Net quark number gain and the QCD phase diagram

See: 2504.06459, 2511.23413

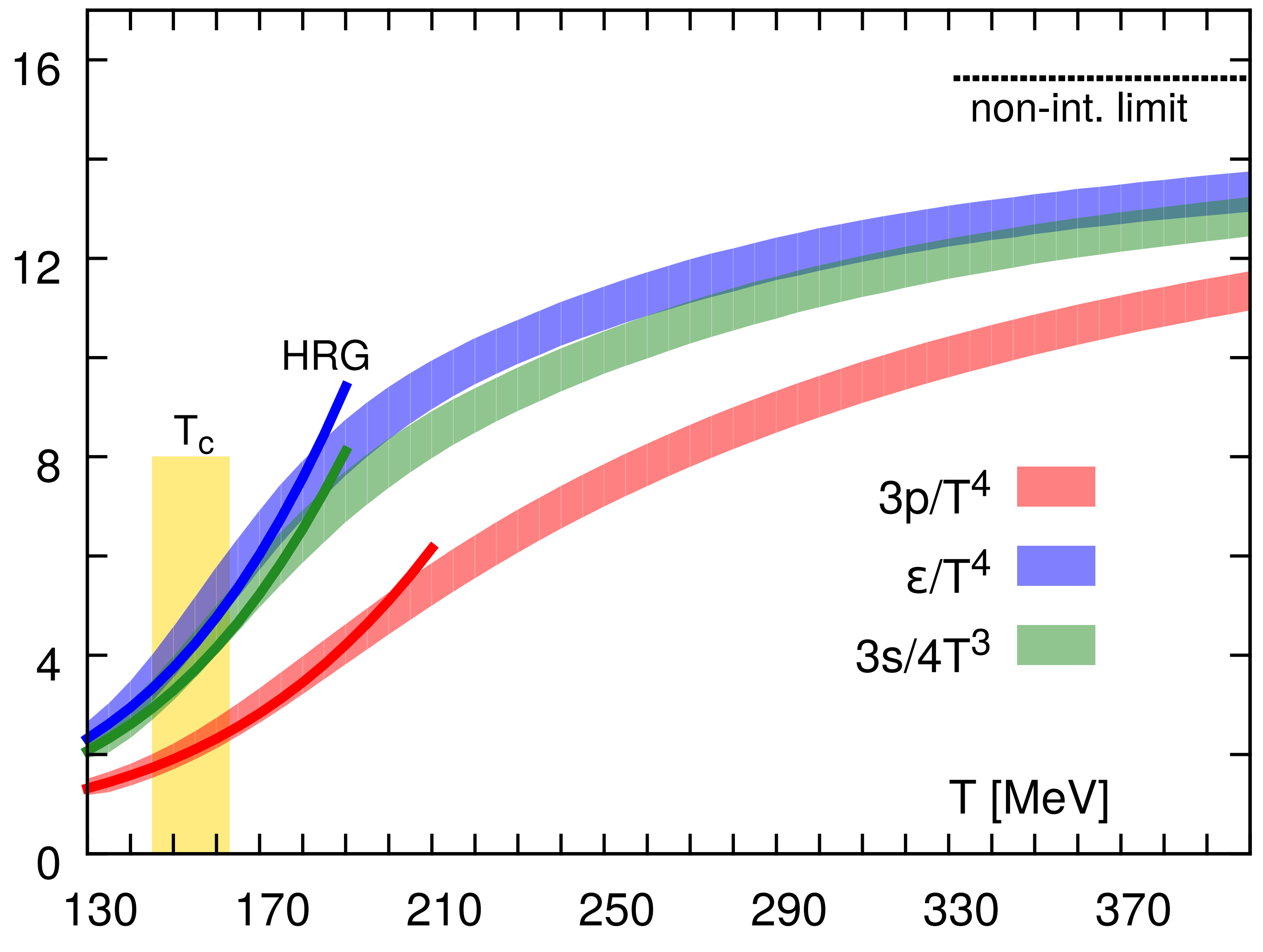
Tomas Mari Surkau and Urko Reinosa

Centre de Physique Théorique, CNRS, École Polytechnique, IP Paris, F-91128 Palaiseau, France.

Our rough understanding

Low $T \rightarrow$ high T: rapid change in degrees of freedom

- **Low T:** Hadron resonance gas.
  - Agrees w. Lattice QCD up to T_c,
  - Predicts freeze-out yields of HIC.



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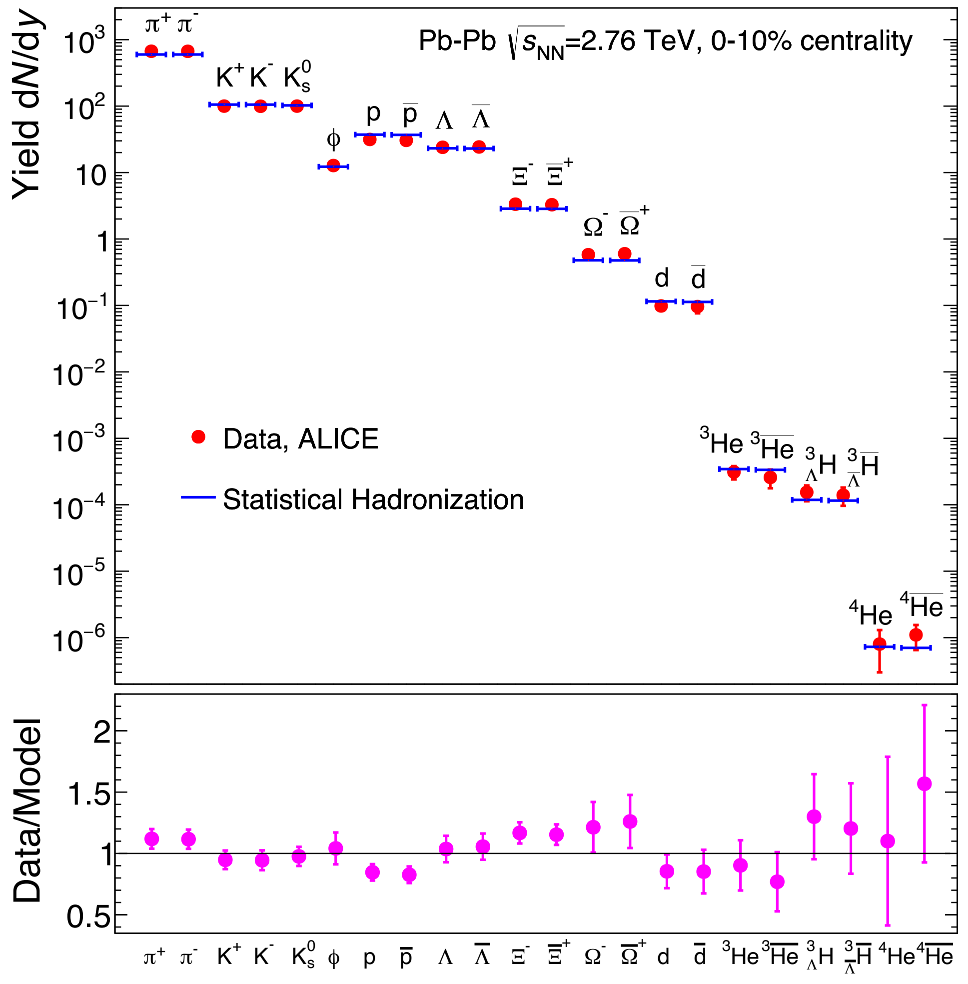

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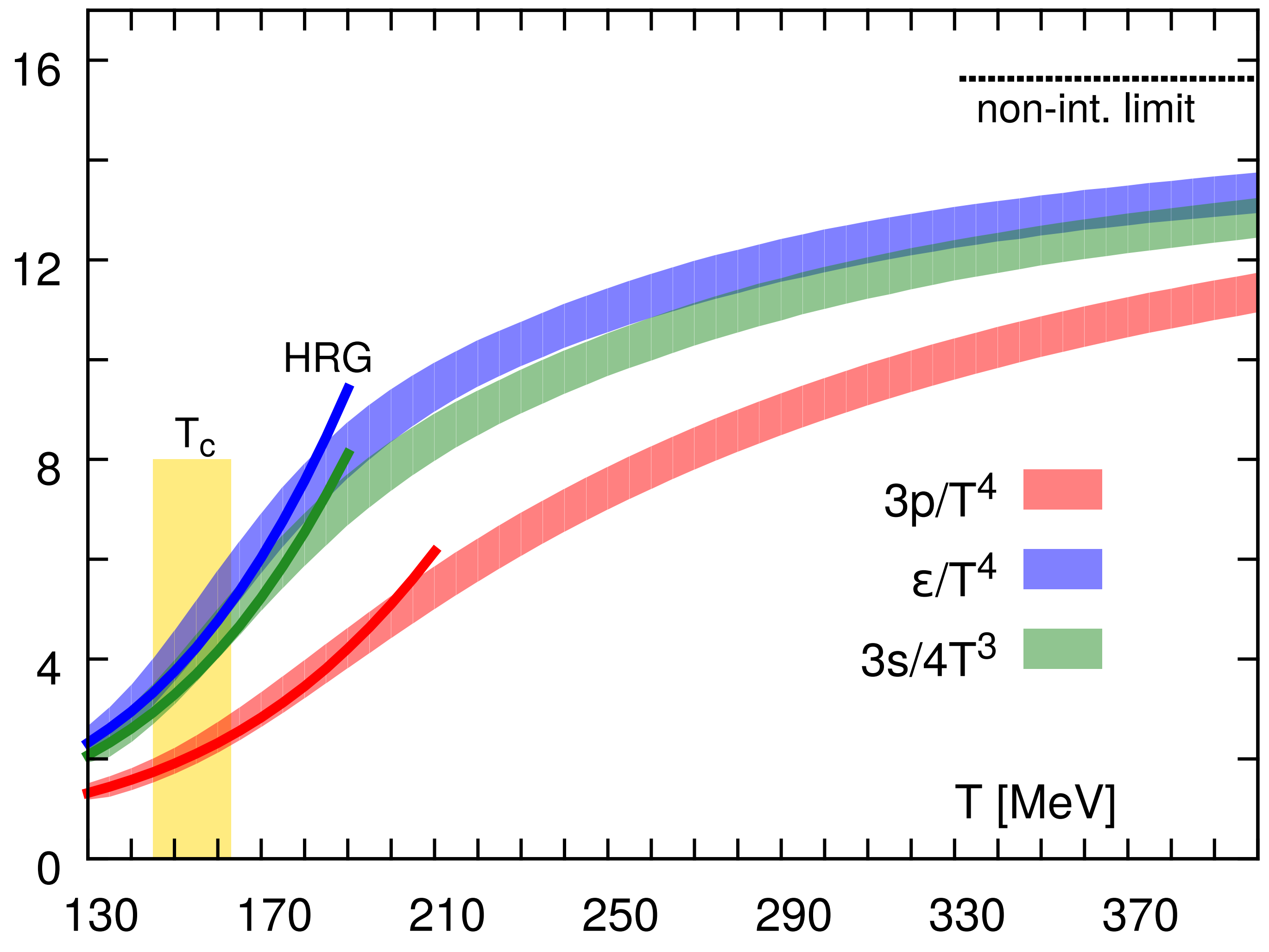

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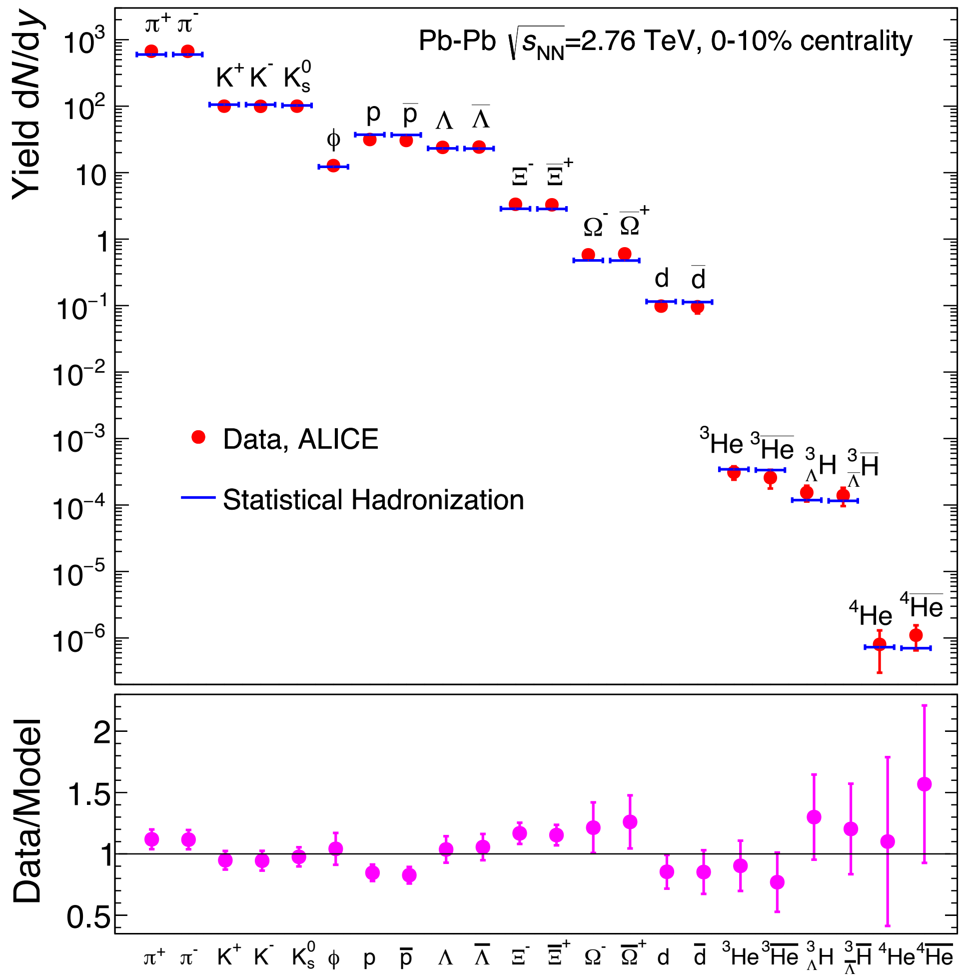

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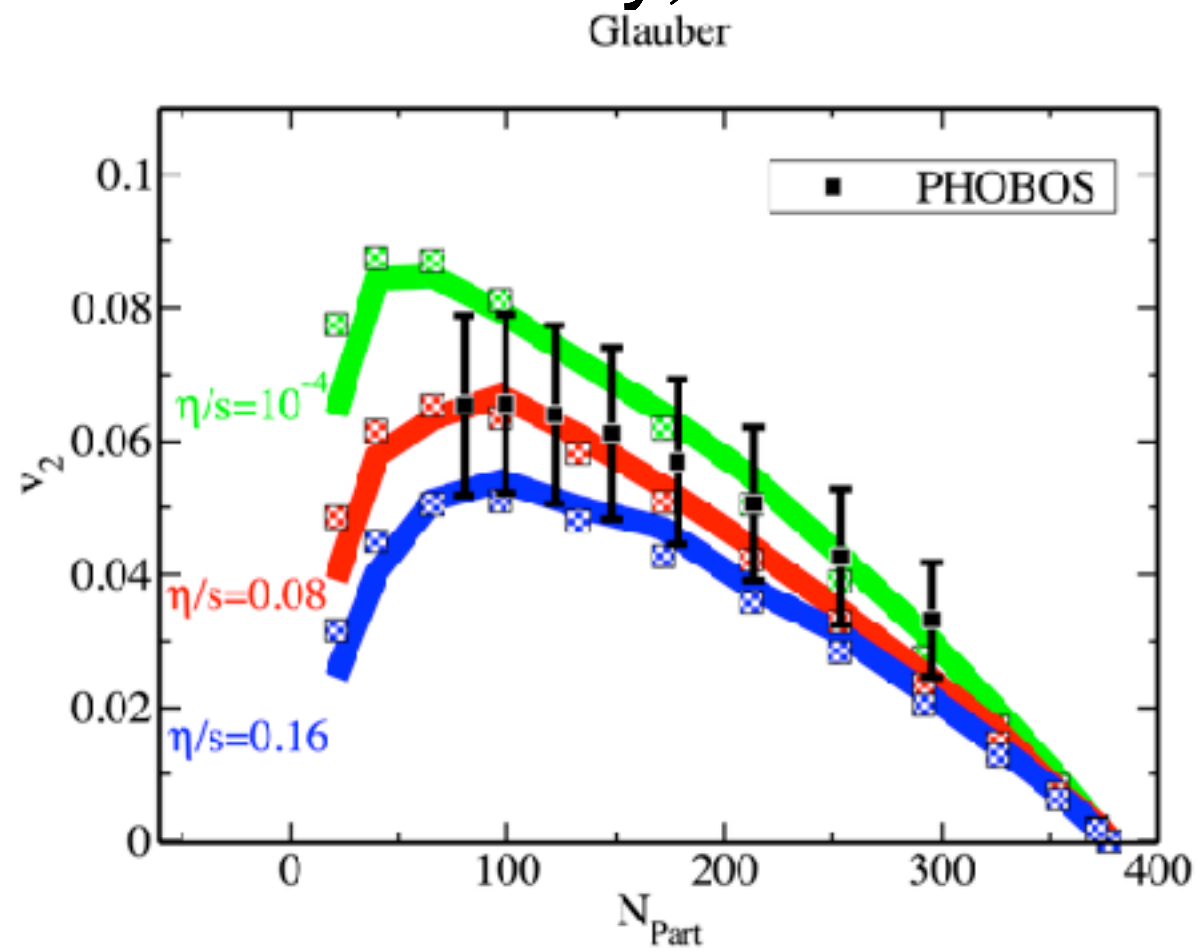
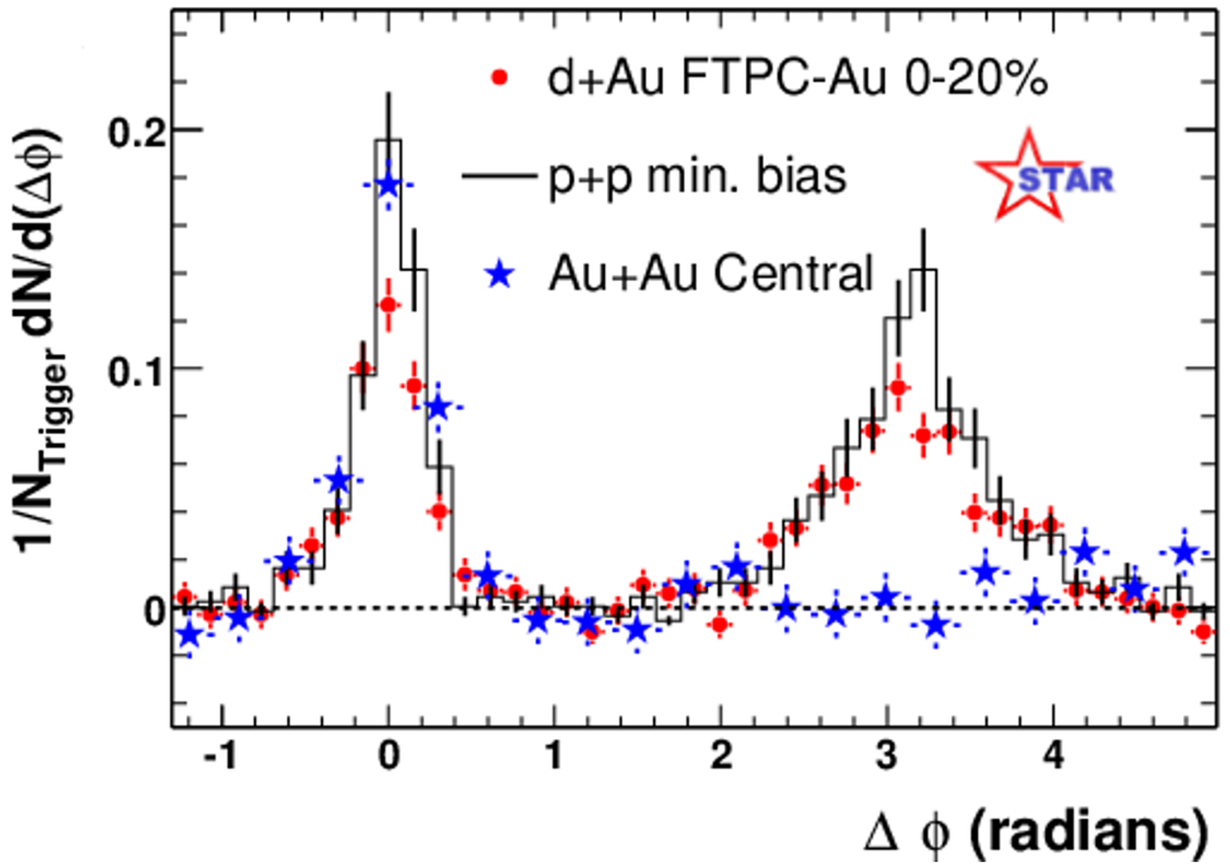
- **High T:** Strongly coupled fluid (~QGP),
  - Analytic crossover from the HRG,
  - HIC: anisotropic flow & jet quenching $\to$ low viscosity, thermalized medium created.



M. Luzum et al., PRC 78, 034915 (2008), [0804.4015]



STAR, PRL 91, 072304 (2003), [nucl-ex/0306024]

($T \to \infty$: asymptotically free ($\alpha_s(2\pi T) \to 0$) parton gas)

The desired understanding

Full QCD phase diagram

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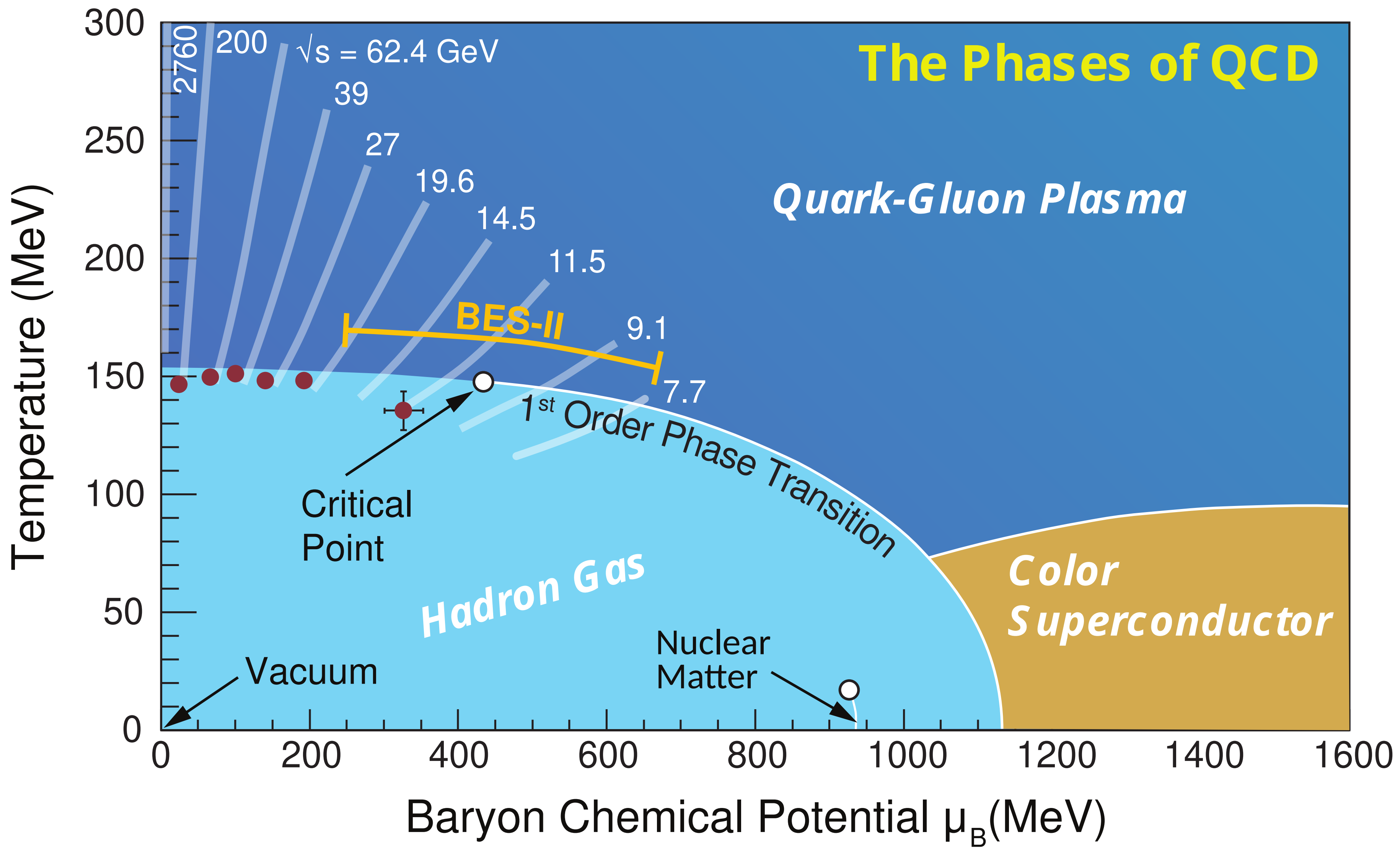
- Expectations(funct.QCD, models):



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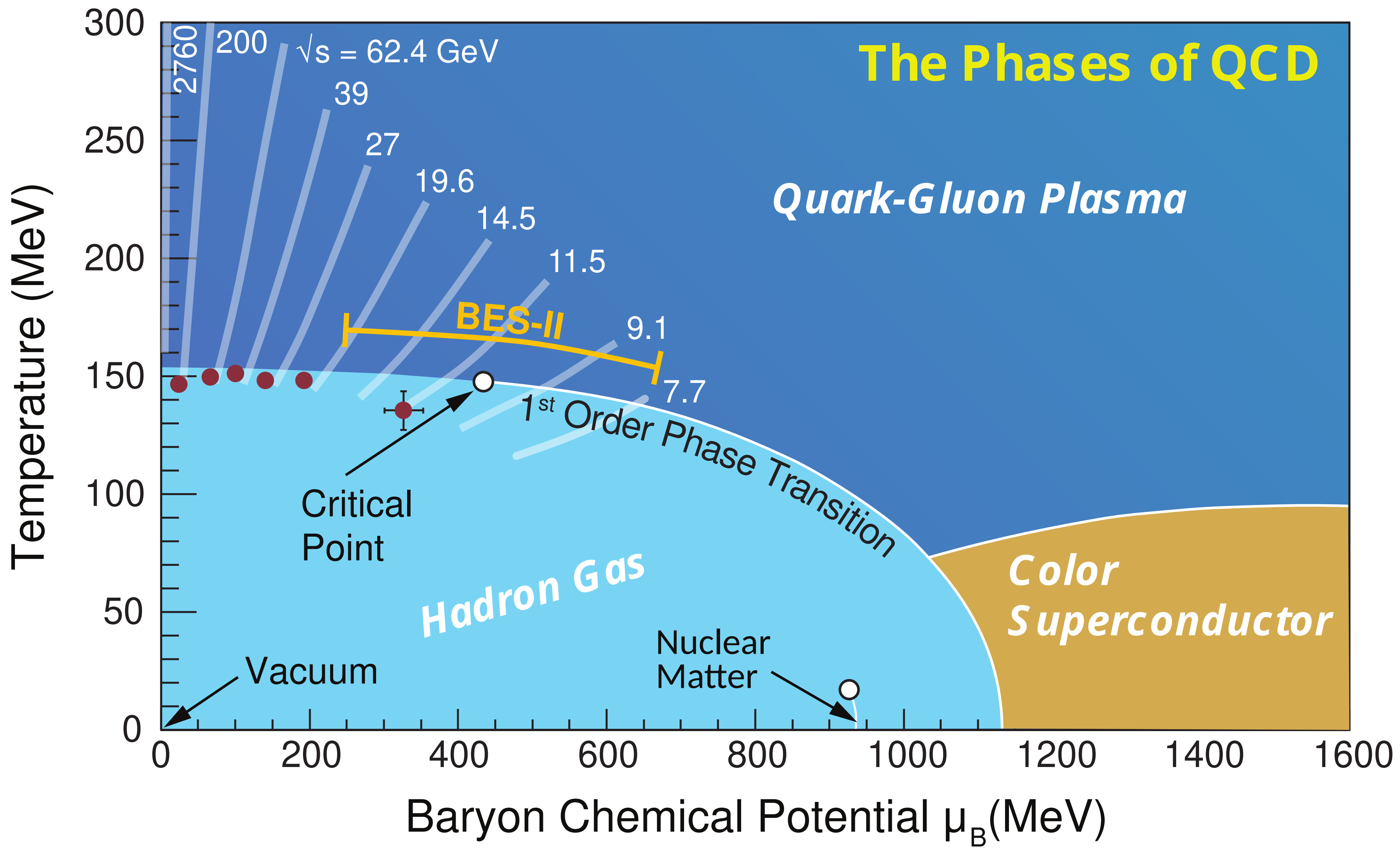

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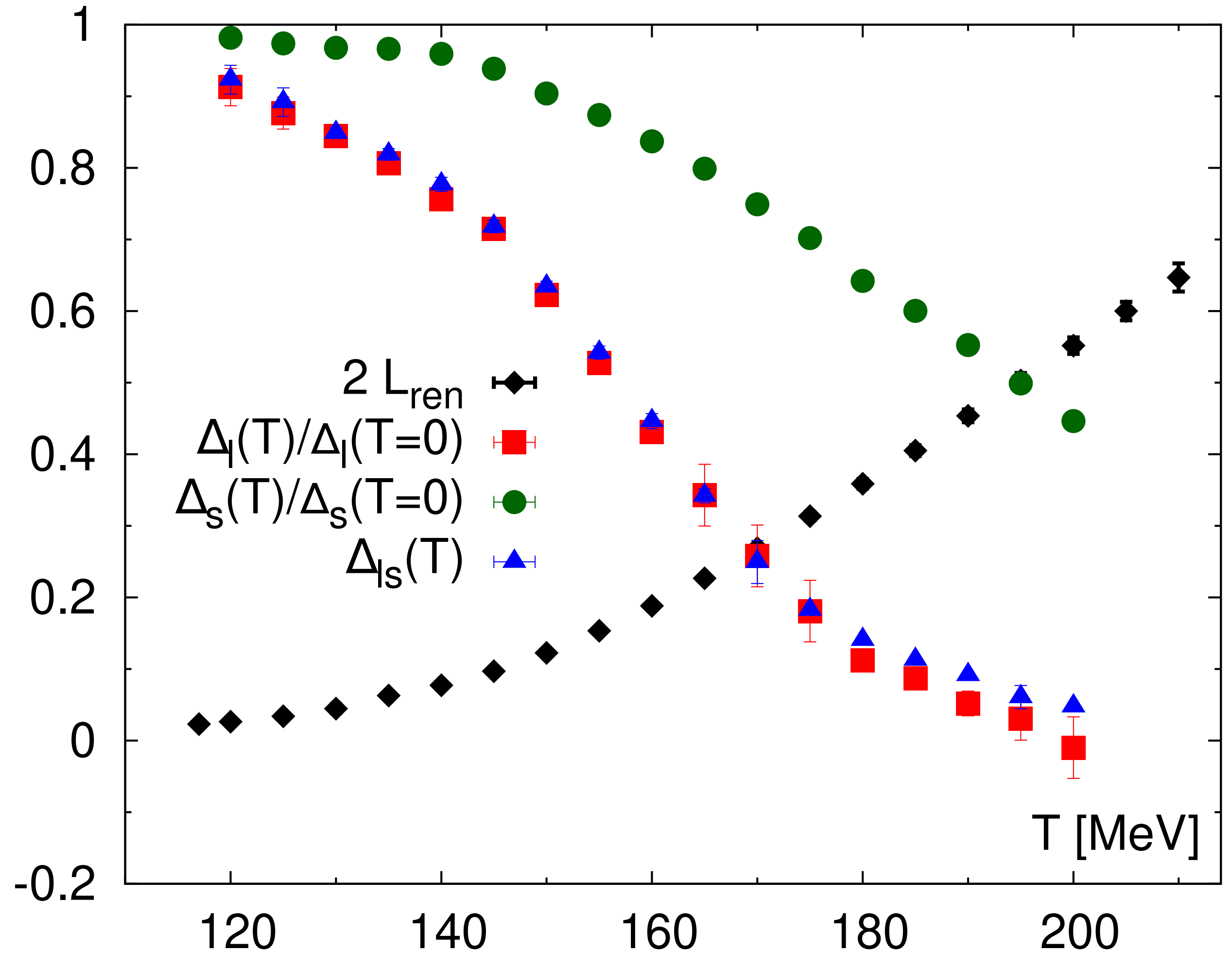
- Crossover $\Rightarrow T_c$ depends on observable,
- Approximate QCD order parameters:
  - Chiral condensates $\langle \bar{q}q \rangle$ (chiral sym. $m_q \to 0$),
  - Polyakov loops $\ell, \bar{\ell}$ (center sym. $m_q \to \infty$).

$\ell \equiv \langle \Phi \rangle$



ℓ

$\langle \bar{s}s \rangle$

$\langle \bar{u}u \rangle (= \langle \bar{d}d \rangle)$

A. Bazavov et al., PRD 87, 094505 (2013), [1301.3943]

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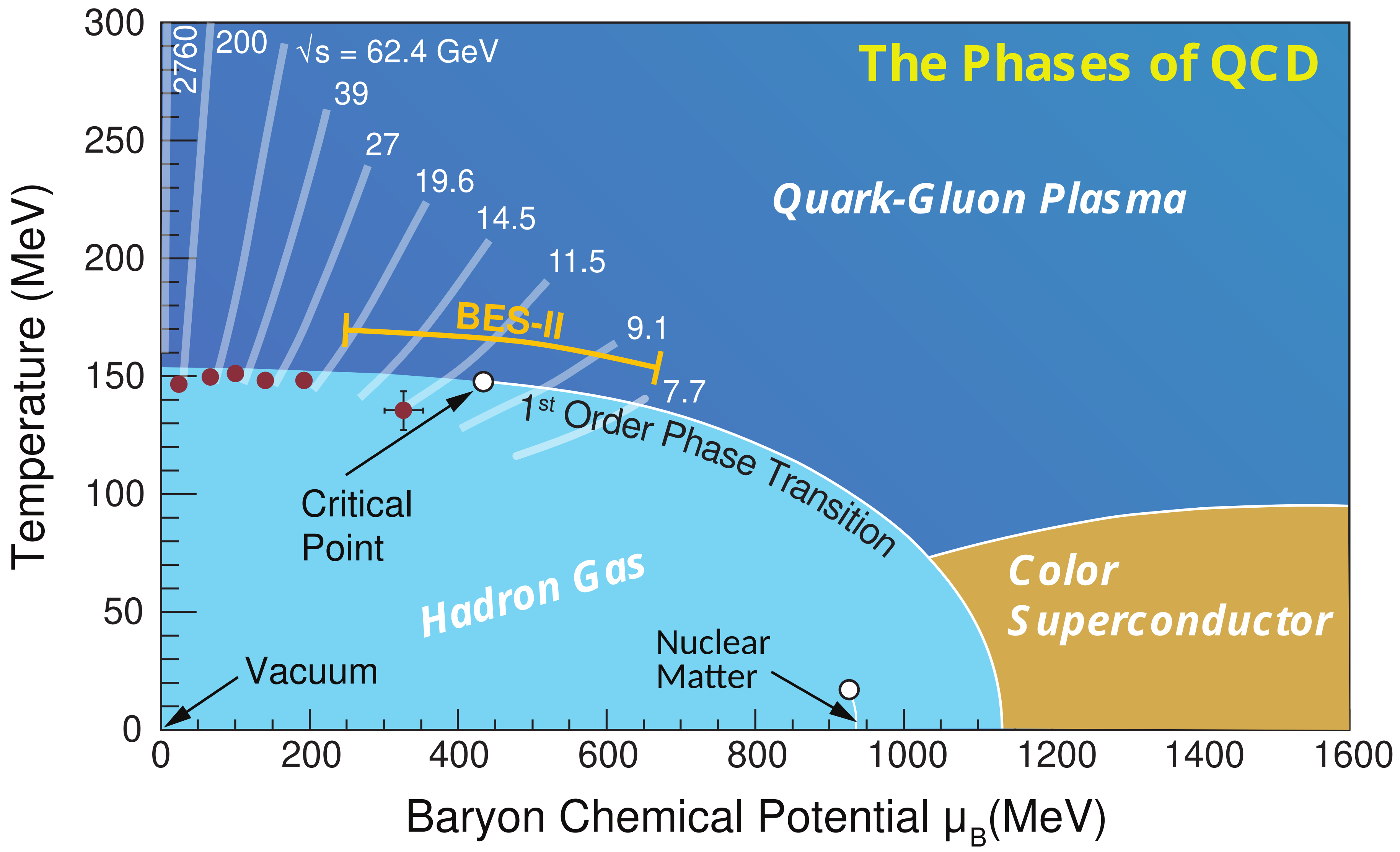

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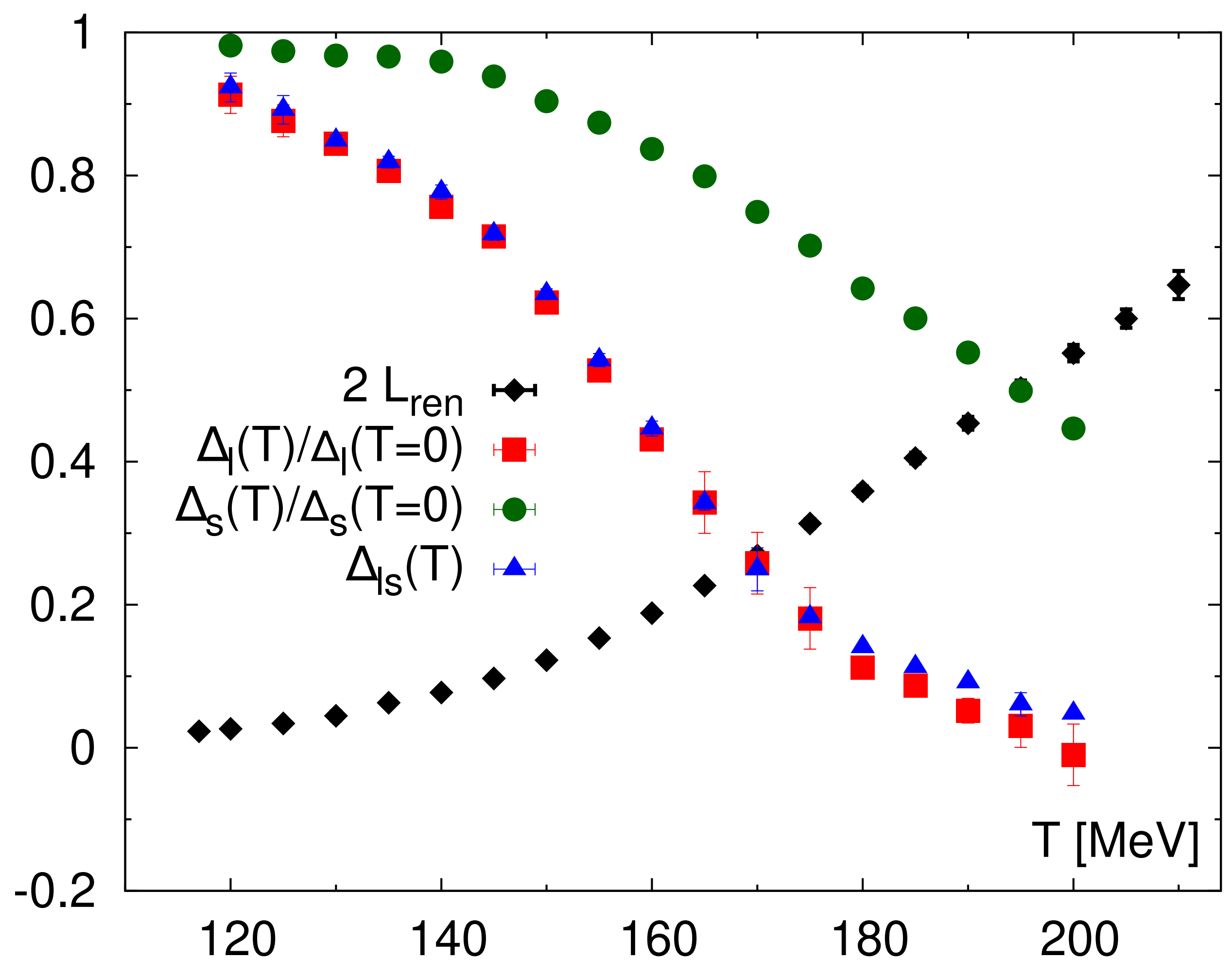

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Use $\ell, \bar{\ell}$ in this talk, best for heavy quarks!

Polyakov loops and confinement

Meaning of the Polyakov loops

- QCD medium with a static quark q:
  Part. fct.: $Z_q = \mathrm{Tr}_q\, e^{-\beta(H-\mu Q)}$, Quark number
- $\ell = Z_q/Z, \quad \bar{\ell} = Z_{\bar{q}}/Z \quad \Rightarrow$
  $-T\ln(\ell) = \Delta F_q, \; -T\ln(\bar{\ell}) = \Delta F_{\bar{q}},$
- $\ell, \bar{\ell}$ measure **free energy increase** related to adding a static quark/anti-quark source to a QCD medium.

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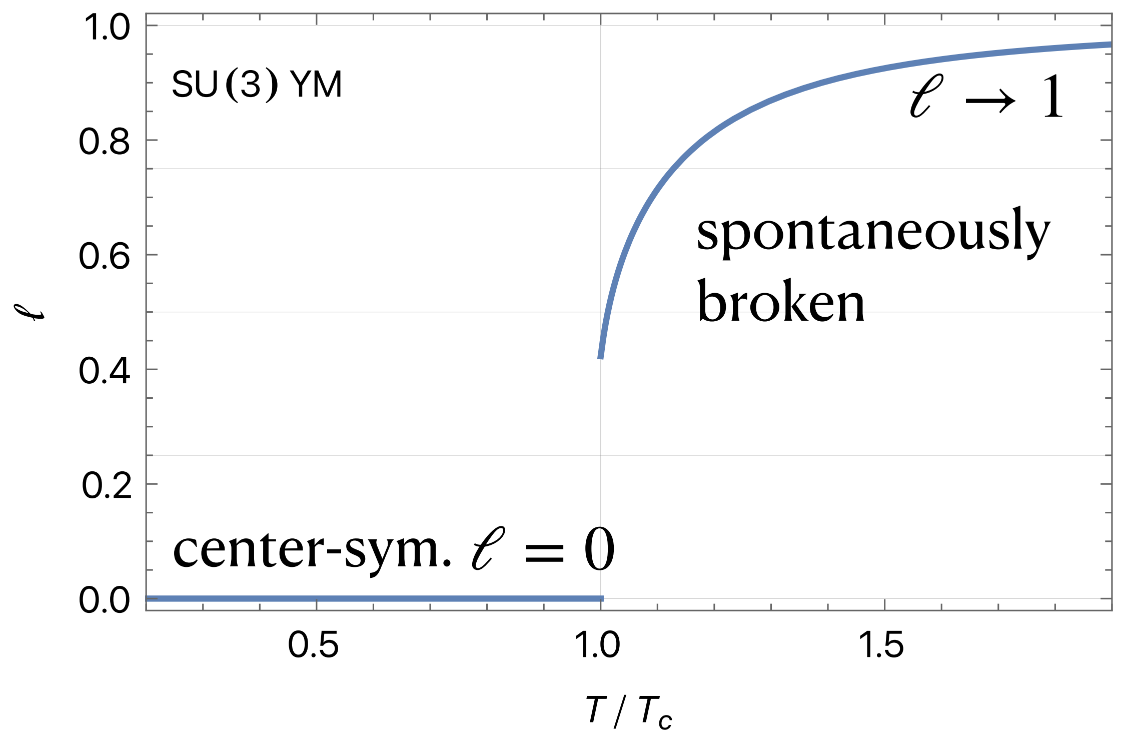
Values of $\ell(T)$

- YM: if $\ell = 0 \Leftrightarrow \Delta F_q = \infty$ →confined q,



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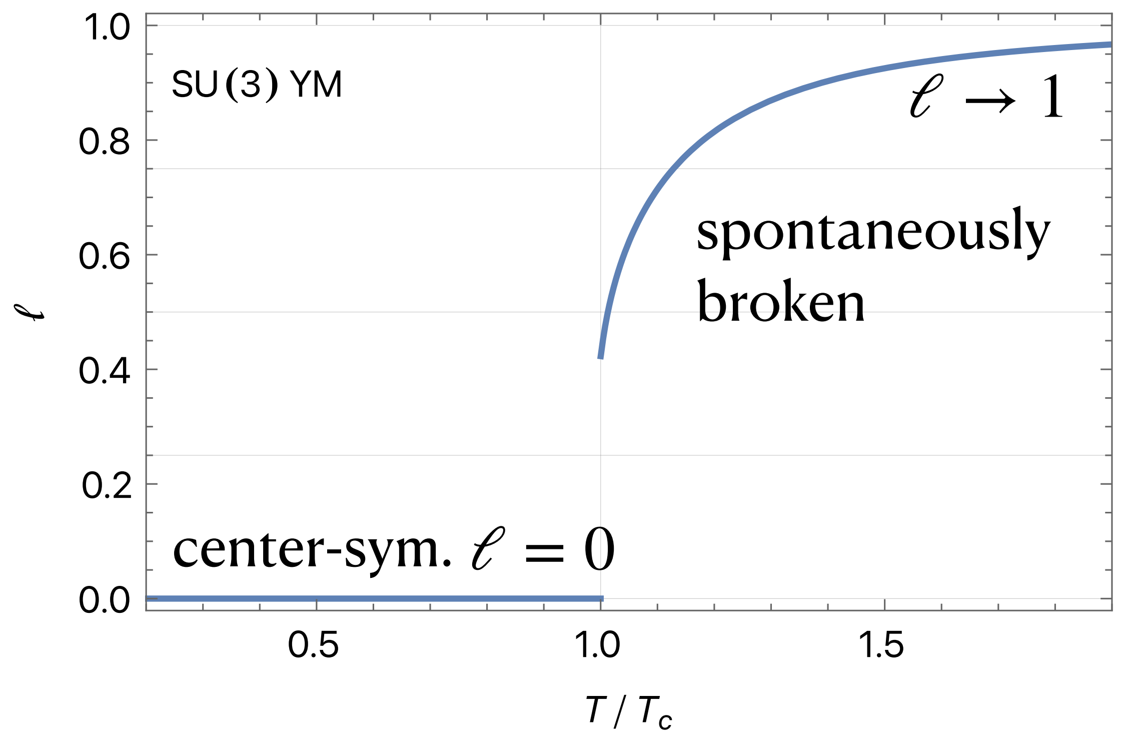

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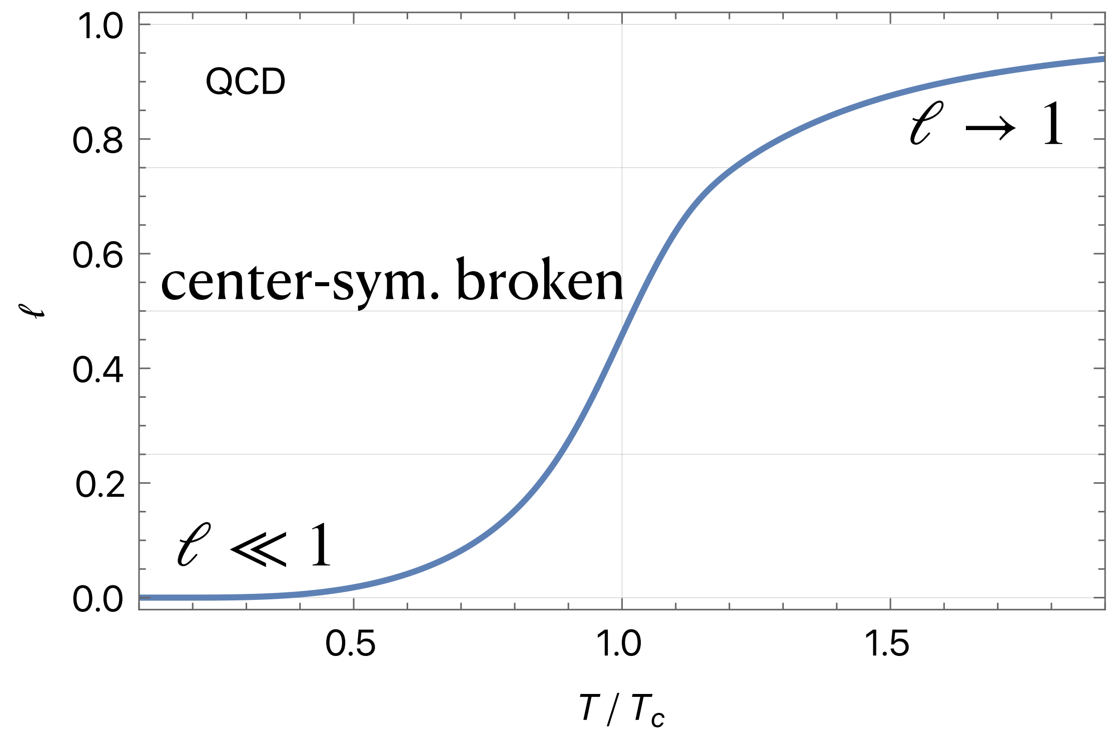
- QCD: $M < \infty$ breaks center-sym.:



Adding a quark is never forbidden?!

ℓ not real order parameter!

But, very high ΔF_q at low T.

Still, how can it be compatible with hadronic dof's?

Thermodynamics with Polyakov loops

Net quark number gain

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  Grand-canonical ensemble: average response $\Delta Q_{q,\bar{q}}$ of medium to $q, \bar{q}$:

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upon bringing q : $\Delta Q_q + 1,$

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- This is not the same as the net quark number **density**, $n_q = -\dfrac{\partial \Omega}{\partial \mu}$.

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Important distinction:

- This is not the same as the net quark number **density**, $n_q = -\dfrac{\partial\Omega}{\partial\mu}$.
- Order parameters can feed back into n_q, $\Omega = \Omega(T, \mu, \ell(T, \mu), \langle\bar{q}q\rangle(T, \mu), \ldots)$, but n_q corresponds to Q-density of the system **without** an added quark.
- $\Delta Q_{q,\bar{q}} \pm 1$, the net quark number gain, is a "global" observable, not a density.

Thermodynamic potential at low T

Polyakov loop potential

heavy quark or chirally
broken constituent mass

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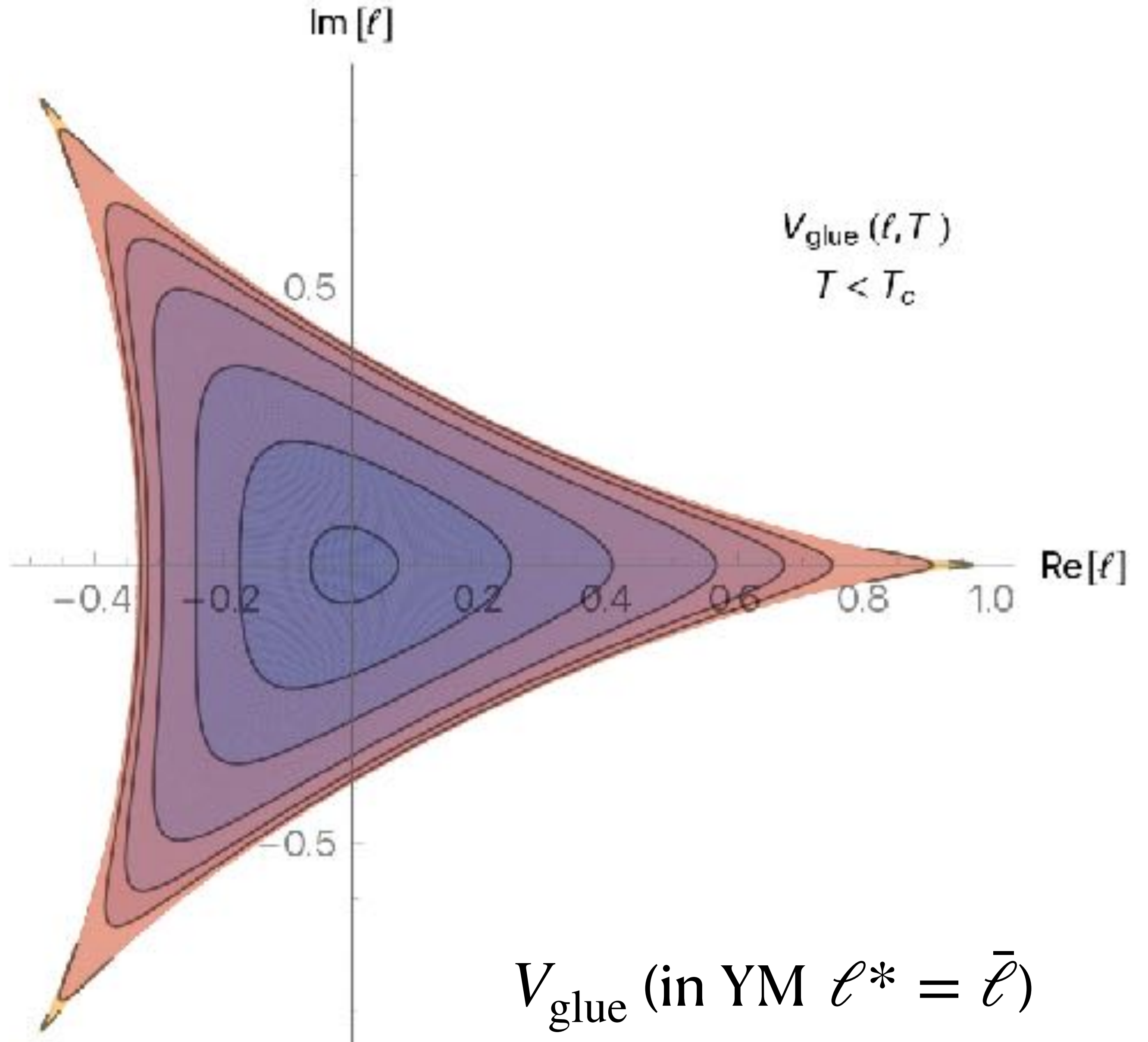
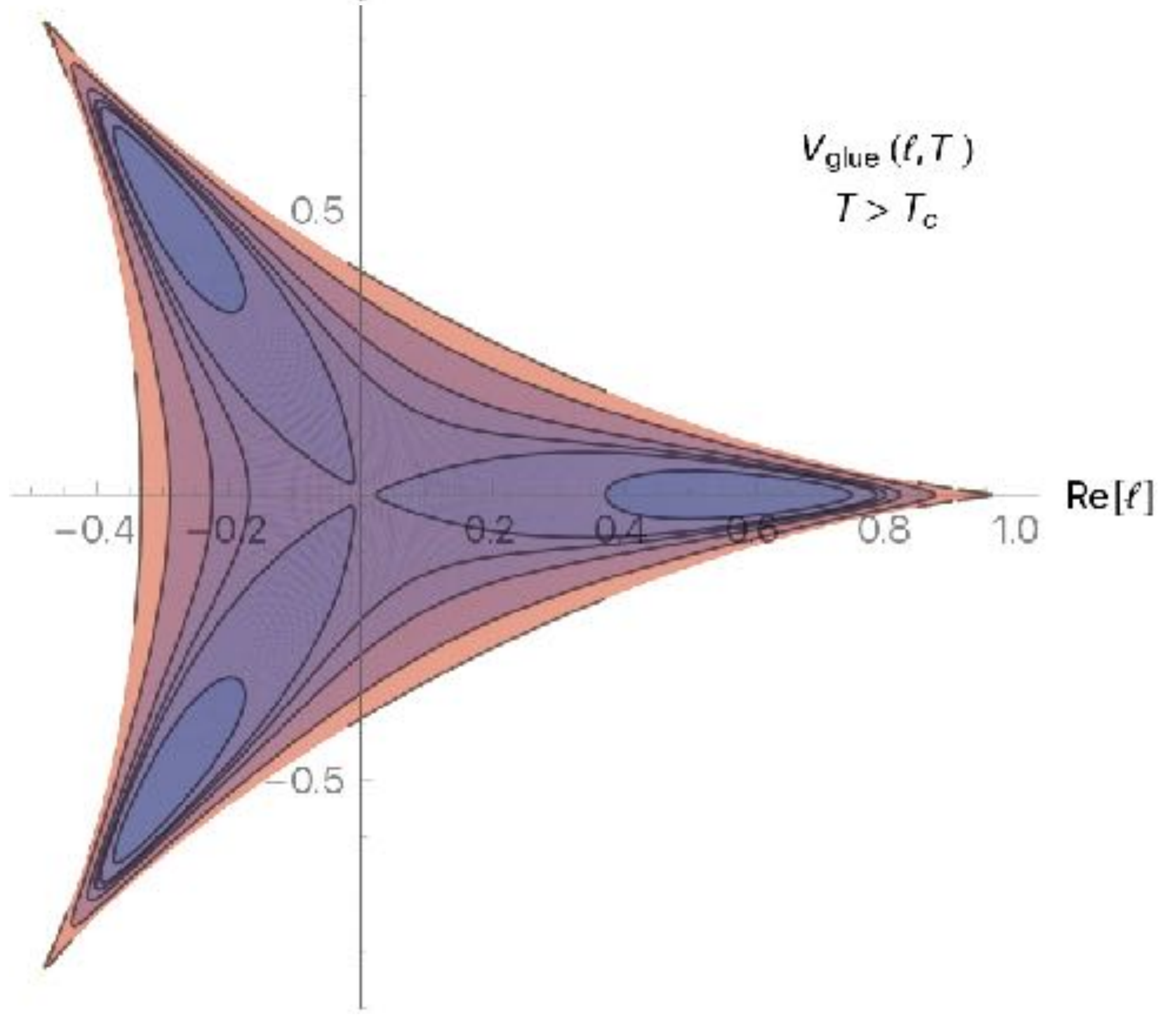
- When $T \ll M$: $\Omega \simeq V_{\text{glue}} + V_{\text{quark}}$, with:
- $V_{\text{glue}}(\ell, \bar{\ell}, T)$ satisfying:
  - **Center-symmetric**: $V_{\text{glue}}(\ell, \bar{\ell}) = V_{\text{glue}}(e^{i2\pi/3}\ell, e^{-i2\pi/3}\bar{\ell})$ (as given by YM),
  - **Confining** at low T: $\ell, \bar{\ell} \xrightarrow{T\to 0} 0$ (as seen on lattice),
  - T-**power-law** behaviour at low T: $V_{\text{glue}} \sim T^{\#}$,



V_{glue} (in YM $\ell^* = \bar{\ell}$)



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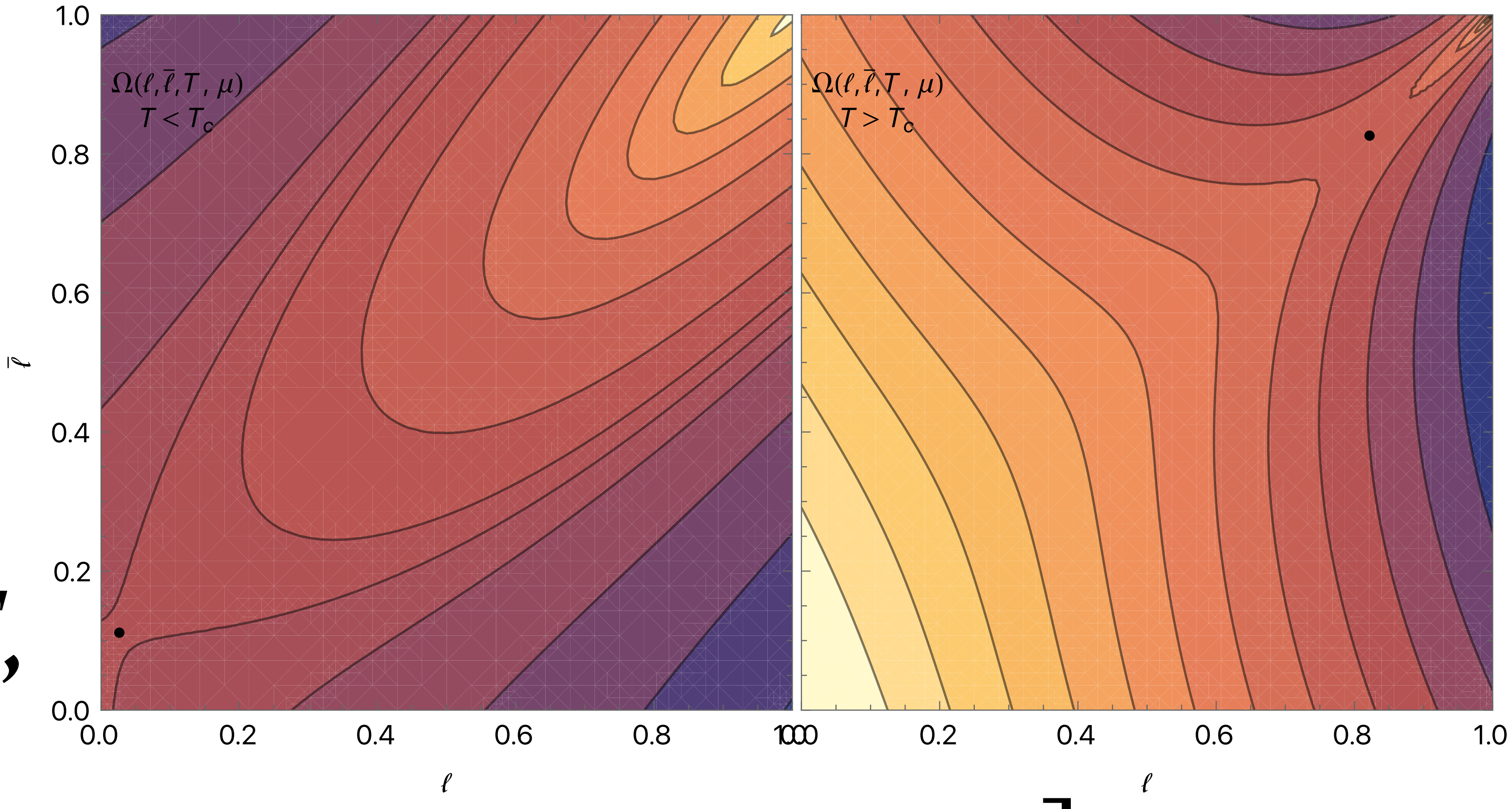

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see e.g. K. Fukushima, Phys. Lett. B 591, 277 (2004), [hep-ph/0310121]

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$$\underset{|\mu| < M}{\overset{\beta M \gg 1,\ \ell, \bar{\ell} \ll 1}{\simeq}} - \left\{ \ell(e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M}) + \bar{\ell}(e^{2\beta\mu} f_{2\beta M} + e^{-\beta\mu} f_{\beta M}) + (e^{3\beta\mu} + e^{-3\beta\mu}) f_{3\beta M}/3 \right\}$$

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- With $f_y\,\pi^2/3TN_f = \displaystyle\int_0^\infty dx\, x^2 e^{-y\sqrt{1+x^2}} \sim 3\sqrt{2\pi}e^{-y}y^{-3/2}$ for $y \gg 1$.

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Calculating ℓ, $\bar{\ell}$ at low T

Chemical potential μ dependence

- $$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\partial \Omega}{\partial \ell} \\ \frac{\partial \Omega}{\partial \bar{\ell}} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} \\ \partial_\ell \partial_{\bar{\ell}} V_{\text{glue}} & 0 \end{pmatrix} \Bigg|_{\ell, \bar{\ell} = 0} \begin{pmatrix} \ell \\ \bar{\ell} \end{pmatrix} \simeq \begin{pmatrix} e^{\beta\mu} f_{\beta M} + e^{-2\beta\mu} f_{2\beta M} \\ e^{-\beta\mu} f_{\beta M} + e^{2\beta\mu} f_{2\beta M} \end{pmatrix},$$

2509.26178

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- $$\Rightarrow \ell \simeq \frac{1}{\partial_\ell\partial_{\bar{\ell}}V_{\text{glue}}\,|_{\ell,\bar{\ell}=0}}(e^{-\beta\mu}f_{\beta m_q} + e^{2\beta\mu}f_{2\beta m_q}), \quad \bar{\ell} \simeq \frac{1}{\partial_\ell\partial_{\bar{\ell}}V_{\text{glue}}\,|_{\ell,\bar{\ell}=0}}(e^{\beta\mu}f_{\beta m_q} + e^{-2\beta\mu}f_{2\beta m_q})$$

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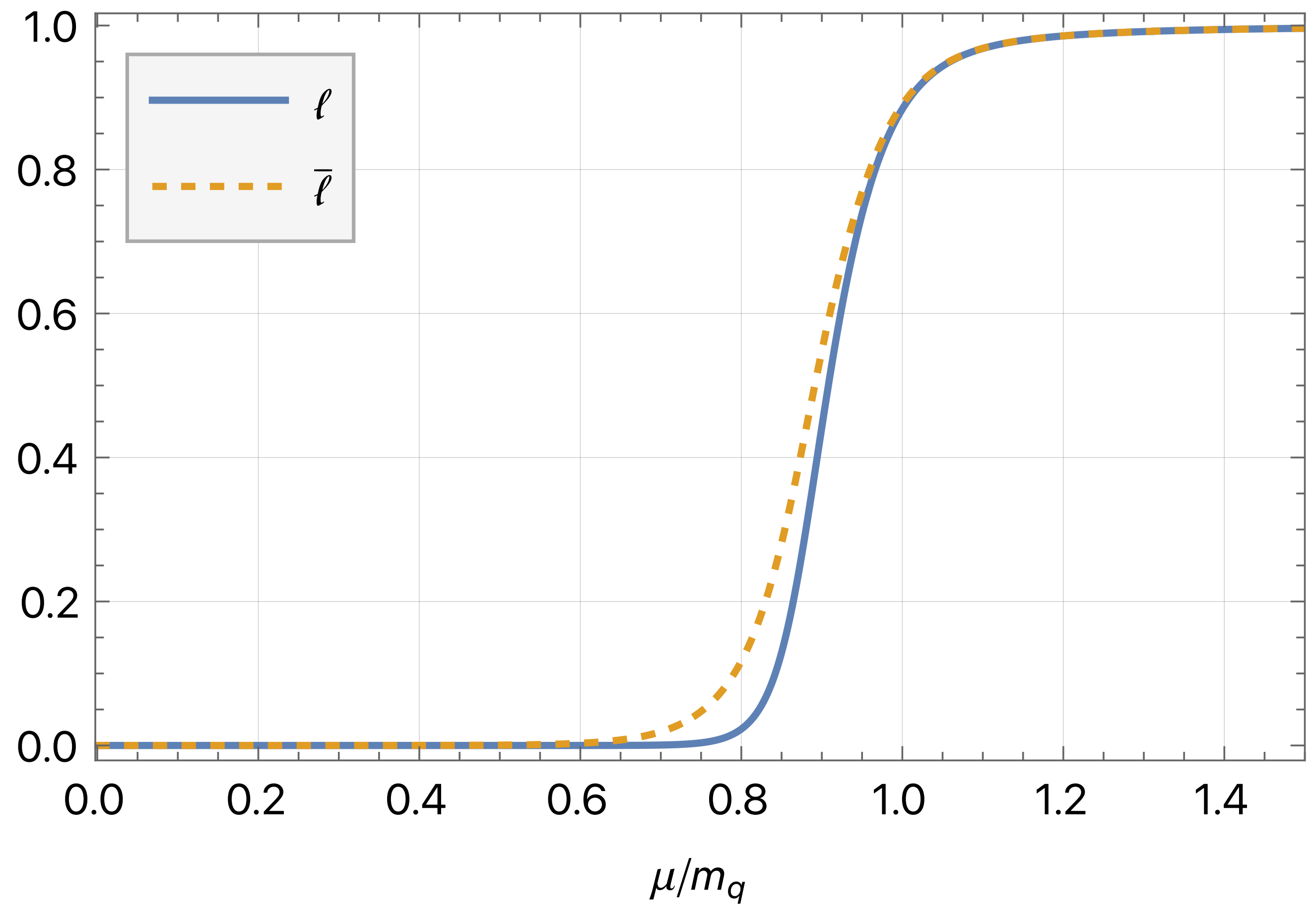

$|\mu| > M$: deconfined (see backup)

2509.26178

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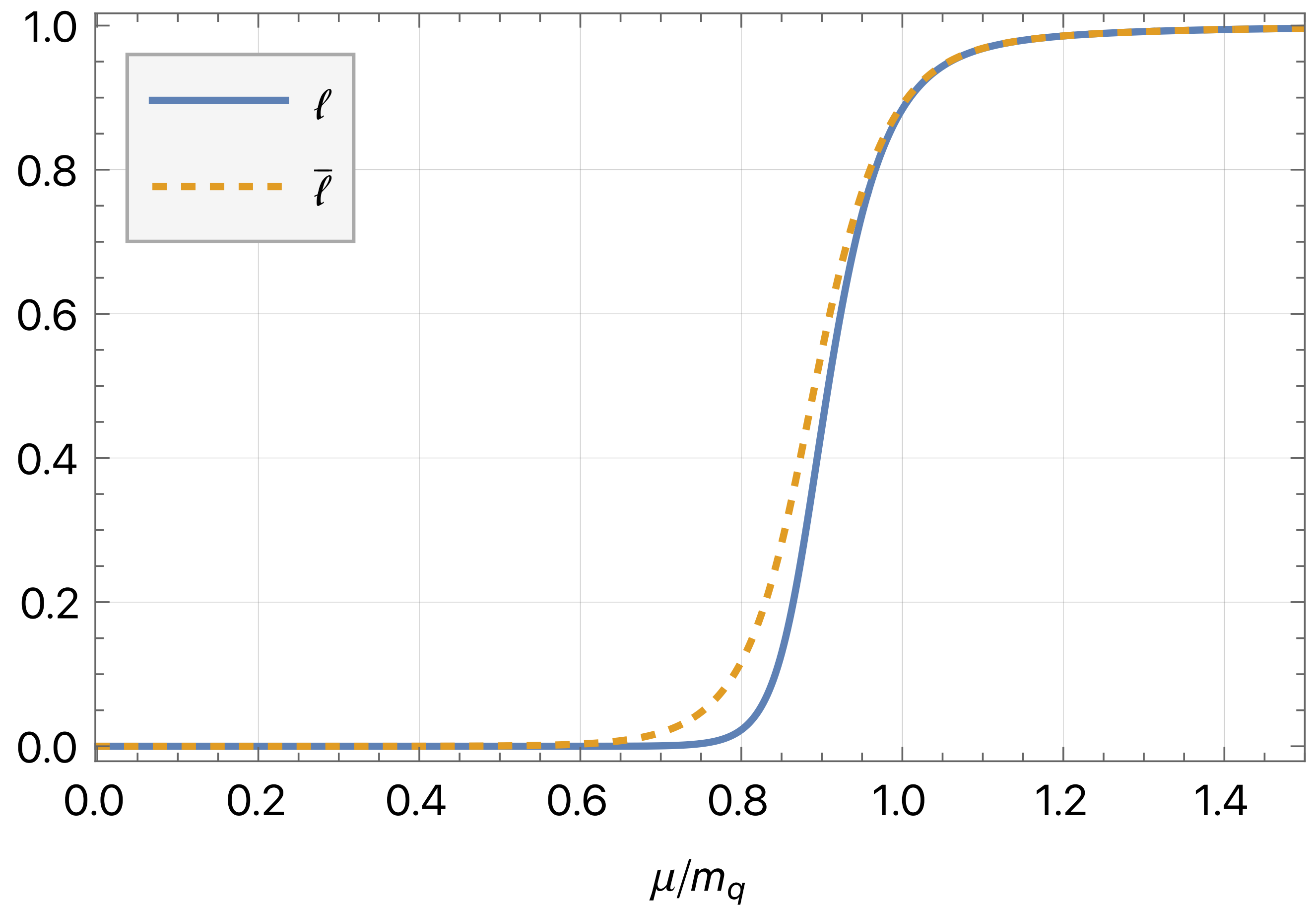
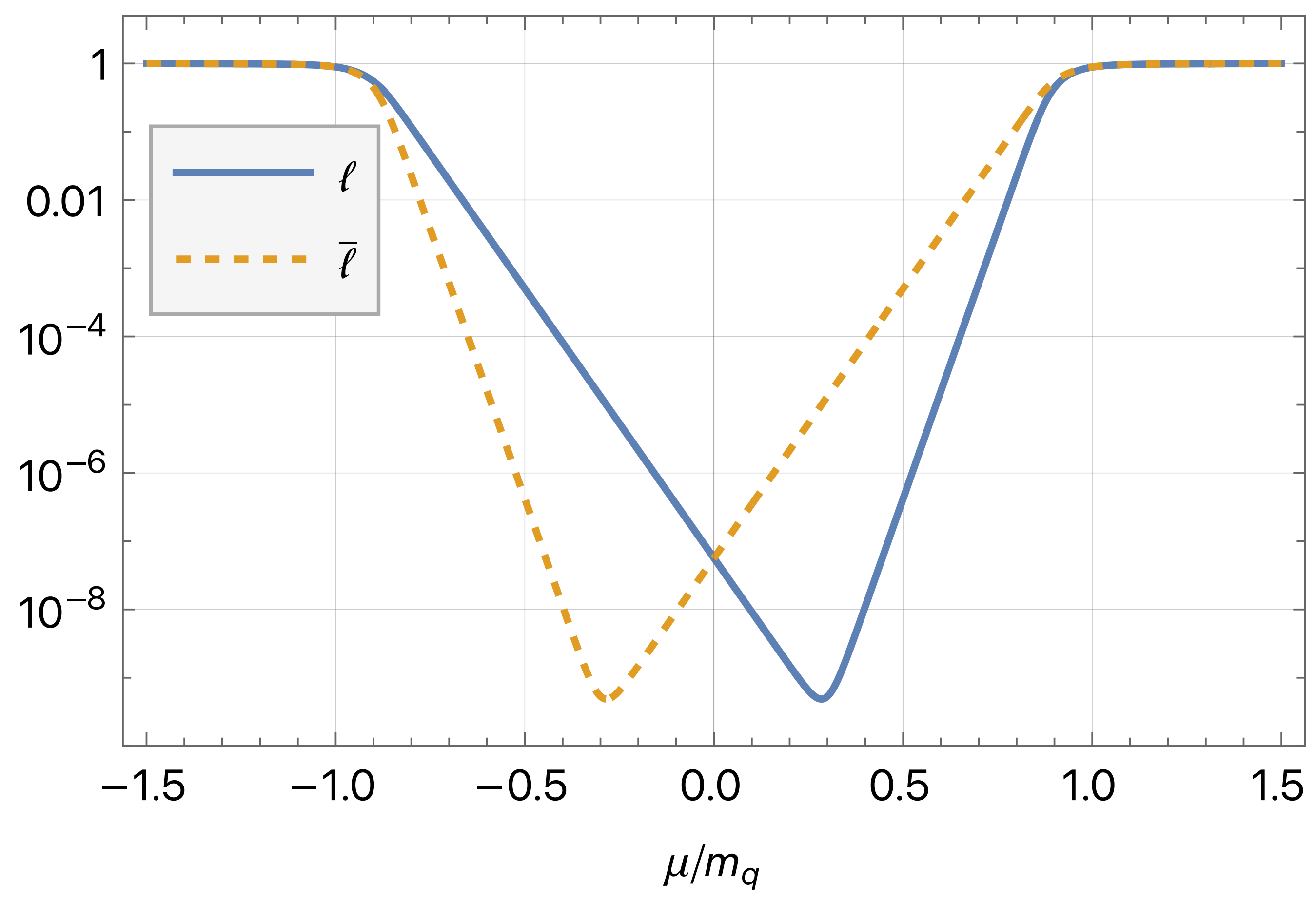


Log-plot shows more structure!



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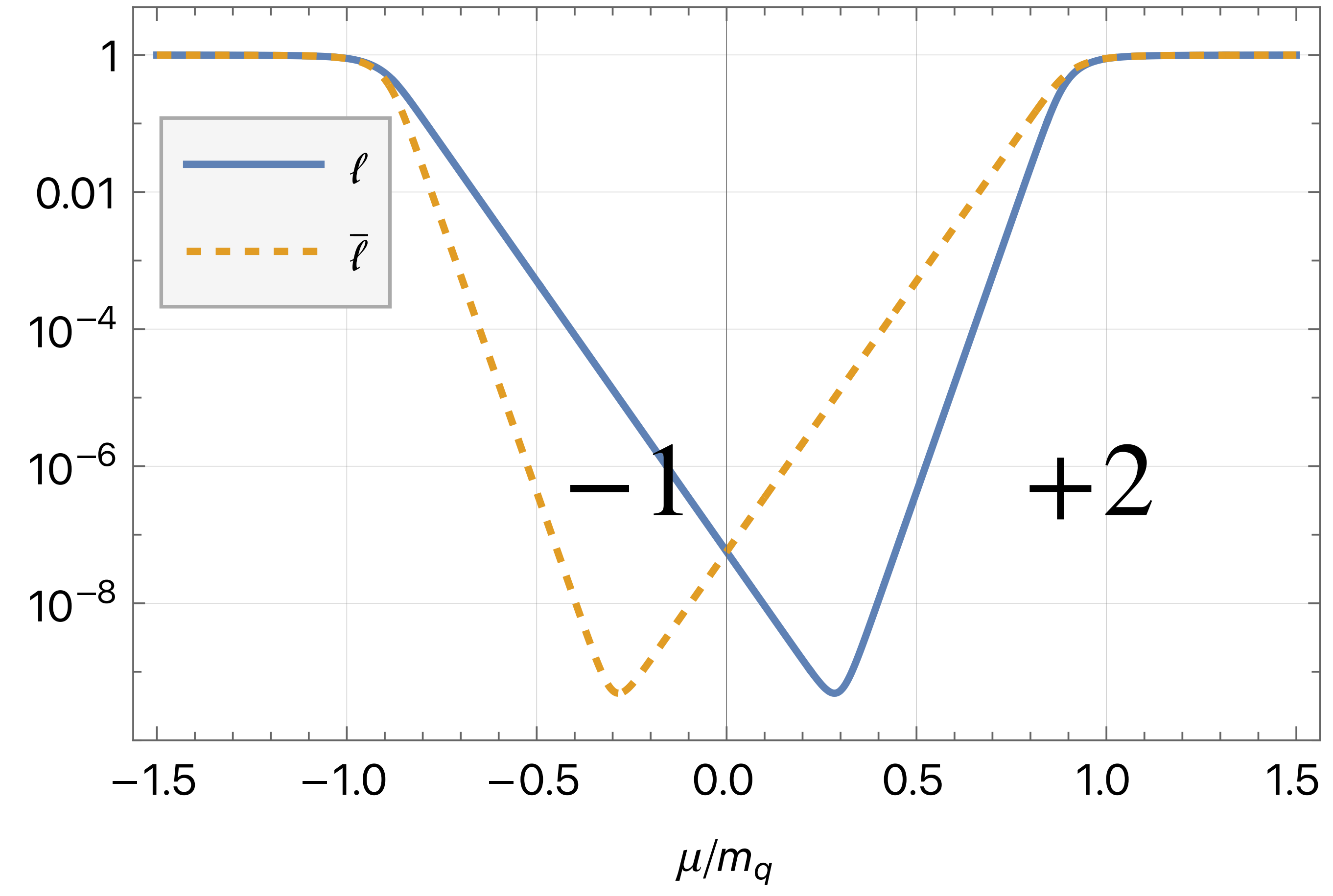
-1 $+2$

μ-derivative



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Net quark number gains $\Delta Q_{q,\bar{q}}$

From Polyakov loop potential

- Then the **net quark number gains** are:
- $$\Delta Q_q + 1 \simeq \frac{3}{1 + e^{-3\beta\mu} f_{\beta M} / f_{2\beta M}},$$
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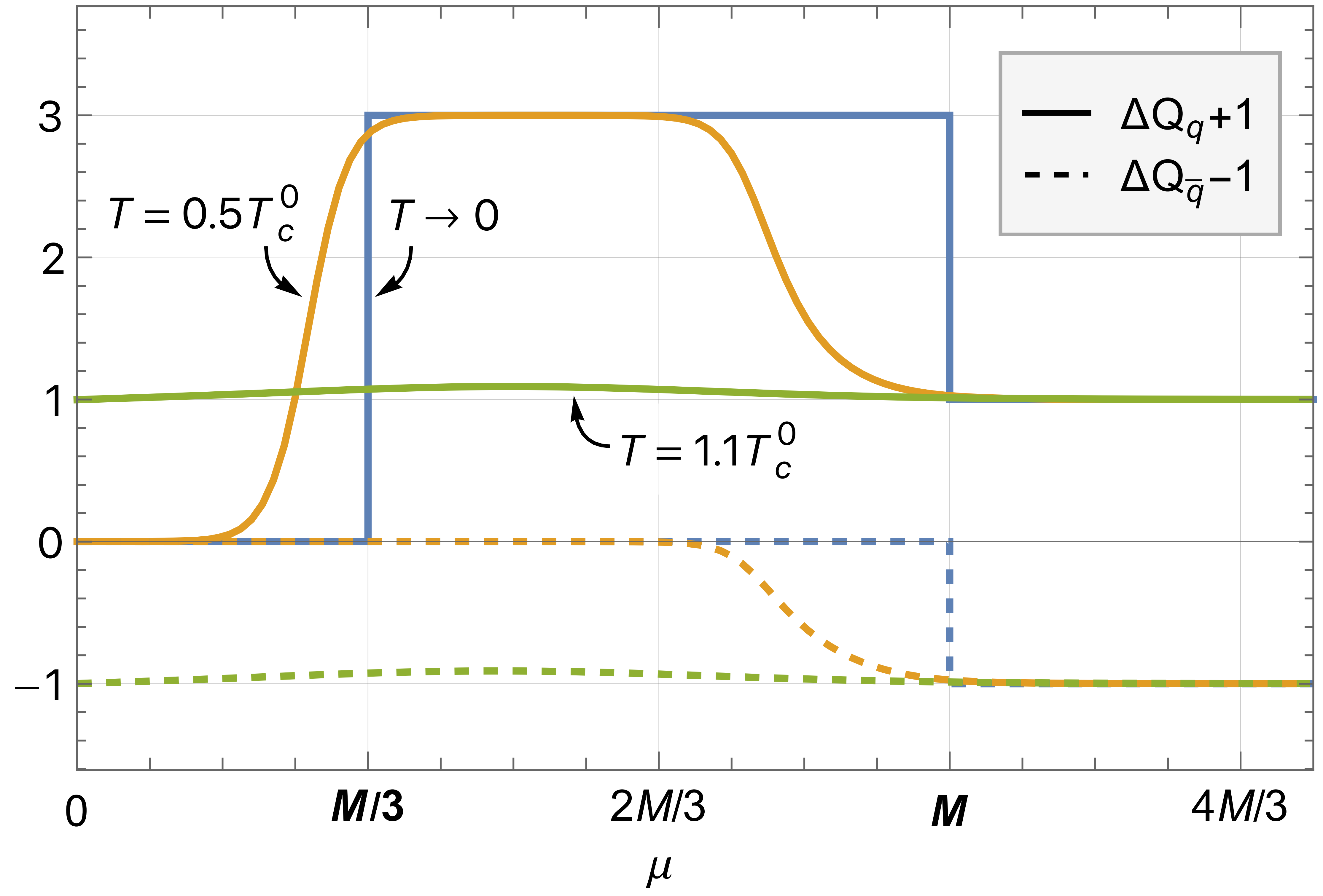
- Independent of V_{glue}.



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Net quark number gains $\Delta Q_{q,\bar{q}}$

From Polyakov loop potential

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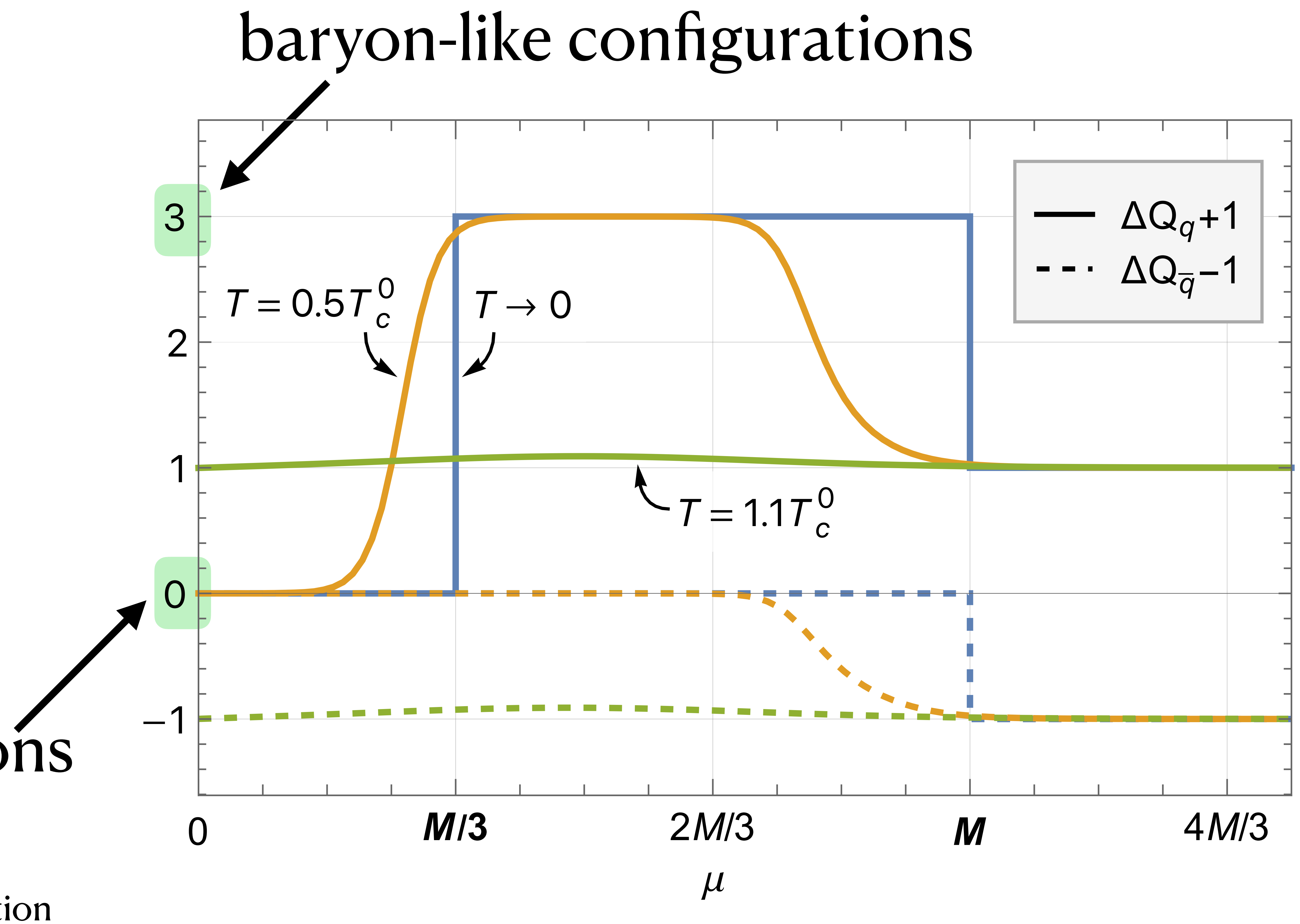

"-like" since don't know color representation

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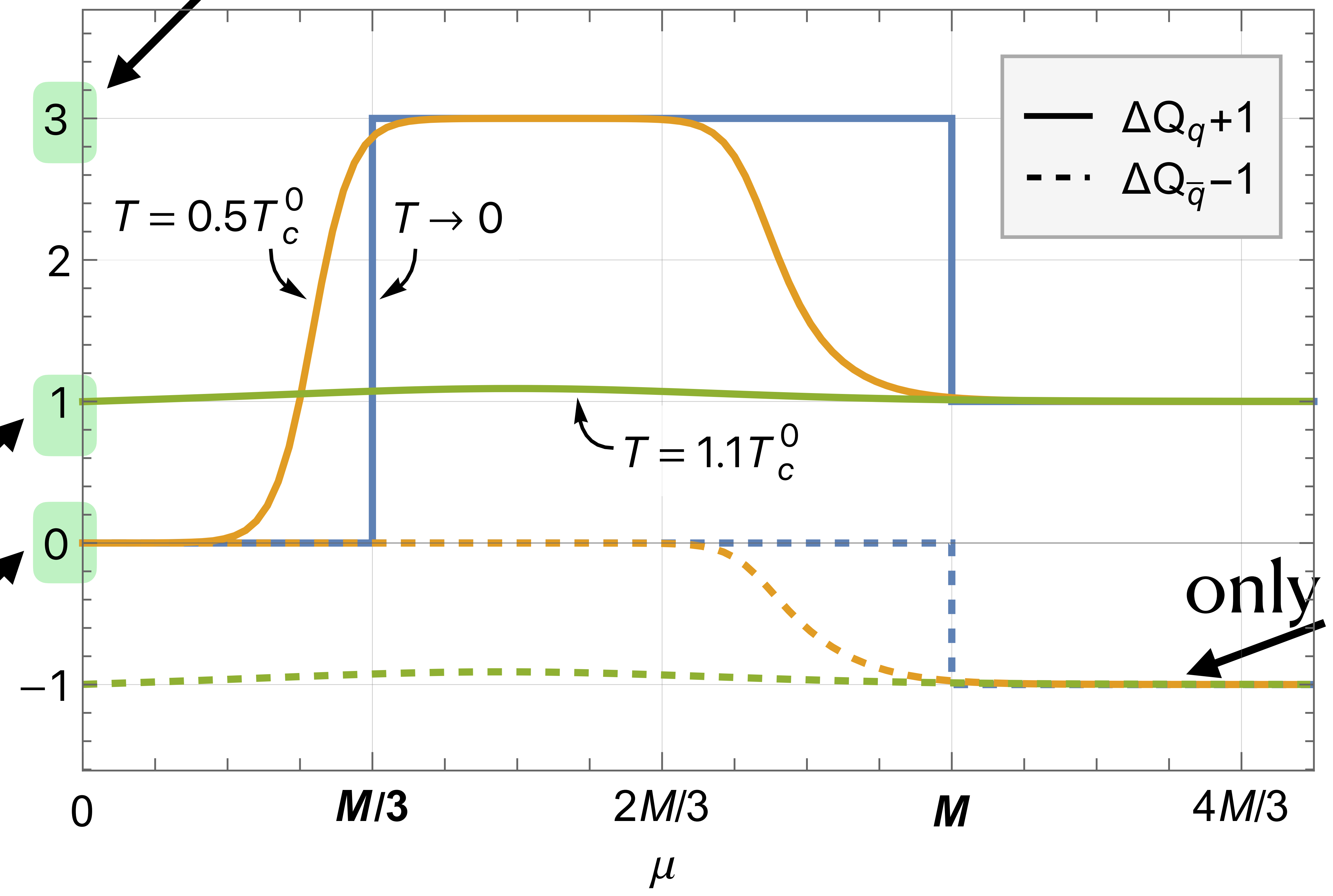

"-like" since don't know color representation

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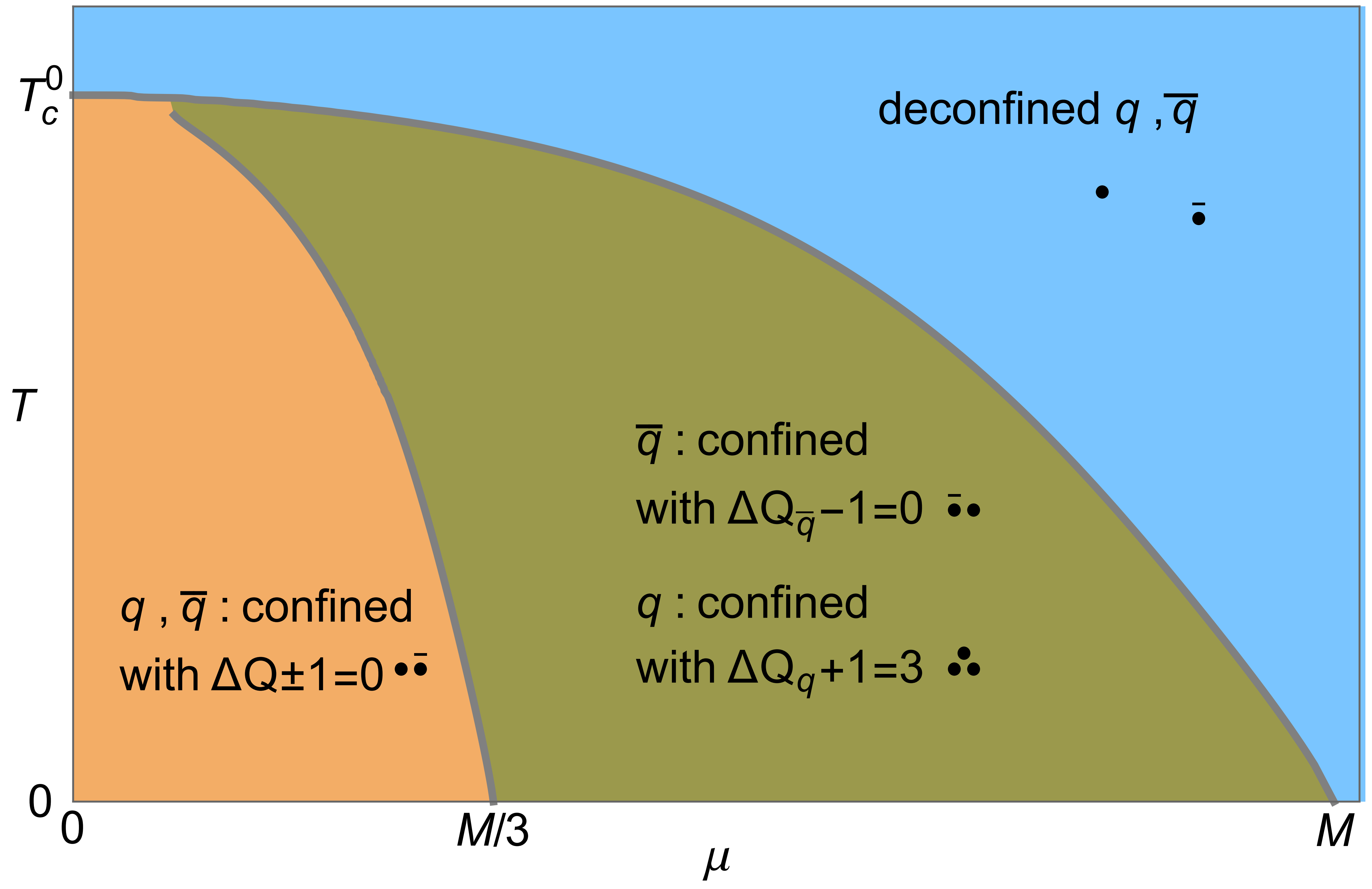
QCD phase diagram

Net quark number of dominant degrees of freedom

- **Net quark number gain** for heavy quark QCD:



QCD phase diagram

Net quark number of dominant degrees of freedom

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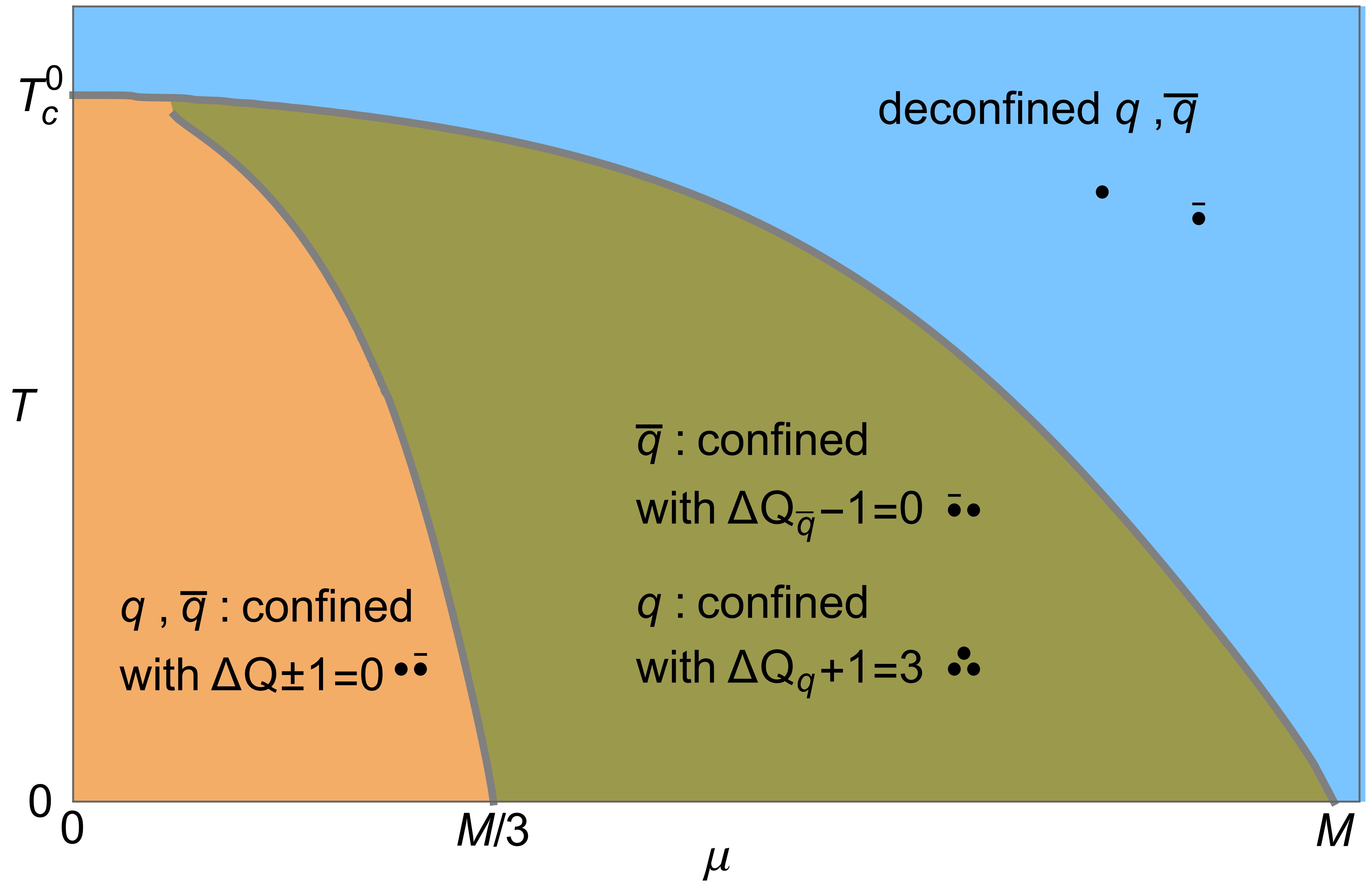
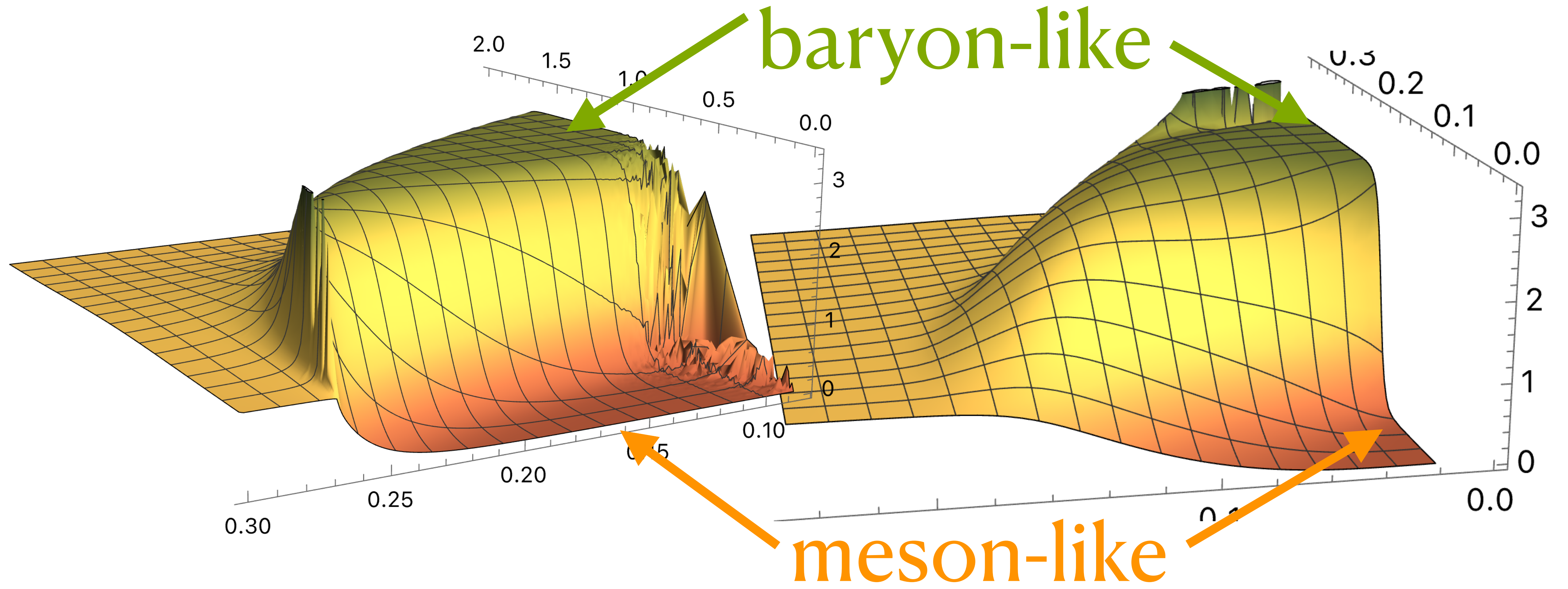


Heavy-quark QCD vs PNJL model:



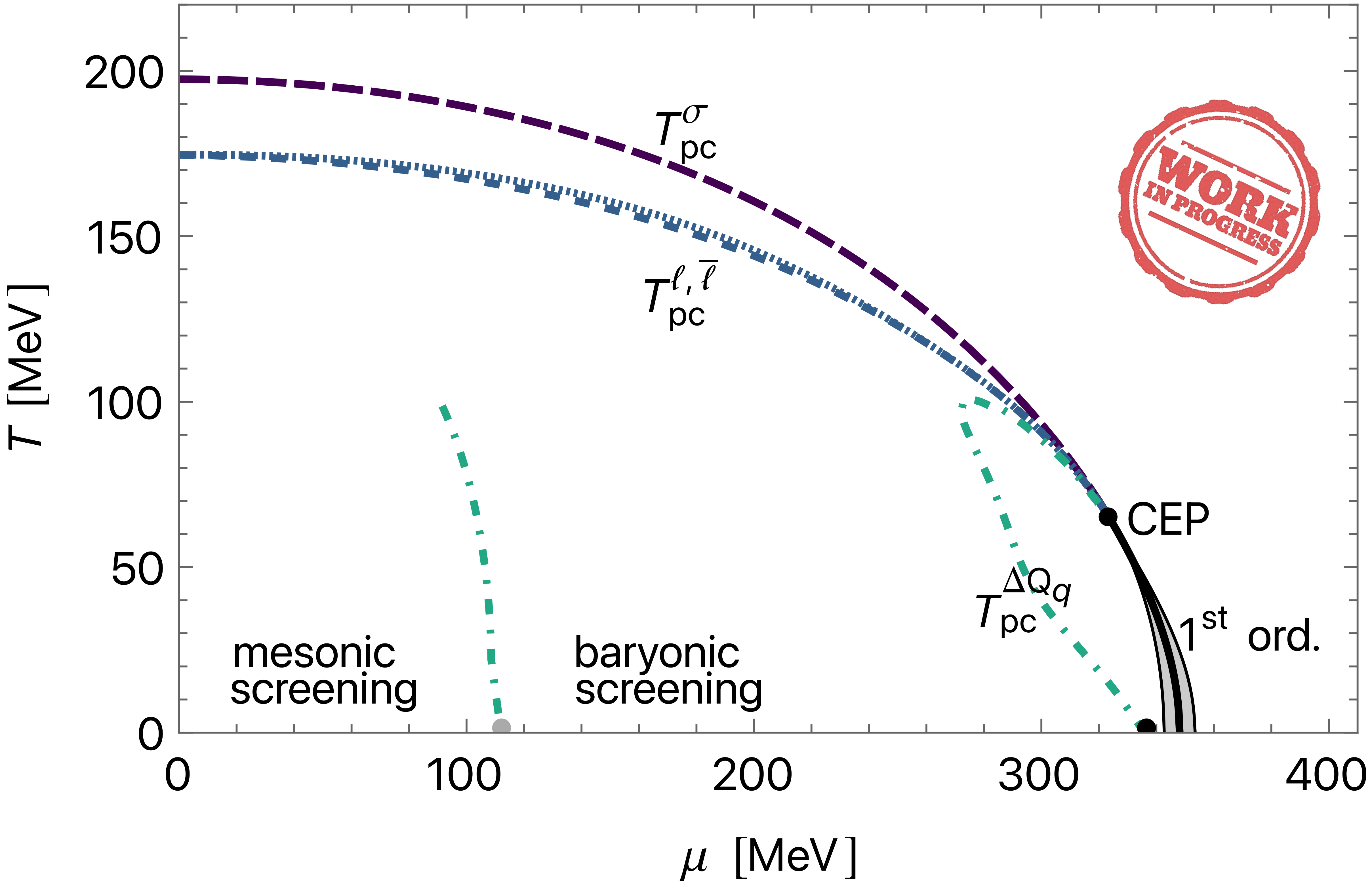
Plateau of $\Delta Q_{q,\bar{q}} \pm 1$ for:

whole confined phase vs only at low T

QCD phase diagram

Net quark number of dominant degrees of freedom

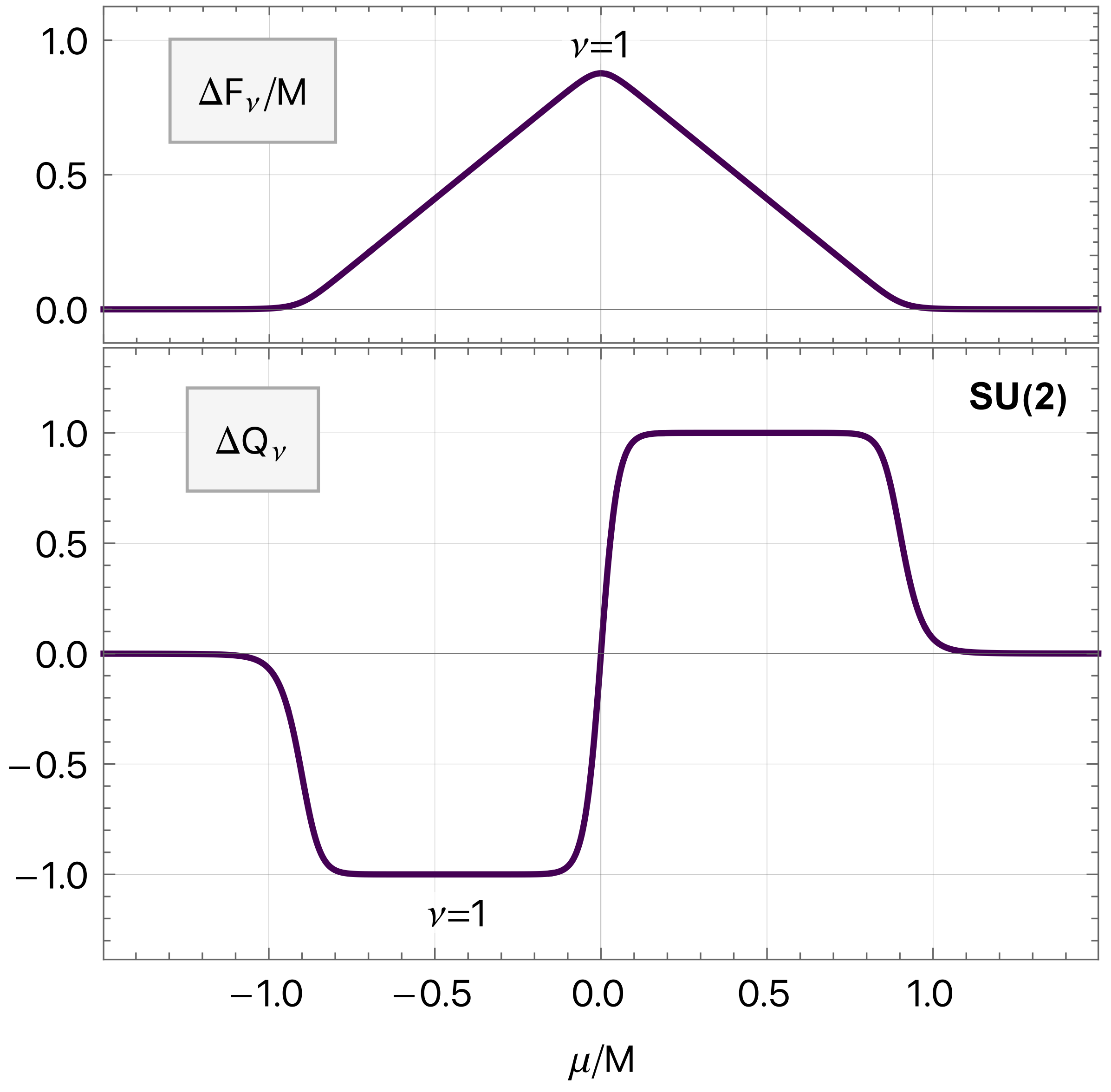
- **Net quark number gain** with chiral SB:

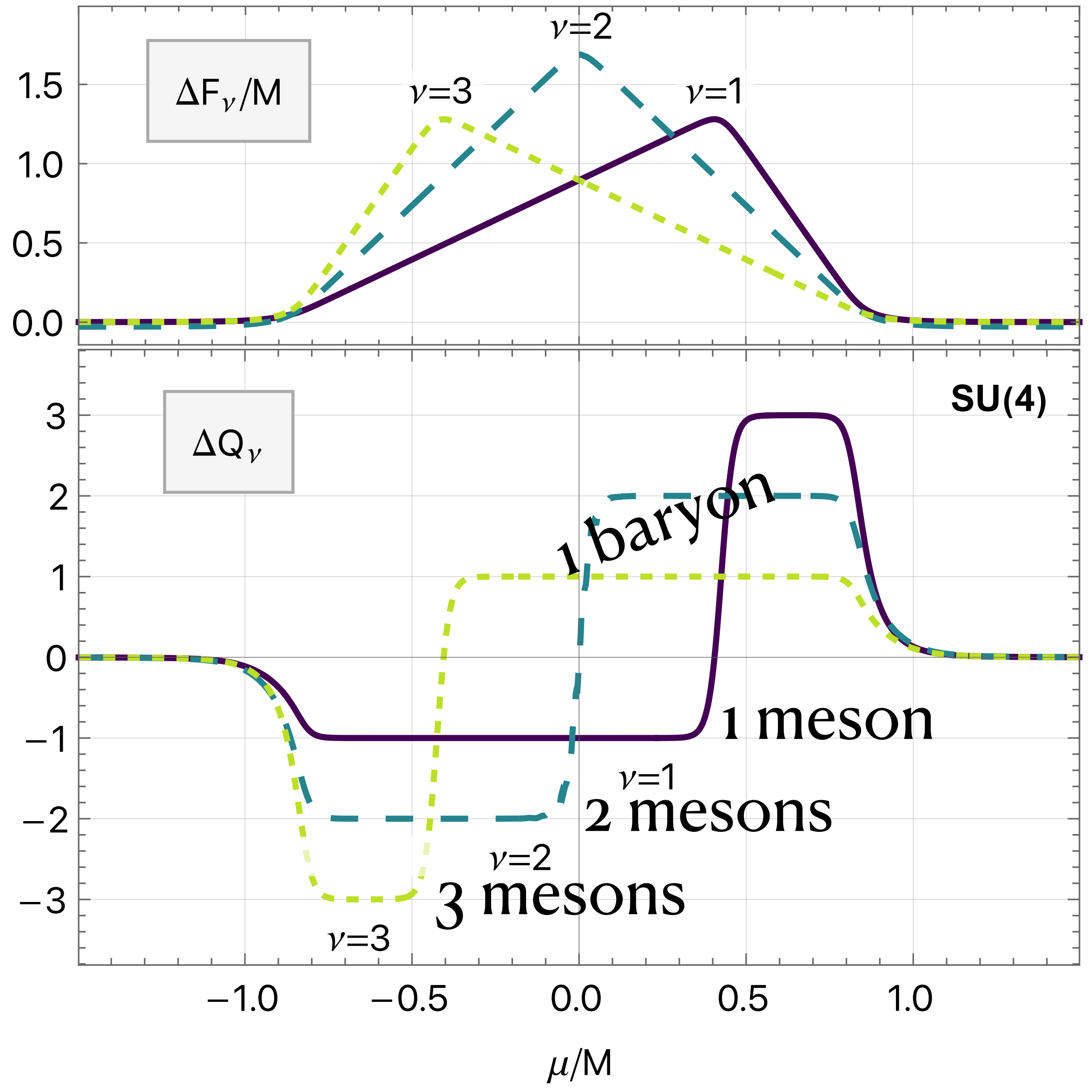


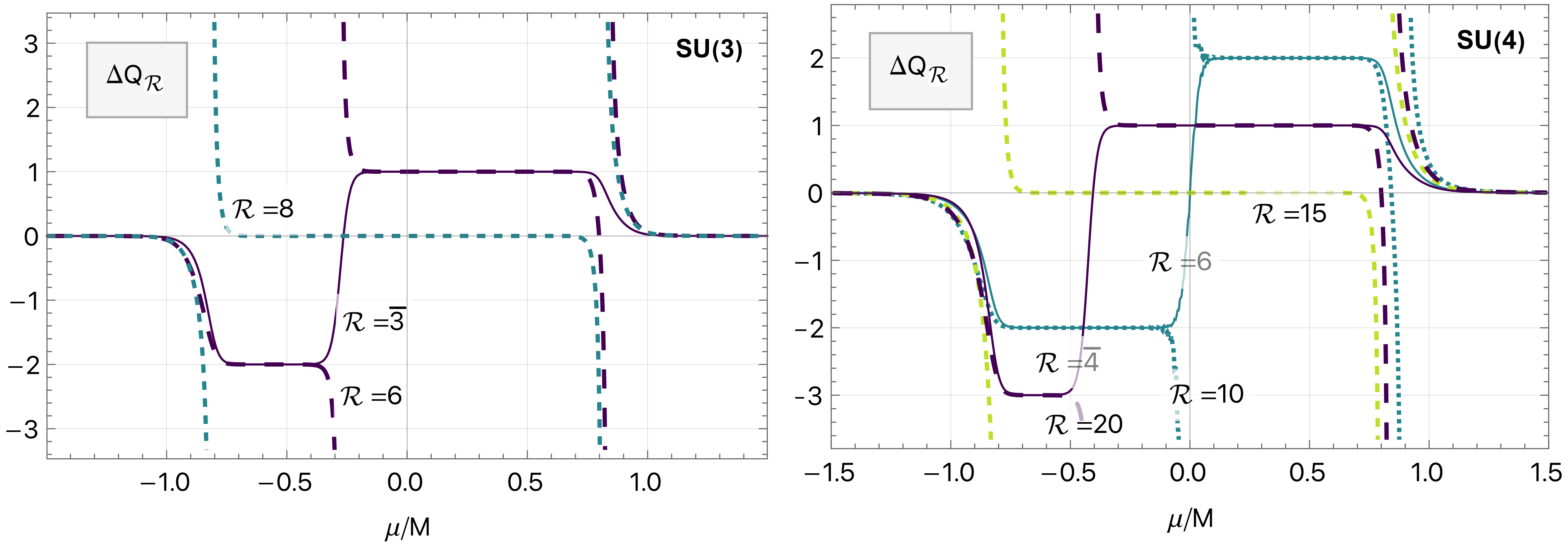
Generalization to SU(N_c)

$(N_c - 1)$ **fundamental Polyakov loops** $\ell_\nu \leftrightarrow$ **adding** ν **quarks**

- $\Delta Q_\nu + \nu$: with the ν added q's get a ν-mesons- or one-baryon-like configuration, $\rightarrow N_c$ quarks
- Change from meson- to baryon-like being favorable at $\mu = m_q(1 - 2\nu/N_c)$,
- Color charges in non-fundamental $\mathscr{R}$: only N_c-ality matters







adjoint repr. not confined, reps. of same N_c-ality behave the same!

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Outlook

Possible applications of the net quark number gain

- Theoretical observable, sensitive to net quark number content of active dof's,
- Shows transitions and critical point $\rightarrow \Delta Q_q + 1 \gg 3$,
- Sensitive to dof's in other phases (diquarks, chiral-spin symm. hadrons at $T > T_c$)?
- The net quark number gain is essentially $\langle \Phi Q \rangle$,
  other combinations of Polyakov loops and conserved charges could be interesting:
  e.g. $\langle \Phi Q^a Q^a \rangle$ color Casimir, or $\langle \Phi I \rangle$ isospin, ...
- Heavy-quark lattice simulations are viable at $\mu \neq 0 \rightarrow$ tests possible.

Thank you! :)

Backup

For questions also: victor-tomas.mari-surkau@polytechnique.edu

Confinement from N_C-ality and in Minkowski

Expectations for $\Delta Q_q + 1$

$\to$No Euclidean path integral needed!

- Consider **heavy-quark QCD** at low T: what states dominate Z_q?
- Heavy-quark, non-relativistic system:

$$H - \mu Q \simeq (N_q + N_{\bar{q}})m_q - \mu(N_q - N_{\bar{q}}),$$

- Dof's with 0 N_c-ality: $N_q - N_{\bar{q}} + 1 = 3k,\ k \in \mathbb{Z}$ $\Rightarrow (N_q, N_{\bar{q}}) = (0,1), (2,0)\ (3,1), \ldots$

$$H - \mu Q \ \simeq \ \underset{(0,1)}{m_q + \mu}, \ \ \underset{(2,0)}{2m_q - 2\mu}, \ \ \underset{(3,1)}{4m_q - 2\mu}, \ldots$$

- If $\mu < m_q/3$ then (0,1) dominates, if $\mu > m_q/3$ then (2,0), never (3,1) and higher.

$\to \Delta Q_q + 1 = 0$ $\to \Delta Q_q + 1 = 3$

Meson Baryon

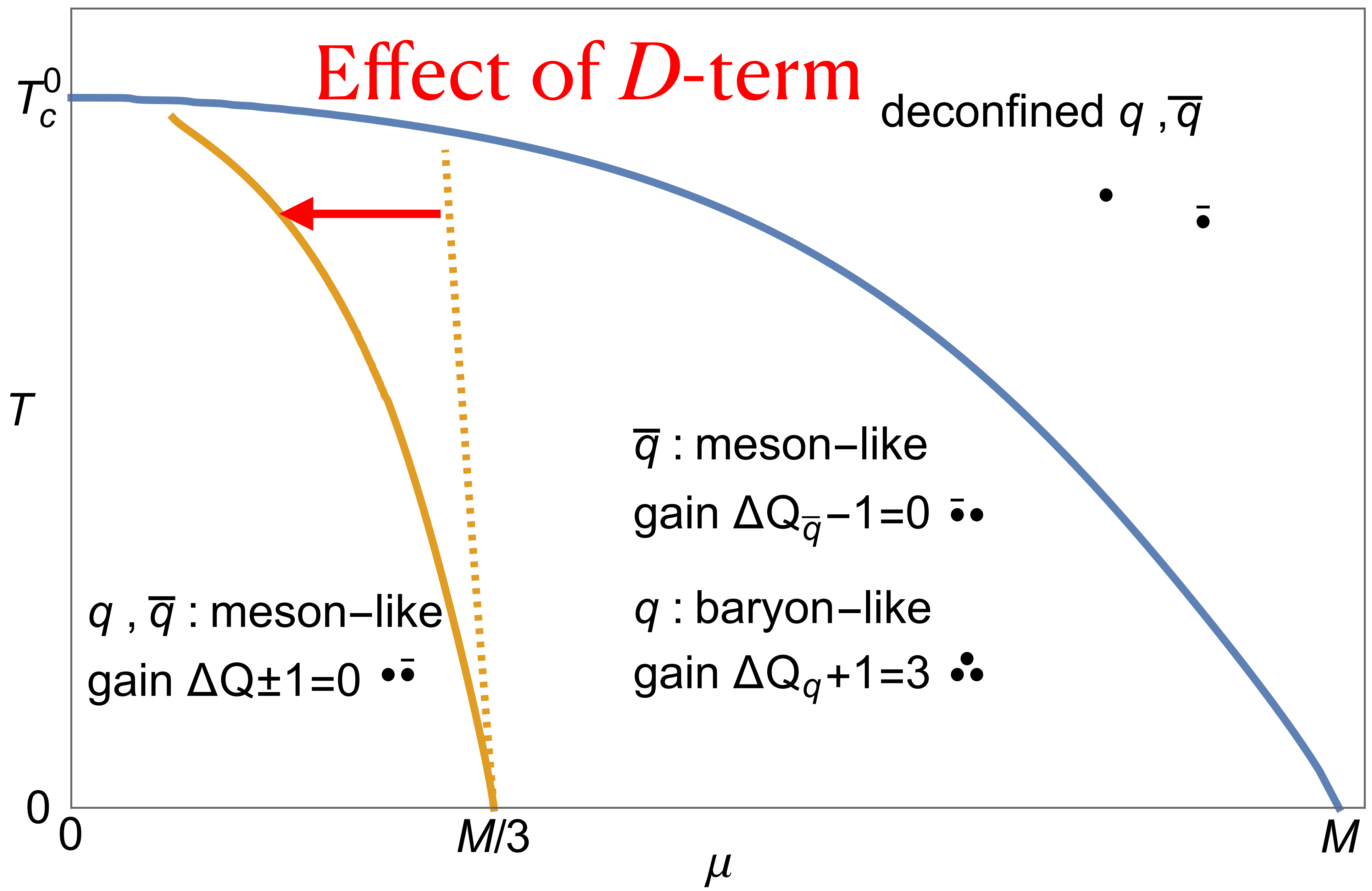
Beyond the linear order in $\ell, \bar{\ell}$

Modifications at finite temperatures

- At non-zero $\mu > 0$: ℓ more suppressed than $\bar{\ell}$, and vice-versa at $\mu < 0$
- In equation with ℓ take into account the order $\frac{1}{2}\bar{\ell}^2 \partial^3_{\bar{\ell}} V_{\text{glue}}$,
- $\Delta Q_q + 1 \simeq \dfrac{3}{1 + D e^{-3\beta\mu} f_{\beta M} / f_{2\beta M}}$, with

$$D = \frac{1 - C \dfrac{\partial^3_{\bar{\ell}} V_{\text{glue}}}{(\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}})^2} f_{2\beta M}}{1 - \frac{1}{2} C \dfrac{\partial^3_{\bar{\ell}} V_{\text{glue}}}{(\partial_\ell \partial_{\bar{\ell}} V_{\text{glue}})^2} f^2_{\beta M} / f_{2\beta M}}$$

$C \equiv 3N_f T M^3$



The case of $|\mu| > m_q$

Heavy quarks deconfine

- The quark potential is only power-law suppressed at low T, no longer exponentially,
- Polyakov loops either $\to 1$, or $\xrightarrow{\text{power-law}} 0$, depending on if V_q or V_glue dominate,
- For heavy quarks and any V_glue: see as deconfined phase, (nuclear liquid in QCD),
- μ-derivatives of $\ln(\ell)$ and $\ln(\bar{\ell})$ vanish,
- $\Delta Q_{q,\bar{q}} \pm 1 \simeq \pm 1, \Delta Q_{q,\bar{q}} = 0$