Thermal Regge Trajectories

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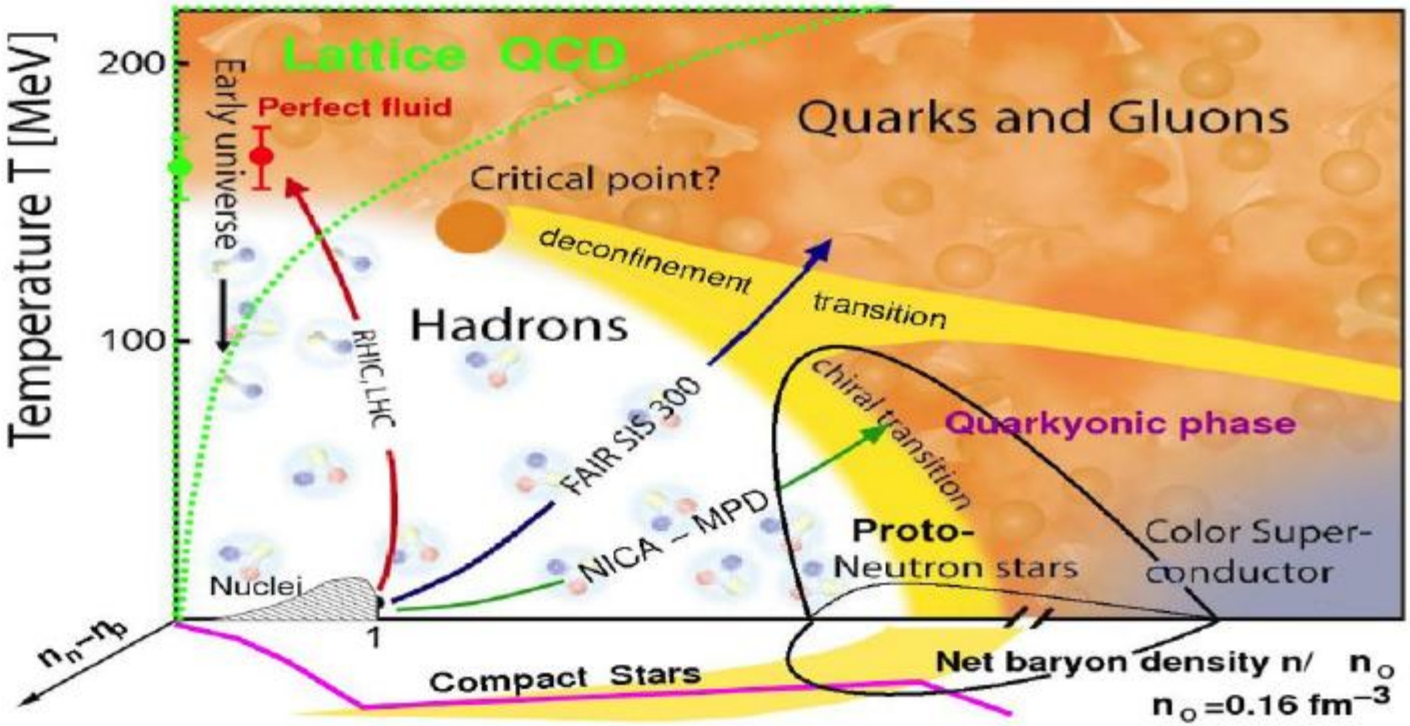
Thermal corrections to Regge trajectories
Phys. Rev. D 112, 056028 (2025) (R.Cádiz, M. Loewe, and R. Zamora)

Relativistic heavy Ion collision experiments (RHIC, LHC, FAIR and NICA in the near future) open a window to explore initial stages of the universe.

A high temperature regime, huge magnetic fields, together with density effects, affect in a dramatic way the physics of strong interactions.

Several phase transitions occur: Deconfinement, Chiral Restoration, transition to a Quarkyonic phase

QCD phase diagram



Here I want to mention some recent results for thermal effects in the Regge behavior of scattering amplitudes.

In general, we refer to an amplitude as reggeized when it becomes analytical in its dependence on J, in the high energy limit,

$$s \to \infty, t \; finite$$

becoming

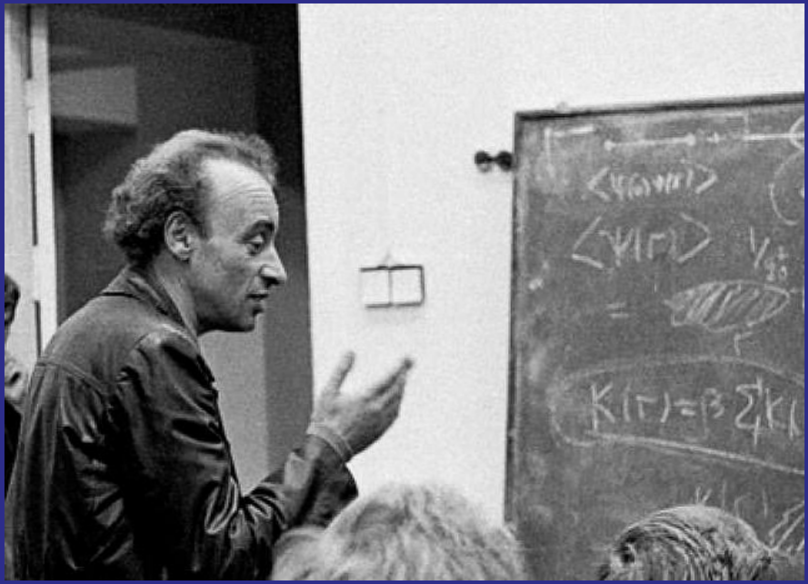
$A(s, t) \sim s^{\alpha(t)}$

This happens, normally, when resummation of diagrams are considered.

See, for example: P. D. Collins: “An Introduction to Regge Theory and High Energy Physics”, Cambridge University Press and references therein. ,

Tulio Regge

Vladimir Gribov



Reggeized gluon (bold corkscrew line) represented as the sum of all leading-lns corrections to the single-gluon exchange amplitude for qq→qq scat

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The Regge limit appears in
the above mentioned situation.

In this limit, the scattering amplitude (according to our present understanding) is dominated by the exchange of effectively composite systems of gluons in QCD, like Pomerons (or odderons).

From the book “Quantum Chromodynamics at High Energy”
by Yuri V. Kovchegov and Eugene Levin

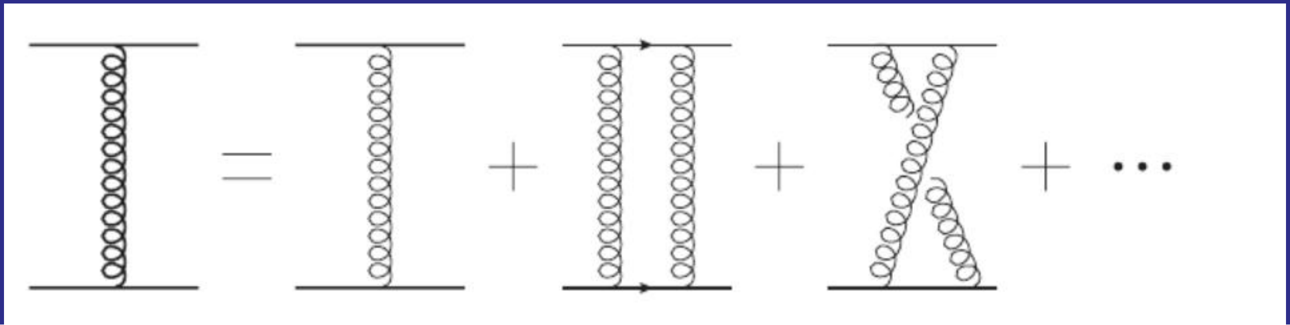


Fig. 3.10. Reggeized gluon (bold corkscrew line) represented as the sum of all leading-ln s corrections to the single-gluon exchange amplitude for $qq \to qq$ scattering.

Essentially, a reggeized gluon is an effective particle Resulting from a quantum superposition of one gluon, two gluons, three gluons…retaining only terms in the amplitude that are leading logatithms.

The effect form such procedure is that instead of writing the usual expresión for the propagator

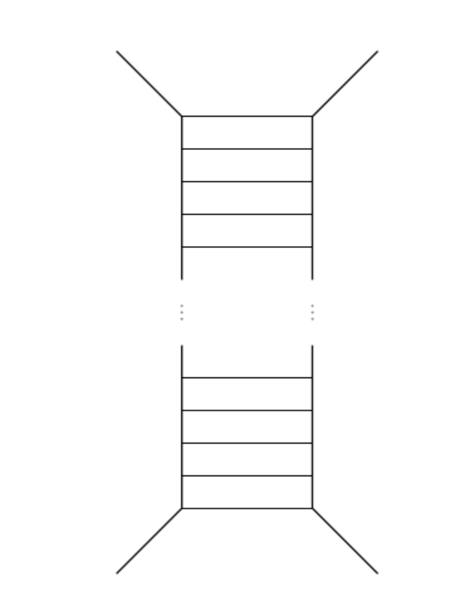
$$D_{\mu\nu} = \frac{ig_{\mu\nu}}{k_\perp^2}$$

you have now

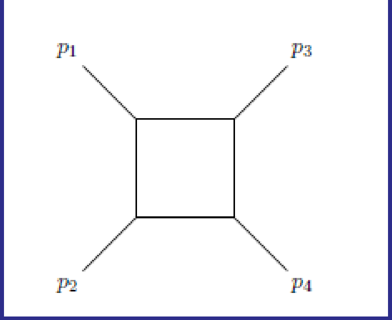
$$D_{(Regge)\mu\nu} = \frac{ig_{\mu\nu} s^{\omega_G(k_\perp)+1}}{k_\perp^2}$$

For our purpose we use (as a first and modest approach) the $\lambda\varphi^3$ theory

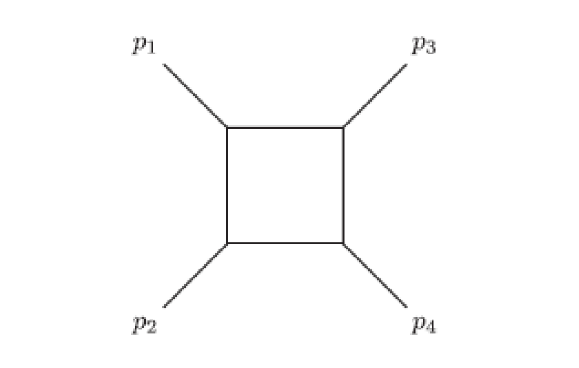
At T = 0, let us considered resummation of a Ladder diagram



The building element is the box diagram



The building blocks of the ladder are box diagrams



$$i\mathcal{M} = \lambda^4 \int \frac{d^4k}{(2\pi)^4} D(k) D(k+p_1) D(k+p_1+p_3) \times D(k-p_2),$$

With $$D(k) = \frac{i}{k^2 - m^2 + i\varepsilon},$$ the usual scalar propagator in Minkowski space

Proceeding in the usual way (Feynman parameters), we have

$$\begin{aligned} i\mathcal{M} &= \lambda^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{k^2 - m^2}\right)\left(\frac{i}{(k+p_1)^2 - m^2}\right)\left(\frac{i}{(k+p_1-p_3)^2 - m^2}\right)\left(\frac{i}{(k-p_2)^2 - m^2}\right) \\ &= \lambda^4 \int \frac{d^4k}{(2\pi)^4} \int \frac{3!\delta(x+y+z+w-1)\,dx\,dy\,dz\,dw}{[x(k^2-m^2) + y((k+p_1)^2 - m^2) + z((k+p_1-p_3)^2 - m^2) + w((k-p_2)^2 - m^2)]^4}. \end{aligned}$$

To continue the calculation, we can manipulate the denominator to isolate the k dependence by the usual completion of squares. After this algebra and with the change of variables $k \to q - yp_1 - z(p_1 - p_3) + wp_2$, we get

$$i\mathcal{M} = 3!\lambda^4 \int_0^1 dx\,dy\,dz\,dw\,\delta(x+y+z+w-1) \cdot \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2-\Delta)^4},$$

$$\Delta = yws - zt(1-z) + 2z(p_1-p_3)(yp_1 - wp_2) - m^2[y+w-1-(y-w)^2],$$

With s and t are the usual Mandelstam variables: $s = (p_1 + p_2)^2$

$t = (p_1 - p_3)^2$

After integrating over q we have

$$\mathcal{M} = \frac{\lambda^4}{96\pi^2} \int \frac{\delta(x+y+z+w-1)dx\,dy\,dz\,dw}{\Delta^2}$$

Remember:

$$\Delta = yws - zt(1-z) + 2z(p_1 - p_3)(yp_1 - wp_2) - m^2[y + w - 1 - (y-w)^2],$$

The philosophy for dealing with this integral is proposed in the book by Collins: Since we are working in the regime where

$$s \rightarrow \infty,$$

the most relevant contribution in the denominator is given by the first term. We can safely neglect other terms containing w and y.

Just focusing on the integrations in w and y we find

$$\int_0^1 \frac{1}{[yws - zt(1-z) + m^2]^2}\, dy\, dw$$
$$= \int_0^1 \frac{1}{[m^2 - zt(1-z)][sy - zt(1-z) + m^2]}\, dy$$
$$= \frac{1}{s[m^2 - zt(1-z)]} \ln\left(\frac{s - zt(1-z) + m^2}{m^2 - zt(1-z)}\right)$$
$$\approx \frac{1}{m^2 - zt(1-z)} \cdot \frac{\ln s}{s}$$

The remaining integrals look like.

The beautiful ln(s) comes from the asymptotic región

$s \longrightarrow \infty$

$$\mathcal{M} = \frac{\lambda^4}{96\pi^2}\left(\frac{\ln s}{s}\right) \int_0^1 \int_0^1 \frac{\delta(x+z-1)}{m^2 - zt(1-z)}\, dx\, dz.$$

Finally, the remaining integrals on x and z can be done. We get

$$\mathcal{M} = \lambda^2 K(t) \left(\frac{\ln s}{s} \right)$$

where

$$K(t) = \frac{\lambda^2}{24\pi^2 \sqrt{t(4m^2 - t)}} \arctan \left(\sqrt{\frac{t}{4m^2 - t}} \right).$$

For the ladder diagram with n rungs, we have then

$$\mathcal{M}_n = \frac{\lambda^2}{s} \frac{[K(t) \ln s]^{n-1}}{(n-1)!}$$

Thus, summing an infinite series of Ladder diagrams yields the asymptotic behavior of the amplitude as

$$\begin{aligned} A(s,t) &= \sum_{n=1}^{\infty} \mathcal{M}_n \\ &= \frac{\lambda^2}{s} \sum_{n=1}^{\infty} \frac{[K(t)\ln s]^{n-1}}{(n-1)!} \\ &= \frac{\lambda^2}{s} \cdot e^{K(t)\ln s} \\ &= \lambda^2 s^{\alpha(t)}, \end{aligned}$$

Identifying, then, the Regge trajectory as

$$\alpha(t) = K(t) - 1.$$

When temperature appears, we have to use the well known prescription

$$\int \frac{d^4q}{(2\pi)^4} f(q) \to \frac{i}{\beta} \sum_n \int \frac{d^3q}{(2\pi)^3} f(\omega_n, \mathbf{q}),$$

ω_n are the bosonic Matsubara frequencies:
$\omega_n = 2\pi n / \beta$ with $\beta = 1/T$

for calculating the box diagram. The result can be decomposed as $\mathcal{M}_0 + \mathcal{M}_T$

$$\mathcal{M}_0 = -\lambda^4 \int_0^1 dx\, dy\, dz\, dw\, \delta(x+y+z+w-1) \times \left(\frac{\partial}{\partial \Delta}\right)^3 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2(\mathbf{q}^2+\Delta)^{1/2}}, \qquad (19)$$

and

$$\mathcal{M}_T = -\lambda^4 \int_0^1 dx\, dy\, dz\, dw\, \delta(x+y+z+w-1) \times \left(\frac{\partial}{\partial \Delta}\right)^3 \int \frac{d^3q}{(2\pi)^3} \frac{1}{(\mathbf{q}^2+\Delta)^{1/2}} e^{-\beta\sqrt{\mathbf{q}^2+\Delta}}. \qquad (20)$$

we have performed the sum over Matsubara frequencies

M_0 is the term we already calculated

$$\mathcal{M} = \lambda^2 K(t) \left(\frac{\ln s}{s} \right)$$

It is posible to show that

$$\mathcal{M}_T = \lambda^2 B(t,\beta)\left(\frac{\ln s}{s}\right)$$

$$B(t,\beta) = -\lambda^2 \int_0^1 dz\, \frac{\beta^2 e^{-\beta\sqrt{tz(1-z)+m}}}{32\pi}$$

Inserting the temperature independent part and summing over an infinite number of rungs we get

$$\begin{aligned} A(s,t,\beta) &= \sum_{n=1}^{\infty} \mathcal{M}_{0,T,n} \\ &= \frac{\lambda^2}{s}\sum_{n=1}^{\infty} \frac{[(K(t)+B(t,\beta))\ln s]^{n-1}}{(n-1)!} \\ &= \frac{\lambda^2}{s}\cdot e^{(K(t)+B(t,\beta))\ln s} \\ &= \lambda^2 s^{\alpha(t,\beta)}, \end{aligned}$$

$$\alpha(t,\beta) = K(t) - 1 + B(t,\beta).$$

Expanding for small values of t

$$\alpha(t) = \alpha_0 + \alpha' t,$$

$$\alpha(t,\beta) = \frac{\lambda^2}{96\pi^2 m^2} - \lambda^2\beta^2 \frac{e^{-m\beta}}{32\pi} + \lambda^2 t \left(\frac{1}{576\pi^2 m^4} - \beta^3 \frac{e^{-m\beta}}{384\pi m} \right).$$

The Regge trajectories for the rho meson

$$\alpha_\rho(t) \approx 0.5 + 0.9t$$

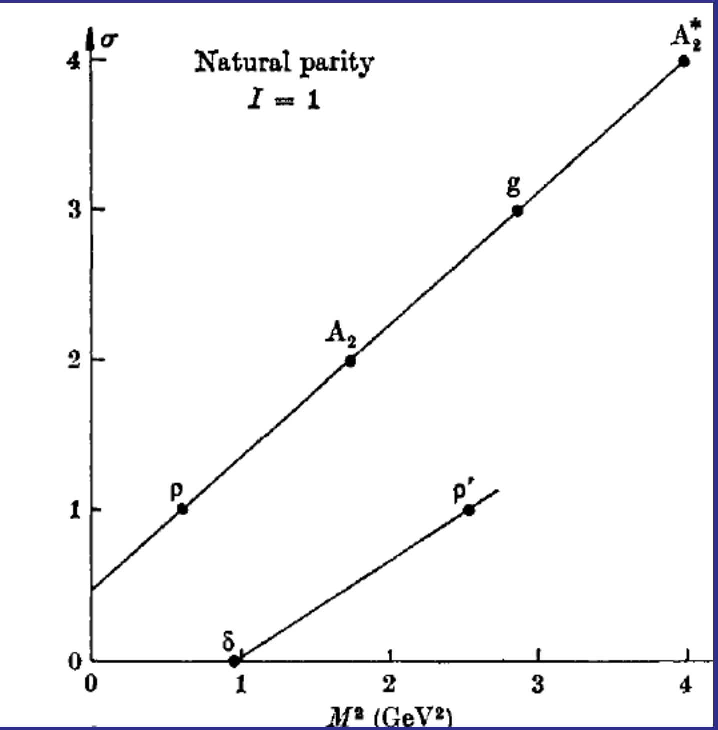


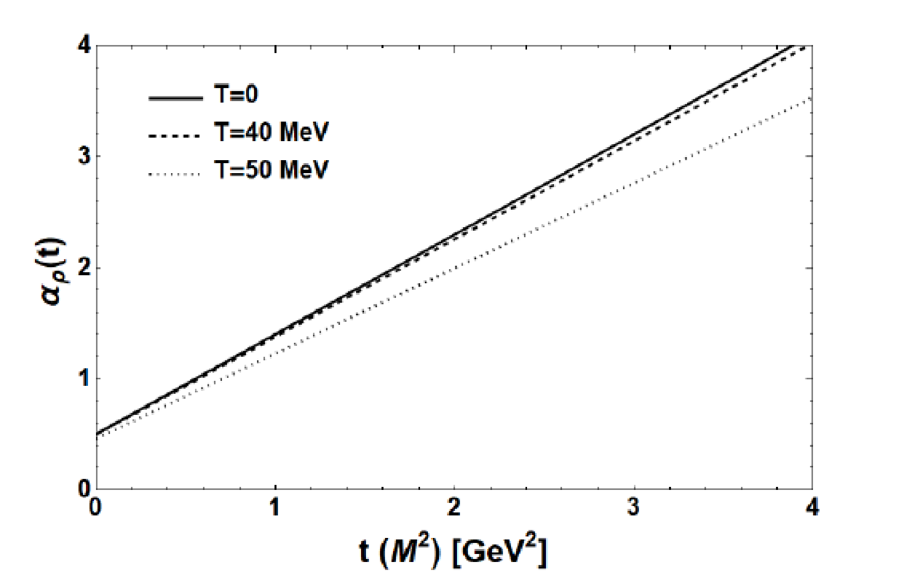
TABLE I. Intercepts α_0 and slopes α' of the linear Regge trajectories $\alpha(t) \approx \alpha_0 + \alpha' t$ for different mesons at $T = 0$.

Meson	α_0	α' [GeV^{-2}]
ρ	0.500 ± 0.002	0.900 ± 0.004
K^*	0.300 ± 0.010	0.900 ± 0.010
ϕ	0.100 ± 0.009	0.900 ± 0.009
π	0.000 ± 0.001	0.800 ± 0.003

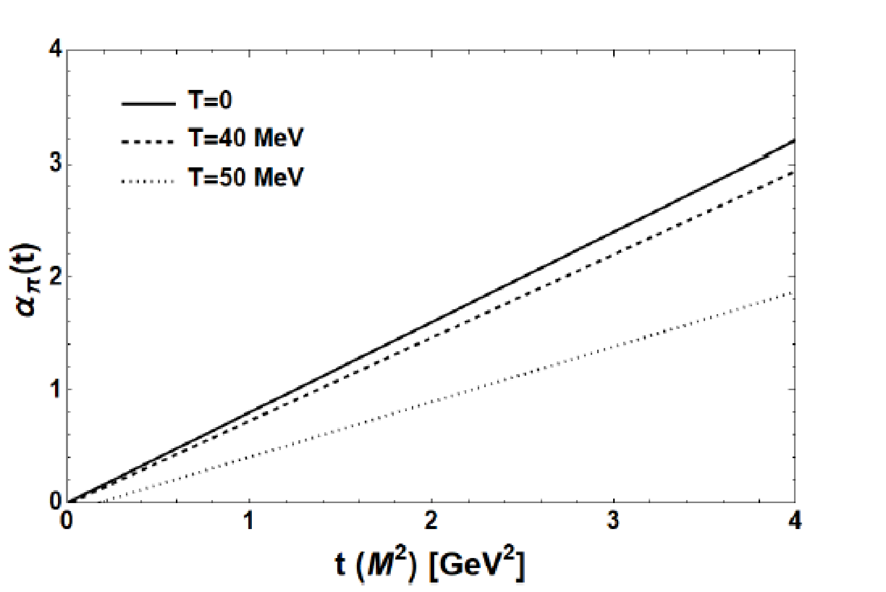
TABLE II. Values of the coupling constant λ and the mass parameter m extracted from the Regge parameters (α_0, α') at $T = 0$ for different mesons.

Meson	λ [GeV]	m [GeV]
ρ	19.869 ± 0.030	0.5270 ± 0.0009
K^*	17.219 ± 0.140	0.4906 ± 0.0042
ϕ	14.570 ± 0.451	0.1630 ± 0.0056
π	14.049 ± 0.140	0.4564 ± 0.0042

The slope diminishes with T



The same happens with other trajectories



This means that the mass increases with T

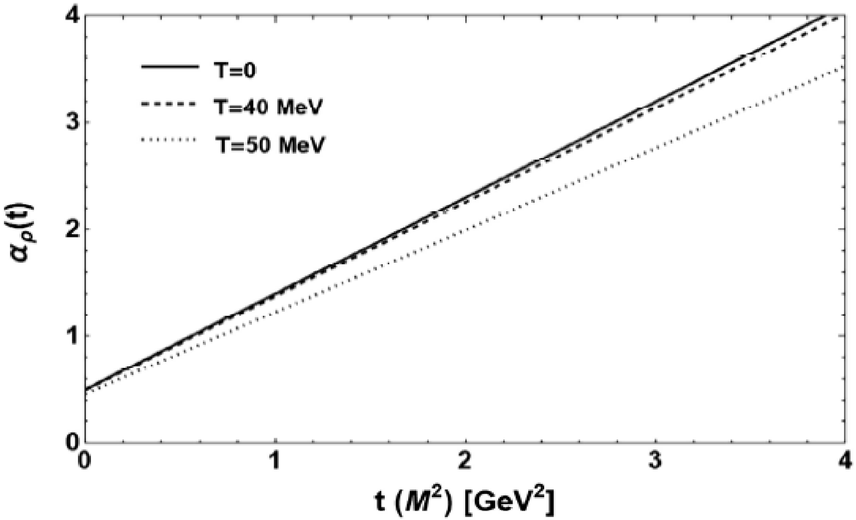


FIG. 3. α_ρ(t) vs t. The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

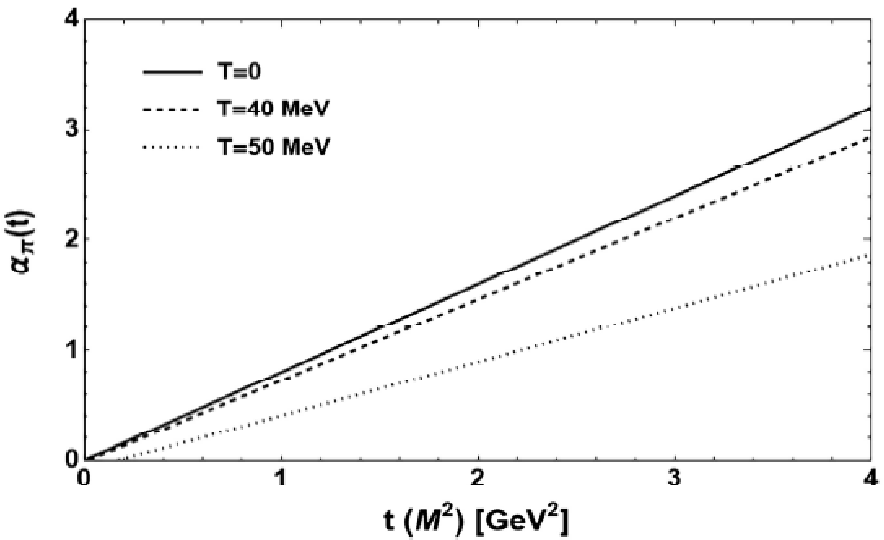


FIG. 4. α_π(t) vs t. The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

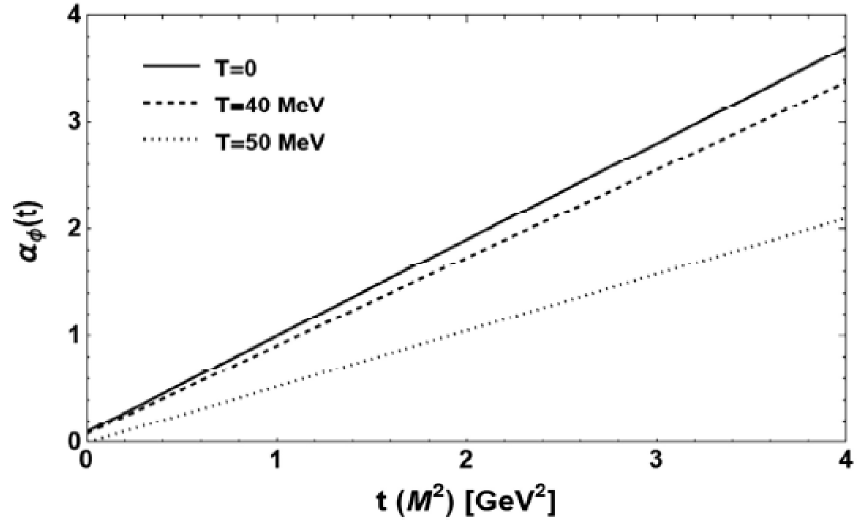


FIG. 5. α_ϕ(t) vs t. The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.

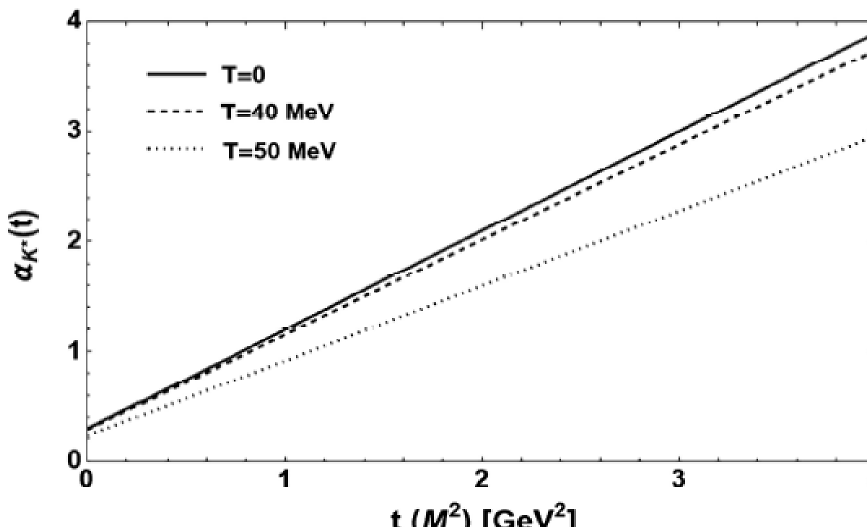
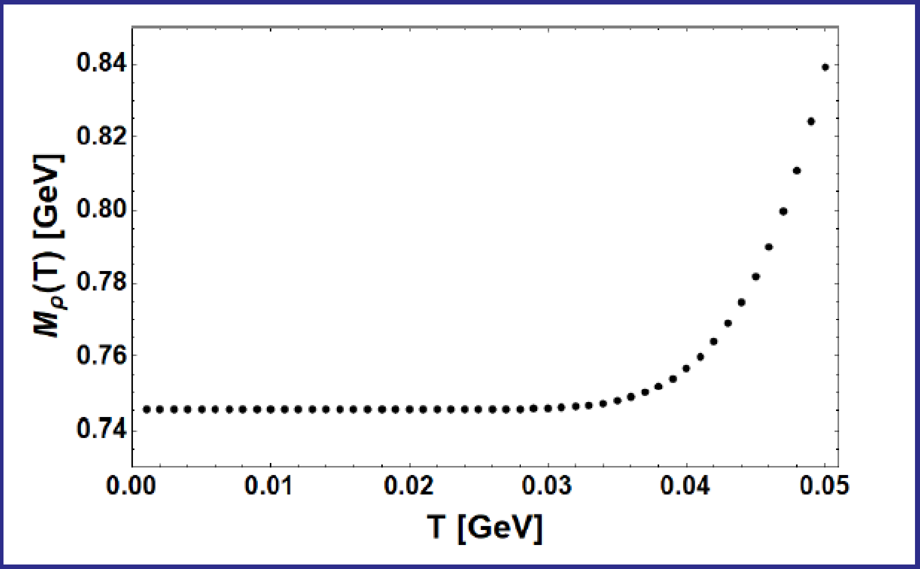


FIG. 6. α_{K^*}(t) vs t. The solid line corresponds to the Regge trajectory at $T = 0$, the dashed line to $T = 40$ MeV, and the dotted line to $T = 50$ MeV.



This behavior is surprisingly similar to the rho-mass evolution obtained from totally different approaches

Temperature dependence of the rho-meson mass and width

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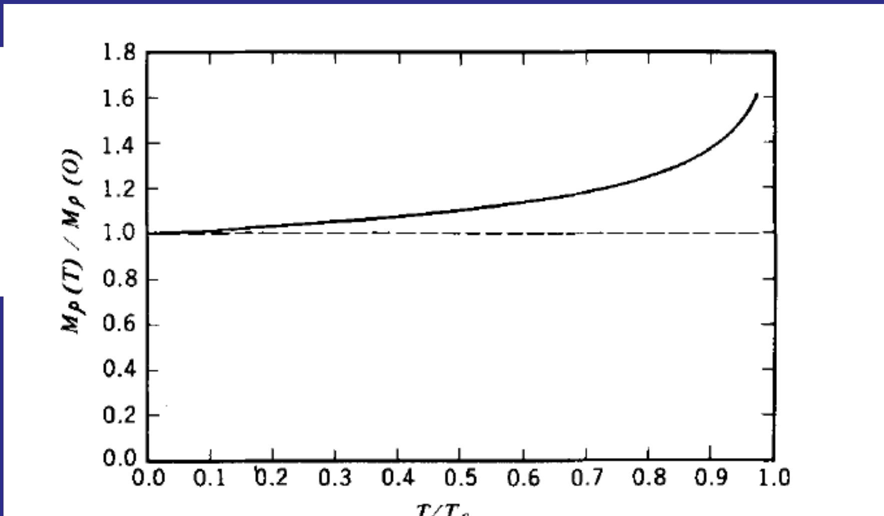

Z. Phys. C 59, 63 (1993)

Our calculations are valid for small temperatures.

In fact, temperature cannot be to high, $T < (t)^{1/2} << S$

We are still in the Regge scenario dominated by the high energy región $s \to \infty$

Possible future work: Exchange of two termal reggeons looking for thermal modifications of branch cuts.

As a kind of conclusión: New approaches or perspectives for physical problems, this time invoking temperature dependence in Regge trajectories are always welcome!!

Thank you!!

Collins technique

$$\int_0^{\varepsilon} \frac{1}{[yws - zt(1-z) + m^2]^2}\, dy\, dw$$

$$= \int_0^{\varepsilon} \frac{\varepsilon}{[m^2 - zt(1-z)][sy\varepsilon - zt(1-z) + m^2]}\, dy$$

$$= \frac{1}{s[m^2 - zt(1-z)]} \ln\left(\frac{s\varepsilon^2 - zt(1-z) + m^2}{m^2 - zt(1-z)}\right)$$

$$\approx \frac{1}{m^2 - zt(1-z)} \cdot \frac{\ln s}{s}$$