Quark gluon vertex from the Curci-Ferrari model: two-loop corrections

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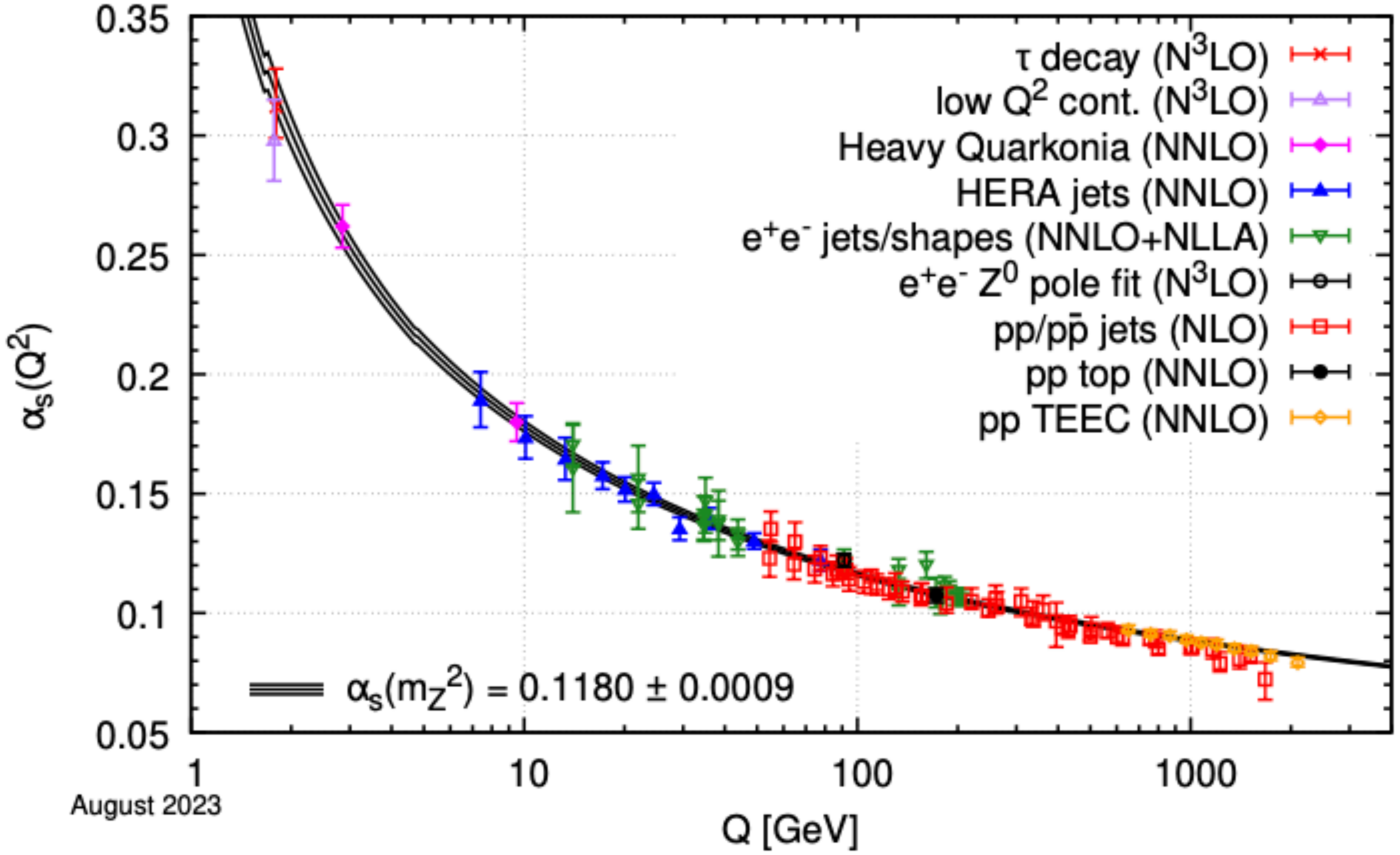
Outline

1. Motivation
2. Correlation functions in Yang-Mills theory
3. Two-point correlation functions in the presence of quarks
4. Quark-gluon vertex at two-loop order
5. Final remarks

Infrared QCD

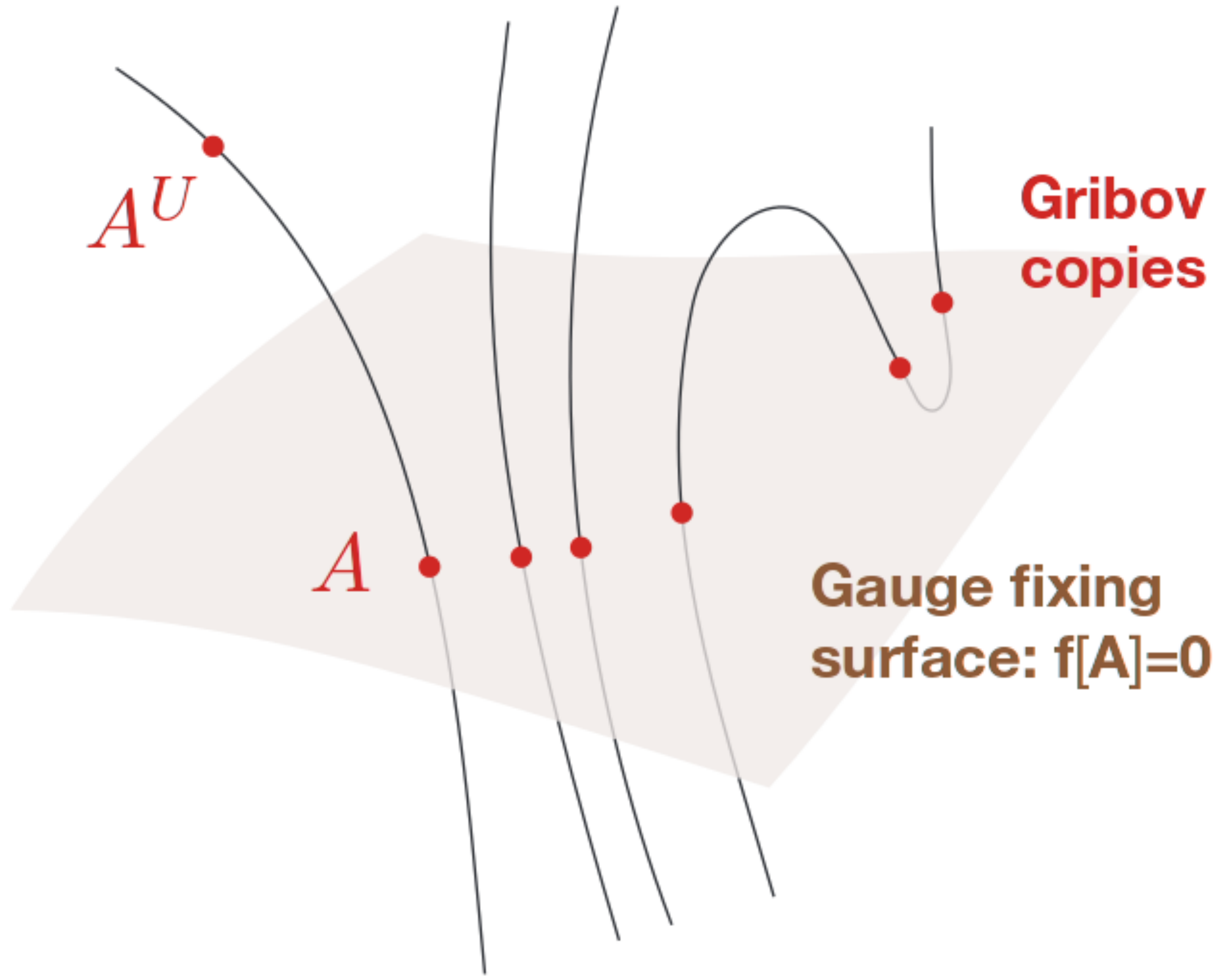
$$\mathscr{L}_{QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\gamma_\mu\partial_\mu - i\mathbf{g}\gamma_\mu A^a_\mu t^a + M_i)\psi_i; \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \mathbf{g}f^{abc}A^b_\mu A^c_\nu$$



Review of Particle Physics - PRD 110.030001 (2024)

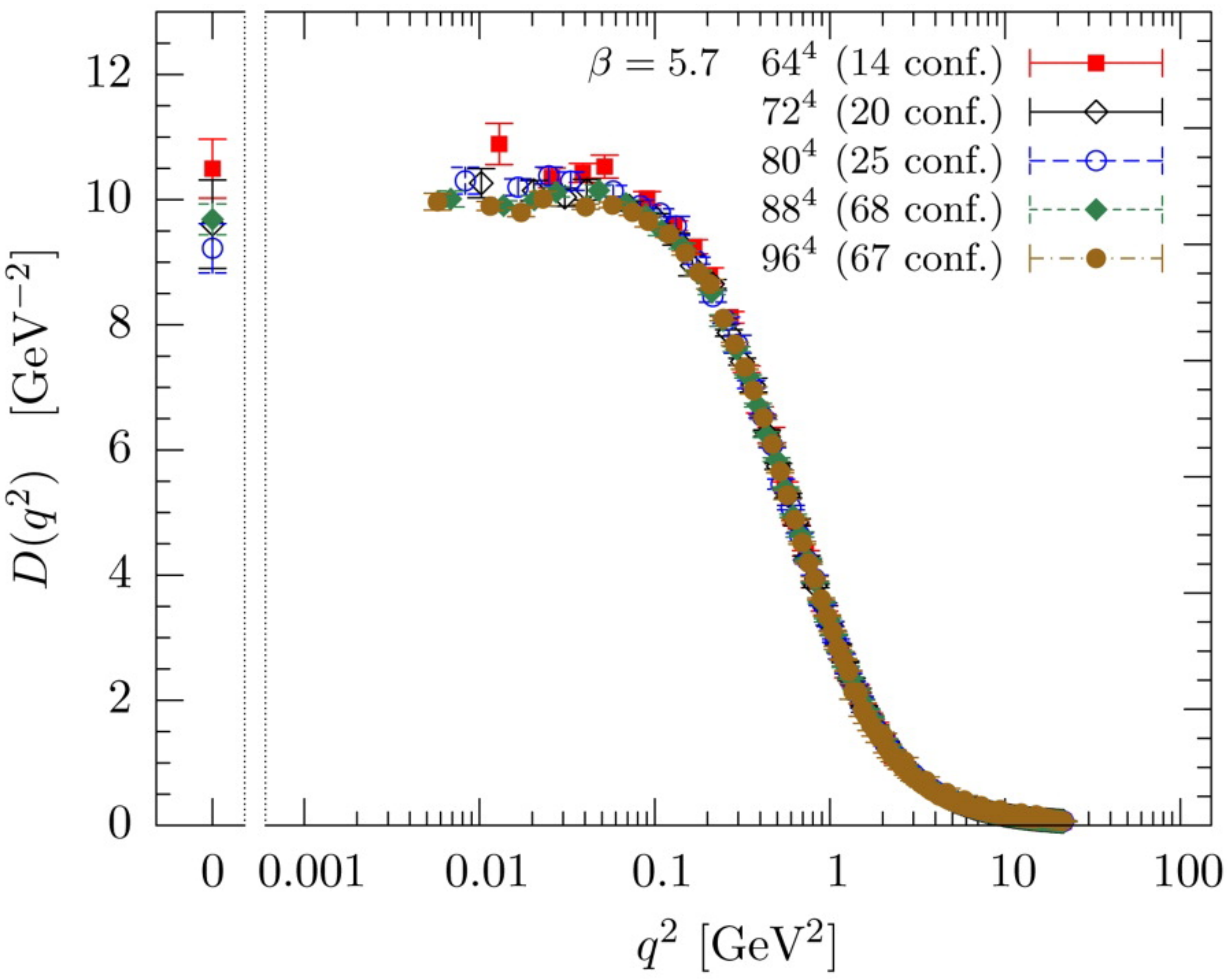
Faddeev-Popov procedure

$$\mathscr{L}_{FP} = \mathscr{L}_{QCD} + ih^a\partial_\mu A^a_\mu + \partial_\mu\bar{c}^a(\partial_\mu c^a + \mathbf{g}f^{abc}A^b_\mu c^c)$$



Faddeev-Popov is not fully justified in the infrared

The Curci-Ferrari model



I.L. Bogolubsky et al. Phys. Lett. B 676 (2009).

$$\mathscr{L}_{CF} = \mathscr{L}_{FP} + \frac{m^2}{2} A_\mu^a A_\mu^a$$ [1]

The model:

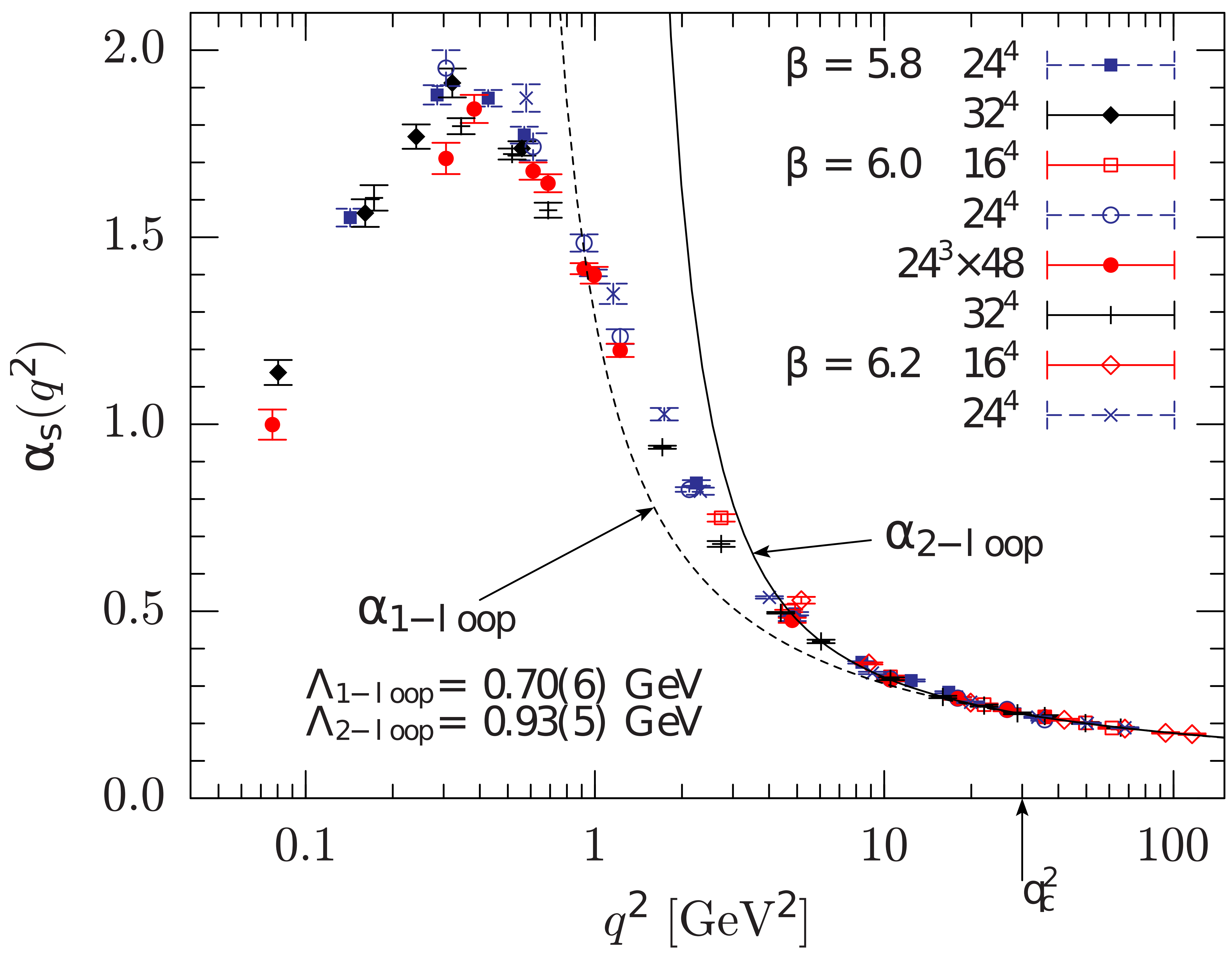
- is a minimal deformation of FP
- is not BRST invariant
- is renormalizable
- retrieves FP in the UV
- is introduced at the gauge fixed level
- is not derived from first principles

[1] G. Curci and R. Ferrari, Nuovo Cim.A 32 (1976) 151-168. M. Tissier and N. Wschebor PRD 82 (2010) 101701.

Perturbation theory - pure gauge

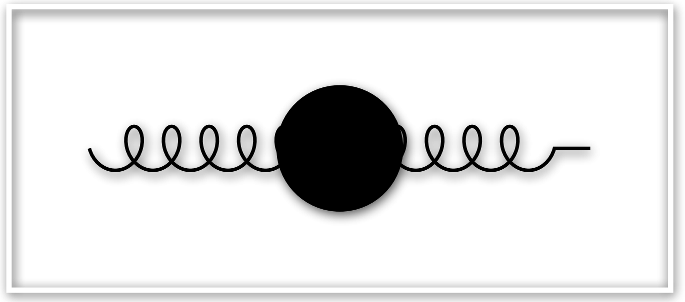
$$\alpha_s = \frac{g^2}{4\pi}$$

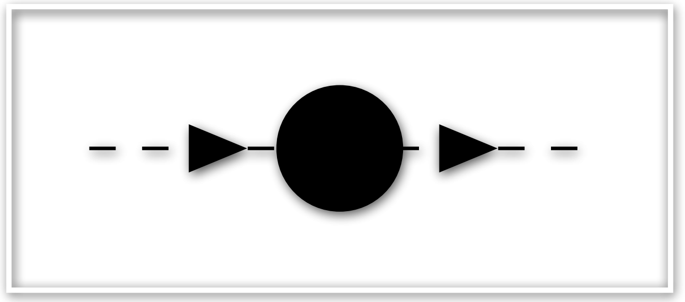
$$\lambda = \frac{\alpha_s N_c}{4\pi}$$



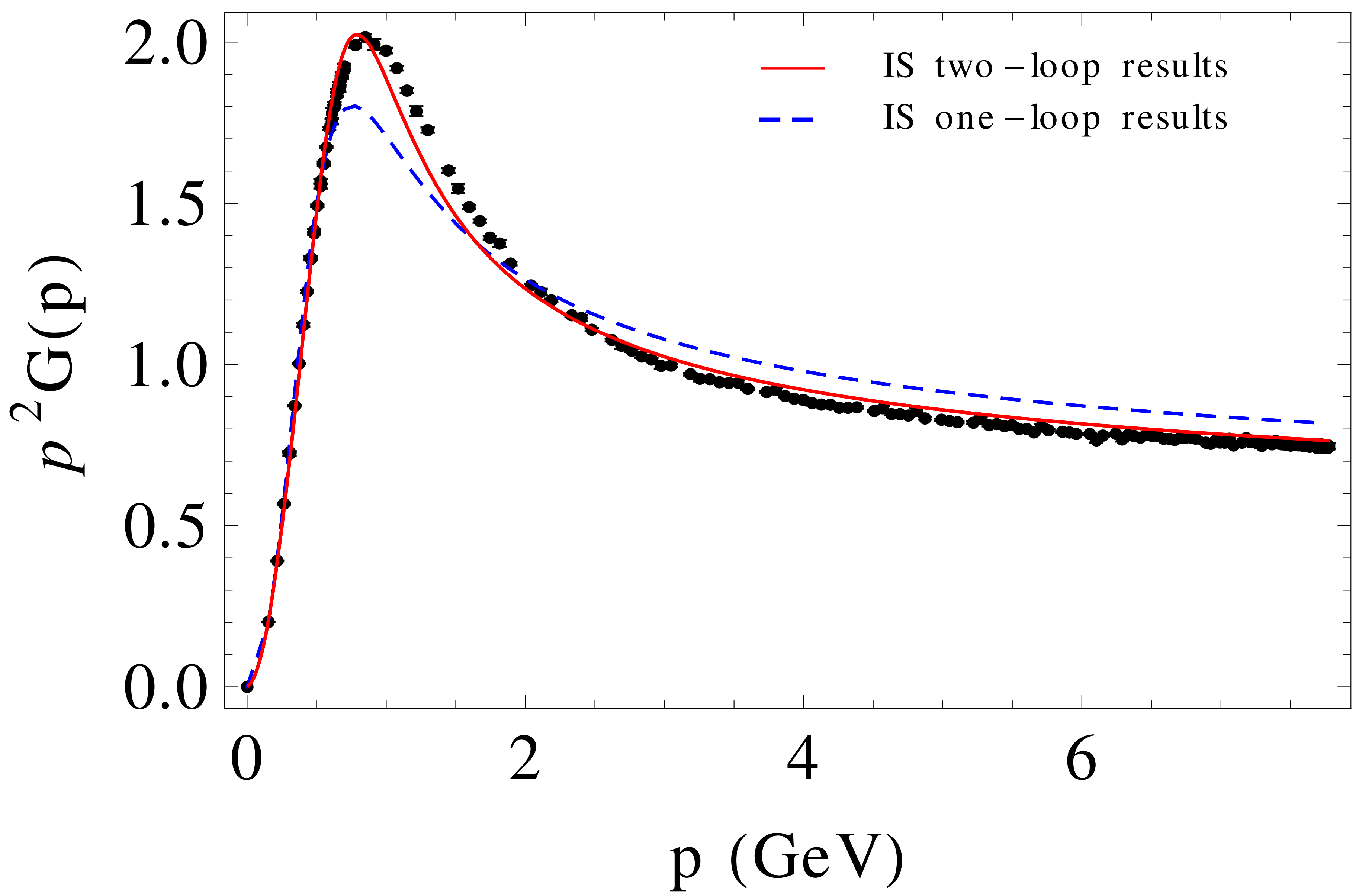
A. Sternbeck et al. Nucl. Phys. B Proc.Suppl. 153 (2006).

Correlation functions



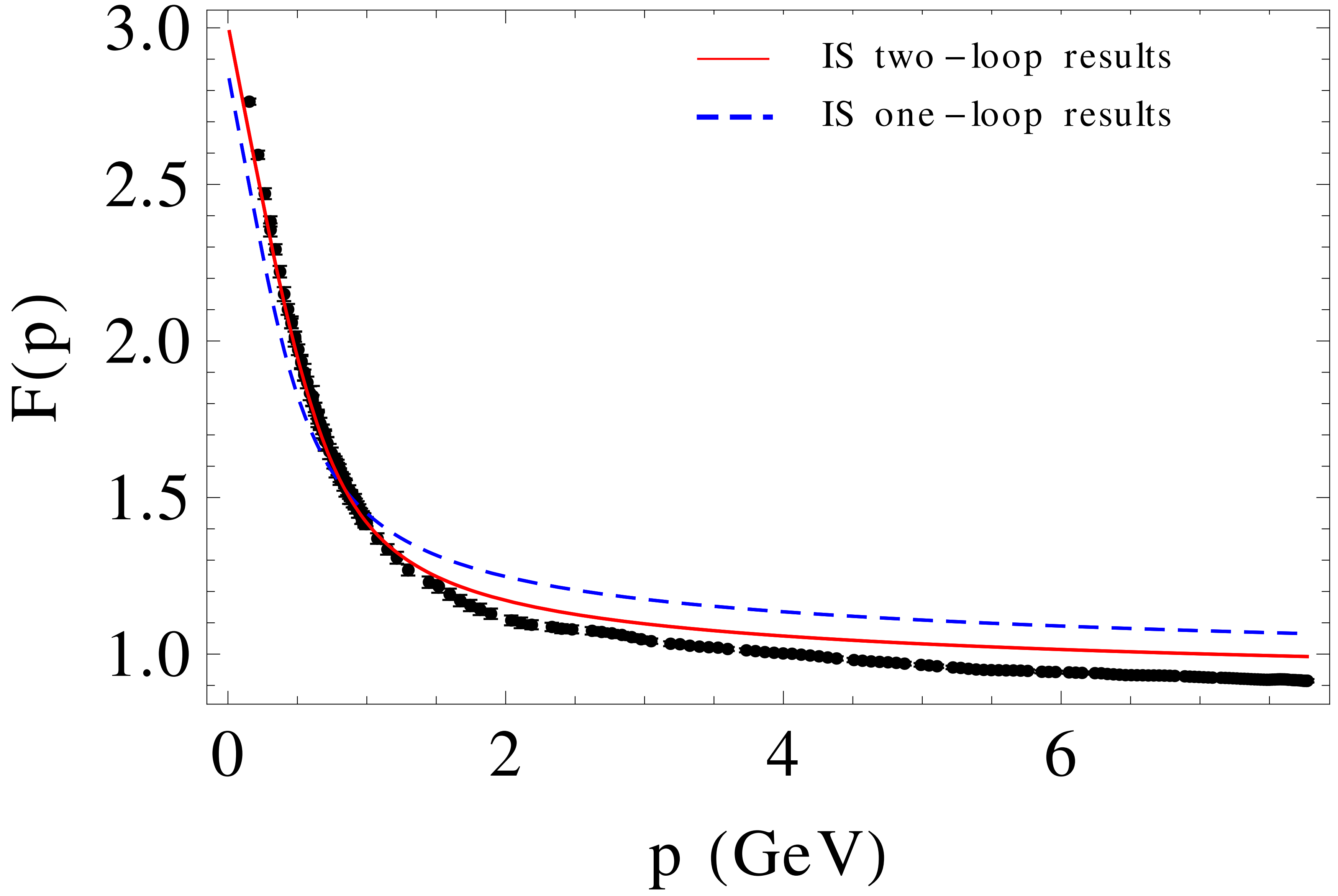


Gluon dressing



Lattice data from D. Dudal, O. Oliveira and P. J. Silva. Annals Phys. 397 (2018).

Ghost dressing



Lattice data from A. G. Duarte, O. Oliveira and P. J. Silva.. Phys. Rev. D 96.9 (2017), 098502.

Plots from J. A. Gracey, M. Peláez, U. Reinosa and M. Tissier. PRD 100, 034023 (2019)

order	λ_0	m_0(MeV)
1 loop	0.30	350
2 loop	0.27	320

Infrared safe renormalization scheme

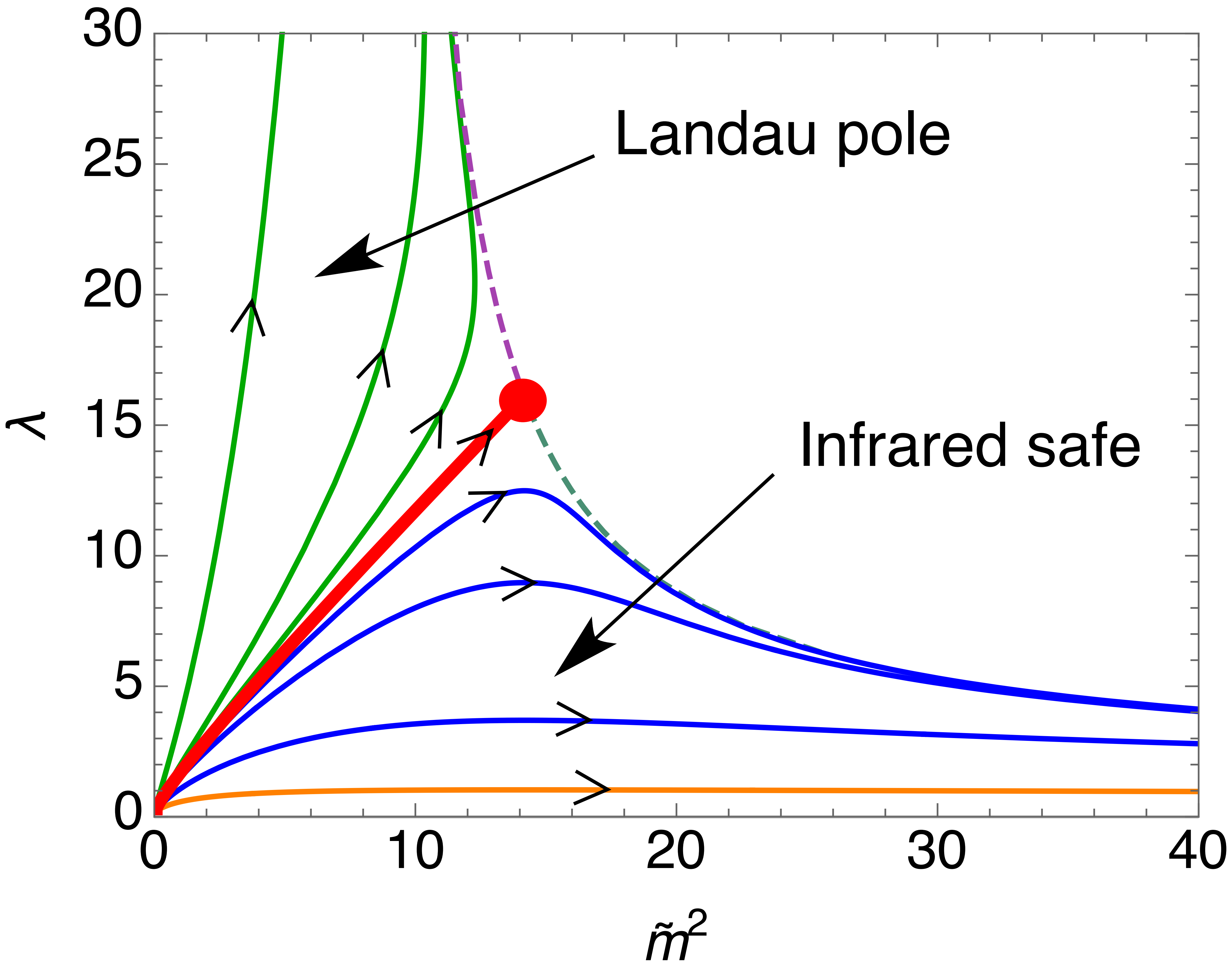
$$\Gamma_A^{(2)}(p=\mu) = m^2 + \mu^2$$

$$\Gamma_{\bar{c}c}^{(2)}(p=\mu) = \mu^2$$

$$Z_g\sqrt{Z_A}Z_c = 1$$

$$Z_{m^2}Z_AZ_c = 1$$

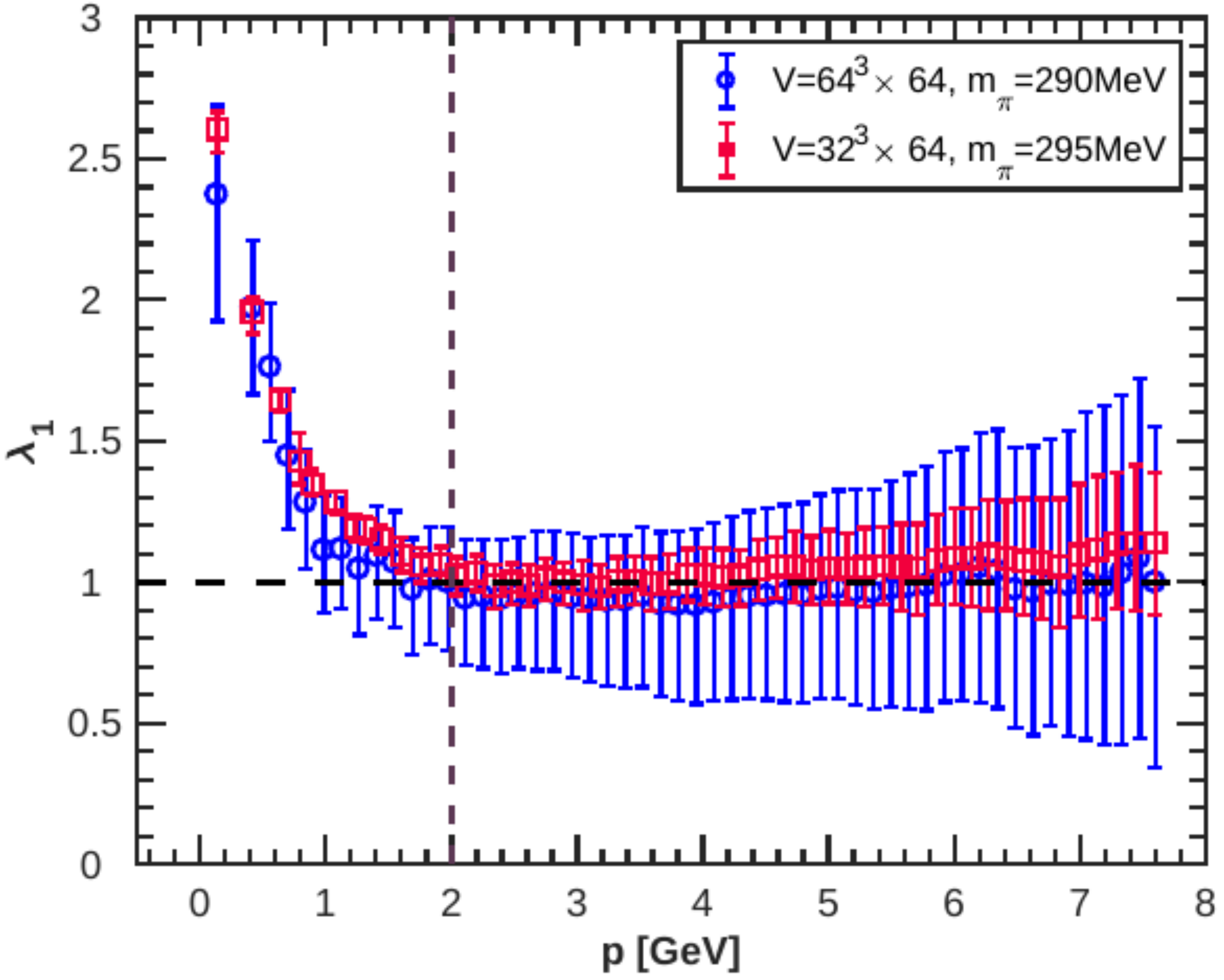
Three- and four-point functions from the CF model are also compatible with lattice simulations



U. Reinosa et at. PRD 96 (2017) 1, 014005.

In the presence of quarks

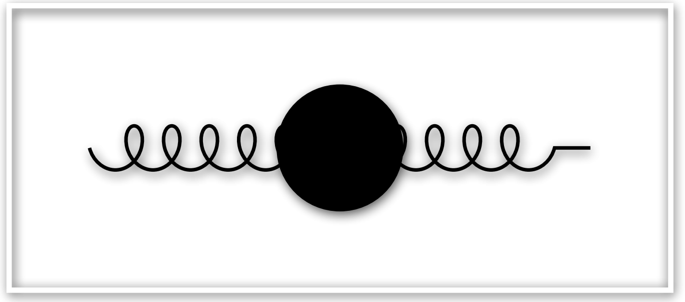
$$\lambda_1 = \frac{g_q}{g_g}$$

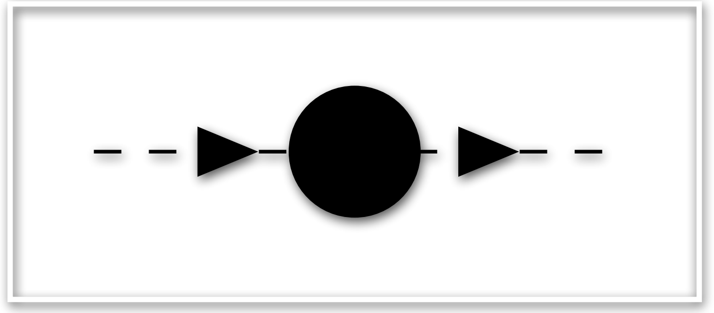


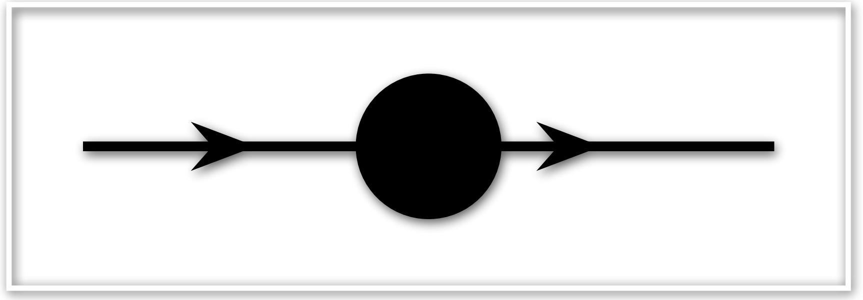
A. Kızılersu et al. PRD 103, 114515 (2021)

- The use of perturbation theory is delicate
- There are phenomena of genuine nonperturbative origin such as spontaneous chiral symmetry breaking

Two-point functions







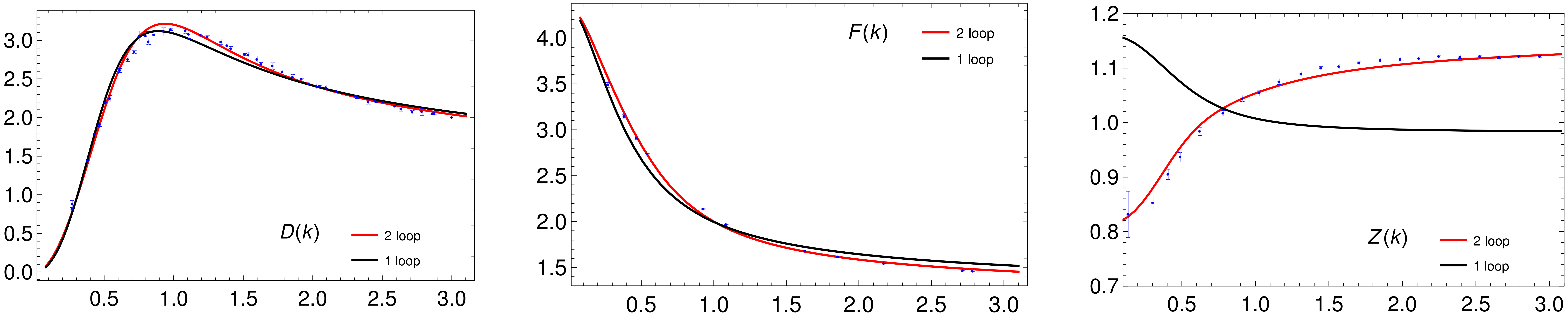
$$S(p) = Z(p)\frac{i\not{p} + \mathbf{1}M(p)}{p^2 + M^2(p)}$$

Two cases:

1. Close to the quiral limit ($m_\pi = 150$ MeV)
2. Heavy quarks ($m_\pi = 422$ MeV)

A) F, D and Z are fitted simultaneously, Nf=2

Gluon (left), ghost (middle) and quark (right) dressing as function of momentum (GeV)



Lattice data from A. Sternbeck et al. PoS LATTICE2012 (2012) (left and middle) and O. Oliveira et al. Phys. Rev. D 99.9 (2019) (right). Lattice data correspond to m_π=150 MeV.

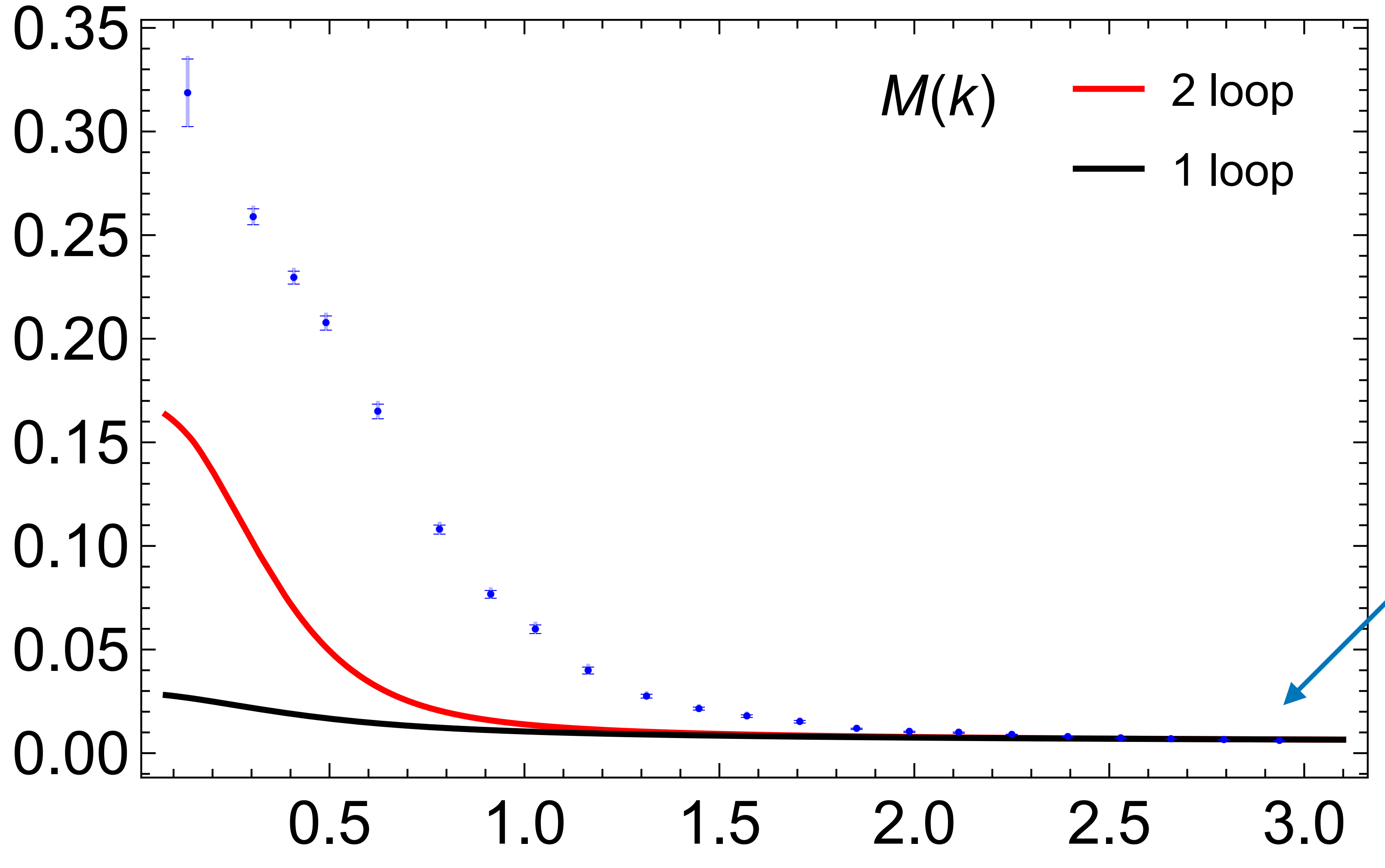
order	λ_0	m_0(MeV)	M_0(MeV)
1 loop	0.33	410	250
2 loop	0.32	370	160

order	χ_D	χ_F	χ_Z
1 loop	3,6%	4,4%	14,9%
2 loop	2,6%	1,5%	1,1%

Plots from **NB**, J. Gracey, M. Peláez and U. Reinosa. PRD 104, 094019 (2021)

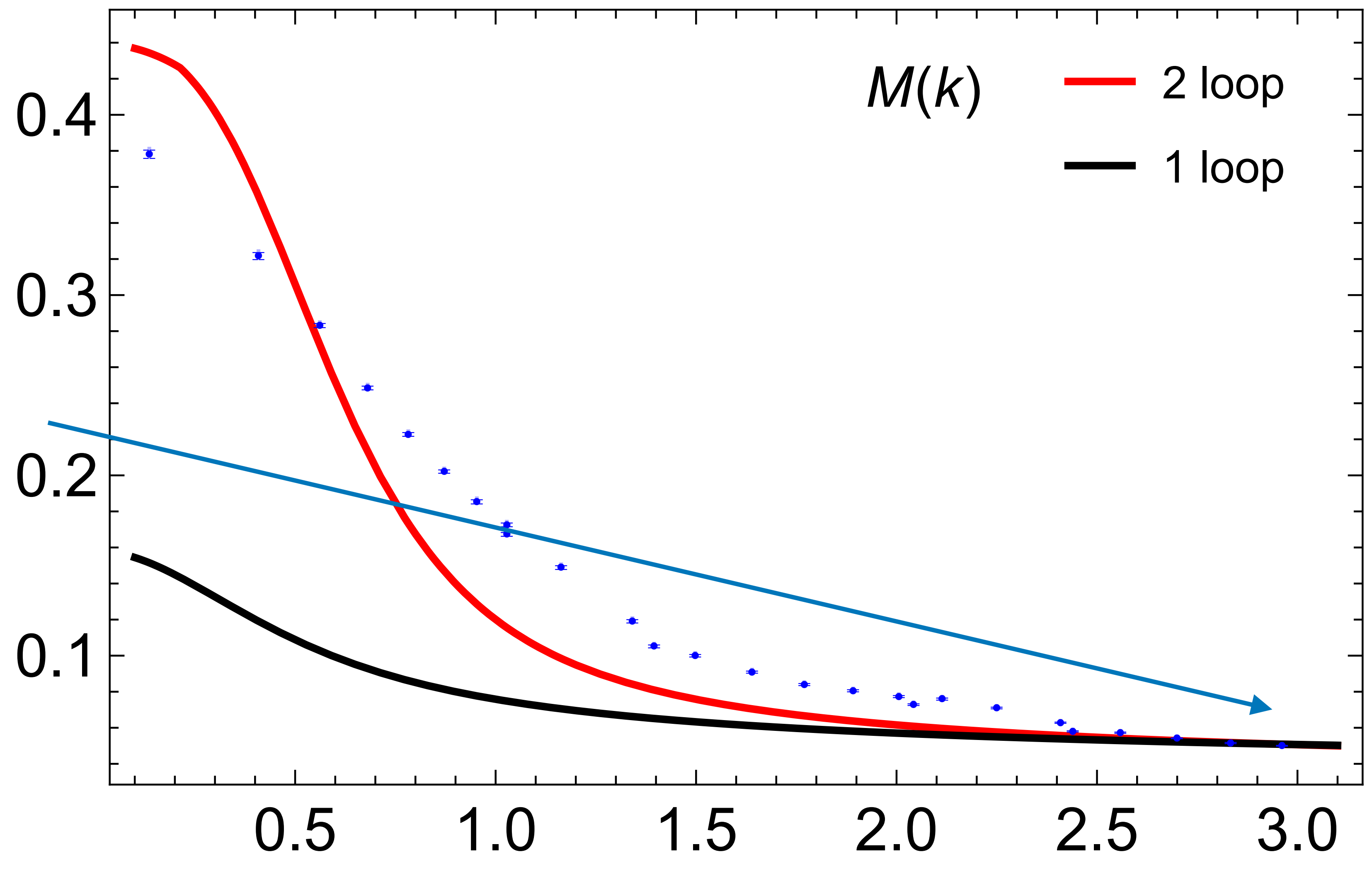
B) F, D, Z and M are fitted simultaneously, Nf=2

M_π=150 MeV



$M(\mu^*) = M_{\text{latt.}}(\mu^*)$

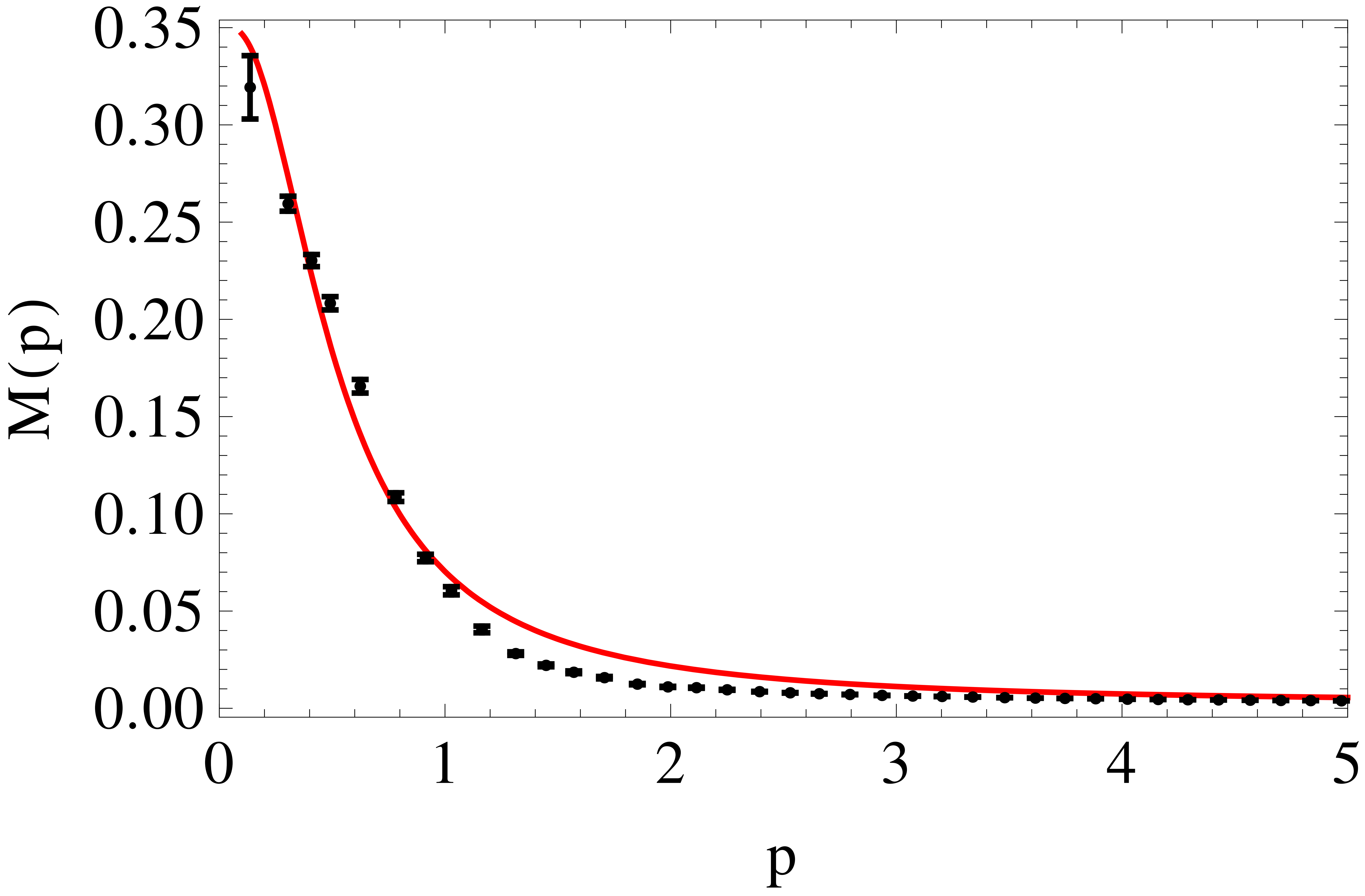
M_π=422 MeV



order	χ_D	χ_F	χ_Z	χ_M
1 loop	3,1%	7,0%	16,9%	50,9%
2 loop	6,6%	3,0%	6,1%	41,8%

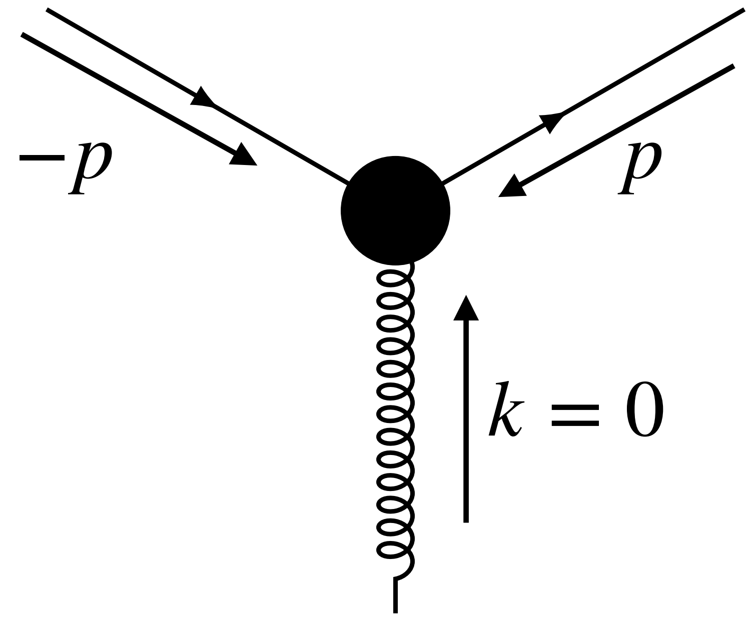
order	χ_D	χ_F	χ_Z	χ_M
1 loop	5,6%	5,3%	12,1%	34,6%
2 loop	4,5%	2,7%	0,9%	16,0%

But… in the case of light quarks the CF model also works well



M. Peláez et al. PRD 103, 094035 (2021)

Quark-gluon vertex in the soft gluon configuration



$$\Gamma_{\bar{\psi}\psi A^a_\mu}(p,r,k) = g t^a \Gamma_\mu(p,r,k)$$

$$\Gamma_\mu(p,-p,0) = -i(\lambda_1(p^2)\gamma_\mu - 4\lambda_2(p^2)\gamma_\alpha p_\alpha p_\mu - 2i\lambda_3(p^2)p_\mu)$$

The vertex emerges as a pure prediction of the CF model

Computation procedure

- We evaluated the diagrams (2 at 1 loop + 37 at 2 loops) and reduced the Feynman integrals to two-loop self-energy master integrals

- Regularization

- Renormalization $\Gamma^R_{\psi\bar{\psi}A^a_\mu}(p,r,k) = t^a Z_\psi \sqrt{Z_A} Z_g g \Gamma^B_\mu(p,r,k)$

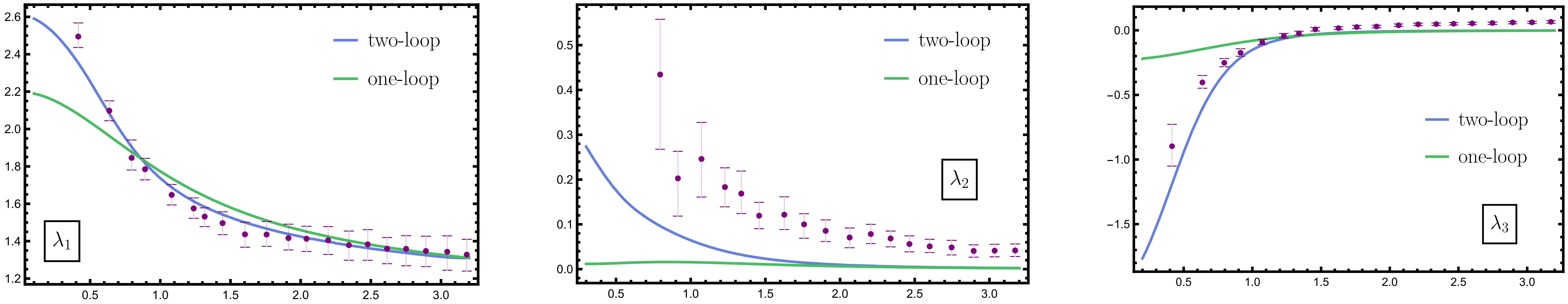
- Renormalization group

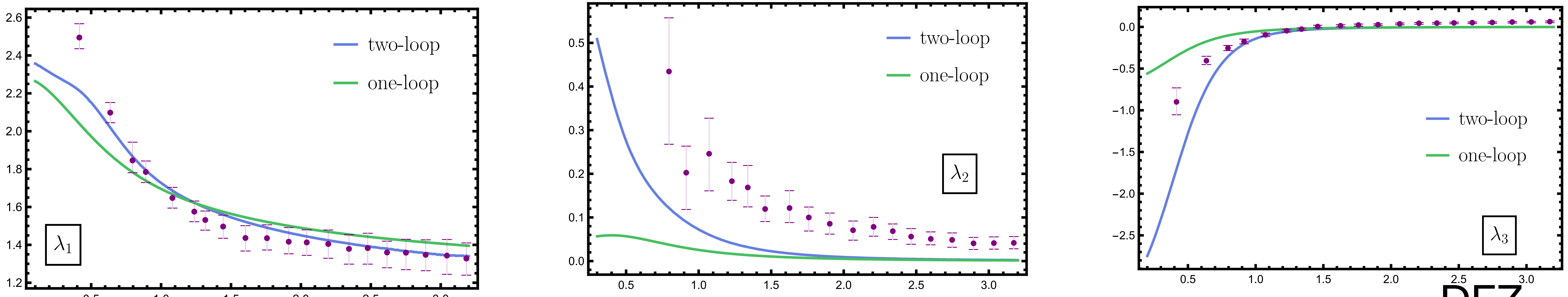
$$\lambda_i^R(g(\mu_0), m(\mu_0), M(\mu_0)) = \frac{Z_\psi \sqrt{Z_A} Z_g}{z_\psi(\mu)\sqrt{z_A(\mu)}} \frac{g(\mu)}{g(\mu_0)} \lambda_i^B(g(\mu), m(\mu), M(\mu))$$

where $\ln z_X = \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_X(g(\mu'), m(\mu'), M(\mu')),$ with $\gamma_X = \frac{d \ln Z_X}{d \ln \mu}$

Preliminary results, M_π=422 MeV, Nf=2

DFZM



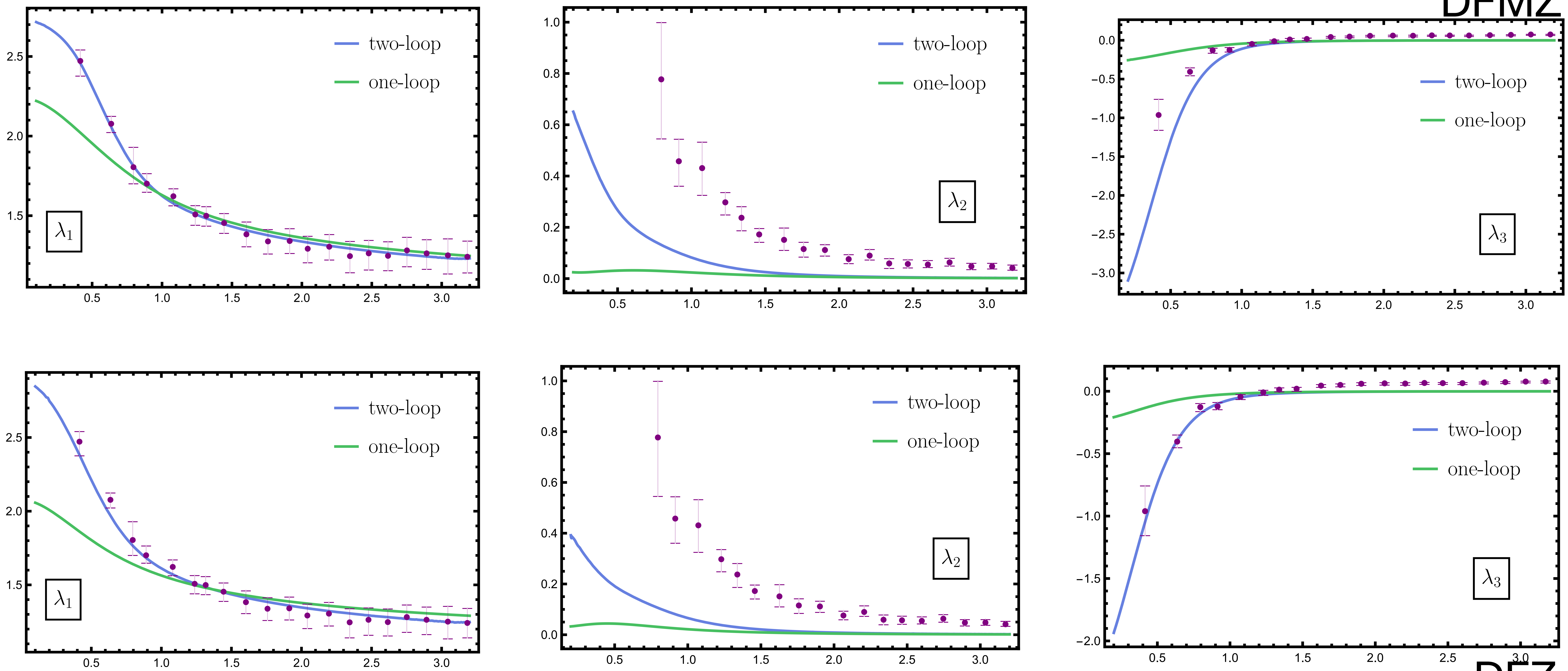


DFZ

Lattice data from A. Kizilersu et al. PRD 103, 114515 (2021)

Preliminary results, M_π=295 MeV, Nf=2

DFMZ



DFZ

Lattice data from A. Kizilersu et al. PRD 103, 114515 (2021)

Analytical asymptotic expressions

$$\lambda_1 \underset{p\to\infty}{\sim} = 1 + a\left(\frac{9}{4} - \mathrm{zc10} + \mathrm{zp10} - \frac{9}{4}\,\mathrm{Log}\left[\frac{p^2}{\mathrm{mub}^2}\right]\right) + a^2\left(\frac{105017}{576} + \frac{1}{16}\,\mathrm{Nf}\left(-159 + 2\pi^2\right) - \frac{9\,\mathrm{za10}}{4} - \frac{9\,\mathrm{za11}}{2} - \frac{9\,\mathrm{za1m1}}{4} - \frac{27\,\mathrm{zc10}}{4} + \mathrm{zc10}^2 - \frac{27\,\mathrm{zc11}}{2} - \frac{27\,\mathrm{zc1m1}}{4} + 2\,\mathrm{zc11}\,\mathrm{zc1m1} - \mathrm{zc20} + \frac{9\,\mathrm{zp10}}{4} - \mathrm{zc10}\,\mathrm{zp10} + \frac{3}{64}\pi^2\left(-53 + 4\,\mathrm{za11} + 12\,\mathrm{zc11} - 4\,\mathrm{zp11}\right) + \frac{9\,\mathrm{zp11}}{2} - \mathrm{zc1m1}\,\mathrm{zp11} + \frac{9\,\mathrm{zp1m1}}{4} - \mathrm{zc11}\,\mathrm{zp1m1} + \mathrm{zp20} + \frac{1}{48}\,\mathrm{Log}\left[\frac{p^2}{\mathrm{mub}^2}\right]\left(-4789 + 268\,\mathrm{Nf} + 108\,\mathrm{za10} + 108\,\mathrm{za11} + 324\,\mathrm{zc10} + 324\,\mathrm{zc11} - 108\,\mathrm{zp10} - 108\,\mathrm{zp11} - 9\left(-159 + 8\,\mathrm{Nf} + 6\,\mathrm{za11} + 18\,\mathrm{zc11} - 6\,\mathrm{zp11}\right)\mathrm{Log}\left[\frac{p^2}{\mathrm{mub}^2}\right]\right) - \frac{91\,\mathrm{Zeta}[3]}{2}\right)$$

$$Z_X = 1 + \frac{g^2}{16\pi^2}\left(\frac{z_{x11} + \epsilon z_{x10} + \epsilon^2 z_{x1m1}}{\epsilon}\right) + \mathcal{O}(g^4)$$

Final remarks

- The CF model is a gluon mass extension of the standard FP action.
- In the pure gauge case it is able to reproduce lattice results by means of standard perturbation theory.
- As for heavy quarks, perturbation theory provides reasonable results.
- As for light quarks, standard perturbation theory is not enough.
- As for the quark-gluon vertex, λ_1 and λ_3 seem to admit a perturbative description. This is not the case of λ_2 which, up to date, has remained elusive for other approaches in the continuum as well.
- Preliminary results for now. Stay tuned for the final results.

Thank you!