# Correlation functions for $n\bar{D}_{s1}(2460)$ and $n\bar{D}_{s1}(2536)$

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# The Quest for Understanding Exotic Hadrons

- Since the discovery of the $X(3872)$ in 2003[1] a wide variety of exotic hadrons have been observed, many of which cannot be easily described as simple $q\bar{q}$ or $qqq$ systems[2].
- These states often appear near two-hadron thresholds, suggesting a molecular or dynamically generated origin[3].
- Understanding their nature is crucial for clarifying the nonperturbative dynamics of QCD and the mechanisms of hadron formation.

[1] S.K. Choi et al. (Belle Collaboration), Phys. Rev. Lett. 91, 262001 (2003). [2] F.-K. Guo et al., Rev. Mod. Phys. 90, 015004 (2018). [3] E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998).

# Probing Hadron Interactions via Correlation Functions

- Direct scattering measurements involving unstable hadrons are not experimentally accessible.
- Instead, femtoscopic correlation functions, first developed in astronomy[7], are now a powerful tool in high-energy nuclear and particle physics[8].
- The correlation function $C(p) = \frac{A(p)}{B(p)}$ measures deviations of the observed pair distribution from an uncorrelated baseline, reflecting the influence of final-state interactions at low relative momentum.
- Experimental collaborations like ALICE[9], STAR[10], and Belle II[11] have already measured hadronic correlation functions such as $p\Lambda$, $pK$, and $p\Xi$.

[7] R. Hanbury Brown and R.Q. Twiss, Nature 177, 27 (1956). [8] G. Goldhaber et al., Phys. Rev. Lett. 3, 181 (1959). [9] ALICE Collaboration, Phys. Lett. B 802, 135225 (2020). [10] STAR Collaboration, Nature 614, 244 (2023). [11] Belle Collaboration, Phys. Rev. D 105, 112011 (2022).

# Why the $n\bar{D}_{s1}$ Systems?

- Over the past decades, several studies have reported the formation of possible three-body hadronic molecules[12][13][15][16].
  - $\phi(2170) \to \phi \underbrace{K\bar{K}}_{f_0(980)}, \quad K(1460) \to K \underbrace{K\bar{K}}_{f_0(980)}, \quad N^*(1710) \to N \underbrace{\pi\pi}_{\sigma(600)}.$
- The $D_{s1}(2460)$ and $D_{s1}(2536)$ can be dynamically generated from $KD^*$ and $K^*D$ interactions in coupled-channel unitarized models [17].
- The neutron is chosen to avoid Coulomb effects, isolating the strong-interaction dynamics and by the attractive $n\bar{K}$ interaction, which dynamically generates the $\Lambda(1405)$[18].

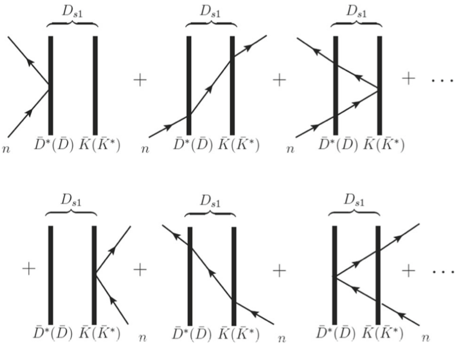
[12] A. Martínez Torres, D. Jido, Y. K. En'yo, Phys. Rev. C 83, 065205 (2011). [13] A. Martínez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, E. Oset, Phys. Rev. D 86, 114011 (2012). [15] B. B. Malabarba, K.P. Khemchandani, A. Martinez Torres, Phys.Rev.D 108, 3, 036010 (2023). [16] A. Martinez Torres, B. B. Malabarba, K.P. Khemchandani, EPJ Web Conf. 301, 03002 (2024). [17] D. Gamermann, E. Oset, D. Strottman, M. J. Vicente Vacas, Phys. Rev. D 76, 074016 (2007). [18] E. Oset, A. Ramos, Nucl. Phys. A 635, 99 (1998). [19] N. Ikeno, E. Oset, arXiv:2507.16367 [hep-ph] (2025).

## Objectives

- Solve the Faddeev equations using the Fixed Center Approximation (FCA) with exact unitarity.
- Compute $T$-matrices and then, observables as scattering lengths and effective ranges.
- Search for subthreshold states.
- Use correlation functions to extract scattering information on the $n\bar{D}_{s1}$ systems.

# Faddeev Equations with Fixed Center Approximation (FCA)

- The FCA simplifies the three-body Faddeev equations by assuming one pair (the cluster) is bound and acts as a fixed center[20].




- The third particle (neutron) interacts with each component of the cluster through multiple scattering.

[20] P. Encarnación, A. Feijoo, E. Oset, Phys. Rev. D 111, 114023 (2025).

# Faddeev Equations with FCA

- The coupled equations for the partition functions $\tilde{T}_{ij}$ are

$$\tilde{T}_{11} = t_1 + t_1 G_0 \tilde{T}_{21}, \quad \tilde{T}_{12} = t_1 G_0 \tilde{T}_{22},$$
$$\tilde{T}_{21} = t_2 G_0 \tilde{T}_{11}, \quad \tilde{T}_{22} = t_2 + t_2 G_0 \tilde{T}_{12}.$$

where $G_0$ is the propagator of the neutron inside the cluster given by

$$G_0(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{M_N}{\omega_N(\vec{q})} \frac{1}{2\omega_C(\vec{q})} \frac{F_C(\vec{q})}{\sqrt{s} - \omega_N(\vec{q}) - \omega_C(\vec{q}) + i\varepsilon},$$

being $F_C(q)$ a form factor and $\omega_N(\vec{q}) = \sqrt{M_N^2 + q^2}$, $\omega_C(\vec{q}) = \sqrt{M_C^2 + q^2}$ the energy of the neutron and cluster, respectively.

# Two-Body Subsystems

- In isospin basis:

$$\left|I_{n\bar{D}_{s1}} = 1/2, I_3 = -1/2\right\rangle = \left|I_N = 1/2, I_3 = -1/2\right\rangle \otimes \left|I_{\bar{D}_{s1}} = 0, I_3 = 0\right\rangle .$$

- The transition amplitude is

$$\left\langle I_{n\bar{D}_{s1}} = 1/2, I_3 = -1/2\right| t \left|I_{n\bar{D}_{s1}} = 1/2, I_3 = -1/2\right\rangle ,$$

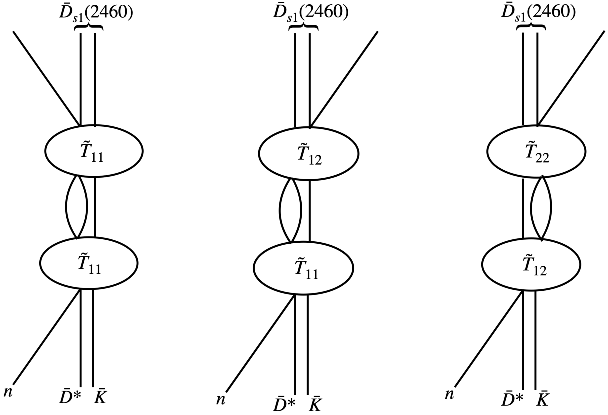
Then

$$n\bar{D}_{s1}(2460) \to t_1 = \frac{3}{4} t^{I=1}_{n\bar{D}^*} + \frac{1}{4} t^{I=0}_{n\bar{D}^*}, \quad t_2 = \frac{3}{4} t^{I=1}_{n\bar{K}} + \frac{1}{4} t^{I=0}_{n\bar{K}}.$$

$$n\bar{D}_{s1}(2536) \to t_1 = \frac{3}{4} t^{I=1}_{n\bar{D}} + \frac{1}{4} t^{I=0}_{n\bar{D}}, \quad t_2 = \frac{3}{4} t^{I=1}_{n\bar{K}^*} + \frac{1}{4} t^{I=0}_{n\bar{K}^*}.$$

# Unitarized FCA

- Are there any others contributions? Yes! Coherent $n$–cluster propagation is included through additional propagators $G_C^{(i)}$. For example,




$\tilde{T}_{11}\, G_C^{(1)}\, \tilde{T}_{11}$, $\quad \tilde{T}_{11}\, G_C^{(1)}\, \tilde{T}_{12}$, $\quad \tilde{T}_{12}\, G_C^{(2)}\, \tilde{T}_{22}$

# Unitarized FCA

- The neutron–cluster propagator differs from $G_0(\sqrt{s})$ due to the form factor

$$G_C^{(i)}(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{M_N}{\omega_N(\vec{q})} \frac{1}{2\omega_C(\vec{q})} \frac{[F_C^{(i)}(\vec{q})]^2}{\sqrt{s} - \omega_N(\vec{q}) - \omega_C(\vec{q}) + i\epsilon},$$

where[22]

$$F_C^{(1)}(\vec{q}) = F_C\left(\frac{M_2}{M_1 + M_2}\vec{q}\right),$$
$$F_C^{(2)}(\vec{q}) = F_C\left(\frac{M_1}{M_1 + M_2}\vec{q}\right).$$

[22] J. Yamagata-Sekihara, J. Nieves, E. Oset, Phys. Rev. D 83, 014003 (2011).

# Unitarized FCA

- The total amplitude is

$$T = \sum_{i,\,j\,=\,1}^{2} \mathbb{T}_{ij}$$
$$= \frac{\tilde{T}_{11} + 2\tilde{T}_{12} + \tilde{T}_{22} + (\tilde{T}_{12}^2 - \tilde{T}_{11}\tilde{T}_{22})\left(G_C^{(1)} + G_C^{(2)}\right)}{1 - \tilde{T}_{11}G_C^{(1)} - \tilde{T}_{22}G_C^{(2)} - (\tilde{T}_{12}^2 - \tilde{T}_{11}\tilde{T}_{22})G_C^{(1)}G_C^{(2)}}.$$

- In terms of the two-body amplitudes, we write

$$T = \frac{t_1 + t_2 + (2G_0 - G_C^{(1)} - G_C^{(2)})t_1 t_2}{1 - G_C^{(1)}t_1 - G_C^{(2)}t_2 - (G_0^2 - G_C^{(1)}G_C^{(2)})t_1 t_2}.$$

- The unitarity condition reads

$$\frac{8\pi\sqrt{s}}{2M_N}\,\mathrm{Im}\{T^{-1}\} = q_{cm}.$$

# Correlation Function

- The correlation function for a source of radius $R$[24]:

$$C_{n\bar{D}_{s1}}(p) = 1 + 4\pi \int_0^\infty dr\, r^2 S_{12}(r) \Big( \big| j_0(pr) + T' G' \big|^2 - j_0^2(pr) \Big),$$

where $S_{12}(r) = \frac{e^{-r^2/4R^2}}{(4\pi R^2)^{3/2}}$, the propagators $G_1(\sqrt{s}, r)$, $G_2(\sqrt{s}, r)$,

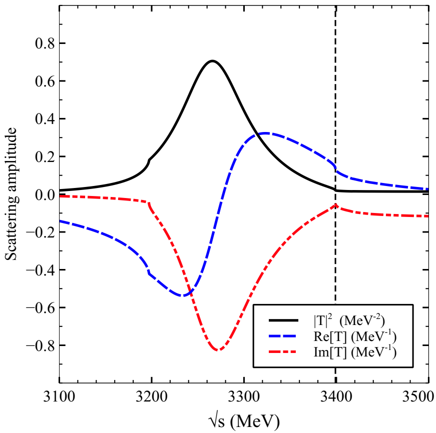
$$G_1(\sqrt{s}, r) = \int \frac{d^3q}{(2\pi)^3} \frac{M_N}{\omega_N(\vec{q})} \frac{1}{2\omega_C(\vec{q})} \frac{j_0(qr) F_C^{(1)}(\vec{q})}{\sqrt{s} - \omega_N(\vec{q}) - \omega_C(\vec{q}) + i\epsilon},$$

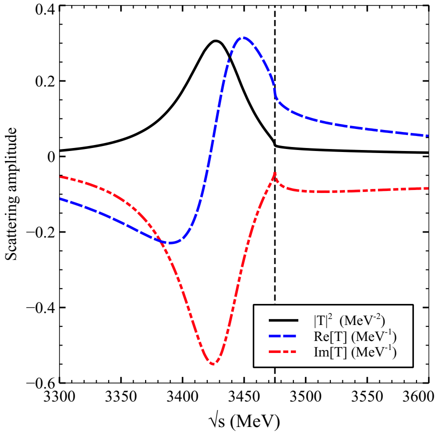
$$G_2(\sqrt{s}, r) = \int \frac{d^3q}{(2\pi)^3} \frac{M_N}{\omega_N(\vec{q})} \frac{1}{2\omega_C(\vec{q})} \frac{j_0(qr) F_C^{(2)}(\vec{q})}{\sqrt{s} - \omega_N(\vec{q}) - \omega_C(\vec{q}) + i\epsilon},$$

and $T' G' = (\mathbb{T}_{11} + \mathbb{T}_{21}) G_1(\sqrt{s}, r) + (\mathbb{T}_{12} + \mathbb{T}_{22}) G_2(\sqrt{s}, r)$.

[24] I. Vidana, A. Feijoo, M. Albaladejo, J. Nieves, E. Oset, Phys. Lett. B 846, 138201 (2023)

# $n\bar{D}_{s1}$ Amplitudes

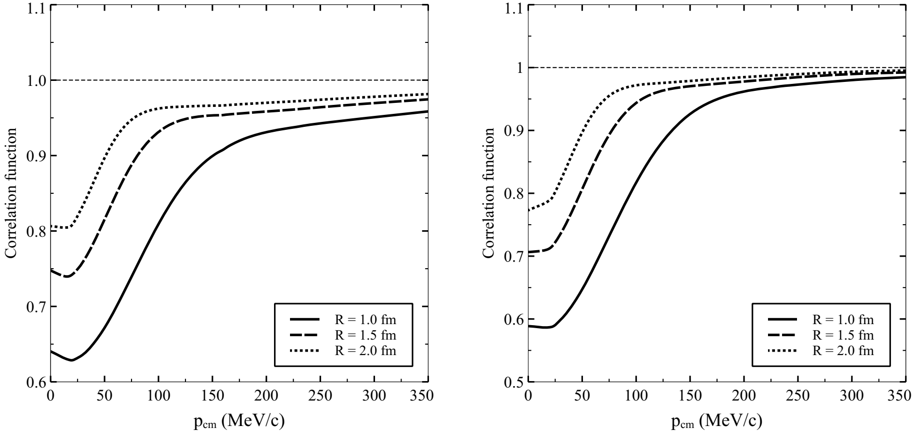



- Scattering parameters:

$$n\bar{D}_{s1}(2460) \to a = (0.59 - i0.21)\,\text{fm}, \quad r_0 = (0.65 - i0.16)\,\text{fm},$$
$$n\bar{D}_{s1}(2536) \to a = (0.71 - i0.18)\,\text{fm}, \quad r_0 = (0.16 + i0.32)\,\text{fm}.$$

# Correlation functions



# Discussion

- Implemented an unitary FCA formalism for $n$–$D_{s1}$ systems.
- Predicted states:
  - $n\bar{D}_{s1}(2460)$: $M \sim 3265$ MeV.
  - $n\bar{D}_{s1}(2536)$: $M \sim 3425$ MeV.
- The behavior of $C(p)$ follows the expected pattern for a state below threshold.
- Future directions:
  - Encourage correlation measurements in ALICE, STAR, and Belle II.

THANK YOU!

# Scattering Parameters

- In quantum mechanics, the scattering amplitude $f_{QM}$ and the $T$-matrix are related via[23]

$$-\frac{8\pi\sqrt{s}}{2M_N}T^{-1} = (f_{QM})^{-1}.$$

- The effective range expansion near threshold:

$$f_{\text{QM}}^{-1} = -\frac{1}{a} + \frac{1}{2}r_0 k^2 - ik,$$

with $k = q_{\text{cm}}$.

- Scattering length and effective range extracted from the unitary $T$-matrix:

$$a = \frac{2M_N}{8\pi\sqrt{s}}T\Big|_{\text{th}}, \qquad r_0 = \frac{1}{\mu}\frac{\partial}{\partial\sqrt{s}}\left(-\frac{8\pi\sqrt{s}}{2M_N}T^{-1} + iq_{\text{cm}}\right)\Big|_{\text{th}}.$$

[23] F. Mandl, G. Shaw, Quantum Field Theory, 2nd edn. (Wiley, Chichester, 2010).