpha_2;\ \ \mu_1, \mu_2;\ \ \lambda_{ijk}$

$$(\beta_{1,11}, \beta_{2,22}, \beta_{1,22}, \beta_{2,11}, \beta_{1,12},\ \beta_{2,12}, \beta_{\tilde{m}_1}, \beta_{\tilde{m}_2}, \beta_{r_1})$$

$$\begin{aligned}
\beta_{i,jk} &= \frac{\partial}{\partial t}\tilde{\lambda}_{i,jk} \\
&= \left(\frac{D-4}{2} - \frac{1}{2}\tilde{\zeta}_1 - \frac{1}{2}\tilde{\zeta}_2\right)\tilde{\lambda}_{i,jk} \\
&\quad + \frac{1}{2}\Big[\left(Z^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{i,i'}\tilde{\lambda}_{i',jk} + \tilde{\lambda}_{i,j'k}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)_{j'j} + \tilde{\lambda}_{i,jk'}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)_{k'k} \\
&\quad + \left(Z^{-1}\frac{\partial}{\partial t}Z_1\right)_{i,i'}\tilde{\lambda}_{i',jk} + \tilde{\lambda}_{i,j'k}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{j'j} + \tilde{\lambda}_{i,jk'}\left(Z_1^{-1}\frac{\partial}{\partial t}Z_1\right)^T_{k'k}\Big] \\
&\quad + \frac{1}{2}\Big[Z^T_{1\ ii'}\left(\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'k^2}}\frac{\partial}{\partial t}\Gamma^{(1,2)}_{AB;i',j'k'}\right)Z_{1j'j}Z_{1k'k} \\
&\qquad\qquad + Z_{1ii'}\left(\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2'k^2}}\frac{\partial}{\partial t}\Gamma^{(1,2)}_{AB;i',j'k'}\right)Z^T_{1\ j'j}Z^T_{1\ k'k}\Big]. \qquad (
\end{aligned}$$

$$\beta_{\tilde{m}_1} = \frac{\partial}{\partial t}\tilde{m}_1 = \left(-2 - \frac{1}{2}\tilde{\zeta}_1 - \frac{1}{2}\tilde{\zeta}_2\right)\tilde{m}_1 - \tilde{m}_1 s'_{11} + \frac{1}{\sqrt{\alpha_1'\alpha_2'k^2}}\frac{\partial}{\partial t}m_{11}$$

$$\beta_{\tilde{m}_2} = \frac{\partial}{\partial t}\tilde{m}_2 = \left(-2 - \frac{1}{2}\tilde{\zeta}_1 - \frac{1}{2}\tilde{\zeta}_2\right)\tilde{m}_2 - \tilde{m}_2 s'_{22} + \frac{1}{\sqrt{\alpha_1'\alpha_2'k^2}}\frac{\partial}{\partial t}m_{22}$$

$$\beta_{r_1} = \frac{\partial}{\partial_t}r_1 = \frac{1}{2}r_1\left(\tilde{\zeta}_1 - \tilde{\zeta}_2\right)$$

In the $D = 4 - \epsilon\ \ \rightarrow \lambda \sim \sqrt{\epsilon}\ \ :\ \mu_i \sim \epsilon$

$$\frac{k^{\frac{D}{2}}}{\sqrt{\alpha_1'\alpha_2' k^2}}\ \partial_t \Gamma^{(1,2)}_{AB;i,jk} =$$
$$\Big(\tilde\lambda_{i;11}\tilde\lambda_{j;11}\tilde\lambda_{1;1k}\ cld(11;11) + \tilde\lambda_{i;22}\tilde\lambda_{j;22}\tilde\lambda_{2;2k}\ cld(22;22)$$
$$+\tilde\lambda_{i;12}\tilde\lambda_{j;12}\tilde\lambda_{2;2k}\ cld(12;12) + \tilde\lambda_{i;21}\tilde\lambda_{j;21}\tilde\lambda_{1;1k}\ cld(21;21)$$
$$+\Big[\tilde\lambda_{i;11}\tilde\lambda_{j;12}\tilde\lambda_{1;2k} + \tilde\lambda_{i;12}\tilde\lambda_{j;11}\tilde\lambda_{2;1k}\Big]\ cld(11;12)$$
$$+\Big[\tilde\lambda_{i;12}\tilde\lambda_{j;22}\tilde\lambda_{1;2k} + \tilde\lambda_{i;22}\tilde\lambda_{j;12}\tilde\lambda_{2;1k}\Big]\ cld(21,22)\Big) + (j \leftrightarrow k)$$

$$csd(11) = \frac{1}{(8\pi)^2}\frac{1}{r_1^2},\ \ csd(22) = \frac{1}{(8\pi)^2}\frac{1}{r_2^2},\ \ csd(12) = 8\frac{1}{(8\pi)^2}\frac{1}{(r_1+r_2)^2}$$

$$cmd(11) = \frac{1}{(8\pi)^2}\frac{1}{r_1},\ \ cmd(22) = \frac{1}{(8\pi)^2}\frac{1}{r_2},\ \ cmd(12) = 4\frac{1}{(8\pi)^2}\frac{1}{r_1+r_2}$$

$$cad(11) = \frac{1}{2}\frac{1}{(8\pi)^2}\frac{1}{r_1^2},\ \ cad(22) = \frac{1}{2}\frac{1}{(8\pi)^2}\frac{1}{r_2^2},\ \ cad(12) = 8\frac{1}{(8\pi)^2}\frac{1}{(r_1+r_2)^3}$$

$$cld(ab,ac) = \frac{8}{(8\pi)^2}\frac{1}{r_a+r_b}\frac{1}{r_a+r_c},$$

Stability matrix

The movement of the coupling constants $\boldsymbol{g_{k,i}}$ in the vicinity of the fixed point is determined by the Stability matrix calculated at the fixed point.

$$M_{ij} = \frac{\partial \beta_i}{\partial \lambda_j}\Big|_{\lambda_{FP}}$$

The eigenvalues of matrix $\boldsymbol{M}$ are $\{\boldsymbol{\theta_i}\}$ and the corresponding eigenvectors be $\boldsymbol{v_i^a}$

Critical Exponents θ_i of the stabilty matrix

$$\boldsymbol{g_{k,i} = g^*{}_i + c_a e^{\theta_i t}\, v_i^a}$$

Linealizations of the Flow close to a FP:
$\theta_i > 0$ define a IR attractor – Critical Surface with relevant behaviour,
Where
$\theta_i < 0$ is UV attractor

Numerical Solution

Two decoupled Pomerons:

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,11} = \sqrt{\epsilon}\frac{r_1}{\sqrt{6}} \quad , \quad \frac{1}{(8\pi)}\tilde{\lambda}_{2,22} = \sqrt{\epsilon}\frac{1}{r_1\sqrt{6}}$$

$$\tilde{m}_1 = \frac{\epsilon}{12} r_1 \ , \ \tilde{m}_2 = \frac{\epsilon}{12}\frac{1}{r_1}.$$

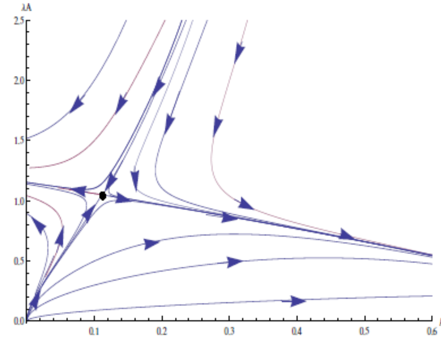


Fig.2: Some trajectories of the flow equation.

Anomalous dimensions:

$$\tilde{\zeta}_1 = -\frac{1}{12}\epsilon \ , \ \tilde{\zeta}_2 = -\frac{1}{12}\epsilon$$

$$\gamma_{ii} \rightarrow (Z_1^{-1}\partial_t Z_1)_{11} = (Z_1^{-1}\partial_t Z_1)_{22} = -\frac{\epsilon}{12}$$

$$\gamma_{ij} \rightarrow (Z_1^{-1}\partial_t Z_1)_{12} = (Z_1^{-1}\partial_t Z_1)_{21} = 0$$

Critical exponents:

$$\epsilon(1.0, 1.0, -0.3333, -0.3333, -0.1667, -0.08333, 0.0)$$

1980 Cardy y Sugar found that the RFT in D=2 is in the same Universality class of "Percolation"

- Two decoupled Pomerons D=2

Bartels, Contreras and Vacca; ***JHEP*** **03 (2016) 201**

- We reproduce the values of the critical exponents universality class of Percolation
- The convergence is under control with the increasing the local truncation

Percolation and Monte Carlo Simulation:
The critical Exponent ν = 0.73 with is related with our
ν = - 1/(most negative eigenvalue) "
(L. Canet, B. Delamotte, N. Wschebor,…)

Pomeron-Odderon Interaction

Bartels, Contreras and Vacca; *Phys. Rev. D* 95 (2017) 1, 014013

arXiv 2001.0599 M. Braum and G. P. Vacca

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,11} = 0.43284\sqrt{\epsilon}\ ,\ \frac{1}{(8\pi)}\tilde{\lambda}_{2,12} = 0.29801\sqrt{\epsilon}$$

$$\tilde{m}_1 = 0.088353\epsilon\ ,\ \tilde{m}_2 = 0.088655\epsilon.$$

Anomalous dimensions:

$$r_1 = 1.06023.$$

$$\gamma_{ii} \to \left(Z_1^{-1}\partial_t Z_1\right)_{11} = -0.08333\epsilon\ ,\ \left(Z_1^{-1}\partial_t Z_1\right)_{22} = -0.08850\epsilon$$

$$\gamma_{ij} \to \left(Z_1^{-1}\partial_t Z_1\right)_{12} = \left(Z_1^{-1}\partial_t Z_1\right)_{21} = 0$$

$$\tilde{\zeta}_1 = -\frac{1}{12}\epsilon\ ,\ \tilde{\zeta}_2 = -\frac{1}{12}\epsilon$$

Critical exponents :

$$\epsilon(1.0, 0.7860, 0.6042, 0.1848, -0.0698, -0.0483, 0.0478).$$

Bartels, Contreras and Vacca; *JHEP* 05 (2024) 032

Effective Potencial:

$$V_{3,k} = i\,\lambda_{1,k}\,\psi_1^{\dagger}(\psi_1 + \psi_1^{\dagger})\psi_1 + i\,\lambda_{2,k}\,\psi_2^{\dagger}(\psi_1 + \psi_1^{\dagger})\psi_2$$

New Solution

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,11} = 0.28868\sqrt{\epsilon}\ ,\ \frac{1}{(8\pi)}\tilde{\lambda}_{2,12} = 0.28868\sqrt{\epsilon}$$

Fixed Point IR

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,22} = -0.28868\sqrt{\epsilon}\ ,\ r_1 = 1.0 \qquad \tilde{m}_1 = 0.08333\ \epsilon\ ,\ \tilde{m}_2 = 0.08333\ \epsilon.$$

Anomalous dimensions:

$$\tilde{\zeta}_1 = -0.083333\epsilon\ ,\ \tilde{\zeta}_2 = -0.083333\epsilon$$

$$\gamma_{ii} \to \left(Z_1^{-1}\partial_t Z_1\right)_{11} = -0.08333\,\epsilon\ ,\ \left(Z_1^{-1}\partial_t Z_1\right)_{22} = -0.08333\,\epsilon$$

$$\gamma_{ij} \to \left(Z_1^{-1}\partial_t Z_1\right)_{12} = \left(Z_1^{-1}\partial_t Z_1\right)_{21} = 0$$

$$\epsilon(-0.1443, 0.0, 0.1443, 0.0, 0.0, 0.2887, -0.0833).$$

Stable Fixed Point in IR

Fixed Point IR

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,11} = 0.303779\,\sqrt{\epsilon}\ ,\ \frac{1}{(8\pi)}\tilde{\lambda}_{2,12} = 0.282231\,\sqrt{\epsilon}$$

$$\frac{1}{(8\pi)}\tilde{\lambda}_{1,22} = -0.283615\,\sqrt{\epsilon}\ ,\ r_1 = 0.950123$$

$$\tilde{m}_1 = 0.086776\,\epsilon\ ,\ \tilde{m}_2 = 0.079550\,\epsilon. \qquad \tilde{\zeta}_1 = -0.0835073\,\epsilon\ ,\ \tilde{\zeta}_2 = -0.0835073\,\epsilon$$

Anomalous dimensions:

$$\gamma_{ii} \to \left(Z_1^{-1}\partial_t Z_1\right)_{11} = -0.087419\epsilon\ ,\ \left(Z_1^{-1}\partial_t Z_1\right)_{22} = -0.079446\epsilon$$

$$\gamma_{ij} \to \left(Z_1^{-1}\partial_t Z_1\right)_{12} = \left(Z_1^{-1}\partial_t Z_1\right)_{21} = 0$$

$$\epsilon(1.0, -0.3649, -0.3113, -0.1505, -0.1427, 0.0840, 0.0661),$$

Unstable Fixed Point in IR

Summary and outlook

Using Numerical analysis of the RFT in the Improved one loop PT

1. We can find different Fixed Points solutions.

2. We find previous solution Pomerons decoupled and Pomeron and Odderon solution

3. We studied the interaction of two Pomeron with triple Pomeron vertex

3. Can interpreted that the Pomeron really mixed between the two BFKL discrete Pomeron around the IR fixed point

4. The one loop Improved PT allow to have an approximate picture that help in the next step to consider the nonperturbative approach FRG

5. We need to go to Physical dimension D = 2

In the future:

Extension to high order Pomeron vertex in FRG
N-Pomeron Interaction in diagonal basis in FRG


Thank you

Analysis of ε-expansion for RFT at one loop

Considering the for N=2 RFT Pomerons

$$S = \int d\tau d^D x \Big[(\bar{Z}_\psi \psi^\dagger)^T \left(i\partial_\tau + \alpha' Z_\alpha \nabla^2 + \alpha' m Z_\mu \right) (\bar{Z}_\psi \psi) \\ -i\alpha' M^{\frac{\epsilon}{2}} Z_{\lambda i;jk} \psi_i^\dagger (\psi_j + \psi_j^\dagger) \psi_k \Big],$$

Th renormalized form of the action at D = 4 - ε, where the M denotes the renormalization scale, we can obtain the Beta function at the one loop and the solution for the Fixed Point have the same structure.

The bare action:

$$S = \int d\tau d^D x \Big[\psi_B^{\dagger T} \left(i\partial_\tau + \alpha'_B \nabla^2 + \alpha'_B \mu_B \right) \psi_B \\ -i\alpha'_B \lambda_{B,i,jk} (\psi_{B,i}^\dagger \psi_{B,j} \psi_{B,k} + \psi_{B,i} \psi_{B,j}^\dagger \psi_{B,k}^\dagger) \Big].$$

The one loop integrals for two point function in the leading ϵ expansión es :

$$i(\Gamma^{(1,1)}_{kl;div})_{ij} = \frac{1}{(r_k+r_l)^2}\frac{\lambda_{i,kl}\lambda_{j,kl}}{(4\pi)^2}\frac{2}{\epsilon}\Big[E - a\frac{r_k r_l}{r_k+r_l}q^2 + a(\mu_k+\mu_l)\Big]$$

with $r_1 = \frac{\alpha'_1}{a}$, $r_2 = \frac{\alpha'_2}{a}$. Summing over k,l we obtain

$$i\Gamma^{(1,1)}_{div} = \frac{1}{\epsilon}\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22}\end{pmatrix} E - \frac{1}{\epsilon}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix} aq^2 + \frac{1}{\epsilon}a\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{pmatrix}$$

with

$$s_{ij} = \frac{1}{(4\pi)^2}\left(\lambda_{i,kl}\lambda_{j,kl}\frac{1}{(r_k+r_l)^2}\right);\;\; a_{ij} = \frac{1}{(4\pi)^2}\left(\lambda_{i,kl}\lambda_{j,kl}\frac{r_k r_l}{(r_k+r_k)^3}\right)$$

$$m_{ij} = \frac{1}{(4\pi)^2}\left(\lambda_{i,kl}\lambda_{j,kl}\frac{(\mu_k+\mu_l)}{(r_k+r_l)^2}\right).$$

To derive the β-function we impose the usual condition

$$M\frac{\partial}{\partial M}\lambda_{B;i,jk} = 0 \qquad\qquad \beta_{i,jk} = M\frac{\partial}{\partial M}\lambda_{i,jk}$$

$$\beta_{i,jk} = 0, \quad \beta_{r_1} = M\frac{\partial}{\partial M}r_1 = 0.$$

Two decoupled Pomerons:

$$\frac{1}{8\pi}\lambda_{1,11} = 0.4082\sqrt{\epsilon} = \sqrt{\epsilon}\,\frac{r_1}{\sqrt{6}} \quad , \quad \frac{1}{8\pi}\lambda_{2,22} = 0.4082\sqrt{\epsilon}/r_1.$$

Anomalous dimensions:

$$\zeta = -\frac{\epsilon}{12}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.08333 \end{pmatrix}$$

$$\gamma = -\frac{\epsilon}{12}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.08333 \end{pmatrix}.$$

Critical exponents:

$$\epsilon(1.0, 1.0, -1/3, -1/3, -1/6, 0.0, 0.0).$$

Pomeron-Odderon Interaction

$$\frac{1}{8\pi}\lambda_{1,11} = 0.4328\sqrt{\epsilon} \ , \ \frac{1}{8\pi}\lambda_{2,12} = 0.2980\sqrt{\epsilon}$$

$$r_1 = 1.0602.$$

$$\epsilon(1.0, 0.6552, 0.6042, 0.1848, 0.0478, -0.0699, -0.0083).$$

$$\zeta = -\frac{\epsilon}{4}\begin{pmatrix} 0.3333 & 0 \\ 0 & 0.3747 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.09367 \end{pmatrix},$$

$$\gamma = -\frac{\epsilon}{4}\begin{pmatrix} 0.3333 & 0 \\ 0 & 0.3540 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.0885 \end{pmatrix},$$

***Bartels, Contreras and Vacca; JHEP* 05 (2024) 032**
Bartels, Contreras and Vacca to appear in arXiv
M. Braun et al arXiv :2311.13870

Another solutions:

$$\begin{aligned} \frac{1}{8\pi}\lambda_{1,11} &= 0.28868\,\sqrt{\epsilon} = 0.28868\,\sqrt{\epsilon} \\ \frac{1}{8\pi}\lambda_{1,22} &= 0.28868\,\sqrt{\epsilon} = -0.28868\,\sqrt{\epsilon} \\ \frac{1}{8\pi}\lambda_{2,12} &= 0.28868\,\sqrt{\epsilon} = 0.28868\,\sqrt{\epsilon} \\ r_1 &= 1.000. \end{aligned}$$

Anomalous dimensions:

$$\zeta = -\frac{\epsilon}{4}\begin{pmatrix} 0.3333 & 0 \\ 0 & 0.3333 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.08333 \end{pmatrix}$$

$$\gamma = -\frac{\epsilon}{4}\begin{pmatrix} 0.3333 & 0 \\ 0 & 0.3333 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08333 & 0 \\ 0 & 0.08333 \end{pmatrix}.$$

Critical exponents:

$$\epsilon(1.0, 5/6, -1/3, -1/3, -1/6, -1/12, 0.0).$$

Another solutions:

$$\frac{1}{8\pi}\lambda_{1,11} = 0.3038\,\sqrt{\epsilon}; \quad \frac{1}{8\pi}\lambda_{1,22} = -0.2836\,\sqrt{\epsilon}$$
$$\frac{1}{8\pi}\lambda_{2,12} = 0.2322\,\sqrt{\epsilon}; \quad r_1 = 0.950$$

Anomalous dimensions:

$$\zeta = -\frac{\epsilon}{4}\begin{pmatrix} 0.3653 & 0 \\ 0 & 0.3015 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.09133 & 0 \\ 0 & 0.07538 \end{pmatrix}$$

$$\gamma = -\frac{\epsilon}{4}\begin{pmatrix} 0.3497 & 0 \\ 0 & 0.3178 \end{pmatrix} = -\epsilon\begin{pmatrix} 0.08742 & 0 \\ 0 & 0.07945 \end{pmatrix}.$$

Critical exponents:

$$\epsilon\,(1.0, -0.3624, -0.3113, -0.1427, -0.0273, -0.0018, 0.0840).$$

The degree of stability of each fixed point is determined by the signs of the eigenvalues of the stability matrix In the phase space, there are directions of instability for each negative eigenvalue on the infrared limit and stabilities for the case of positive eigenvalues. For the last solutions the eigenvalues are negative, then the fixed point is IR unstable, and we will not consider those solutions.


Thank you