CENTER-SYMMETRIC LANDAU GAUGE MEETS THE LATTICE

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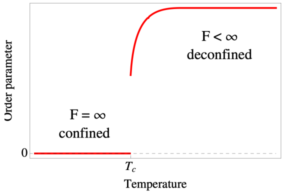
INTRODUCTION

- At some very high temperature T_c, hadrons become free quarks and gluons $\rightarrow$ quark-gluon plasma.
- An order parameter for this transition is the Polyakov loop:

$$\ell \sim \langle P e^{i \int_0^\beta d\tau A_0(\tau, x)} \rangle \sim e^{-\beta F}$$



- Under center symmetry, $\ell \rightarrow Z_N \ell$, so deconfinement is signaled by a broken center symmetry (in pure Yang-Mills).
- Confirmed by lattice data: second order transition for $SU(2)$, first order for $SU(3)$.

ENCODING OF THE TRANSITION

Polyakov loop:

$$\ell \sim \langle P e^{i \int_0^\beta d\tau A_0(\tau, x)} \rangle \sim e^{-\beta F}.$$

Because the Polyakov loop is related to A_0, it is expected that the transition is encoded in (the tower of)

$$\langle A_0 \rangle, \langle A_0 A_0 \rangle, \ldots, \langle A_0^n \rangle.$$

For the appropriate choice of gauge, can the transition be reflected in the lowest order correlators?

LANDAU GAUGE CORRELATOR

In principle:

- $\langle A \rangle$ is found by minimizing the effective action $\Gamma[A]$. It represents the state of the system . $\langle A_0 \rangle \to$ order parameter.
- The two-point correlator derives from the effective action

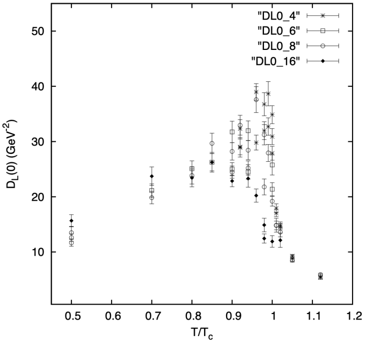
$$1 \Big/ \left.\frac{\partial^2 \Gamma}{\partial A^2}\right|_{A=\langle A \rangle} = \langle AA \rangle_c,$$

so for $SU(2)$, $\langle A_0 A_0 \rangle$ should diverge at T_c.

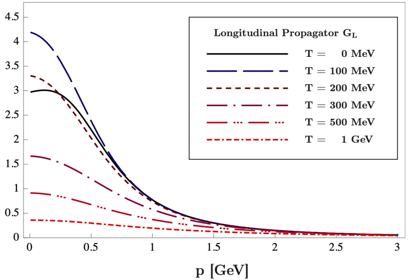
In practice:

- In the Landau gauge, $\partial_\mu A_\mu = 0$, then $\langle A_0 \rangle = 0$. $\to$ no order parameter.
- No evidence of divergence of $\langle A_0 A_0 \rangle$ was found on the (gauge-fixed) lattice or in the continuum.

$SU(2)$ LANDAU GAUGE CORRELATORS



Electric susceptibility (zero momentum longitudinal propagator)



Longitudinal gluon propagator

*T. Mendes and A. Cucchieri, PoS LATTICE2014, 183 (2015).

*L. Fister and J. M. Pawlowski, [arXiv:1112.5440 [hep- ph]](2012).

BACKGROUND FIELD GAUGES

- In the Landau gauge the effective action is not explicitly center-symmetric because $\Gamma[A] \neq \Gamma[A^U]$.
- Introducing a background field $\bar{A}$, the effective action is gauge-invariant $\Gamma_{\bar{A}}[A] = \Gamma_{\bar{A}^U}[A^U]$. Landau-deWitt gauge:

$$\bar{D}_\mu(A_\mu - \bar{A}_\mu) = 0, \text{ with } \bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right].$$

- We propose the Center-symmetric Landau gauge, which fixes $\bar{A} = \bar{A}_c$. Then $\Gamma_{\bar{A}_c}[A] = \Gamma_{\bar{A}_c}[A^{U^c}]$.

OUR SETUP

- We work in the Landau-deWitt gauge with a background field $\bar{A}_\mu$:

$$\bar{D}_\mu(A_\mu - \bar{A}_\mu) = 0, \text{ with } \bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right].$$

- We take $\bar{A}$ and $\langle A \rangle$ in the temporal direction, $\propto \delta_{\mu 0}$, and along the diagonal color directions (σ^3 for $SU(2)$, (λ^3, λ^8) for $SU(3)$), so that $\Gamma[A, \bar{A}] \propto V(A, \bar{A})$.
- The center-symmetric values for $\bar{A}$ and $\langle A \rangle$ are found by Weyl chambers. For example in $SU(2)$, $\langle A \rangle = \delta_{\mu 0} \frac{T}{g} r \frac{\sigma^3}{2}$ with $r \to 2\pi - r$, then $r_c = \pi$.
- We fix $\bar{A} = \bar{A}_c$: Center-symmetric Landau gauge.
Center-symmetric phase when $\langle A \rangle = A_c$, $\to$ order parameter.

CURCI-FERRARI MODEL

We have computed $\langle A \rangle$ and $\langle A(0,p)A(0,-p) \rangle$ up to first loop order in the finite temperature Curci-Ferrari model:

$$S = S_{YM} + S_{gf} + \int_{x,\tau} \frac{m^2}{2} (A_\mu^a - \bar{A}_\mu^a)^2$$

Several motivations:

- Perturbative gauge-fixed Yang-Mills breaks down at low energies (Landau pole, Gribov copies...), there is no analytical model for this region.
- A gluon mass term seems to dominate the (unknown) gauge-fixed action in the IR; decoupling behaviour on the lattice. CF could be an effective model.
- The CF model is renormalizable, avoids the Landau pole and lifts the degeneracy between Gribov copies. Perturbative window into non-perturbative region.

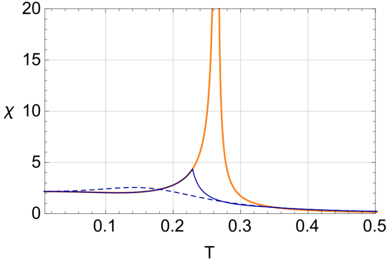
RESULTS - $T_c(MeV)$

	Lattice	**CF-CS, 1-lp**[1]	FRG-BG[2]	CF-BG, 1-lp[3]	CF-BG, 2-lp[4]
SU(2)	295	**265**	230	238	284
SU(3)	270	**267**	275	185	254

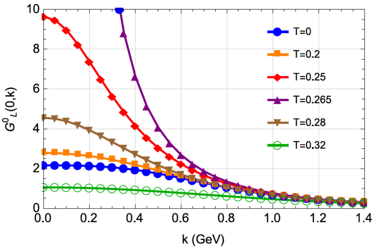
BG: Background effective action
CS: Centersymmetric Landau gauge
.

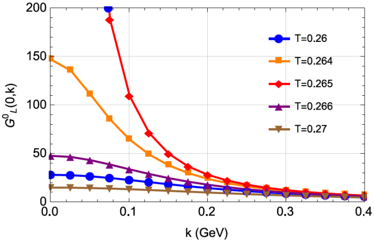
[1] **DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022)**
[2] L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010
[3] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.
[4] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

RESULTS: SU(2) GLUON PROPAGATOR



Landau gauge
Background field effective action
Centersymmetric Landau gauge

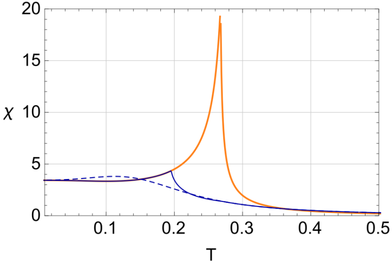


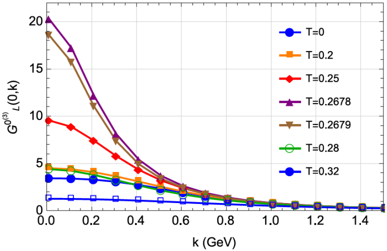


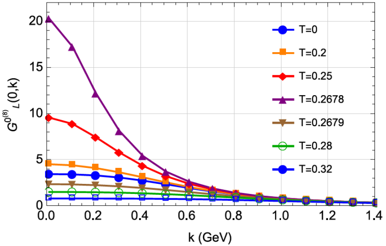
m=0.68 GeV, μ=1 GeV, g=7.5

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

RESULTS: SU(3) GLUON PROPAGATOR





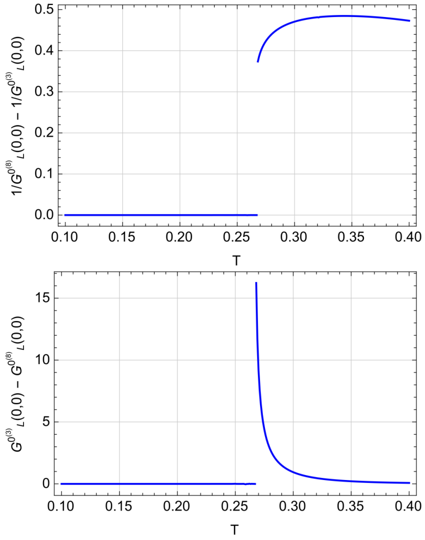


Landau gauge
Background field effective action
Centersymmetric Landau gauge

m=0.54 GeV, μ=1 GeV, g=4.9

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

SU(3) PROPAGATOR DIFFERENCE



CS LANDAU GAUGE MEETS THE LATTICE

The center-symmetric Landau gauge can be implemented on the lattice by maximizing

$$F = \sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\left[g_c^\dagger(\mu) U_\mu(x)\right]$$

with

$$g_c^\dagger(\mu) = e^{iaT\bar{r}_j t_j \delta_{\mu 4}} = e^{iag\bar{A}_{c,\mu}\delta_{\mu 4}}.$$

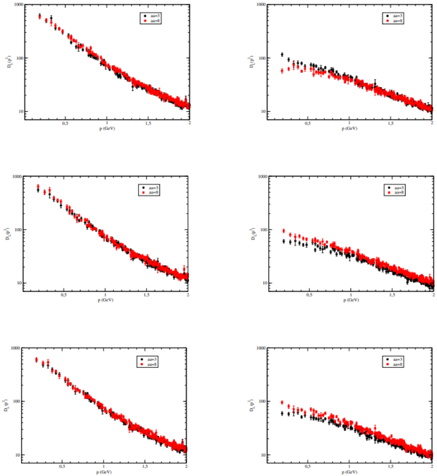
This is invariant under $U_\mu(n) \to \tilde{g}(n) U_\mu^\dagger(n) \tilde{g}^\dagger(n+\hat{\mu})$ with

$$\tilde{g}(n) = e^{i\pi\frac{\lambda_4}{2}} e^{i\pi\frac{\lambda_1}{2}} e^{-i\frac{n_4}{L_4}\pi\left(\lambda_3 + \frac{\lambda_8}{\sqrt{3}}\right)}$$

Here, a is lattice spacing and $aT = 1/L_t$.

CS LANDAU GAUGE MEETS THE LATTICE

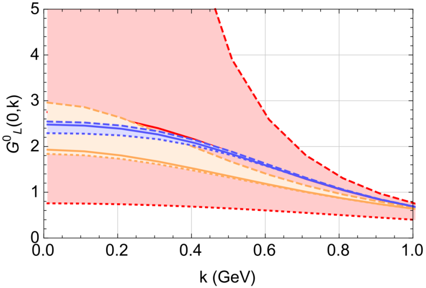
$64^3 \times L_t$ lattice volume with $T < T_c\,(T = 243\text{MeV}, L_t = 8)$ and $T > T_c\,(T = 324\text{MeV}, L_t = 6)$.



CONCLUSION AND OUTLOOK

- We have performed, for the first time, calculations of the gluon one-and two-point correlator in the center-symmetric Landau gauge.
- We find a good agreement with lattice data for T_c.
- For $SU(2)$, the deconfinement transition is signaled by a divergence of the longitudinal gluon propagator for $k \to 0$.
- For $SU(3)$, the difference between the propagators in the neutral color mode is an order parameter for the transition, and this has been confirmed by lattice data.
- Ideas for future works: transversal propagator, higher order correlation functions, $SU(2)$ on the lattice, $SU(2)$ on the lattice.

DISCUSSION

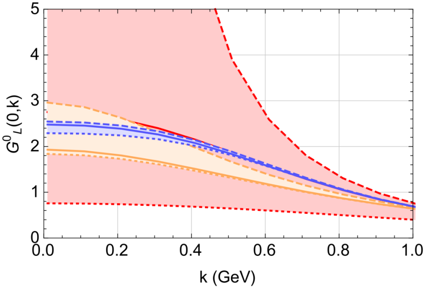


Landau gauge

Self-consistent Landau gauge

Center-symmetric Landau Gauge

DISCUSSION



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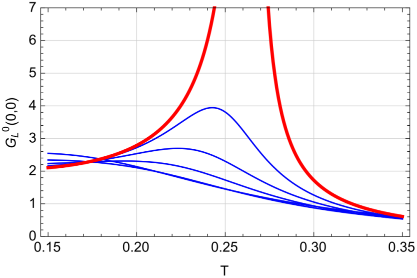

DISCUSSION