Real-time Dynamics and Flux Tube Fluctuation in SU(2) Gauge Theory on a Quantum Computer

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Outline

Lattice QCD

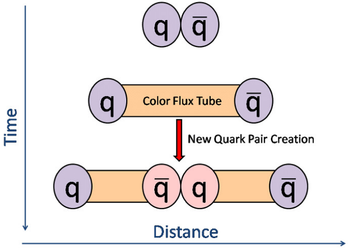
- Spacetime is discretized into a lattice.
- Lagrangian approach, formulated in Euclidean spacetime by performing a Wick rotation $t \to -i\tau$.
- Uses the Monte Carlo method: field configurations are generated with probability weight e^{-S_E}.
- Highly successful at computing static and equilibrium properties, such as:
  - The hadron mass spectrum
  - Euclidean correlation functions
  - Finite temperatures

Limitations

- It fails for finite baryon density due to the **sign problem**, e^{-S_E} becomes complex, Monte Carlo inapplicable.

- Real time processes (Minkowski space), weights e^{iS} are complex, oscillate, Monte Carlo inapplicable.

- This makes it difficult to access:
  - Particle production
  - Time evolution of an initial state
  - High density matter
  - Nonequilibrium problems



Real time: Hamiltonian approach

- Spatial coordinates are discretized, but time remains continuous.
- The dynamics (time evolution) is given by

$$|\psi(t)\rangle = e^{-iHt}|\psi(o)\rangle$$

- Need to fix the gauge and work directly with the gauge -invariant Hilbert space.
- It can be mapped directly onto quantum computers

Quantum Simulation

- Nowadays, quantum computers have small numbers of qubits (10^2), and are noisy.
- We must first establish the foundations of what we want to simulate on these devices using small systems.
  - Consider just two plaquettes
  - Just two colors: $SU(2)$
  - Consider static quarks
- Objectives:
  - Study the pure gauge excitation traveling on the lattice.
  - Add a static pair quark-antiquark and simulate the flux-tube fluctuation.

Continuum QCD Hamiltonian

In temporal gauge $A_0 = 0$, the Hamiltonian density for static quarks is

$$\mathcal{H} = \text{Tr}\left(E^2 + B^2\right) + m_0 \bar{q} q. \tag{1}$$

and the Gauss law constrain the physical states to obbey

$$(D_i E_i^a - \rho^a)|\textbf{physical}\rangle = 0. \tag{2}$$

where:

- $\rho^a = g q^\dagger T^a q$
- $D \cdot \mathbf{E} = \nabla \cdot \mathbf{E} - ig[A_i, E_i]$

Space discretization

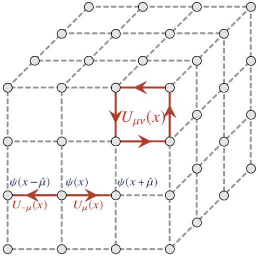


Figure: Discretization of the space into a cubic lattice

- Quarks live on the sites
- Gauge fields $A^a_\mu(x)$ are defined on the links connecting the nearest neighboring sites via the link variable define by

$$U_\mu(x) = P \exp\left(ig \int_x^{x+a\hat{u}} dx_\mu \, T^a A^a_\mu(x) \right) \quad (3)$$

Kogut-Susskind Hamiltonian

- Hamiltonian :

$$H = \frac{g^2}{2} \sum_{\text{links}} \mathbf{E}_L^2 + \frac{1}{2g^2} \sum_p \left(2 - \mathrm{Tr}\big(U_p + U_p^\dagger\big)\right) + \sum_n m_0 \bar{q}_n q_n .$$

- Plaquette $U_{\mu\nu} \equiv U_p$ is product of four links around an elementary square.
- Gauss law at node n:

$$\left(\sum_{l \in out(n)} \mathbf{E}_L^a(l) - \sum_{l \in in(n)} \mathbf{E}_R^a(l) + q_n^\dagger T^a q_n \right) |\text{physical}\rangle = 0$$

Quantum Computing: Basic Concepts

Qubits

- A qubit is a two-level quantum system:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad |\alpha|^2 + |\beta|^2 = 1.$$

- Basis states: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Multi-Qubit States

- Example basis: $|00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bigotimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |01\rangle, |10\rangle, |11\rangle$.
- Enables entanglement.

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Quantum Computing: Basic Concepts

Quantum Gates

- Gates are unitary operators: $|\psi\rangle \to U|\psi\rangle$.
- Examples:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

- Two-qubit entangling gate: CNOT$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

Implementation workflow*

1. Fix gauge $A_0 = 0$: build physical gauge-invariant basis using Gauss law.
2. Compute Hamiltonian matrix in that basis.
3. Decompose Hamiltonian into Pauli strings for quantum hardware.
4. Prepare initial state; apply Trotterized evolution; measure.
5. Compare with exact classical evolution to validate.

*Based on: S.A. Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D 106, 074502 (2022)

Two-plaquette system

Now, to compute the Hamiltonian matrix:

- Truncate SU(2) link representations to $j = 0,\ 1/2$ to keep finite-dimensional link Hilbert spaces.

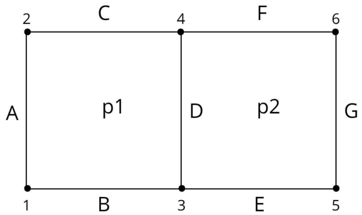


Figure: A lattice with 2 plaquettes and 7 links

$$H = \frac{g^2}{2}\left(\sum_{\text{links}} \mathbf{E}_L^2 + 2x\sum_p \mathrm{Tr}\, U_p\right) + \sum_n m_0 \bar{q}_n q_n. \tag{4}$$

where $x = 1/g^4$.

Pure gauge system

From Gauss' law we can have 4 states

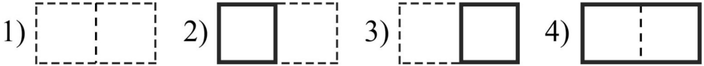


Figure: Solid lines: $j = 1/2$, energy is present. Dashed lines: $j = 0$, no energy is present

We encode these states onto two qubits:

$$1) \rightarrow |00\rangle, \quad 2) \rightarrow |10\rangle, \quad 3) \rightarrow |01\rangle, \quad 4) \rightarrow |11\rangle$$

Hamiltonian for a pure gauge system

With our considerations, the Hamiltonian will be a 4×4 matrix given by

$$\frac{2}{g^2}H = \begin{pmatrix} 0 & -2x & -2x & 0 \\ -2x & 3 & 0 & -x \\ -2x & 0 & 3 & -x \\ 0 & -x & -x & \frac{9}{2} \end{pmatrix} \tag{5}$$

$$= \frac{3}{8}(7 - 3Z_0 - Z_0Z_1 - 3Z_1) - \frac{x}{2}(3 + Z_1)Y_0 - \frac{x}{2}(3 + Z_0)Y_1 \tag{6}$$

We can now simulate real-time evolution. Choosing our initial state to be the state that have a left plaquette excitation (state 2).

$$|\psi(t)\rangle = e^{-iHt}|2\rangle \tag{7}$$

Pure gauge excitation evolution

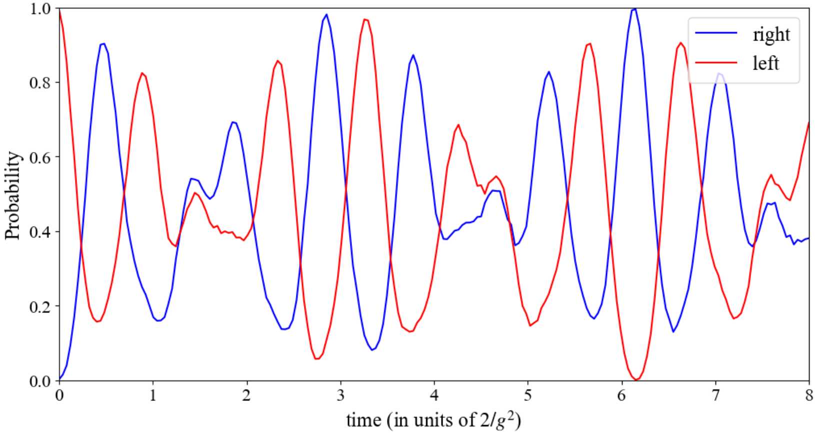


Figure: Time evolution of a left plaquette excitation for x=2.0

Adding a static quark–antiquark pair

Now, we introduce static color charges at the corners of our lattice. And from Gauss' law we have the following states

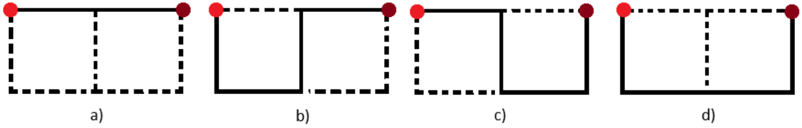


Figure: Solid line means a $j = 1/2$ flux of energy is present, while the dashed line a $j = 0$, red dot (dark red) represents a quark (antiquark).

We encode these states onto two qubit of the following way

$$a) \to |00\rangle, \quad b) \to |10\rangle, \quad c) \to |01\rangle, \quad d) \to |11\rangle \tag{8}$$

Fux-tube Hamiltonian

Now the Hamiltonian matrix for this system is

$$\frac{2}{g^2}H = \begin{pmatrix} \frac{3}{2} & -x & -x & 0 \\ -x & 3 & 0 & x \\ -x & 0 & 3 & x \\ 0 & x & x & 3 \end{pmatrix} + \frac{4}{g^2}m_0$$

$$= \frac{21}{8} + \frac{4}{g^2}m_0 - \frac{3}{8}(Z_0 + Z_0 Z_1 + Z_1) - x(Z_1 Y_0 + Z_0 Y_1)$$

Choosing our initial state the state a:

$$|\psi(t)\rangle = e^{-iHt}|a\rangle$$

Flux-tube fluctuation

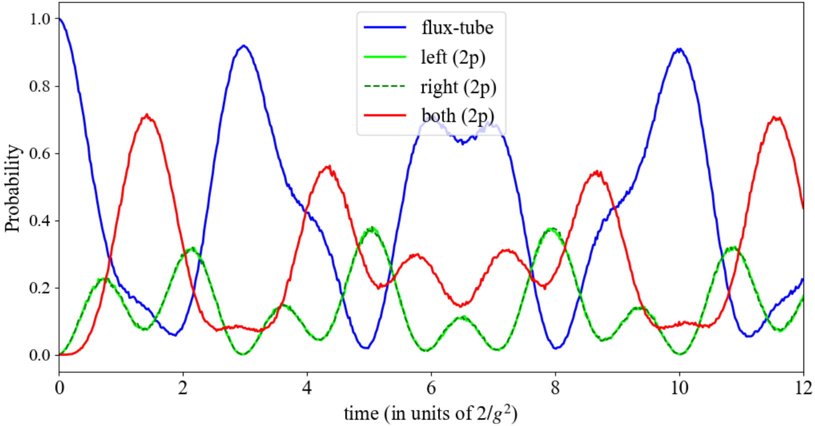


Figure: Flux-tube fluctuation for $x = 1.0$

Conclusions and Outlook

- The Hamiltonian formulation of lattice gauge theory provides a natural framework for studying **real-time dynamics**.
- Quantum hardware can faithfully reproduce the unitary time evolution, offering a promising platform to explore non-equilibrium phenomena in gauge theories.
- Our simulation show the time evolution of the flux-tube configuration.

Outlook:

- Incorporate **dynamical fermions** to investigate phenomena such as string breaking and hadronization in real time.
- Improve circuit optimization, error mitigation, and scalable algorithms to extend real-time evolution on near-term hardware.

Thank you

Questions?

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