Finite Temperature in Quarkyonic matter: Challenges and difficulties.

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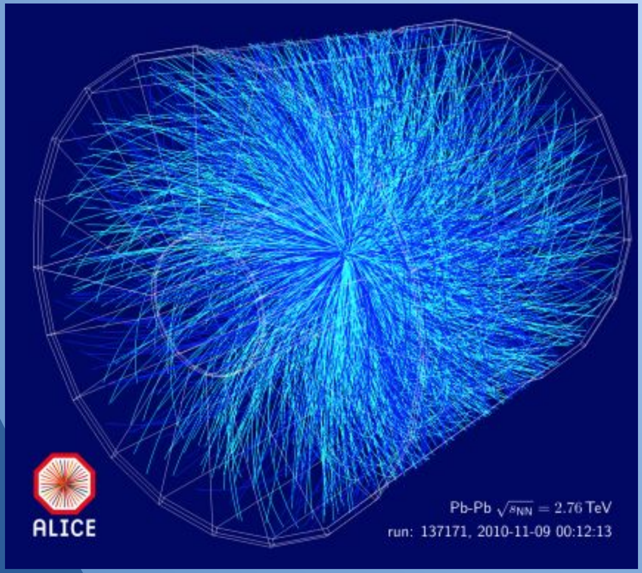
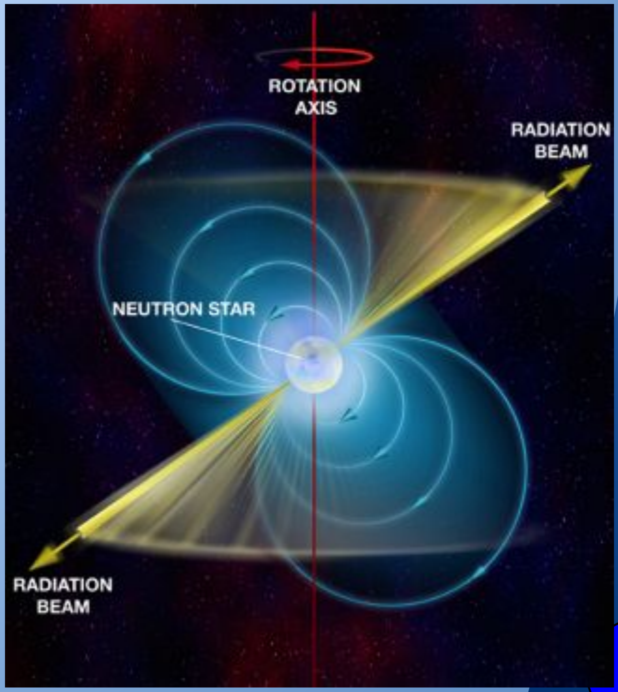
Outline

- Motivation
- Quarkyonic Matter
- Symmetry and Asymmetry Case
- Finite Temperature
- Final Remarks

Motivation

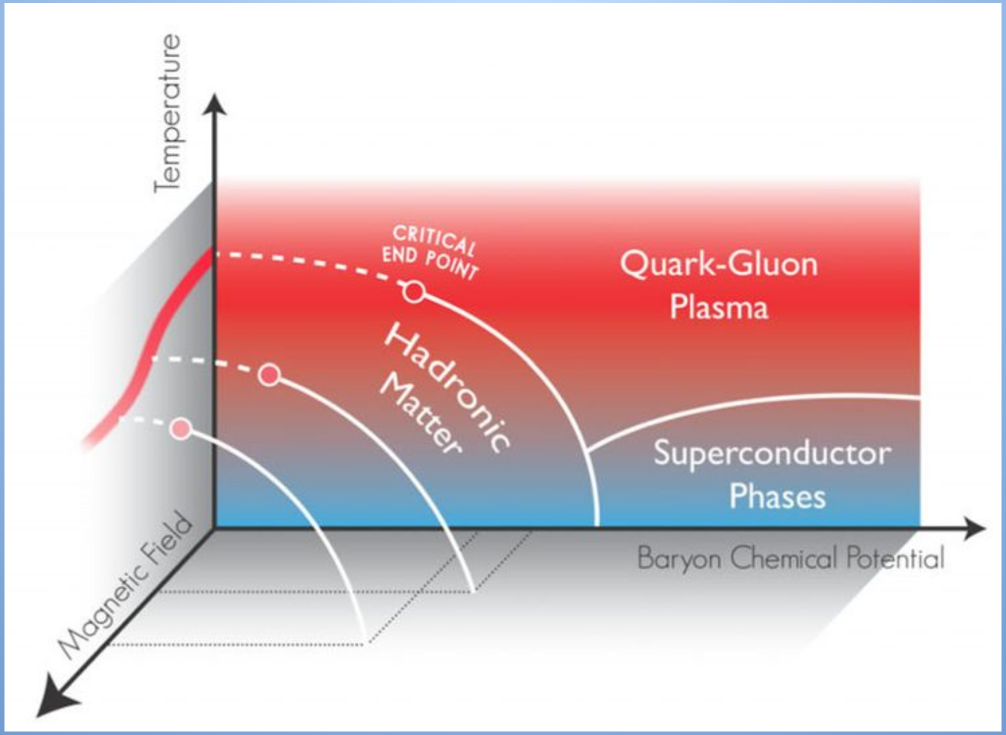
- QCD under extreme conditions (temperature, finite density and magnetic fields) plays an important role in understanding the transitions that took place in the early universe.





Motivation

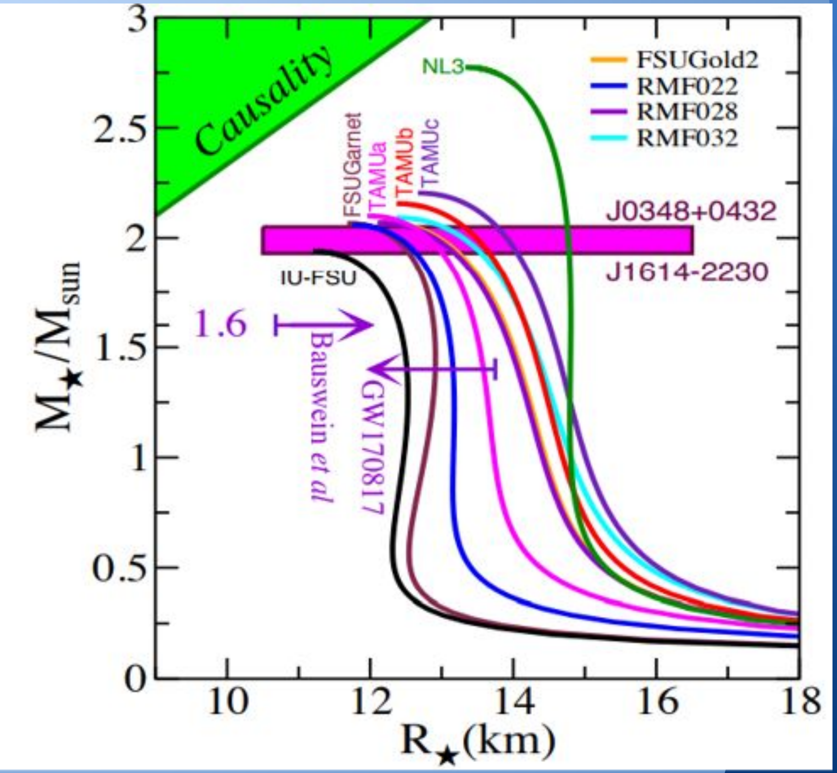
- One way to illustrate the different phases of strongly interacting matter we seek to study is by means of the phase diagram



Motivation

- Observation and analysis of GW170817: Important clues to understand cold and dense matter.

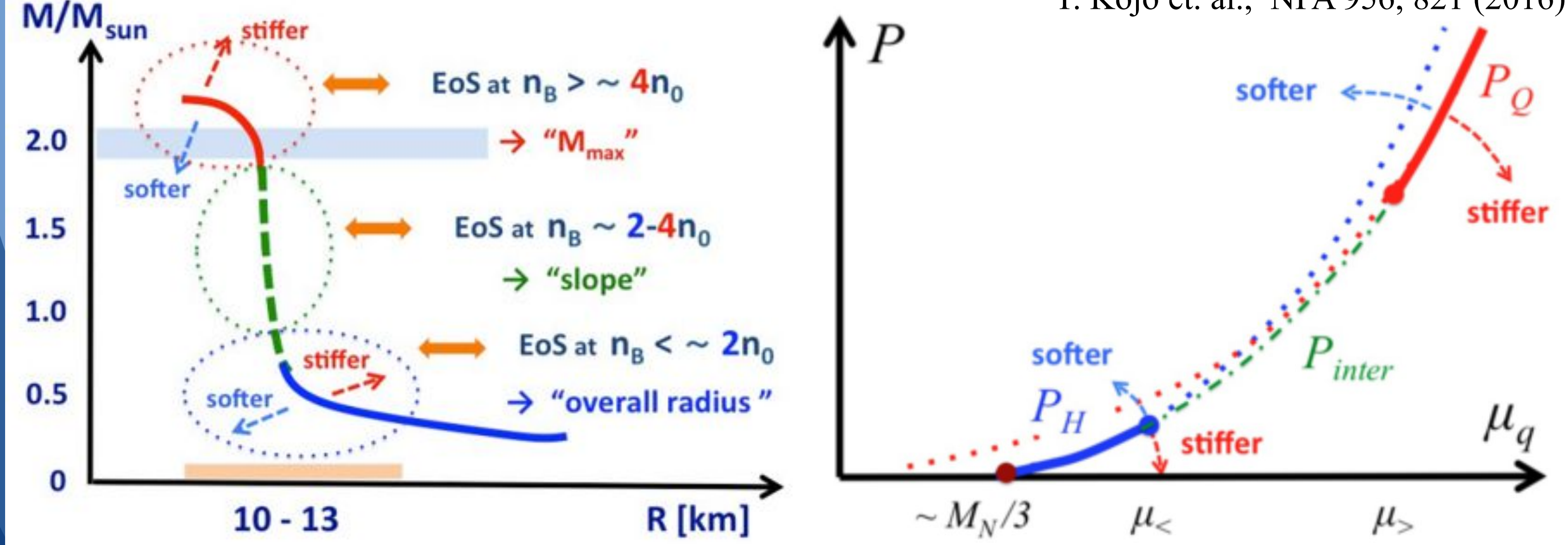
- The structure of a neutron star (NS) is determined by the Tolman-Oppenheimer-Volkoff Equation (TOV).



F. J. Fattoyev et. al, PRL 120, 172702 (2018)

Motivation

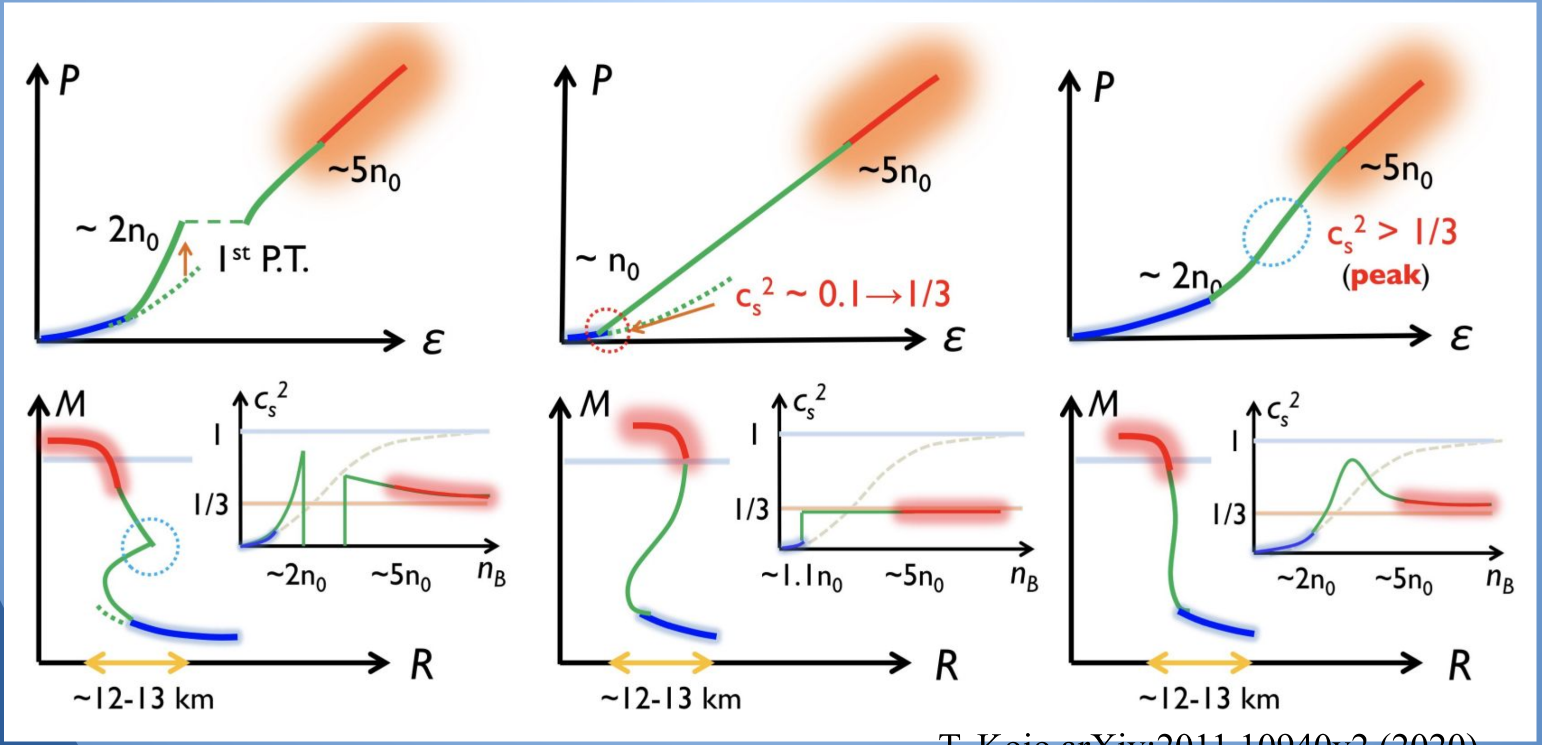
- Description of Equation of State (EoS) of dense QCD matter:
  - Around saturation density: Nuclear experiments.
  - Very high density limit: Asymptotic freedom allows perturbative calculation.



T. Kojo et. al., NPA 956, 821 (2016)

Motivation

- TOV and EoS can give some insight about the transition quark-nucleon matter.



T. Kojo arXiv:2011.10940v2 (2020)

Motivation

- The square of the speed of sound is usually defined as

$$c_\chi^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_\chi$$

where χ denotes the parameter fixed in the calculation of the speed of sound.

- According to the properties on the propagation medium, it may be more useful to keep one quantity fixed rather than another.

Motivation

- This are the expression that we are using on this work,

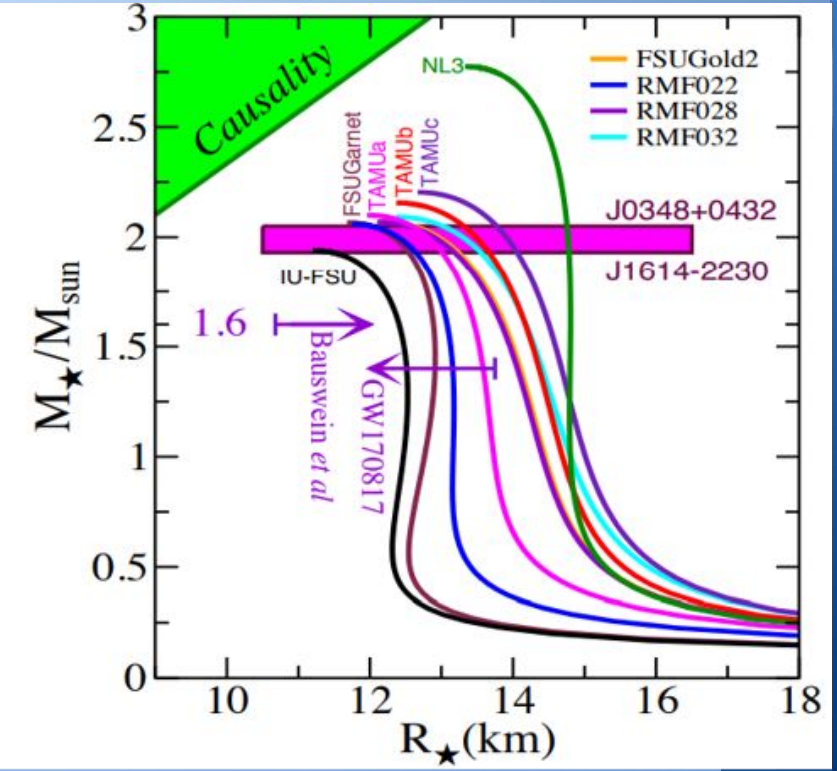
$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{T=0} = \frac{\rho_B}{\mu_B}\left(\frac{\partial \rho_B}{\partial \mu_B}\right)^{-1}$$

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_s = \frac{\rho_B \chi_{TT} - s\chi_{\mu T}}{\mu_B(\chi_{TT}\chi_{TT} - \chi_{TT})}$$

Motivation

- EoS should be hard enough to support $2M_\odot$ and soft enough to satisfy $R_{1.4} \leq 13.5$ km.
- This is also reflected in sound velocity, that should increase rapidly and can be greater then its conformal value $c_s^2 \geq 1/3$.

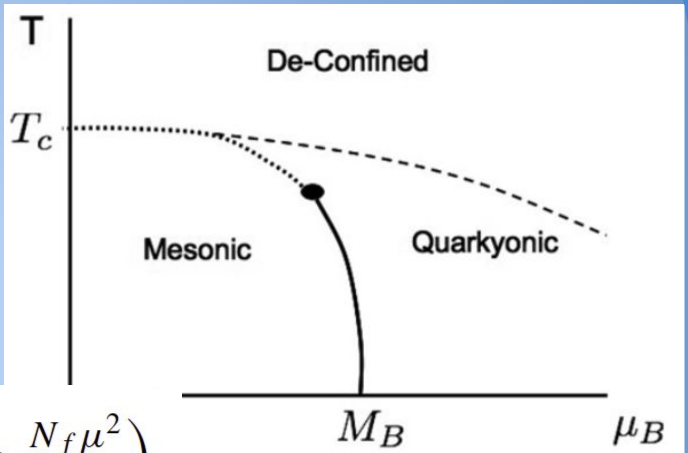
Any suggestions?



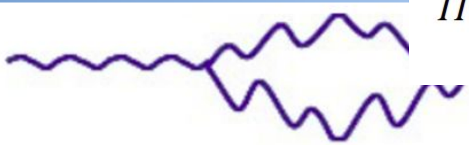
F. J. Fattoyev et. al, PRL 120, 172702 (2018)

Quarkyonic Matter

Phase of dense matter, argued from large Nc approximation and model computations.



$$\Pi^{\mu\mu}(0) = g^2\left(\left(N_c + \frac{N_f}{2}\right)\frac{T^2}{3} + \frac{N_f\mu^2}{2\pi^2}\right).$$



Gluon loop

→ $g^2 N_c T^2 \sim T^2$;

→Dynamics not affected by quarks;

→Debye screening at large distances.

Quark loop

→ $\sim \mu_Q^2 g^2 \Rightarrow$ Supressed by $1/N_c$ at large N_c.

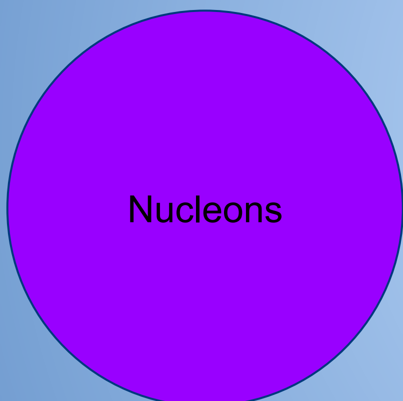
→High density limit: $\mu_Q \gg \Lambda_{QCD}$, so quarks are important when $\mu_Q \sim N_c^{1/2}\Lambda_{QCD}$.

→Debye screen mass $m_D \simeq g\,\mu_Q$

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)

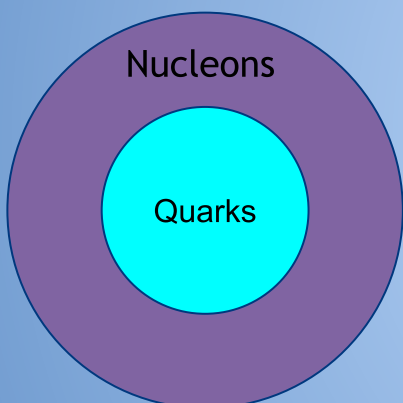


- For $k_F^B < \Lambda_{\mathrm{QCD}}$: Quarks confined in nucleons.

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)

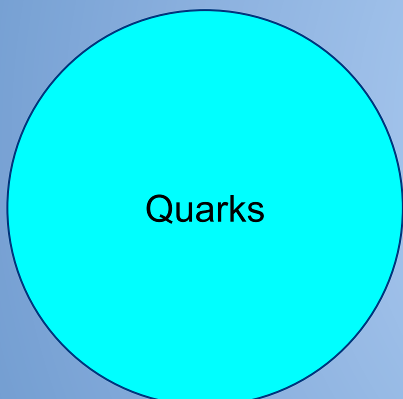


- For $k_F^B < \Lambda_{\mathrm{QCD}}$: Quarks confined in nucleons.
- For $\Lambda_{\mathrm{QCD}} \leq k_F^B \leq N_c \Lambda_{\mathrm{QCD}}$: Quarks starts to take low phase space, and a shell-like structure is formed.

Quarkyonic Matter

Nuclear → Quarkyonic

(at few times ρ_0)



- For $k_F^B < \Lambda_{\rm QCD}$: Quarks confined in nucleons.
- For $\Lambda_{\rm QCD} \leq k_F^B \leq N_c \Lambda_{\rm QCD}$: Quarks starts to take low phase space, and a shell-like structure is formed.
- For $k_F^B \simeq N_c^{3/2} \Lambda_{\rm QCD}$: Confinement disappears.

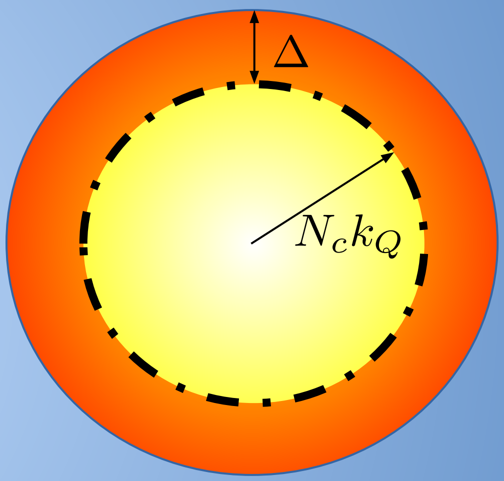
- Total baryon density has smooth behavior and chemical potential for confined states enhance suddenly, then pressure suddenly increases. *This is not an usual phase transition!*

Symmetry matter case

$$n_B = \frac{2}{3\pi^2}\left(k_{\rm FB}^3 - (k_{\rm FB}-\Delta)^3\right)$$

$$\epsilon(n_B) = 4\int_{N_c k_{\rm FQ}}^{k_{\rm FB}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_n^2}\,,$$

$$k_Q = k_F/N_c \qquad m = M/N_c$$



Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2 = \dfrac{N_f}{3\pi^2}k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2\sqrt{k^2+m^2} \end{cases}$$

This value of the shell is a phenomenological input that ensure that the nucleon density approximately saturates when baryons dominate, and the second term is needed to ensure that the $c_s^2 < 1$.

$$\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa\frac{\Lambda}{N_c^2}$$

Mc Lerran and Reddy, Phys. Rev. Lett. 122, 122701 (2019)

Asymmetry matter case

- To describe neutron star matter we need to impose local charge neutrality, for this reason, we will approximate matter to consist of only neutrons. At a given baryon density n_B, the neutron Fermi momenta is denoted by k_{FB} and the up and down quark Fermi momenta are denoted by k_{Fu} and k_{Fd}, respectively.

$$k_{Fd} = \frac{k_{FB} - \Delta}{3}; \qquad k_{Fu} = 2^{\frac{1}{3}} k_{FB};$$

Mc Lerran and Reddy, Phys. Rev. Lett. 122, 122701 (2019)

Asymmetry matter case

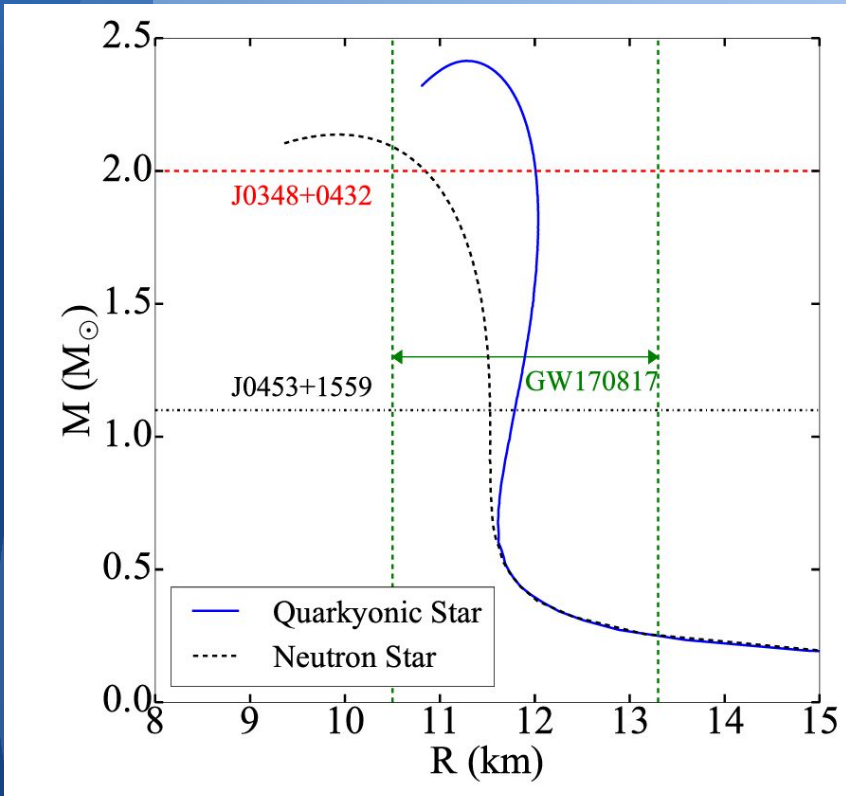
- we will include nuclear interactions by adapting a fit by Ref: Gandolfi, Eur. Phys. J. A50, 10 (2014), where the energy density due to interactions was well approximated by

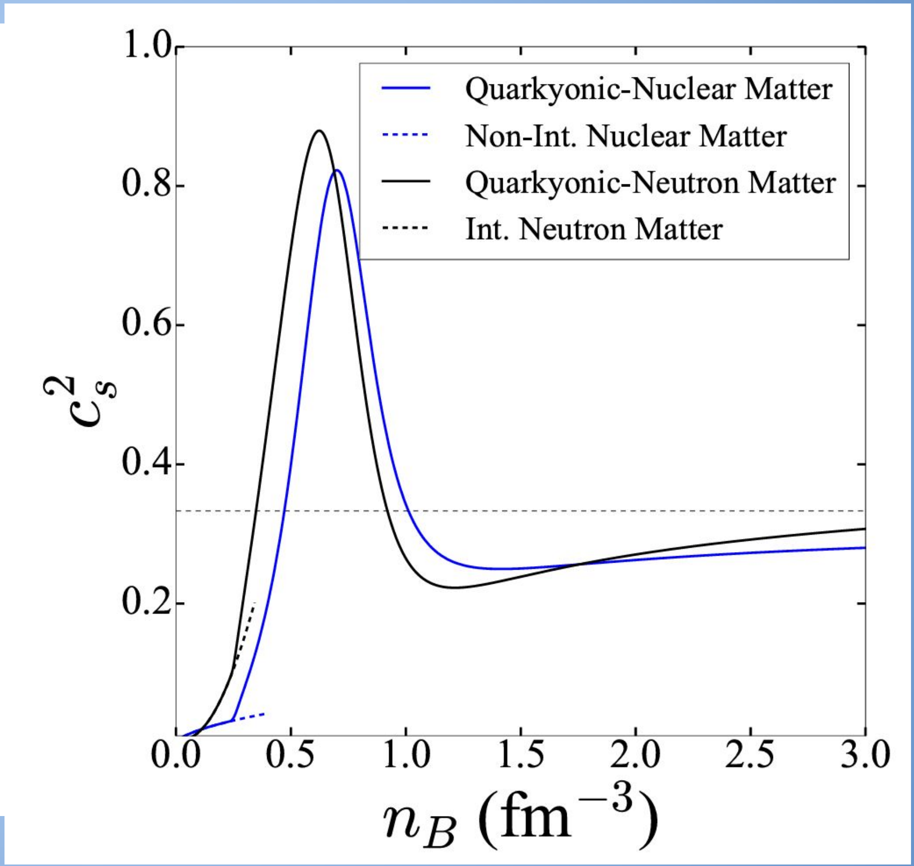
$$V_n(n_n) = \tilde{a}\, n_n \left(\frac{n_n}{n_0}\right) + \tilde{b}\, n_n \left(\frac{n_n}{n_0}\right)^2$$

with a = 28.6 ± 1.2 MeV, b = 9.9 ± 3.7 MeV. The chemical potential and pressure are given by:

$$\mu = \frac{\partial \epsilon}{\partial n_B}; \quad P = -\epsilon + \mu_B n_B$$

Speed of Sound and TOV

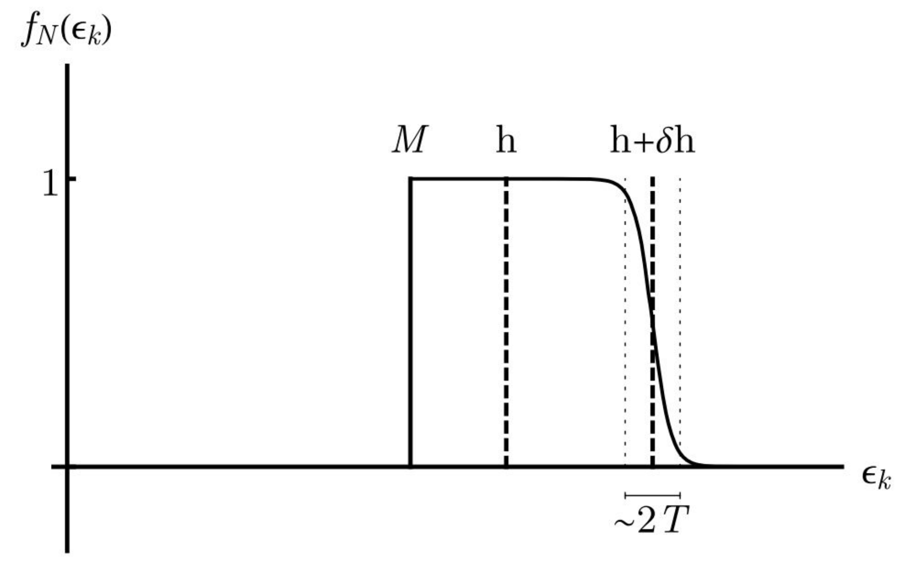




Good agreement with sound velocity obtained from an equation of state extracted from neutron stars properties.

Finite temperature case

- To implement finite temperature we are trying to do it in the simplest way by softening the boundary of the shell and replacing the step function by fermi-distribution, with,



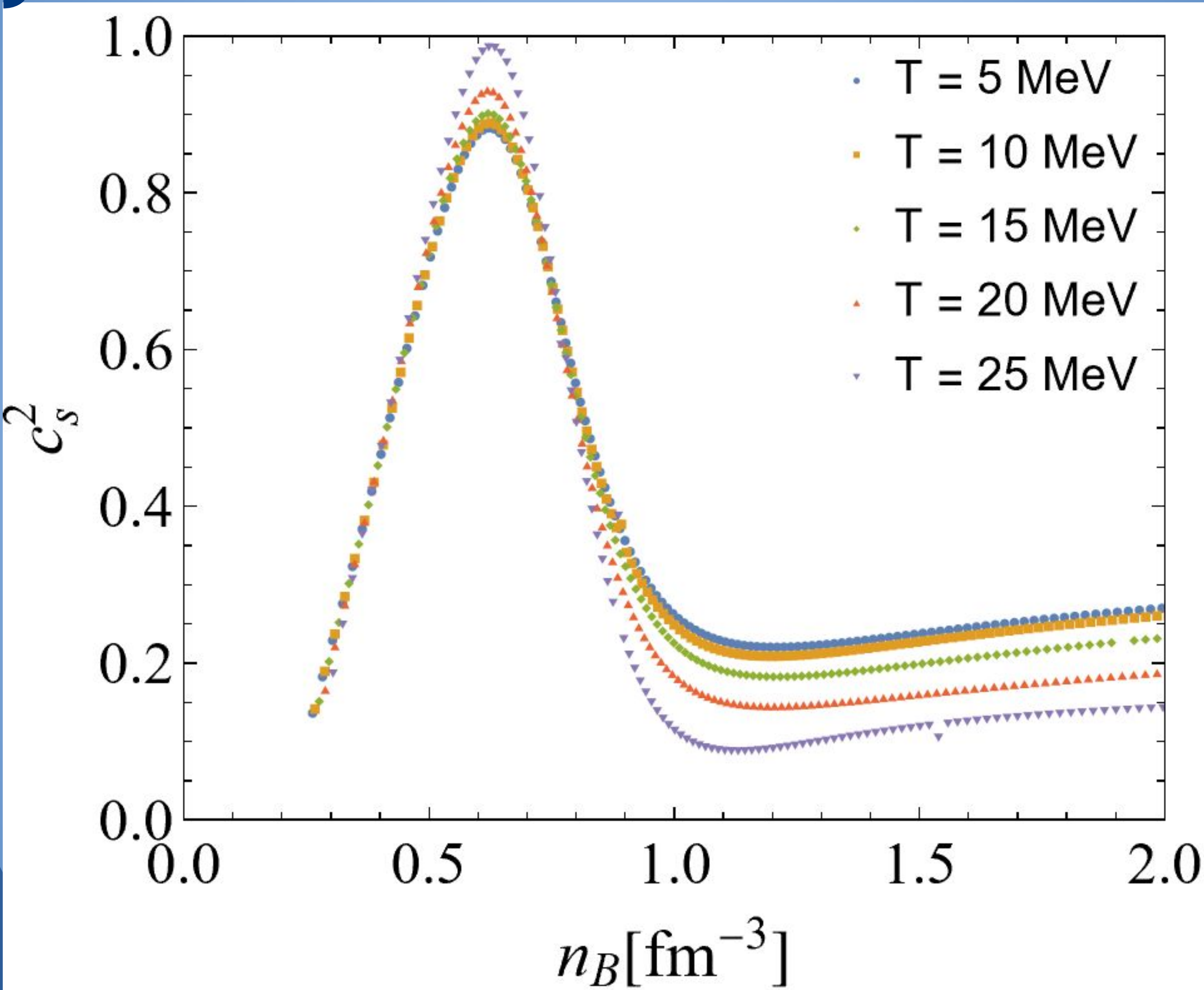
$$g(\epsilon, \mu) = \frac{1}{e^{\frac{\epsilon-\mu}{T}} + 1}$$

and

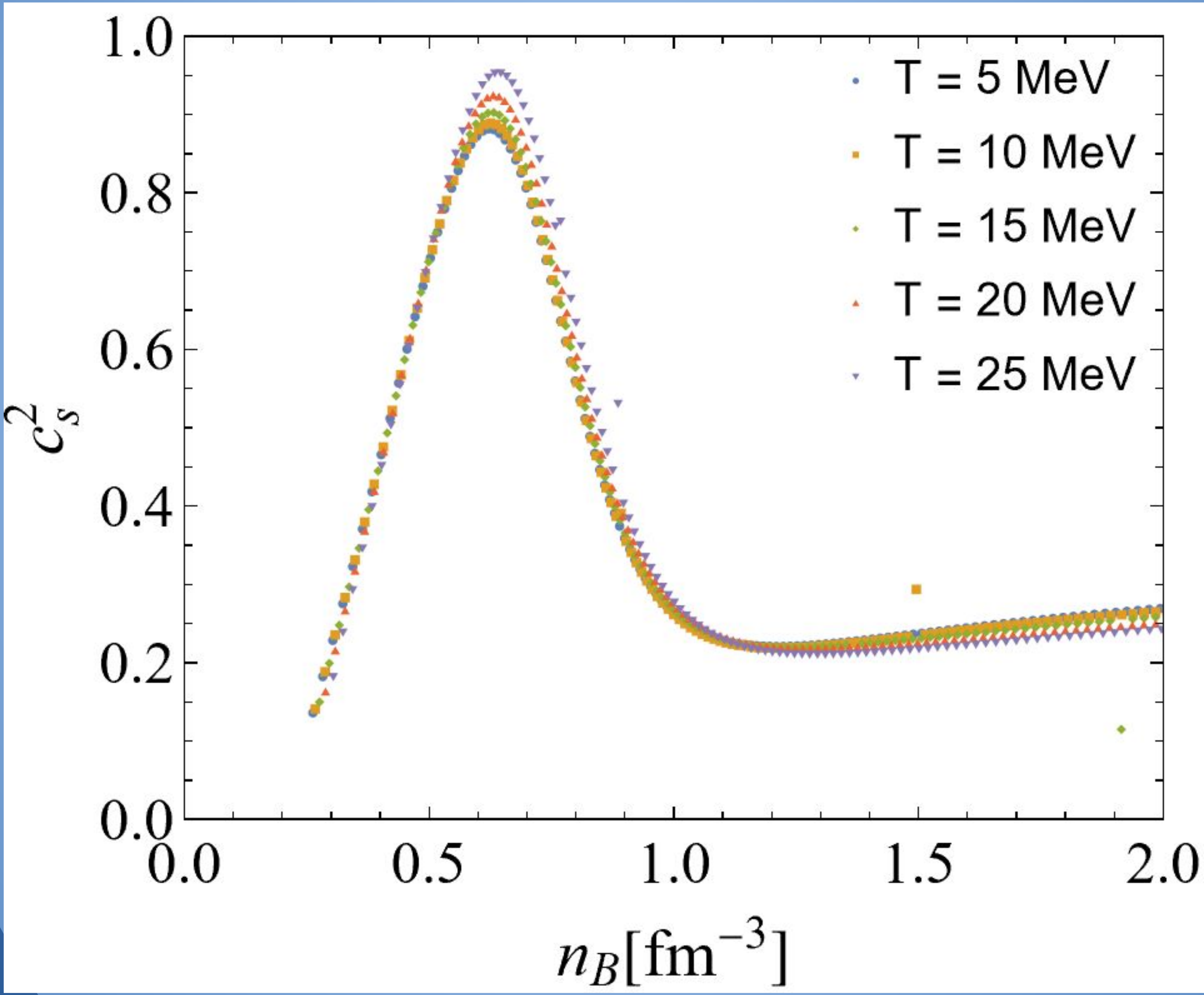
$$f_Q(k) = g(\epsilon_Q(k), k_{kB}/N_c)\theta\left(\frac{k_{kB} - \Delta}{N_c} - \epsilon_Q(k)\right)$$

$$f_N(k) = g(\epsilon_N(k), k_{kB})\theta\left(\epsilon_N(k) - (k_{kB} - \Delta)\right)$$

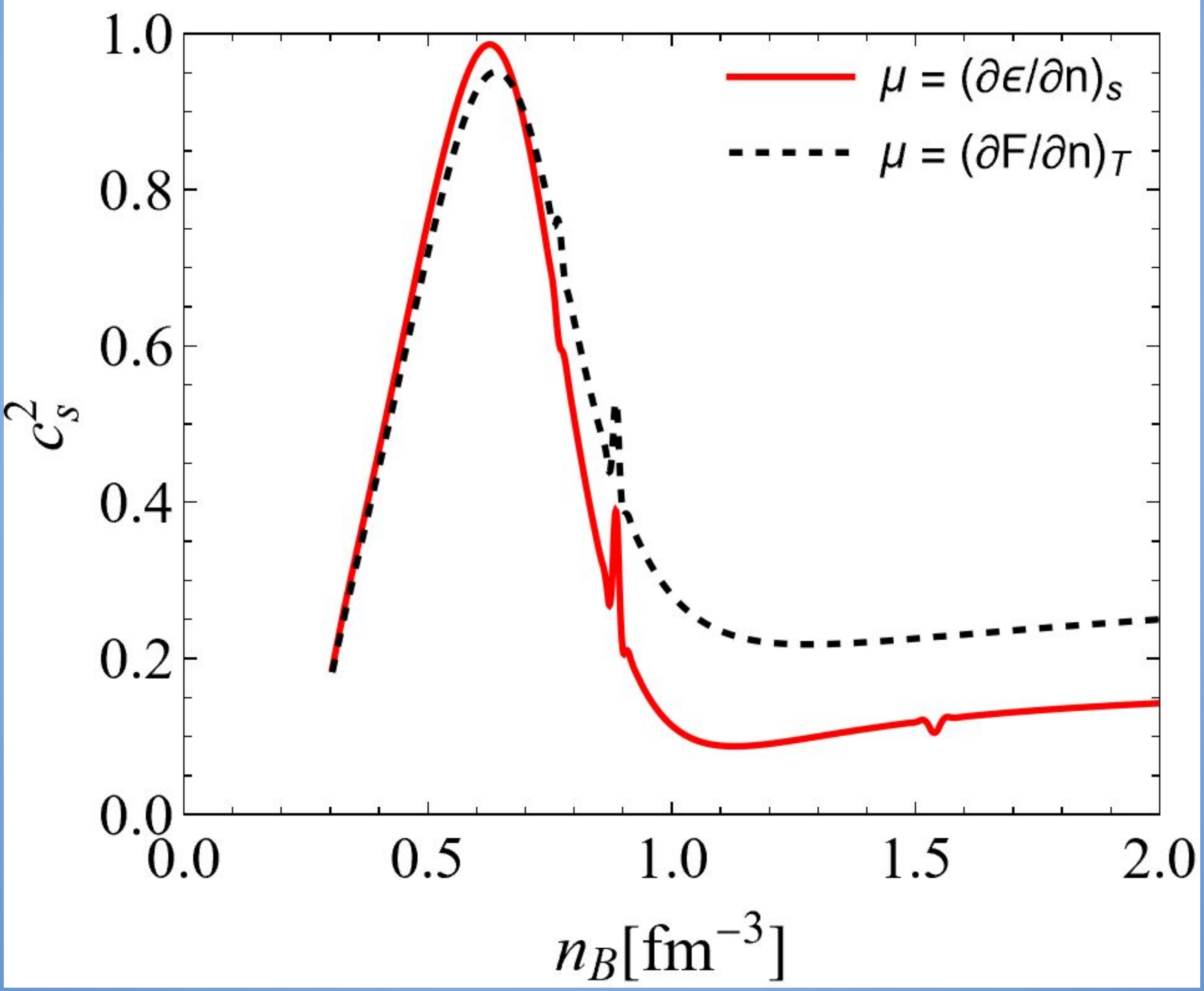
Asymmetric Case



Student approach



but…



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
c_s^2
$n_B[\mathrm{fm}^{-3}]$
$\mu = (\partial\epsilon/\partial n)_s$
$\mu = (\partial F/\partial n)_T$

Conclusions

- Analysis of GW data have been providing very important insights about the properties of dense QCD matter.

- We tried to extend the phenomenological approach of two-flavor dense Quarkyonic matter and address its implications for neutron stars.

- The extension to finite temperature is not possible in the simple way that we suggested because, we don’t have from first principles the energy distribution for the quarkyionic matter.

For the future…

- There are other possibilities for include finite temperature to quarkyonic matter.
- we could try to use this method and use some machine learning techniques to obtain an "energy" distribution that allows to obtain the chemical potential as similar as some resolution.
- Use a more refined model that will allow you to have actual fermi-distribution to include the temperature, like the Quarkyonic effective field theory that is in development with Larry McLerran, Dyana Duarte and William Tavares.

THANKS FOR WATCHING!