The Minkowskian Curci-Ferrari infrared-safe model

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Introduction

Infrared-safe behaviour: Curci-Ferrari model

Minkowskian Curci-Ferrari Model

Analytic (Complex-valued) IR-Safe Flow

Conclusions and Outlook

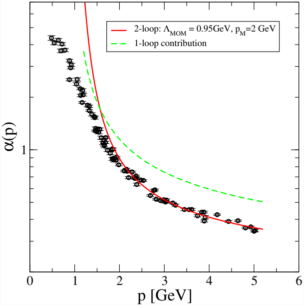
Introduction

Motivation for studying the Infrared Regime of QCD

- **Quark mass Generation**: The infrared regime is crucial for understanding the mechanism of mass generation for quarks. The spontaneous breaking of chiral symmetry, essential for hadron physics, takes place in this regime.
- **Confinement**: Quarks and gluons are never found in isolation.

Challenges in Infrared QCD

- **Non-perturbative regime**: Standard perturbative techniques are not applicable.
- **Complexity**: High computational complexity in dealing with low-energy QCD phenomena.
- **Lattice QCD**: Requires sophisticated numerical methods to study QCD on a discretized space-time lattice.

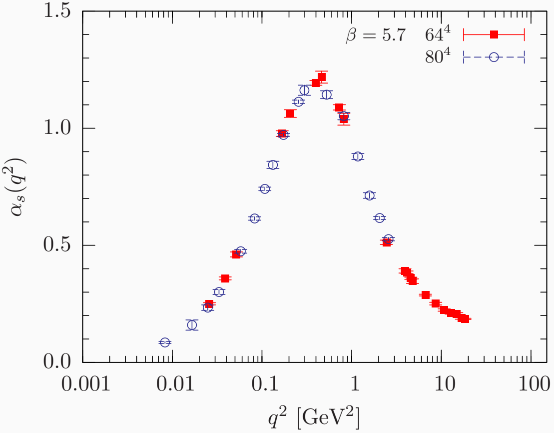


[*Bloch et al, Nucl.Phys. B687 (2004)*]

Standard perturbative correlation functions diverge in the IR.

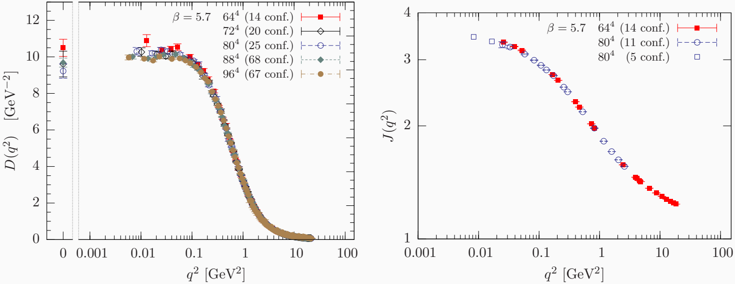
What do the numerical simulations observe?

The coupling constant (scheme dependent) has no Landau pole...



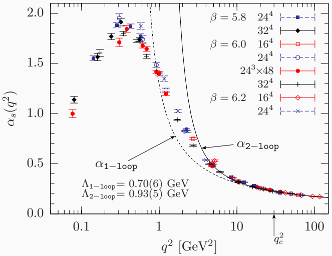
[I. L. Bogolubsky et al. Phys.Lett.B 676 (2009)]

Gluon propagator and ghost dressing function in $SU(3)$.



$SU(3)$ gluon propagator and ghost dressing function from [I. L. Bogolubsky et al. Phys.Lett.B 676 (2009)]

How large is the coupling in infrared regime of QCD?



[A. Sternbeck et al, Nucl.Phys.B Proc.Suppl. 153 (2006) 185-190]

In fact the expansion parameter

$$\lambda = \frac{N\alpha}{4\pi} \sim 0.3$$

is not large!

Some kind of perturbation theory should be possible.

Infrared-safe behaviour: Curci-Ferrari model

The model: Massive gluons (Curci-Ferrari)

Simplest Lagrangian with massive gluon behaviour

- Add a gluon mass term:

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + ih^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \mathbf{\frac{m^2}{2} A^a_\mu A^a_\mu}$$

[G. Curci and R. Ferrari, Nuovo Cim. A 32, 151 (1976).]

- Cons:
  - We don't know how to generate this Lagrangian.
  - One free parameter to be adjusted.
  - The usual construction of the physical space does not apply.

- Pros:
  - It still has a modified-BRST symmetry which allows to prove renormalizability. [G. Curci and R. Ferrari, Nuovo Cim. A 35, 1 (1976)]
  - The mass term regularizes the infrared.
  - It is possible to use an infrared safe renormalization scheme.
  - Perturbation theory can be implemented even in the infrared
  - Feynman rules are identical to usual ones, except for a massive gluon propagator still transverse in Landau gauge.
  - Could be related to de Gribov problem.
- In the last decade perturbative one and two-loop calculation of several correlation functions have been done and compared successfully with lattice simulations.

Renormalization Scheme

Infrared safe scheme:

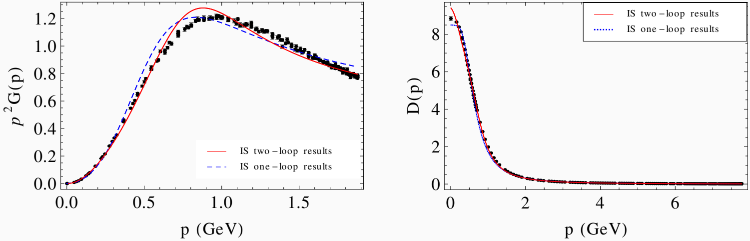
$$\Gamma^{(2)}_{AA}(p=\mu,\mu) = \mu^2 + m^2(\mu),$$
$$\Gamma^{(2)}_{C\bar{C}}(p=\mu,\mu) = \mu^2,$$
$$Z_g\sqrt{Z_A}Z_c = 1,$$
$$Z_{m^2}Z_AZ_c = 1$$

$$\beta_g(g,m^2) = \mu\frac{dg}{d\mu}\Big|_{g_0,m_0^2},$$
$$\beta_{m^2}(g,m^2) = \mu\frac{dm^2}{d\mu}\Big|_{g_0,m_0^2},$$
$$\gamma_A(g,m^2) = \mu\frac{d\log Z_A}{d\mu}\Big|_{g_0,m_0^2},$$
$$\gamma_c(g,m^2) = \mu\frac{d\log Z_c}{d\mu}\Big|_{g_0,m_0^2}.$$

- From the propagators we can compute $\beta-$functions.
- In the infrared-safe scheme, the gluon and the ghost propagators are given explicitely in terms of the running parameters.

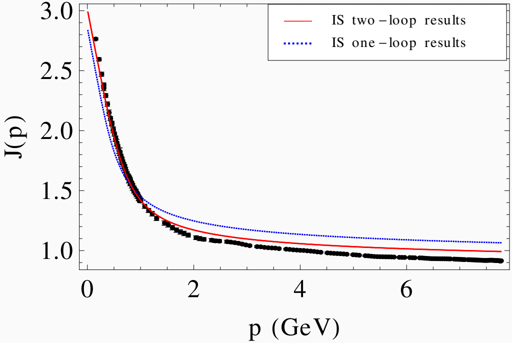
$$D(p) = \frac{g^2(\mu_0)}{m^4(\mu_0)} \frac{m^4(p)}{g^2(p)} \frac{1}{p^2 + m^2(p)}, \qquad J(p) = \frac{m^2(\mu_0)}{g^2(\mu_0)} \frac{g^2(p)}{m^2(p)}$$

Massive behaviour of the gluon propagator



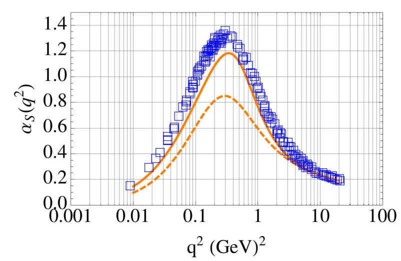
[Gracey, MP, Reinosa, Tissier Phys.Rev.D 100 (2019) 3, 034023]

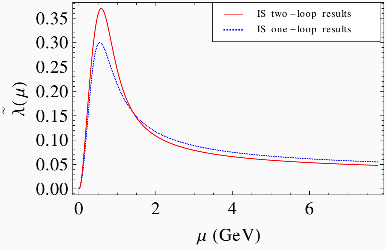
Massless ghosts



[Gracey, MP, Reinosa, Tissier Phys.Rev.D 100 (2019) 3, 034023]

Moderate coupling constant

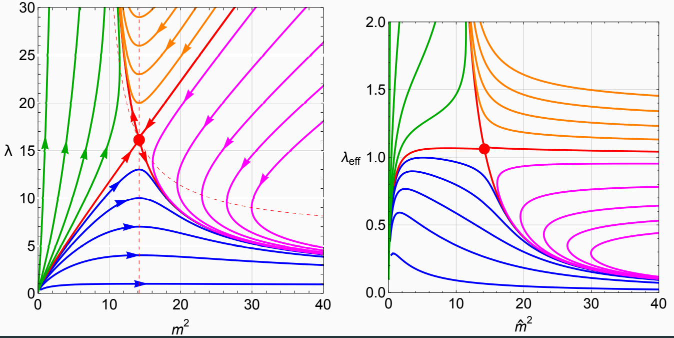




[Gracey, MP, Reinosa, Tissier Phys.Rev.D 100 (2019) 3, 034023]

Euclidean Model: Effective Coupling

- An effective coupling $\lambda_{eff} = \lambda/(1+\tilde{m}^2)$ provides a more sensible measure of perturbative validity.
- The flow diagram in the $(\tilde{m}^2, \lambda_{eff})$ plane (Figure 2) highlights the behavior of this effective coupling.
- At the non-trivial fixed point, the effective coupling $\lambda_{eff} \approx 1.06$.



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- very good description of pure-gauge correlation functions can be obtained at one and two-loops.
- But... All these results are in the Euclidean space, why is this interesting? We have in information of the infrared regime from Euclidean lattice simulations.
- Could be possible for us to use this information to explore also the Minkowski region? [S. Oribe, MP, U. Reinosa Phys.Rev.D 112 (2025) 1, 014005]

Minkowskian Curci-Ferrari Model

Minkowskian CF Model: Formulation & Self-Energies

- The Minkowskian model is obtained from the Euclidean version via formal replacements: $x_0 \to ix^0$, $\partial_0 \to -i\partial^0$, etc.
- Minkowskian self-energies $\Pi(p^2)$ are obtained by analytic continuation of Euclidean self-energies $\hat{\Pi}(-p^2)$.
- Key relation for any $p^2 \in \mathbb{R}$: $\Pi(p^2) = \hat{\Pi}(z = p^2 + i0^+)$.
- This method bypasses direct Minkowskian calculations.

Minkowskian Model: Real-valued Renormalization Factors

- An IR-safe scheme using real-valued renormalization factors $Z_X(q^2)$ is defined for the Minkowskian model.
- Renormalization conditions:

$$\begin{aligned} \text{Re } \tilde{G}^{-1}(p^2 = q^2; q^2) &= q^2 \\ \text{Re } \tilde{G}_A^{-1}(p^2 = q^2; q^2) &= q^2 - m^2 \end{aligned}$$

- In the **space-like region** ($q^2 < 0$), the flow is identical to the Euclidean flow, with $Q^2 = -q^2$.
- In the **time-like region** ($q^2 > 0$), a similar but distinct flow is observed.

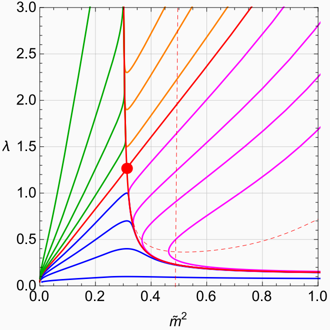


Figure 2: One-loop time-like Minkowskian flow in the real-valued IR-safe scheme, in the plane $(\tilde{m}^2, \lambda)$.

Real-valued Renormalization: Time-like Analysis

- The time-like flow also exhibits four types of theories and fixed points, structurally similar to the Euclidean case.
- UV fixed point: $(m^2 = 0, \lambda = 0)$.
- The effective coupling λ_{eff} at the time-like fixed point is ≈ 0.97.
- **Crucial Limitation:** In this real-valued scheme, the space-like and time-like flows are **not connected**.

Analytic (Complex-valued) IR-Safe Flow

Analytic IR-Safe Flow: Complex-valued Factors

- To connect space-like and time-like flows, we investigate using **complex-valued renormalization factors** $Z_X(\omega)$.
- This implies that the renormalized parameters (g^2, m^2) can become complex.
- The analytic flow is solved by integrating along a slightly imaginary trajectory $(z = q^2 + i0^+)$.
- Branchcuts of the beta functions must be managed by adding/subtracting $2\pi i$ to logarithms.

Analytic IR-Safe Flow: Complex Coupling Results

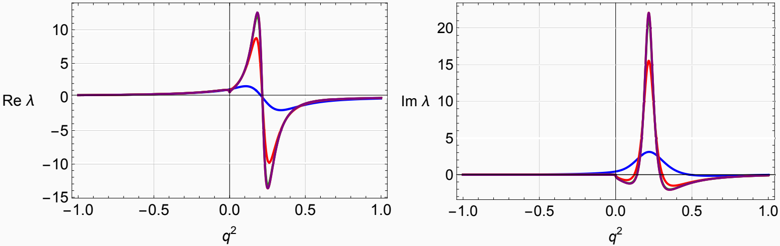


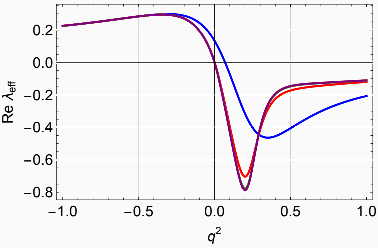
Figure 3: Real and imaginary parts of the coupling from space-like to time-like running scales as one makes the $i0^+$ smaller and smaller: 10^{-1} (blue), 10^{-2} (red), 10^{-3} (green), 10^{-4} (orange), 10^{-5} (purple).

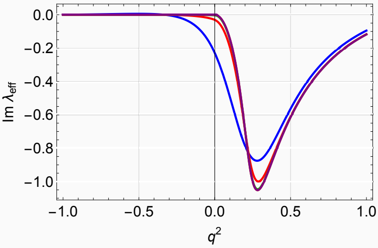
Analytic IR-Safe Flow: Complex Coupling Results

- The coupling is **not symmetric** between space-like and time-like regions.
- Values of λ in the time-like region can be significantly larger, sometimes exceeding the perturbative boundary ($\lambda = 1$).
- The evolution of the complex coupling is tracked across the phase space.

Analytic IR-Safe Flow: Effective Coupling Results

- The effective coupling λ_{eff} in the complex scheme shows different behavior. Real and imaginary parts of λ_{eff} in modulus remain at most slightly above 1, even in the time-like region.
- This suggests that semi-perturbative applications might still be valid.
- This scheme successfully **connects the space-like and time-like regions**, allowing the continuation of Euclidean trajectories.





Conclusions and Outlook

Conclusions and Outlook

- **Real-valued IR-safe scheme:**
  - Euclidean flow matches Minkowskian space-like flow.
  - Time-like flow is distinct but also shows bounded coupling.
  - **Cannot connect** space-like and time-like regions.
- **Complex-valued IR-safe scheme:**
  - Allows analytical connection between space-like and time-like regions.
  - Effective coupling λ_{eff} remains moderate (modulus close to 1).
  - Supports semi-perturbative applications in Minkowskian domain.
- **Future Work:**
  - Detailed study of gluon and ghost two-point correlators in the time-like region.

Thanks