

The proton GPDs from quarkonia-photon pairs

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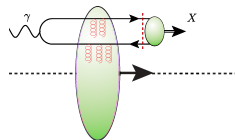
Factorization at high energies

- All factorization approaches are based on set of assumptions, limited range of validity
 - ▶ Different schemes **complement each other, don't duplicate**

- **Previous talk:** very high energies ($x_B \leq 10^{-2}$)

- ▶ Eikonal picture, Color Glass Condensate

★ “frozen” partons fly through the gluonic field of the target



- **This talk:** moderate energies, partonic picture

- ▶ Application: low- and middle-energy runs at EIC ($\sqrt{s_{ep}} \lesssim 100$ GeV)

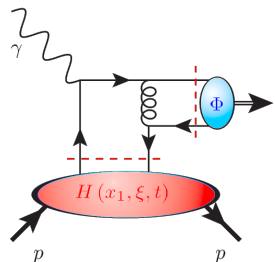
- ▶ Physical amplitudes:

–Convolutions of factorized partonic distributions:

$$\mathcal{A} = \int dx_1 dx_2 \dots C(x_1, x_2, \xi \dots) H(x_1, \xi, t) \Phi(x_2) \dots$$

–All hadronic properties encoded in $H(x_1, \xi, t)$, $\Phi(x_2) \dots$

–Factorization theorems require that final-state hadrons are well-separated kinematically to suppress soft exchanges between them



(Generalized) Partonic distributions

Cross-sections controlled by the **Generalized Parton Distributions**:

[PDG 2024, Sec 18.6]

*Leading twist-2: 8 GPDs for each flavor, with different projectors \mathbf{r} , $\mathcal{F}^{(\mathbf{r})}$

$$\int \frac{dz}{2\pi} e^{ix\bar{P}^+z} \left\langle P' \left| \bar{\psi} \left(-\frac{z}{2} \right) \mathbf{r} e^{i \int d\zeta n \cdot A} \psi \left(\frac{z}{2} \right) \right| P \right\rangle = \bar{U}(P') \mathcal{F}^{(\mathbf{r})} U(P)$$

*For gluons use operators $G^{+\alpha} G_{\alpha}^+$, $G^{+\alpha} \tilde{G}_{\alpha}^+$, $\mathbb{S} G^{+i} G^{+j}$ in left-hand side

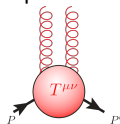
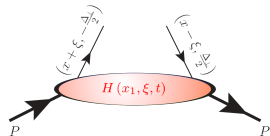
GPDs \Rightarrow matrix elements of any physical observable represented by bilinear field operators

Example: Energy-momentum tensor

$$\mathcal{T}^{\mu\nu} = -F^{\mu\alpha} F^{\nu}_{\alpha} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} \bar{\psi} \gamma^{\{\mu} i D^{\nu\}} \psi$$

$$\langle P' | \mathcal{T}^{\mu\nu} | P \rangle =$$

$$\bar{U}(P') \left[A(t) \frac{\gamma^{\mu} \bar{P}^{\nu} + \gamma^{\nu} \bar{P}^{\mu}}{2} + B(t) \frac{\bar{P}^{\mu} i \sigma^{\nu\alpha} \Delta_{\alpha} + \bar{P}^{\nu} i \sigma^{\mu\alpha} \Delta_{\alpha}}{2M_N} + D(t) \frac{g^{\mu\nu} \Delta^2 - \Delta^{\mu} \Delta^{\nu}}{4M} \right] U(P)$$



$$\bar{P} = \frac{P + P'}{2}$$

$$\Delta = P' - P$$

Relation to GPDs:

$$\int_{-1}^{+1} dx \, x H(x, \xi, t) = A(t) + \xi^2 D(t), \quad \int_{-1}^{+1} dx \, x E(x, \xi, t) = B(t) - \xi^2 D(t),$$

Rest frame of the hadron: Fourier images of A, B, D related to pressure, shear forces ...

Phenomenological constraints

$$\text{Physical observable : } \mathcal{A} = \int dx_1 dx_2 \dots C(x_1, x_2, \xi \dots) H(x_1, \xi, t) \Phi(x_2) \dots$$

–The model-independent extraction of GPDs from data \Rightarrow “deconvolution”, classical inverse problem

–For all $2 \rightarrow 2$ processes coef. function $C(x, \xi)$ strongly peaked at line $x = \pm \xi \Rightarrow$

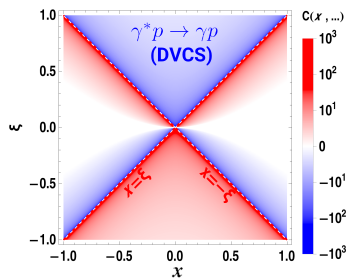
*White-colored regions almost do not contribute to observable ...

*Similar behavior for all $2 \rightarrow 2$ processes, and after we take into account NLO corrections, ...

\Rightarrow ***“Shadow GPD” problem:** [PRD 108 (2023) 3, 036027]

–Poor sensitivity to behavior of GPDs in some regions

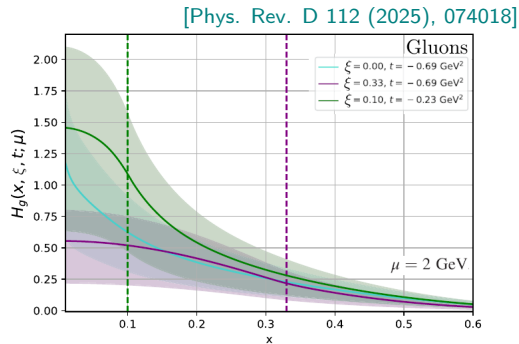
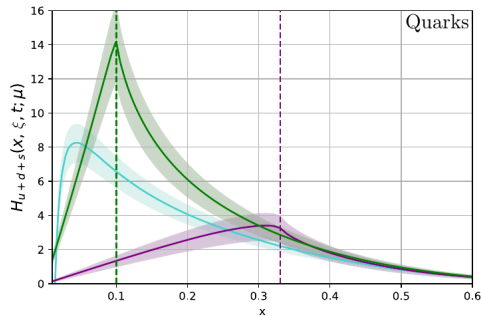
\Rightarrow Cross-sections constrain GPDs but usually don't allow to fix them uniquely



\Rightarrow We need various channels (preferably with different topology) to understand better the GPDs

What do we know about GPDs now ?

Typical GPD uncertainties:



- Unpolarized quark GPDs are reasonably well constrained
 - * Phenomenological parametrizations: GUMP, GK, KM, JAM ...
 - * More ambiguities for polarized/transversity GPDs, but there is a lot of developments in that direction
- Gluon GPDs are poorly constrained even for unpolarized case
 - * Sizeable uncertainty in observables that get dominant contribution from gluons, more data are needed

Novel tools for femtography: $2 \rightarrow 3$ processes

Process:

[Recent review: [arXiv:2511.20402](#)]

$$\gamma^{(*)} + p \rightarrow h_1 + h_2 + p$$

Various states h_1, h_2 considered in the literature:

$\ell \bar{\ell}$ (DDVCS)

[PRD 107 (2023), 094035]

$\gamma\pi, \gamma\rho$

[JHEP 03 (2023) 241; PRD 107 (2023) 9, 094023 2]

$\gamma\gamma$

[JHEP 08 (2022) 103]

$\gamma\gamma^* \rightarrow \gamma\bar{\ell}\ell$

[Phys. Rev. D 103 (2021) 114002]

$\pi\rho$

[Phys.Lett.B 688 (2010) 154-167]

$\gamma\eta, \gamma\eta'$

[2511.19720]

Main advantage:

–Invariant mass $M_{h_1 h_2} \Rightarrow$ additional hard scale, can modify the coefficient function used to probe the GPDs

*Factorization is justified even in photoproduction regime

Our suggestion: $2 \rightarrow 3$ processes with heavy mesons:

– $h_1, h_2 = \gamma + \text{quarkonium } (\eta_c, \chi_c, J/\psi)$ (this talk)

–**Hard scales:** quark mass m_Q , invariant mass $M_{\gamma Q}$ (\approx transverse momenta $\mathbf{p}_\gamma \approx -\mathbf{p}_Q$).

*Dominant contribution from gluon GPDs in ERL kinematics.

*

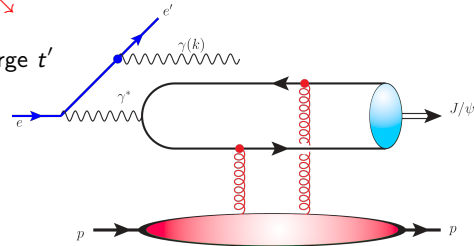
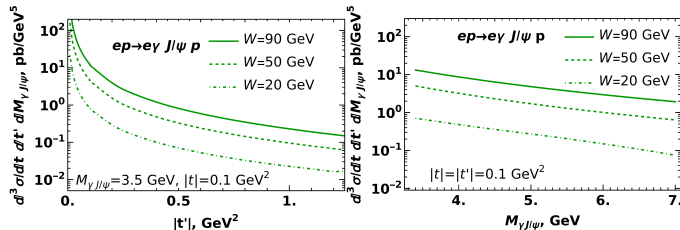
Photoproduction of $J/\psi \gamma$ pairs

Most “popular” quarkonium: J/ψ (light, clean experimental signature)

—leading order contribution in α_s : “Bethe-Heitler”-style process \rightarrow

* Photon emission from **incoming or outgoing charged particles**

** Photon γ^* is virtual, $p_{\gamma^*}^2 = t' = \mathcal{O}(M_{J/\psi}^2)$, suppressed at large t'



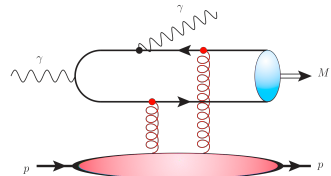
* This mechanism probes the same gluon GPD as simple J/ψ photoproduction ...

Photon emission from heavy lines: vanishes for $J/\psi \gamma$

—At chosen order in α_s , color structure is trivial, $g \sim \gamma$

—Similar result for any C -odd meson

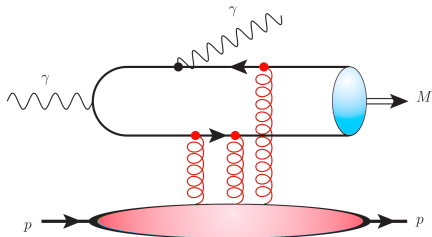
\Rightarrow Relevant only for C -even quarkonia (η_c, χ_c, \dots)



Photoproduction of $J/\psi \gamma$ pairs

Next-to-leading order: $J/\psi + \gamma$ may proceed:

Via 3-gluon exchange (3-point twist-3 GPDs)



—poorly known at present (dimensionality curse)

—**most likely very small:**

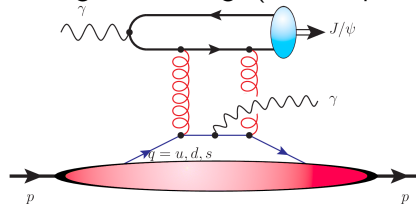
*Formally, $\alpha_s(m_c)g(x) < 1$

*recall negative “odderon” searches



Will be disregarded in what follows

Via 2 gluon exchange (valence quark GPD)



—18 diagrams at (N)LO order

*The C-parity forbids emission of photon from charm, only from light quarks

* t -channel gluons are NOT collinear: large p_{\perp} , $u' = (p_{J/\psi} - p_{\gamma_{in}})^2$

*Probe the GPD of valence (light) quarks

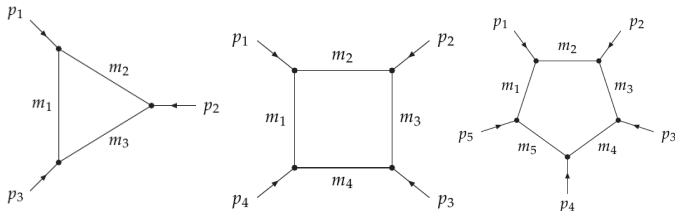
$$\frac{2}{3}H_u(x, \xi, t) - \frac{1}{3}H_d(x, \xi, t) - \frac{1}{3}H_s(x, \xi, t)$$

Photoproduction of $J/\psi \gamma$: technical details

Evaluation was done analytically in FeynCalc

–result expressed as a superposition of Passarino-Veltman functions $C_{...}, D_{...}, E_{...}$

(also known as 3,4,5-point functions)



* Can be expressed via $\text{Li}_2(\dots)$, logarithms and rational expressions (very lengthy ...)

* Used PackageX to single out possible UV and IR parts (`PaXEvaluateUV[]`, `PaXEvaluateIR[]`)

** UV divergences don't appear at all, IR divergences vanish exactly in the final result

* For numerical evaluations used LoopTools (Colliers) libraries

–Cross-sections are smaller than for BH, suffers from numerical instabilities in PaVe-functions

$\Rightarrow J/\psi \gamma$ might be challenging for GPD studies.

$\gamma p \rightarrow \gamma \eta_c p$ and $\gamma p \rightarrow \gamma \chi_c p$ photoproduction

We'll focus on the **C-even charmonia** now: η_c, χ_c

- The quarkonium η_c is the **lightest**, expect to have the largest cross-section
 - ... Wave functions/LDMEs related to those of J/ψ in the heavy quark mass limit, smaller uncertainties
- The quarkonium χ_c is the **lightest P-wave**, 3 spin states ($J = 0, 1, 2$) with very close masses
 - ... Very narrow states, decay primarily into $J/\psi \gamma$: experimentally very clean signal, easy for detection
 - ... Can study various spins, helicity projections, independent tests
 - ... The ratio $\sigma_{\gamma\chi_{c2}}/\sigma_{\gamma\chi_{c1}}$ can be used for tests of NRQCD LDME relation in the heavy quark mass limit:

* Lesson from **hadroproduction of χ_c @ LHC**

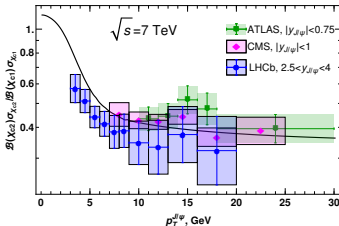
Description of $\sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$ requires at least one of the two assumptions:

- 1) Sizable Color Octets (CO) LDMEs
- 2) Heavily broken Heavy Quark Spin Symmetry (HQSS)

(Open question as of now)

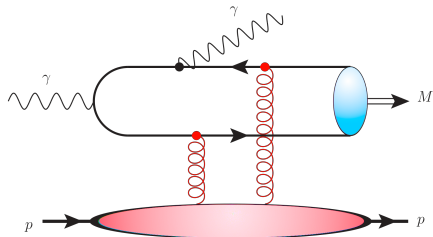
How photoproduction of $\gamma\chi_c$ can help?

- * There is no color octet contribution
 - * Other mechanisms are suppressed (provided $M_{\gamma\chi_c} > M_{\psi(2S)}$)
- \Rightarrow Deviation of $\sigma_{\gamma\chi_{c2}}/\sigma_{\gamma\chi_{c1}}$ from theoretical prediction would be clear signal that HQSS is broken



LO: Exclusive photoproduction of $\gamma\eta_c$ and $\gamma\chi_c$ pairs

Hierarchy of scales: $M_{\gamma\eta_c} \sim M_{\gamma\chi_c} \sim 2m_c \gg m_p, \Lambda_{\text{QCD}}$



Summation over all possible gluon and photon attachments is implied

—Full NLO expression is available for η_c ; is very lengthy

* Expressed in terms of Passarino-Veltman functions B_{\dots} , C_{\dots} , D_{\dots} , E_{\dots} , F_{\dots}

—At leading order (LO): 24 diagrams

* Many diagrams are related to each other due to C -parity, permutation of t -channel gluons ($x \leftrightarrow -x$ symmetry in coef. function)

—At NLO >500 diagrams;

* Get also contributions from (sea) light quarks

* Evaluation is highly repetitive, parallelizable, done in FeynCalc



Focus only on LO now

Coefficient functions for $\gamma\eta_c$ and $\gamma\chi_c$ photoproduction (LO)

$$C_{\gamma\eta_c}^{(++)} \left(r = \frac{M_{\gamma\eta_c}}{M_{\eta_c}}, \alpha = \frac{p_{\eta_c}^+}{p_{\gamma(\text{in})}^+}, \zeta = \frac{x}{\xi} \right) = \frac{4\bar{\alpha}\mathfrak{E}_{\eta_c}}{\xi^2\alpha^2(1+\bar{\alpha})^2(r^2-1)(\bar{\alpha}+1/r^2)} \times$$

$$\times \frac{1}{(\zeta+1-i0)(\zeta-1+i0)(\zeta+\kappa-i0)(\zeta-\kappa+i0)} \left[\alpha^2(1+\bar{\alpha})(\zeta^2+1) + \right.$$

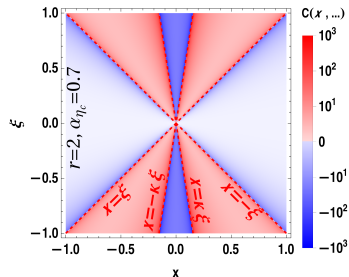
$$+ \frac{(\alpha^2-4)\zeta^4 - 2(\alpha^3-4\alpha+2)\zeta^2 - \alpha(2\alpha^2+\alpha-4)}{(\zeta+1-i0)(\zeta-1+i0)r^2} +$$

$$\frac{(1+\bar{\alpha})^2\zeta^4 + \alpha(\alpha^2+3\alpha-12)\zeta^2 + \alpha(3\alpha^2+8\alpha-4)}{(\zeta+1-i0)(\zeta-1+i0)r^4} - \frac{4(\zeta^2-3\alpha)}{(\zeta+1-i0)(\zeta-1+i0)r^8}$$

$$\left. + \frac{-\alpha((\alpha-4)\zeta^2+11\alpha+8)+8\zeta^2+4}{(\zeta+1-i0)(\zeta-1+i0)r^6} - \frac{4}{(\zeta+1-i0)(\zeta-1+i0)r^{10}} \right]$$

- Similar (relatively simple) expressions for other helicity components $C_{\gamma\eta_c}^{(\lambda,\sigma)}$, $C_{\gamma\chi_c}^{(\lambda,\sigma,H)}$, ...
- ... For χ_c the poles are at the same position, but of higher (2nd) order:
for P -wave take derivative over small relative momentum
- ... Helicities affect numerators, but not denominators (position of poles)

Coefficient function for $\gamma\eta_c$ and $\gamma\chi_c$ photoproduction (LO)



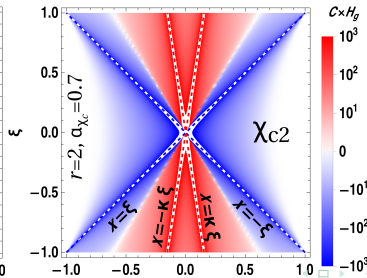
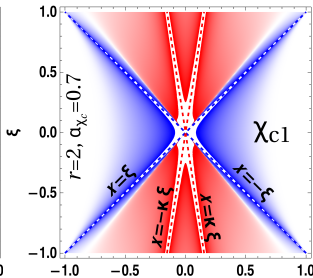
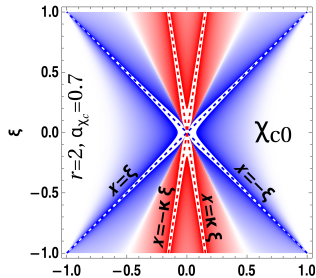
Location of poles in coef. function:

- Classical $x = \pm\xi \mp i0$ ($\zeta = \pm 1 \mp i0$)
- **New poles** at $x = \pm\kappa\xi \mp i0$ ($\zeta = \pm\kappa \mp i0$), where

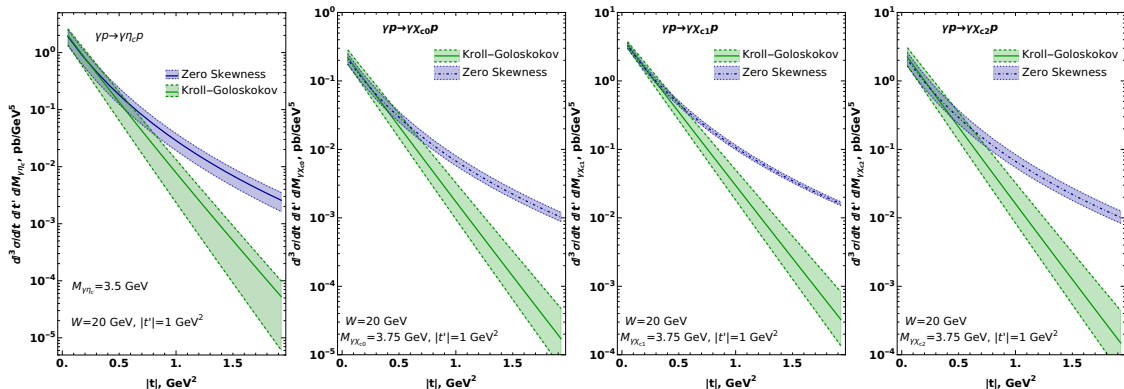
$$\kappa = \frac{1}{r^2} \frac{2 - \alpha r^2}{2 - \alpha}, \quad |\kappa| \leq 1 \quad (\text{ERBL kinematics})$$

...Values of $r = M_{\gamma\eta_c}/M_{\eta_c} \geq 1$ and $\alpha = p_{\eta_c}^+/q^+ \in (1/r^2, 1)$ in the physically allowed range

– For $\chi_c\gamma$ poles are of second order, much more pronounced (coef. function vanishes rapidly)



t -dependence of the cross-section

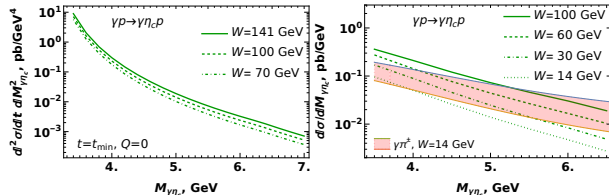


- Dependence on t is from GPD, not coef. functions. Almost the same for all ...
- Compare Kroll-Goloskokov and Zero Skewedness parametrizations for definiteness
- Width of the band = uncertainty due to NLO corrections, found varying fact. scale $\mu \in (0.5-2)M_{\gamma\chi_c}$
 - * Reasonable at low energies W , grows rapidly due to BFKL logs $\sim \ln(1/x)$ in GPD parametrization
- Differential cross-sections $\sim 1 - 10$ pb for $\eta_c\gamma$, $\chi_{c\gamma}$

Dependence on invariant mass $M_{\gamma Q}^2$ and variable t'

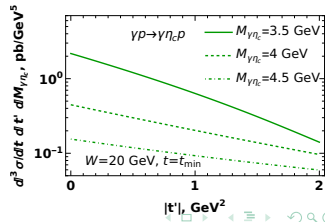
Invariant mass $M_{\gamma Q}^2 = (k_\gamma + p_Q)^2$:

- Determines $x \sim \xi \sim M_{\gamma Q}^2/W^2$, so at large $M_{\gamma Q}$ strong decrease of the cross-section due to decrease of gluon GPDs/PDFs
- ... $M_{\gamma Q}^2$ also appears in the coef. functions, but this has only minor overall effect
- ... Similar dependence for all quarkonia, and even for $\gamma\pi^\pm$ (!) if compared at the same invariant mass $M_{\gamma M}$



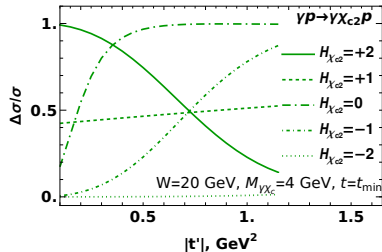
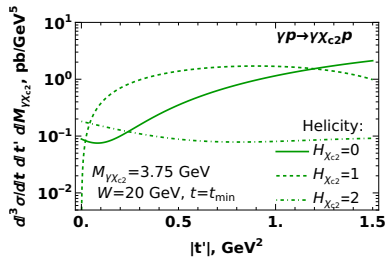
Momentum transfer to the photon $t' = (k_\gamma - q_\gamma)^2$:

- t' determine angles between γ , quarkonium in final state and collision axis
- ... dependence is encoded in coef. function, depends a lot on spin J of quarkonium, helicity (for $J \neq 0$).



Understanding polarizations in $\gamma\eta_c, \gamma\chi_c$

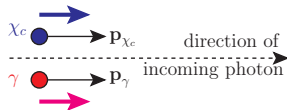
- Helicity flip of the target suppressed, controlled by small GPDs \tilde{H}, \dots (negligible)
- Can study contributions with or without helicity flip of the final state photon



- Assume incoming photon has helicity “+”
- $\Delta\sigma/\sigma$ is a fraction of helicity flip contribution
- Suppression at extremes for some components is due to angular momentum conservation:

$$t' = 0:$$

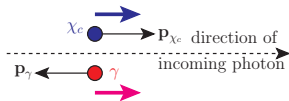
- final-state χ_c, γ move in the same direction



$$H_{\chi_c} + H_{\gamma} = H_{in}$$

$$|t'| = |t'|_{max} = M_{\gamma\chi_c}^2 - M_{\chi_c}^2:$$

- final-state χ_c, γ move in opposite directions,



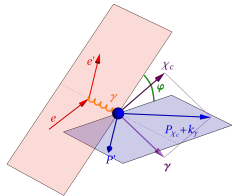
$$H_{\chi_c} - H_{\gamma} = H_{in}$$

Angular asymmetries in electroproduction ($ep \rightarrow ep\gamma Q$)

Strong polarization dependence \Rightarrow observable even if detectors do not measure polarization:
Leptonic tensor $L_{\lambda, \bar{\lambda}}$ is not diagonal in helicity basis,

$$d\sigma(ep \rightarrow ep\gamma\eta_c) = L_{\lambda, \bar{\lambda}} A^{(\bar{\lambda})*} A^{(\lambda)} d\Omega \neq \underbrace{\text{Flux}(e \rightarrow e\gamma^*) \times d\sigma(\gamma^* p \rightarrow p' \eta_c \gamma)}_{\text{equivalent photons approx.}}$$

—leads to dependence on angle φ between leptonic and hadronic planes:



$$\begin{aligned} \frac{d\sigma_{ep \rightarrow ep\gamma\eta_c}}{d\Omega} &= \frac{d\sigma^{(T)}}{d\Omega} + \epsilon \frac{d\sigma^{(L)}}{d\Omega} + \sqrt{\epsilon(1+\epsilon)} \left(\cos \varphi \frac{d\sigma^{(LT)}}{d\Omega} + \sin \varphi \frac{d\sigma^{(L'T)}}{d\Omega} \right) \\ &+ \epsilon \cos 2\varphi \frac{d\sigma^{(TT)}}{d\Omega} + \epsilon \sin 2\varphi \frac{d\sigma^{(T'T)}}{d\Omega} \approx \frac{d\sigma^{(T)}}{d\Omega} (1 + c_2 \cos 2\varphi + s_2 \sin 2\varphi) \end{aligned}$$

—Longitudinal (L) is suppressed at small- Q ; $\epsilon \approx (1-y)/(1-y+y^2/2)$.
 T, T' is interference of γ^* with opposite helicities in $A^{(\bar{\lambda})*}, A^{(\lambda)}$

Definition of φ in collider kinematics:

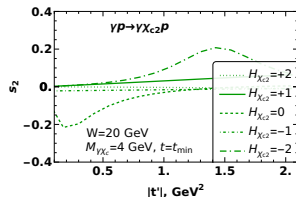
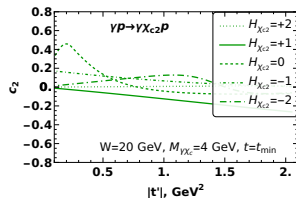
- Assume \hat{z} points in direction of collision axis, and φ_p, φ_e are azimuthal angles of recoil electron and proton
- The angle φ is given by $\varphi = \varphi_e - \varphi_p$

Results for angular asymmetry in $\gamma\chi_c$, $\gamma\eta_c$

T, T' interference terms allow to probe the real and imaginary parts:

$$\frac{d\sigma^{(TT)}}{d\Omega} \sim \text{Re} \left(\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)*} \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right), \quad \frac{d\sigma^{(T'T)}}{d\Omega} \sim \text{Im} \left(\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)*} \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right)$$

–Easy to study: just extract φ -dependence of the recoil proton w.r.t. leptonic plane



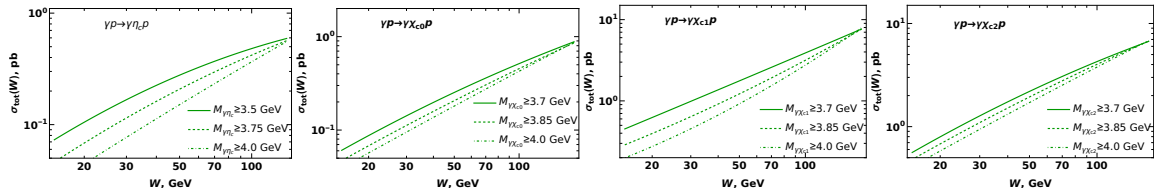
$$\frac{d\sigma_{ep \rightarrow ep \gamma \chi_c}}{d\Omega} \approx \frac{d\sigma^{(T)}}{d\Omega} (1 + c_2 \cos 2\varphi + s_2 \sin 2\varphi)$$

$$c_2 \approx \frac{2\epsilon \text{Re} \left(\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)*} \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right)}{\left| \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right|^2 + \left| \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)} \right|^2}, \quad s_2 \approx \frac{2\epsilon \text{Im} \left(\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)*} \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right)}{\left| \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)} \right|^2 + \left| \mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)} \right|^2}$$

–A sizable asymmetry is possible if both components $\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(+,+)}$ and $\mathcal{A}_{\gamma p \rightarrow \chi_c \gamma p}^{(-,+)}$ (w/ and w/o photon helicity flip) have comparable magnitudes.
... Is satisfied for η_c , χ_{c0} and for helicity component $H_{\chi_c} = 0$ helicity of χ_{c1}, χ_{c2}

–Asymmetries decrease as a function of W (smaller angles)

Yields and counting rates for $\gamma\eta_c$ and $\gamma\chi_c$ photoproduction



- Power law dependence on invariant energy $\sim W^\alpha$ ($W \equiv \sqrt{s_{\gamma p}}$)
... Reflects x -dependence $\sim x^{-\nu}$ implemented in the GPD model ($\alpha \approx 4\nu$)
- Cutoff on minimal invariant mass $M_{\gamma\eta_c}$ to exclude feed-down contributions

Expected production and counting rates:

	σ_{tot}	Production rates		Decay channel	Combined branching	Counting rates	
		N	dN/dt			N_d	dN_d/dt
η_c	49 fb	4.9×10^3	42/day	$\eta_c(1S) \rightarrow K_S^0 K^+ \pi^-$	2.6%	127	32/month
χ_{c0}	31 fb	3.1×10^3	27/day	$\chi_c \rightarrow J/\psi \gamma$ $J/\psi \rightarrow \mu^+ \mu^-$	0.08 %	2.5	0.65/month
χ_{c1}	230 fb	2.3×10^4	199/day		2 %	460	120/month
χ_{c2}	250 fb	2.5×10^4	216/day		1.1 %	280	73/month
$\sqrt{s_{\text{sep}}} = 141 \text{ GeV}, \int dt \mathcal{L} = 100 \text{ fb}^{-1}, \mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$							

Summary

Photoproduction of quarkonia-photons may be interesting for GPD studies

- The $\chi_c\gamma$ and $\eta_c\gamma$ photoproduction are sensitive to GPD $H_g(x, \xi, t)$
- The $J/\psi \gamma$ is sensitive to:
 - * At LO: gluon GPD, convoluted with usual coefficient function of DV $J/\psi P$ (dominant at small t')
 - * At NLO: valence quark GPD combination

$$\frac{2}{3}H_u(x, \xi, t) - \frac{1}{3}H_d(x, \xi, t) - \frac{1}{3}H_s(x, \xi, t)$$

* Dominant at large t' , cross-sections and counting rates comparable to those of $\eta_c\gamma$, $\chi_c\gamma$

Thank You for your attention!