Measuring SU(3) monopole witn non-Abelian charge in lattice QCD

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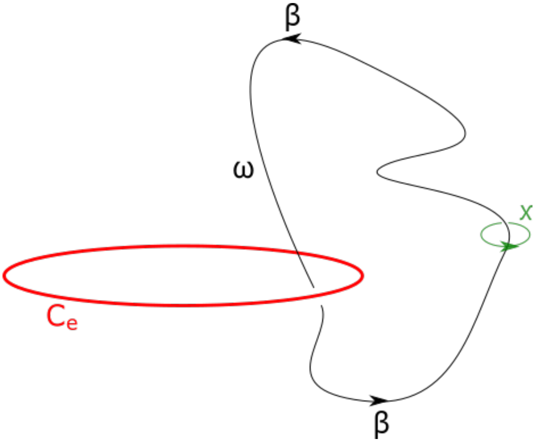
Center vortex and Wilson loop

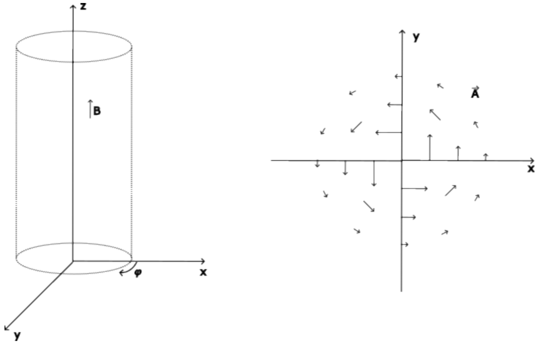
- $\mathcal{W}_{\mathcal{C}_e}[A_\mu] = \frac{1}{N}\,\mathrm{tr}\left(P\left\{e^{i\int_{\mathcal{C}_e} dx_\mu\, A_\mu(x)}\right\}\right)$
- $\langle \mathcal{W}_{\mathcal{C}_e} \rangle = \int [DA]\, \mathcal{W}_{\mathcal{C}_e}[A_\mu]\, e^{-S_{YM}}$
- Center vortices are worldsheets/worldlines in 4D/3D
- $\mathcal{W}_{\mathcal{C}_e} = e^{i\frac{2\pi}{3}}$



Center vortex line linking a Wilson loop.

Simplest example

- $A_\mu = \frac{1}{g}\partial_\mu \varphi \beta \cdot T$
- $\vec{B} = \frac{1}{g}\nabla \times (\nabla\varphi)\beta \cdot T = \frac{\hat{z}}{g}\delta^{(2)}(x,y)\beta \cdot T$
- $\partial_\mu \varphi$ gives the vorticity
- $\beta \cdot T = \beta|_q T_q$ gives the center nature
- Can be created with a singular-valued gauge transformation: $A_\mu = \frac{i}{g} S \partial_\mu S^{-1}$, $S = e^{i\varphi\beta\cdot T}$
- $\varphi \to \chi$ changes the vortex core location



Straight vortex around the z axis.

About the β's

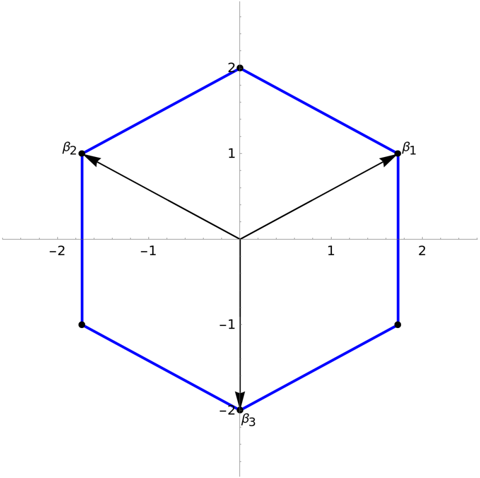


Figure: Weights of the fundamental representation

- There are three of them: $\beta_1, \beta_2, \beta_3$.
- $\beta_1 = (\sqrt{3}, 1)$, $\beta_2 = (-\sqrt{3}, 1)$, $\beta_3 = (0, -2)$.
- Together with $-\beta_1, -\beta_2, -\beta_3$, they define the <u>fundamental hexagon</u>.
- Notice $\beta_1 + \beta_2 + \beta_3 = 0$, which enables 3-matching configurations.

About the β's

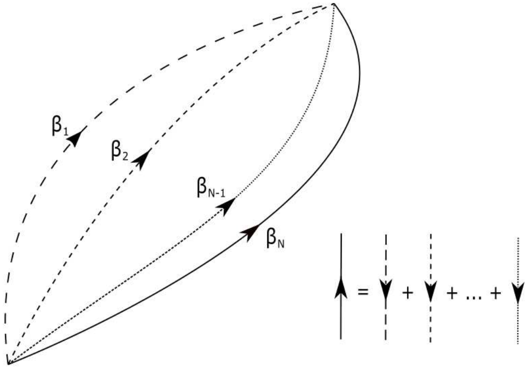


Figure: N-matching configuration.

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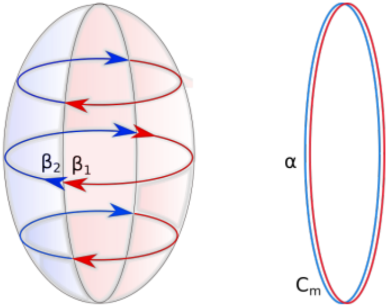
Monopoles

- Monopoles are worldlines/instantons in 4D/3D
- Must be attached to a Dirac string or....
- A non-oriented vortex, carrying weights β_1 and β_2.
- The monopole carries the charge $\alpha = \beta_1 - \beta_2$, which is a root.
- Can also be created with a singular-valued gauge transformation:

  $A_\mu = \frac{i}{g} S \partial_\mu S^{-1}$, $S = e^{i\varphi\beta_1 \cdot T} e^{i\sqrt{N}\theta T_\alpha}$
- $S(\theta = 0) = e^{i\varphi\beta_1 \cdot T}$ and $S(\theta = \pi) = e^{i\varphi\beta_2 \cdot T}$



Non-oriented vortex with a monopole in the middle. From Oxman (2018)

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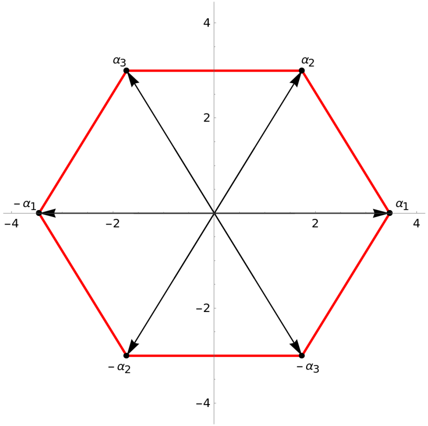


Roots of $SU(3)$

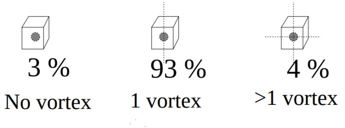
What about in the Lattice?

How are monopoles and vortices detected?

$U(1)$ deGrand-Toussaint

- Link variables $U_\mu(x) = e^{i\theta_\mu(x)}$.
- Try to measure the flux through a unit cube: $m = \frac{1}{2\pi} \sum_{\text{unit cube}} \varepsilon_{ijk} \partial_i f_{jk}$
- If $f_{\mu\nu} = \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$, $m = 0$.
- Monopole flux is exactly canceled by the attached Dirac string.
- DeGrand-Toussaint algorithm: $f_{\mu\nu} = \bar{f}_{\mu\nu} + 2\pi n_{\mu\nu}$
- $\bar{f}_{\mu\nu} \in [-\pi, \pi)$ and $n_{\mu\nu} \in \mathbb{Z}$
- $2\pi n_{\mu\nu}$ is the discrete version of the Dirac string and m for $\bar{f}$ is the monopole charge.

Monopoles in $SU(2)$

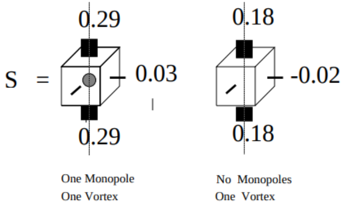


Correlation between monopoles and vortices measured this way. From Ambjørn, Giedt, Greensite (1999)

- Field has 4 components $U_\mu = u_0 \mathbb{I} + i\vec{u} \cdot \vec{\sigma}$
- Maximal Abelian Gauge: $\mathrm{Tr}\left(U_\mu \sigma_3 U_\mu^\dagger \sigma_3\right)$
- Abelian projection:

$$U_\mu \to C_\mu = \begin{pmatrix} e^{i\theta_\mu(x)} & 0 \\ 0 & e^{-i\theta_\mu(x)} \end{pmatrix}$$

- Apply dGT algorithm to $\theta_\mu(x)$



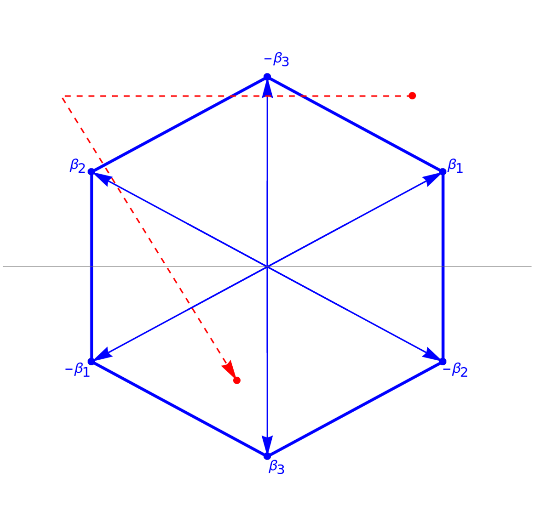
Collimation of monopoles/vortices measured this way. From Ambjørn, Giedt, Greensite (1999)

Usual method for $SU(3)$

- Following the same procedure: $U_\mu \to C_\mu = \begin{pmatrix} e^{i\theta_\mu^{(1)}(x)} & 0 & 0 \\ 0 & e^{i\theta_\mu^{(2)}(x)} & 0 \\ 0 & 0 & e^{i\theta_\mu^{(3)}(x)} \end{pmatrix}$
- $\theta_\mu^{(1)} + \theta_\mu^{(2)} + \theta_\mu^{(3)} = 0$
- Suganuma and Sakumichi (2018) applied dGT to each $\theta_\mu^{(1)}$, $\theta_\mu^{(2)}$, $\theta_\mu^{(3)}$ independetly.
- Lead to three charges (m_1, m_2, m_3) with weak correlation $m_3 = -m_1 - m_2$ or $m_3 = -m_1 - m_2 \pm 1$.
- Gives no information about root charges like $\alpha = \beta_1 - \beta_2$.

Algebraic method for $SU(3)$

- Different parametrization
  $U_\mu \to C_\mu = e^{iv_1 T_1 + iv_2 T_2} = e^{i\phi^{(1)}\alpha_1^S \cdot T + i\phi^{(2)}\alpha_2^S \cdot T}$.
- $U_{\mu\nu} = e^{if^1_{\mu\nu} T_1 + if^2_{\mu\nu} T_2}$.
- No constraints over $\phi^{(1)}$ and $\phi^{(2)}$.
- Bonati and D'Elia (2013) applied dGT to this parametrization.
- Lead to two independent charges (m_1, m_2)
- $(1, 0)$ are associated with α_1, $(0, 1)$ with α_2 and $(1, 1)$ with $\alpha_1 + \alpha_2 = \alpha_3$.



Example of Monopole with charge (1, 1), i.e. α_3

For example

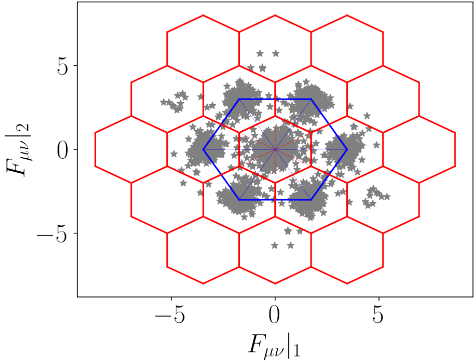


Figure: Values of $(f^1_{\mu\nu}, f^2_{\mu\nu})$ for all plaquettes of a certain lattice configuration

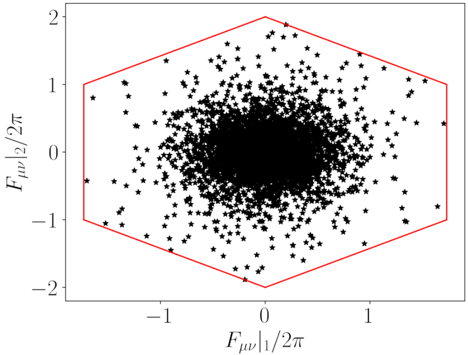
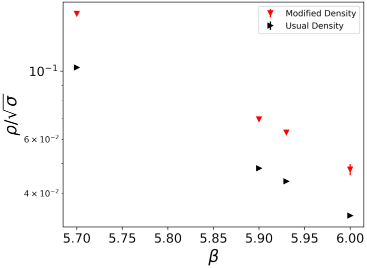


Figure: Values of $(\bar{f}^1_{\mu\nu}, \bar{f}^2_{\mu\nu})$ for all plaquettes of a certain lattice configuration

Monopole Density

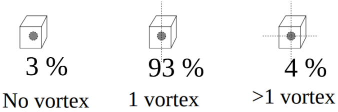


Density of monopoles using both methods for different values of β

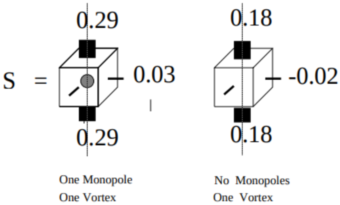
β	a [fm]	V	N^{config}	ρ_{usual}	$\rho_{\text{algebraic}}$
5.70	0.177(1)	8^4	750	0.04069(8)	0.0609(3)
5.90	0.117(1)	12^4	100	0.0126(1)	0.0182(2)
5.93	0.109(2)	16^4	599	0.01064(2)	0.01535(6)
6.00	0.098(1)	16^4	750	0.00745(3)*	0.01053(4)

*Value obtained from Tucker and Stack (2002)

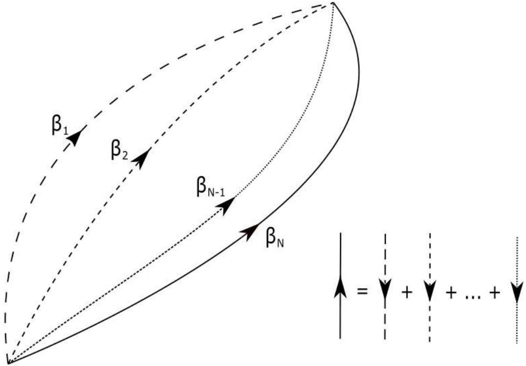
What about...



Monopole/vortex correlation?



Collimation?



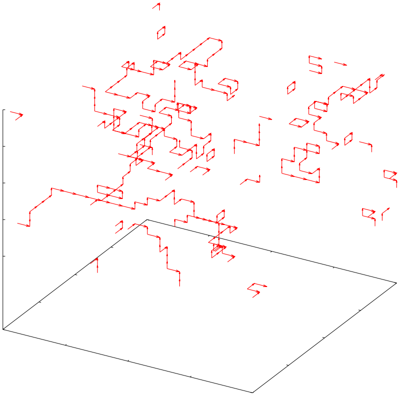
Matchings?

Vortices in the lattice

- The standard procedure starts by fixing the gauge
- Maximal Center Gauge: $\sum_{x,\mu} |\mathrm{Tr} U_\mu(x)|^2$
- Next, center projection:

  $U_\mu \to Z_\mu \in \{1, e^{i\frac{2\pi}{N}}, ..., e^{2\pi i\frac{N-1}{N}}\}$
- Evaluate the plaquete:

  $Z_{\mu\nu}(x) = Z_\mu(x) Z_\nu(x+\mu) Z^*_\mu(x+\nu) Z^*_\nu(x)$
- If $Z_{\mu\nu} \neq 1$, there is a vortex piercing the plaquete (P-vortex)
- How do you distinguish vortices with β_1, β_2,...,β_N? They all have $Z_{\mu\nu} = e^{i\frac{2\pi}{N}}$.

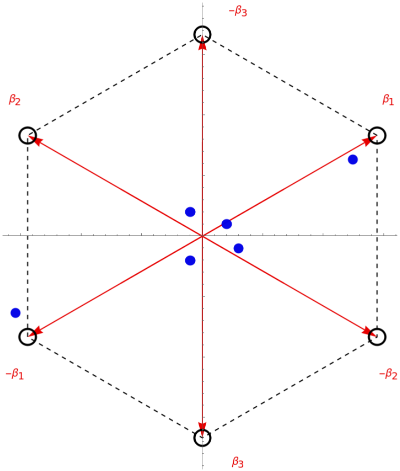


An example of vortices measured this way.

Back to Algebraic method

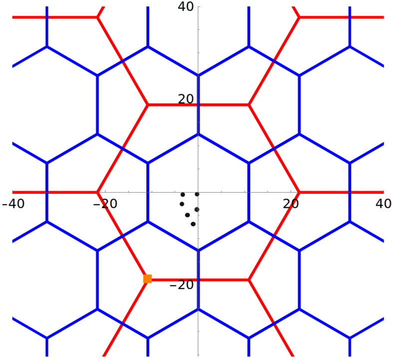
- Maximal Abelian Gauge fixing
- Abelian projection $U_\mu = C_\mu \in U(1)^{N-1}$
- Evaluate the plaquete:

  $C_{\mu\nu}(x) = C_\mu(x) C_\nu(x+\mu) C^*_\mu(x+\nu) C^*_\nu(x)$
- From $C_{\mu\nu}$, extract the values of $f_{\mu\nu}$ for a unit cube.
- $f_{\mu\nu}$ is invariant under the residual $U(1)^2$ symmetry of the MAG.

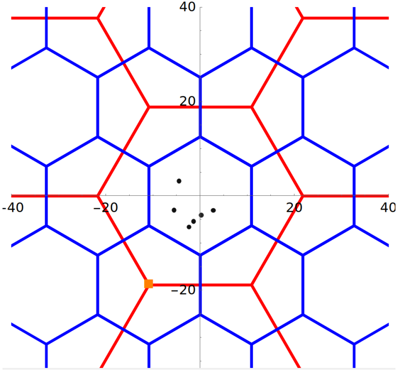


Best case scenario for a collimated flux in a unit cube representing two vortices with charge β_1,β_2 and a monopole with $\beta_2 - \beta_1$.

Flux Distribution



Flux distribution of 1x1x1 cube around a monopole



Flux distribution of 3x3x3 cube around a monopole

Next steps

- Bigger lattice, more configurations. (not a panacea, though!)
- Fine-tuning of the method guided by particular and well-understood configurations.
- Alternate definitions of flux without an Abelian projection.
- Understand the role of rotations.

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Special thanks

You, for your attention!