

Unitarization Schemes at High Energies

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This talk is based on:

- EGSL, M.G. Ryskin, *Phys. Rev. D* **110** (2024) 094007
- M. Maneyro, EGSL, M. Peláez, *Phys. Rev. D* **110** (2024) 074011
- EGSL, M.G. Ryskin, V.A. Khoze, *Phys. Rev. D* **110** (2024) 014002

Outline

■ Unitarization Schemes

⇒ Born amplitudes: Pomeron and Odderon inputs

■ The tension between the TOTEM and ALFA/ATLAS measurements

■ Results

■ Conclusion and Perspectives

- In order to describe the observed increase of $\sigma_{tot}(s)$, the Pomeron should have a supercritical intercept given by $\alpha_P(0) = 1 + \epsilon$ with $\epsilon > 0$
⇒ the behavior of the total cross section for $\alpha_P(0) > 1$ betokens the violation of the Froissart-Martin limit at some energy scale
- It is expected that unitarity can be enforced in high-energy hadron-hadron interactions by the inclusion of the exchange series $P + \overline{P}P + \overline{P}PP + \dots$
⇒ $\alpha_P(0)$ is an effective power representing n -Pomeron exchange processes, $n \geq 1$

Despite the advances in understanding the nature of the Pomeron in the last decades, we still need to learn how to fully compute the contributions from multiple-Pomeron exchange processes with $n \geq 3$

On the other hand, it is well-established that some unitarization schemes sum appropriately rescattering diagrams representing the exchange of several particular multiparticle states

- ⇒ these schemes are primarily based on phenomenological arguments
- ⇒ they are effective procedures for taking into account many of the properties of unitarity in the s -channel or,
at the very least,
for preventing the Froissart-Martin bound for σ_{tot} from being violated
- We focus on two key unitarization schemes: the **eikonal** and the **U -matrix** approaches

Unitarization Schemes

- Certain distinctive features of the high-energy $\mathcal{A}(s, t)$ are better illuminated when examined in the impact parameter **b -representation**:

$$2 \operatorname{Im} H(s, b) = |H(s, b)|^2 + G_{in}(s, b) \quad (1)$$

⇒ $G_{in}(s, b)$ is the inelastic overlap function

- After the integration over two-dimensional impact parameter space:

$$\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{in}(s)$$

where

$$\sigma_{tot}(s) = \frac{4\pi}{s} \operatorname{Im} \mathcal{A}(s, t=0) = 2\pi \int_0^\infty b \, db \, 2 \operatorname{Im} H(s, b)$$

$$\sigma_{el}(s) = \frac{\pi}{s^2} \int_{-\infty}^0 dt |\mathcal{A}(s, t)|^2 = 2\pi \int_0^\infty b db |H(s, b)|^2$$

$$\sigma_{in}(s) = 2\pi \int_0^\infty b db \left(2 \operatorname{Im} H(s, b) - |H(s, b)|^2 \right)$$

□ After defining a function

$$\rho(s, b) = \frac{\operatorname{Re} H(s, b)}{\operatorname{Im} H(s, b)}$$

and solving the quadratic equation for $\operatorname{Im} H(s, b)$ resulting from (1):

$$\operatorname{Im} H(s, b) = \frac{1 \pm \sqrt{1 - (1 + \rho^2) G_{in}(s, b)}}{1 + \rho^2} \quad (2)$$

⇒ We see that $G_{in}(s, b)$ must fit onto the interval

$$0 \leq G_{in}(s, b) \leq (1 + \rho^2)^{-1}$$

where we have required $\text{Im } H(s, b)$ be real.

- The construction of unitarized scattering amplitudes relies on two formal steps:

Step 1. The choice of a Born term $\mathcal{F}(s, t)$ with the crossing-even and crossing-odd parts defined as

$$\mathcal{F}^\pm(s, t) = \frac{1}{2} \left[\mathcal{F}^{pp}(s, t) \pm \mathcal{F}^{\bar{p}p}(s, t) \right]$$

- The correspondent crossing-even and crossing-odd Born amplitudes in \mathbf{b} -space are given by

$$\chi^\pm(s, b) = \frac{1}{s} \int_0^\infty q \, dq J_0(bq) \mathcal{F}^\pm(s, -q^2)$$

Step 2. Consists of writing the scattering amplitude $H_{\bar{p}p}^{pp}(s, b)$ in terms of the Born amplitudes $\chi_{\bar{p}p}^{pp}(s, b)$

⇒ Once this is done, $\mathcal{A}_{\bar{p}p}^{pp}(s, t)$ is finally obtained from the inverse Fourier-Bessel transform of $H_{\bar{p}p}^{pp}(s, b)$:

$$\mathcal{A}_{\bar{p}p}^{pp}(s, t) = s \int_0^\infty b \, db J_0(bq) H_{\bar{p}p}^{pp}(s, b)$$

Eikonal Unitarization

- The eikonal unitarization corresponds to the solution of equation (2) with the minus sign

⇒ The **eikonal scheme (Es)** leads us to the relation

$$H(s, b) = i \left[1 - e^{i\chi(s, b)} \right]$$

so that

$$\mathcal{A}_{[Es]}(s, t) = is \int_0^\infty b \, db \, J_0(bq) \left[1 - e^{i\chi(s, b)} \right]$$

- In the **Es** there is an upper limit on the imaginary part of $H(s, b)$,

$$0 \leq \text{Im}H(s, b) \leq (1 + \rho^2)^{-1}$$

Eikonal Unitarization

- Solving the Unitarity Eq. (1) for $G_{in}(s, b)$ in terms of $\chi(s, b)$ yields

$$G_{in}(s, b) = 1 - e^{-2 \operatorname{Im} \chi(s, b)}$$

- ⇒ The positivity condition on $G_{in}(s, b)$ and the upper limit on $\operatorname{Im} H(s, b)$ restrict the imaginary part of $\chi(s, b)$ over

$$0 \leq \operatorname{Im} \chi(s, b) \leq -\frac{1}{2} \ln \left(\frac{\rho^2}{1 + \rho^2} \right)$$

- ⇒ In the limit of a perfectly absorbing profile $H(s, b)$ and $\chi(s, b)$ are purely imaginary

- ⇒ In this limit we have the asymptotic result $\sigma_{el}/\sigma_{tot} = 1/2$

U-matrix Unitarization

- The *U*-matrix unitarization corresponds to the solution of the unitarity equation (2) with the plus sign
- ⇒ The *U*-matrix scheme (*Us*) leads us to the relation

$$H(s, b) = \frac{\chi(s, b)}{1 - i\chi(s, b)/2}$$

so that

$$\mathcal{A}_{[Us]}(s, t) = is \int_0^\infty b db J_0(bq) \left[\frac{2\chi(s, b)}{\chi(s, b) + 2i} \right]$$

- In the *Us* the $\text{Im}H(s, b)$ is constrained to lie in the interval

$$(1 + \rho^2)^{-1} \leq \text{Im}H(s, b) \leq 2(1 + \rho^2)^{-1}$$

U-matrix Unitarization

- In the black disc and $\rho \rightarrow 0$ limits we have

$$\text{Im}H(s, b) = 2$$

and

$$|H(s, b)|^2 = 4$$

- ⇒ These results lead us to the asymptotic behavior $\sigma_{el}/\sigma_{tot} = 1$
- ⇒ Thus $H(s, b)$ may exceed the black disc limit in this approach

Born Input Amplitudes

■ The input Born amplitudes are associated with Reggeon exchange amplitudes

⇒ The corresponding amplitudes in the b -space are given by

$$\chi_i(s, b) = \frac{1}{s} \int \frac{d^2 q}{2\pi} e^{i\mathbf{q} \cdot \mathbf{b}} \mathcal{F}_i(s, t)$$

where $i = -, +, \mathbb{P}$, and \mathbb{O} .

⇒ The physical amplitudes in b -space are obtained by summing of all possible exchanges:

$$\chi_{\bar{p}p}^{pp}(s, b) = \chi_{\mathbb{P}}(s, b) + \chi_{+}(s, b) \pm \chi_{-}(s, b) \pm \xi_{\mathbb{O}} \chi_{\mathbb{O}}(s, b)$$

⇒ Here $\chi_+(s, b)$ ($\chi_-(s, b)$) is the $C = +1$ ($C = -1$) Reggeon contribution

⇒ $\chi_{\mathbb{P}}(s, b)$ ($\chi_{\mathbb{O}}(s, b)$) is the Pomeron (Odderon) contribution

⇒ $\xi_{\mathbb{O}}$ is the Odderon phase factor

□ $\xi_{\mathbb{O}}$ is associated with the positivity property

□ However, unlike Pomeron, the Odderon is not constrained by positivity requirements

From a theoretical standpoint, this implies that it is not possible to determine the phase of the Odderon mathematically

- Specifically, the Born amplitude for each single exchange is

$$\mathcal{F}_i(s, t) = \beta_i^2(t) \eta_i(t) \left(\frac{s}{s_0} \right)^{\alpha_i(t)}$$

⇒ $\beta_i^2(t)$ is the elastic proton-Reggeon vertex

⇒ $\alpha_i(t)$ is the Regge trajectory

⇒ $\eta_i(t) = -ie^{-i\frac{\pi}{2}\alpha_i(t)}$ is the odd-signature factor

⇒ $\eta_i(t) = -e^{-i\frac{\pi}{2}\alpha_i(t)}$ is the even-signature factor

⇒ $s_0 \equiv 1 \text{ GeV}^2$ is an energy scale

■ For Reggeons with positive charge-conjugation:

$$\beta_+(t) = \beta_+(0) \exp(r_+ t/2)$$

and

$$\alpha_+(t) = 1 - \eta_+ + \alpha'_+ t$$

□ Similarly, the Reggeons with negative charge-conjugation are described by the parameters $\beta_-(0)$, r_- , η_- , and α'_-

■ For Pomeron exchange we adopt

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t + \frac{m_\pi^2}{32\pi^3} h(\tau)$$

where $\alpha_{\mathbb{P}}(0) = 1 + \epsilon$ and

$$h(\tau) = -\frac{4}{\tau} F_\pi^2(t) \left[2\tau - (1 + \tau)^{3/2} \ln \left(\frac{\sqrt{1 + \tau} + 1}{\sqrt{1 + \tau} - 1} \right) + \ln \left(\frac{m^2}{m_\pi^2} \right) \right] \quad (3)$$

with $\epsilon > 0$, $\tau = 4m_\pi^2/|t|$, $m = 1 \text{ GeV}$, and $m_\pi = 139.6 \text{ MeV}$

$\Rightarrow F_\pi(t)$ is the form factor of the pion-Pomeron vertex:

$$F_\pi(t) = \beta_\pi / (1 - t/a_1)$$

$\Rightarrow \beta_\pi$ specifies the value of the pion-Pomeron coupling

\Rightarrow we take the additive quark model relation $\beta_\pi/\beta_P(0) = 2/3$

The third term on the right-hand side of (3) corresponds to pion-loop insertions and is generated by t -channel unitarity

- We investigated two different forms for the proton-Pomeron vertex
- The first vertex, specifying our “Model I”, is given by

$$\beta_{\mathbb{P}}(t) = \beta_{\mathbb{P}}(0) \exp\left(\frac{r_{\mathbb{P}} t}{2}\right)$$

- The second proton-Pomeron vertex, referred to as “Model II”, has the power-like form

$$\beta_{\mathbb{P}}(t) = \frac{\beta_{\mathbb{P}}(0)}{(1 - t/a_1)(1 - t/a_{\mathbb{P}})} \quad (4)$$

⇒ Note that the parameter a_1 in (4) is the same as the one in the expression for $F_{\pi}(t)$

⇒ we fix this parameter at $a_1 = m_{\rho}^2 = (0.776 \text{ GeV})^2$

- The total cross section, the elastic differential cross section, and the ρ parameter are expressed in terms of the physical amplitude $\mathcal{A}_{\bar{p}p}^{pp}(s, t)$,

$$\sigma_{tot}^{pp, \bar{p}p}(s) = \frac{4\pi}{s} \operatorname{Im} \mathcal{A}_{\bar{p}p}^{pp}(s, t = 0)$$

$$\frac{d\sigma^{pp, \bar{p}p}}{dt}(s, t) = \frac{\pi}{s^2} \left| \mathcal{A}_{\bar{p}p}^{pp}(s, t) \right|^2$$

$$\rho^{pp, \bar{p}p}(s) = \frac{\operatorname{Re} \mathcal{A}_{\bar{p}p}^{pp}(s, t = 0)}{\operatorname{Im} \mathcal{A}_{\bar{p}p}^{pp}(s, t = 0)}$$

together with the replacements $\mathcal{A}_{\bar{p}p}^{pp}(s, t) = \mathcal{A}_{[Es]}^{pp, \bar{p}p}(s, t)$ or $\mathcal{A}_{[Us]}^{pp, \bar{p}p}(s, t)$, where

$$\mathcal{A}_{[Es]}^{pp,\bar{p}p}(s, t) = is \int_0^\infty b \, db \, J_0(bq) \left[1 - e^{i\chi_{\bar{p}p}^{pp}(s, b)} \right]$$

and

$$\mathcal{A}_{[Us]}^{pp,\bar{p}p}(s, t) = is \int_0^\infty b \, db \, J_0(bq) \left[\frac{2\chi_{\bar{p}p}^{pp}(s, b)}{\chi_{\bar{p}p}^{pp}(s, b) + 2i} \right]$$

The Odderon input

- The Born amplitude for the Odderon contribution is represented as

$$\mathcal{F}_{\mathbb{O}}(s, t) = \beta_{\mathbb{O}}^2(t) \eta_{\mathbb{O}}(t) \left(\frac{s}{s_0} \right)^{\alpha_{\mathbb{O}}(t)}$$

where $\eta_{\mathbb{O}}(t) = -ie^{-i\frac{\pi}{2}\alpha_{\mathbb{O}}(t)}$

- In the formulation of “**Model III**”, we employ an exponential form factor for the proton-Odderon vertex:

$$\beta_{\mathbb{O}}(t) = \beta_{\mathbb{O}}(0) \exp \left(\frac{r_{\mathbb{O}} t}{2} \right)$$

with $r_{\mathbb{O}} = r_{\mathbb{P}}/2$

The Odderon input

- In the formulation of “Model IV”, we adopt the power-like form for the proton-Odderon vertex:

$$\beta_O(t) = \frac{\beta_O(0)}{(1 - t/m_\rho^2)(1 - t/a_O)}$$

with $a_O = 2a_P$

⇒ The relationship between a_O and a_P that must satisfy the constraint $a_O \geq a_P$ to avoid non-physical amplitudes when using a power-like form factor

The Odderon input

- From the standpoint of QCD (at the lowest order) the $C = +1$ amplitude arises from the exchange of two gluons and the $C = -1$ amplitude from the exchange of three gluons
- Extensive theoretical studies have been directed towards uncovering corrections to these results, particularly in higher orders
- In this scenario, the leading-log approximation allows for the summation of certain higher-order contributions to physical observables in high-energy particle scattering processes
- ⇒ This approach was widely used in the study of the QCD-Pomeron through the BFKL equation

The Odderon input

- ⇒ In BFKL equation terms of the order $(\alpha_s \ln(s))^n$ are systematically summed at high energy (large s) and small strong coupling α_s
- ⇒ The simplistic notion of bare two-gluon exchange gives way to the BFKL Pomeron, which, in an alternative representation, can be seen as the interaction of two reggeized gluons with one another
- Beyond the BFKL Pomeron, the most elementary entity within perturbative QCD is the exchange involving three interacting reggeized gluons
- The evolution of the three-gluon Odderon exchange as energy increases is governed by the BKP equation
- ⇒ A bound state solution of this Odderon equation was obtained with the intercept $\alpha_0(0) = 1$

The Odderon input

- Based on these QCD findings, we adopt in this work the simplest conceivable form for the **Odderon trajectory**:

$$\alpha_{\odot}(t) = 1$$

Results

- The LHC has released exceptionally precise measurements of diffractive processes
- These measurements, particularly the total and differential cross sections obtained from ATLAS and TOTEM Collaborations, enable us to determine the Pomeron and Odderon parameters accurately
- ⇒ However, these experimental results unveil a noteworthy tension between the TOTEM and ATLAS measurements
- ⇒ For instance, when comparing the TOTEM and the ATLAS result for σ_{tot}^{pp} at $\sqrt{s} = 8 \text{ TeV}$, the discrepancy between the values corresponds to 2.6σ

Results

■ In order to systematically explore the tension between **TOTEM** and **ATLAS** results, we perform global fits to pp and $\bar{p}p$ forward scattering data and to pp differential cross-section data while considering two distinct datasets, one with **TOTEM** measurements and the other with **ATLAS** measurements

□ The two data ensembles can be defined as follows:

Ensemble A: $\sigma_{tot}^{pp, \bar{p}p}$ data + $\rho^{pp, \bar{p}p}$ data + **ATLAS** data on $\frac{d\sigma}{dt}$ at 7, 8, and 13 TeV;

Ensemble T: $\sigma_{tot}^{pp, \bar{p}p}$ data + $\rho^{pp, \bar{p}p}$ data + **TOTEM** data on $\frac{d\sigma}{dt}$ at 7, 8, and 13 TeV

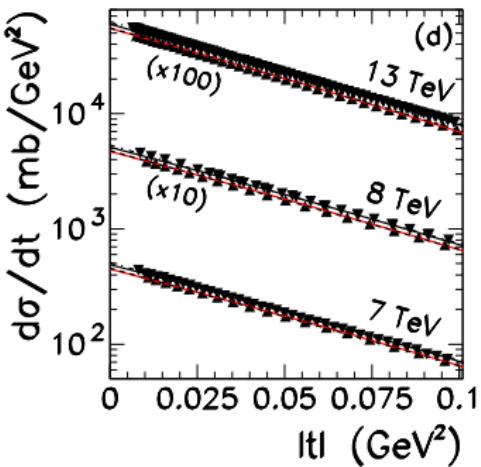
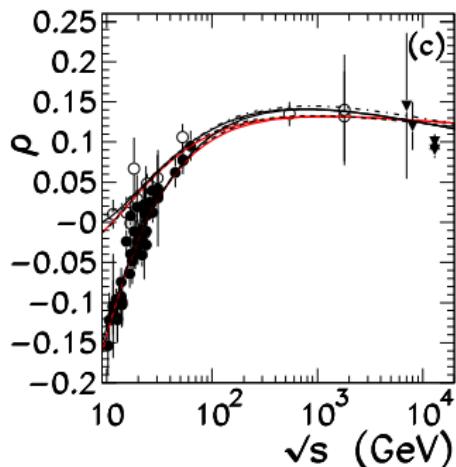
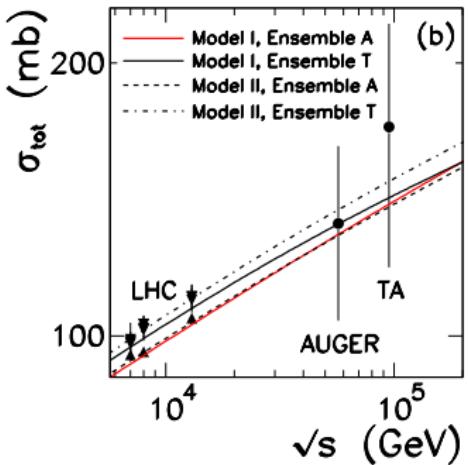
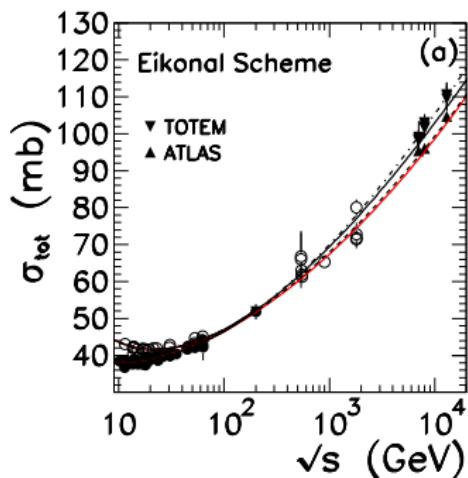
⇒ We carry out global fits to the two distinct ensembles using a χ^2 fitting procedure, where χ^2_{min} follows a χ^2 distribution with ν DoF

⇒ We adopt an interval $\chi^2 - \chi^2_{min}$ corresponding to a 90% confidence level (CL).

Pomeron Analysis

Table: Pomeron parameter values obtained in global fits to Ensembles A and T after the eikonal unitarization.

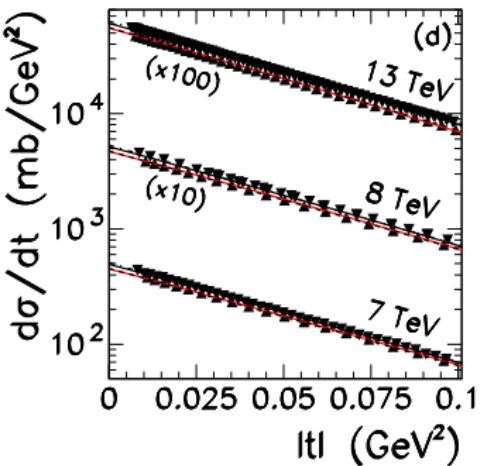
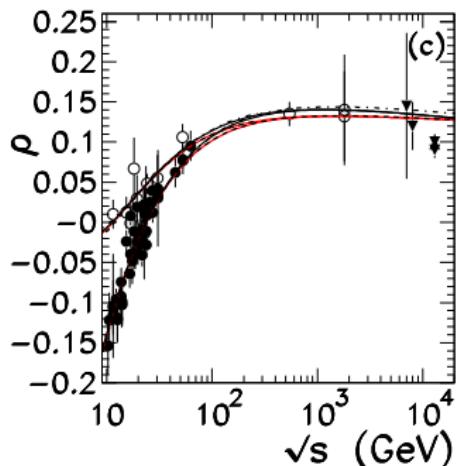
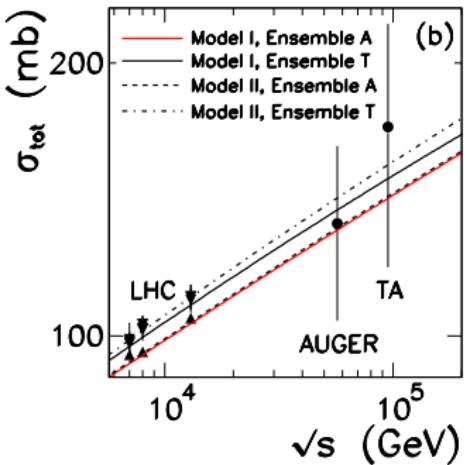
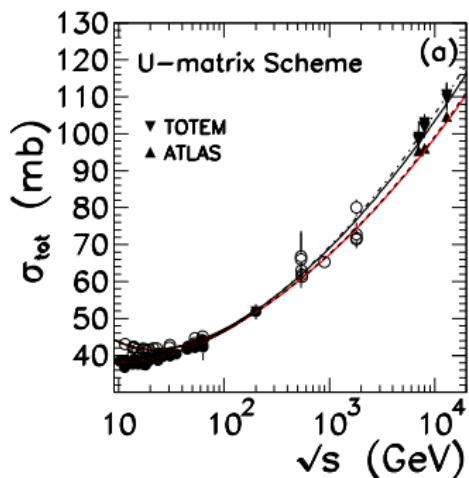
	Ensemble A		Ensemble T	
	Model I	Model II	Model I	Model II
ϵ	0.1014 ± 0.0033	0.1112 ± 0.0013	0.1248 ± 0.0027	0.1336 ± 0.0023
α'_P (GeV $^{-2}$)	0.2938 ± 0.0022	0.1148 ± 0.0076	$0.56 \times 10^{-9} \pm 0.11$	0.009 ± 0.040
$\beta_P(0)$	2.154 ± 0.063	1.999 ± 0.023	1.814 ± 0.043	1.742 ± 0.028
r_P (GeV $^{-2}$)	2.375 ± 0.019	—	7.448 ± 0.087	—
a_P (GeV $^{-2}$)	—	0.829 ± 0.081	—	0.499 ± 0.084
ν	226	226	350	350
χ^2/ν	0.86	0.83	0.74	0.65



Pomeron Analysis

Table: Pomeron parameter values obtained in global fits to Ensembles A and T after the *U*-matrix unitarization.

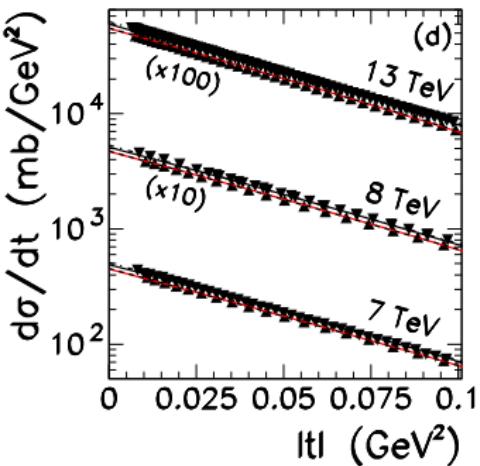
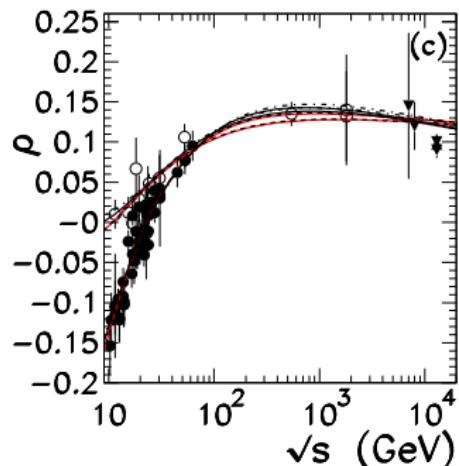
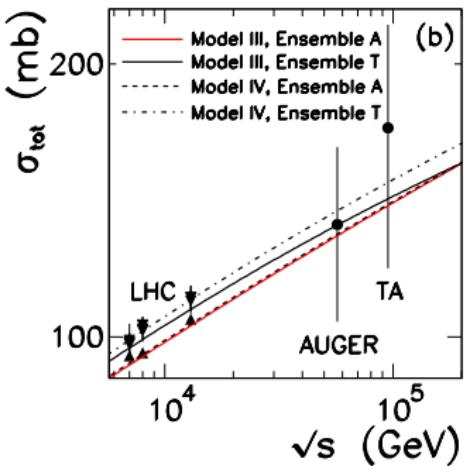
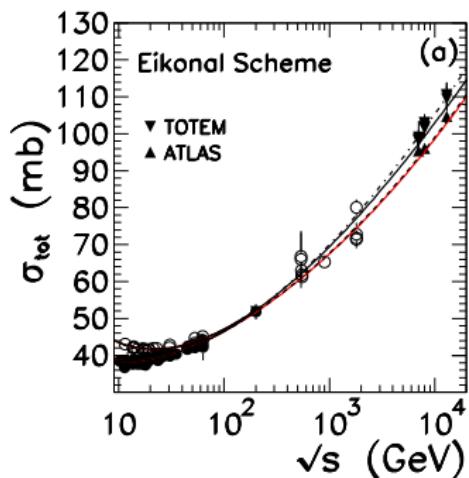
	Ensemble A		Ensemble T	
	Model I	Model II	Model I	Model II
ϵ	0.0911 ± 0.0037	0.0981 ± 0.0029	0.1129 ± 0.0048	0.1150 ± 0.0070
α'_P (GeV $^{-2}$)	0.4425 ± 0.0085	0.2728 ± 0.0089	0.05 ± 0.14	0.10 ± 0.12
$\beta_P(0)$	2.271 ± 0.075	2.140 ± 0.056	1.926 ± 0.085	1.92 ± 0.11
r_P (GeV $^{-2}$)	0.1051 ± 0.0061	—	7.2 ± 2.8	—
a_P (GeV $^{-2}$)	—	40 ± 20	—	0.62 ± 0.49
ν	226	226	350	350
χ^2/ν	0.85	0.86	0.71	0.64



Pomeron \oplus Odderon Analysis

Table: Pomeron and Odderon parameter values obtained in global fits to Ensembles A and T after the eikonal unitarization. We show the results with $\xi_0 = -1$.

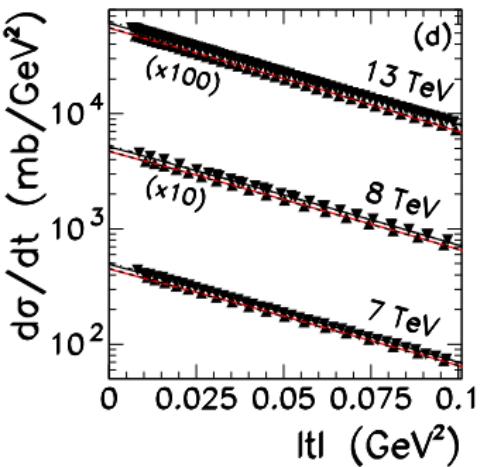
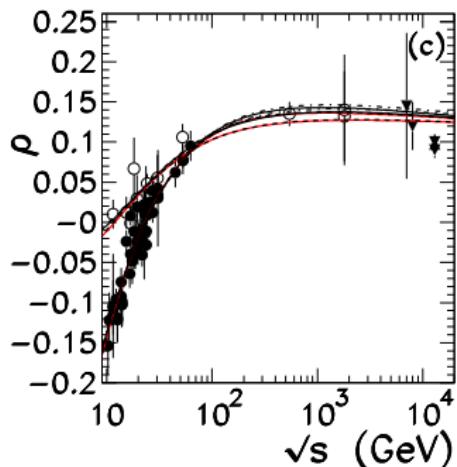
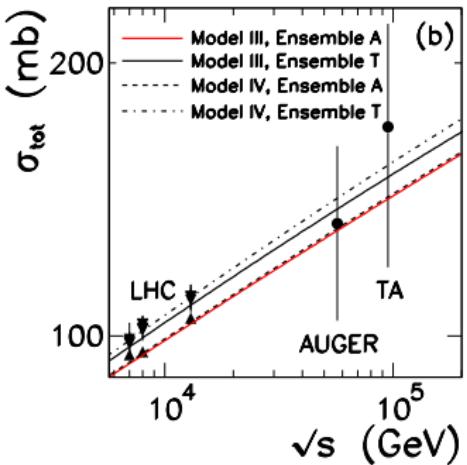
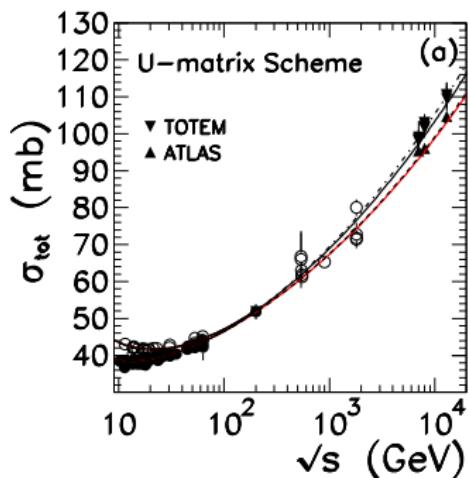
	Ensemble A		Ensemble T	
	Model III	Model IV	Model III	Model IV
ϵ	0.1017 ± 0.0043	0.1043 ± 0.0026	0.1247 ± 0.0048	0.1335 ± 0.0041
α'_P (GeV $^{-2}$)	0.283 ± 0.036	0.242 ± 0.012	$0.94 \times 10^{-4} \pm 0.059$	0.01 ± 0.11
$\beta_P(0)$	2.146 ± 0.083	2.116 ± 0.011	1.815 ± 0.080	1.744 ± 0.035
r_P (GeV $^{-2}$)	2.58 ± 0.68	—	7.45 ± 0.13	—
a_P (GeV $^{-2}$)	—	31 ± 11	—	0.50 ± 0.16
$\beta_0(0)$	0.47 ± 0.24	0.40 ± 0.17	0.31 ± 0.24	0.27 ± 0.20
ν	225	225	349	349
χ^2/ν	0.84	0.80	0.73	0.65



Pomeron \oplus Odderon Analysis

Table: Pomeron and Odderon parameter values obtained in global fits to Ensembles A and T after the *U-matrix unitarization*. We show the results with $\xi_0 = -1$.

	Ensemble A		Ensemble T	
	Model III	Model IV	Model III	Model IV
ϵ	0.0938 ± 0.0045	0.0978 ± 0.0047	0.1115 ± 0.0035	0.1148 ± 0.0060
α'_P (GeV $^{-2}$)	0.364 ± 0.029	0.273 ± 0.031	0.10 ± 0.15	0.106 ± 0.098
$\beta_P(0)$	2.215 ± 0.075	2.146 ± 0.066	1.951 ± 0.063	1.919 ± 0.093
r_P (GeV $^{-2}$)	1.57 ± 0.58	—	6.2 ± 3.0	—
a_P (GeV $^{-2}$)	—	40 ± 24	—	0.63 ± 0.41
$\beta_0(0)$	0.44 ± 0.20	0.23 ± 0.15	0.32 ± 0.18	0.27 ± 0.18
ν	225	225	349	349
χ^2/ν	0.83	0.84	0.71	0.64



Conclusions and Perspectives

- The presence of the **Odderon** immediately impacts the behavior of total cross sections, particularly generating different growth patterns for $\sigma_{tot}^{pp}(s)$ and $\sigma_{tot}^{\bar{p}p}(s)$ at high energies
- ⇒ With an asymptotic non-zero crossing-odd term $\mathcal{A}^-(s, t)$ in the scattering amplitude, it is possible to demonstrate that $|\Delta\sigma|$ can be at most $|\Delta\sigma| = k \ln s$ in the limit $s \rightarrow \infty$, where k is a constant
- After introducing the **Odderon**, the **eikonal scheme** demonstrates a slight advantage over the **U -matrix scheme**, mirroring the scenario where the **Pomeron** is the sole asymptotically dominant entity
- We observe that for an **Odderon** with a phase factor $\xi_0 = +1$, all eight $\beta_0(0)$ values obtained are consistent with zero (errors significantly surpassing central values)

Conclusions and Perspectives

- ⇒ Consequently, the remaining parameters assume values very closely resembling the scenario where the **Pomeron** dominates the scattering amplitude
- The **Odderon phase** is well-defined and is equal to $\xi_0 = -1$
- An ongoing analysis focusing solely on high-energy data, considering exclusively the contributions from **Pomeron** and **Odderon**, is imperative to ascertain the stability of the **Odderon phase factor**
- Ongoing investigations involving a **two-channel model** are underway, focusing on the study of **eikonal** and ***U*-matrix** unitarization schemes within the context of our analysis

THANK YOU

Positivity

- The phase factor is associated with the positivity property
- ⇒ However, unlike **Pomeron**, the **Odderon** is not constrained by positivity requirements
- ⇒ From a theoretical standpoint, this implies that it is not possible to determine the phase of the **Odderon** mathematically
- This issue can be succinctly grasped: in the forward direction the physical amplitudes $\mathcal{F}_{\bar{p}p}^{pp}(s)$ can be written as $\mathcal{F}_{\bar{p}p}^{pp}(s) = F^+(s) \pm F^-(s)$
- Considering that the only relevant contributions are those arising from the **Pomeron** and the **Odderon** exchanges, we can write the symmetric and antisymmetric amplitudes as $F^+(s) = R_{\mathbb{P}}(s) + iI_{\mathbb{P}}(s)$ and $F^-(s) = R_{\mathbb{O}}(s) + iI_{\mathbb{O}}(s)$

□ From the optical theorem, we have $s\sigma_{tot}^{pp,\bar{p}p}(s) = 4\pi \operatorname{Im} \mathcal{F}_{\bar{p}p}^{pp}(s) > 0$, which implies that

$$\operatorname{Im} \mathcal{F}_{\bar{p}p}^{pp}(s) = I_{\mathbb{P}}(s) \pm I_{\mathbb{O}}(s) > 0$$

and, in turn,

$$I_{\mathbb{P}}(s) > |I_{\mathbb{O}}(s)|$$

As a consequence,

$$I_{\mathbb{P}}(s) = \frac{s}{2} \left[\sigma_{tot}^{pp}(s) + \sigma_{tot}^{\bar{p}p}(s) \right] > 0$$

while

$$I_{\mathbb{O}}(s) = \frac{s}{2} \left[\sigma_{tot}^{pp}(s) - \sigma_{tot}^{\bar{p}p}(s) \right]$$

is not bound by the same positivity requirements