Finiteness of the Yang-Mills-Chern-Simons action by taking into account gauge copies

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Quantum Action Principle

The Quantum Action Principle allows us to identify if a symmetry is preserved or not during renormalization, i.e., if it remains a symmetry of the quantum theory. It can be summarized as

$$\mathcal{W}^a\Gamma = \Delta^a \cdot \Gamma, \tag{1}$$

In a similar manner, we can define the dependence on the parameters of the theory as

$$\frac{\partial}{\partial\lambda}\Gamma = \Delta \cdot \Gamma, \tag{2}$$

The renormalization of parameters and fields are expressed by invariant counterterms which are added to the classical action $\Gamma^{(0)} \to \Gamma^{(0)} + \varepsilon\Gamma^{ct}$, so that

$$\mathcal{W}_\Gamma\Gamma^{ct} = 0. \tag{3}$$

If the perturbation can be reabsorbed by a redefinition

$$\Gamma^{(0)}[\phi, \lambda, \rho] + \varepsilon\Gamma^{ct}[\phi, \lambda, \rho] = \Gamma^{(0)}[\phi_0, \lambda_0, \rho_0] + O(\varepsilon^2), \tag{4}$$

we say that the theory is multiplicatively renormalizable.

Yang-Mills-Chern-Simons

The tree-level Yang-Mills-Chern-Simons[1] action for a general semi-simple gauge group

$$S_{YMCS} = \int d^3x \left[-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{m}{2} \epsilon^{\mu\rho\nu} \left(A^a_\mu \partial_\rho A^a_\nu + \frac{2g}{3} f^{abc} A^a_\mu A^b_\rho A^c_\nu \right) \right] . \tag{5}$$

It is a well-known result that YMCS theory is finite, i.e., it receives no quantum corrections on its fields and parameters. This can be summarized as a trivial quantum insertion

$$\Delta \cdot \Gamma = 0 \tag{6}$$

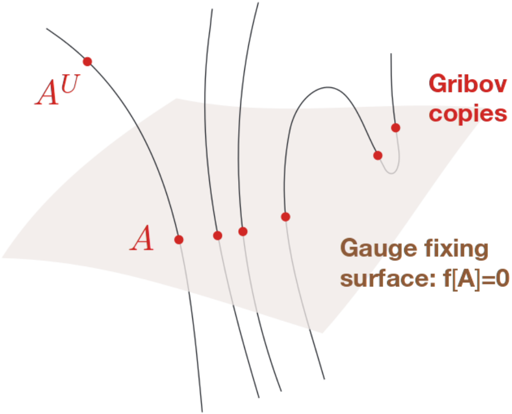
This finite character comes from the absence of local BRST-invariance of the YMCS action[2].

[1]For a detailed review of Chern-Simons theory, see G. Dunne, arXiv:9902.115

[2]Del Cima *et al*. Lett. Math. Phys., 47:265, 1999; Barnich. JHEP, 12:003, 1998.

The Gribov Problem

Gauge-fixing amounts to choosing one representative per orbit, necessary to avoid overcounting in the functional integral. It can be introduced via the Faddeev-Popov procedure, so that



$$\int \mathcal{D}A\; e^{iS} = \int \mathcal{D}\omega\mathcal{D}A\; \delta(F[A']) \left|\det\left(\frac{\delta F[A']}{\delta\omega}\right)\right| e^{iS}. \tag{7}$$

The Faddeev-Popov procedure is able to do this provided that

- The gauge condition $F[A]$ is ideal;
- The Faddeev-Popov determinant is positive;

In general, it is not possible to define a continuous gauge condition which fixes the gauge[3]. One way to see this is to note that the Faddeev-Popov operator develops zero-modes

$$\mathcal{M}^{ab}\omega^b = -\partial_\mu D_\mu^{ab}\omega^b = 0, \tag{8}$$

One way to deal with this problem is to restrict the integration over the gauge fields to a region Ω free of copies

$$\Omega = \{A_\mu^a, \partial_\mu A_\mu^a = 0 | \mathcal{M}^{ab} > 0\}, \tag{9}$$

called the Gribov region.

[3] V. N. Gribov. Nucl. Phys. B, 139:1, 1978; I. M. Singer. Commun. Math. Phys., 60:7–12, 1978.

Another way is to introduce a horizon term which effectively restricts the integration to the Gribov region[4]

$$H(A) = g^2 \int dxdy\ f^{abc} A^b_\mu(x) [\mathcal{M}^{-1}]^{ad}(x,y) f^{dec} A^e_\mu(y). \tag{10}$$

These procedures are equivalent to one another and introduce self-consistently a mass scale to the theory, called the Gribov mass γ, so that the gluon propagator is modified into

$$\langle T A^a_\mu A^b_\nu \rangle(k) = \delta^{ab} \frac{k^2}{k^4 + \gamma^4} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right). \tag{11}$$

It also displays an enhanced 1-loop ghost propagator, proportional to $1/k^4$. The horizon term can be localized by introducing proper auxiliary fields[5].

[4] D. Zwanziger. Nucl. Phys. B, 321:591–604, 1989.
[5] D. Zwanziger. Nucl. Phys. B, 323:513–544, 1989.

Refined Gribov-Zwanziger (RGZ)

Due to results from the lattice[6] showing

- a non-vanishing gluon propagator at null momentum;
- a non-enhanced ghost propagator;

It was necessary to introduce condensates to the theory[7]

$$\frac{m^2}{2}\int dx A_\mu^a A_\mu^a \quad \text{and} \quad M^2\int dx(\bar{\varphi}_\mu^{ab}\varphi_\mu^{ab} - \bar{\omega}_\mu^{ab}\omega_\mu^{ab}), \tag{12}$$

leading to a modified gluon propagator

$$\langle A_\mu^a A_\nu^b\rangle(k) = \delta^{ab}\frac{k^2+M^2}{(k^2+m^2)(k^2+M^2)+\gamma^4}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \tag{13}$$

which agrees with the lattice data.

[6] Cucchieri, Mendes. PoS, LATTICE2007:297, 2007.; Bogolubsky *et al* . PoS, LATTICE2007:290, 2007.
[7] Dudal *et al*. Phys. Rev. D, 78:065047, 2008.

A critical feature of the GZ action is that it presents a soft break of BRST symmetry[8]

$$sS_{GZ} = g\gamma^2 f^{abc} \int dx \left(A^a_\mu \omega^{bc}_\mu - D^{ad}_\mu c^d (\bar{\varphi} + \varphi)^{bc}_\mu \right) . \tag{14}$$

We can restore BRST symmetry[9] defining Ω in terms of $A^{h,a}_\mu$, which minimizes the functional $||A||^2 = \frac{1}{2} \int A^a_\mu A^a_\mu$

$$\begin{aligned} A^{h,a}_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \Bigg(A^a_\nu - ig \left[\frac{1}{\partial^2} \partial A^a, A^a_\nu \right] \\ + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A^a, \partial_\nu \frac{1}{\partial^2} \partial A^a \right] + O(A^3) \Bigg) , \end{aligned} \tag{15}$$

These fields are transverse and BRST-invariant, and they allow for the investigation of Gribov copies in linear covariant gauges.

[8] D. Zwanziger. Nucl. Phys. B, 399:477–513, 1993.
[9] Capri *et al*. Phys. Rev. D, 92(4):045039, 2015.

YMCS within the Gribov Horizon

Our goal now is to prove that the restriction to the Gribov region does not affect the finite character of the YMCS theory. We start with a localized RGZ modification of the YMCS action in linear covariant gauges[10]

$$
\begin{aligned}
S_{YMCS}^{RGZ} = S_{YMCS} + S_{GF}^{LC} &- \int d^3x \, \left(\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab}(A^h) \omega_\mu^{bc}\right) \\
&+ \int d^3x \; g\gamma^2 f^{abc} A_\mu^{h,a} (\bar{\varphi} + \varphi)_\mu^{bc} + \frac{M^2}{2} \int d^3x \; A_\mu^{h,a} A_\mu^{h,a} \\
&- \mu^2 \int d^3x \, \left(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}\right) + \int d^3x \left(\tau^a \partial_\mu A_\mu^{h,a} - \bar{\eta}^a \mathcal{M}^{ab}(A^h) \eta^b\right),
\end{aligned} \tag{16}
$$

[10] Daniel O. R. Azevedo and Antonio D. Pereira, Phys. Rev. D 111, 085028 (2025)

First, we introduce external sources coupled to the non-linear BRST variations and the composite field A^h

$$S_{ext} = \int d^3x \, \left(-\Omega^a_\mu D^{ab}_\mu c^b + \frac{g}{2} L^a f^{abc} c^b c^c + K^a g^{ab}(\xi) c^b + \mathcal{J}^a_\mu A^{h,a}_\mu \right), \tag{17}$$

and sources coupled to the condensates[11],

$$S_{cond} = \int d^3x \, \left[J \left(A^{h,a}_\mu A^{h,a}_\mu \right) - \rho \left(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \right) \right]. \tag{18}$$

which should attain the physical values

$$J|_{phys} = \frac{M^2}{2}, \quad \rho|_{phys} = \mu^2. \tag{19}$$

in the end.

[11] K. Knecht and H. Verschelde. Phys. Rev. D, 64:085006, 2001.

We also modify the Gribov mass term

$$S_{\gamma^2} \equiv \int d^3x \; g\gamma^2 f^{abc} A^{h,a}_{\mu} \left(\bar{\varphi} + \varphi\right)^{bc}_{\mu} \, , \tag{20}$$

with the introduction of new sources[12]

$$\begin{aligned} S_{\gamma^2} \equiv \int d^3x \; \Big[& M^{ai}_{\mu} D^{ab}_{\mu}(A^h)\varphi^{bi} + V^{ai}_{\mu} D^{ab}_{\mu}(A^h)\bar{\varphi}^{bi} + N^{ai}_{\mu} D^{ab}_{\mu}(A^h)\omega^{bi} \\ & + U^{ai}_{\mu} D^{ab}_{\mu}(A^h)\bar{\omega}^{bi} - M^{ai}_{\mu} V^{ai}_{\mu} + N^{ai}_{\mu} U^{ai}_{\mu} \Big] \, , \end{aligned} \tag{21}$$

which return the original term in the limit

$$\begin{aligned} & M^{ab}_{\mu\nu}|_{phys} = V^{ab}_{\mu\nu}|_{phys} = \gamma^2 \delta_{\mu\nu} \delta^{ab} , \\ & N^{ab}_{\mu\nu}|_{phys} = U^{ab}_{\mu\nu}|_{phys} = 0. \end{aligned} \tag{22}$$

[12] D. Zwanziger. Nucl. Phys. B, 323:513–544, 1989.; Dudal *et al*. Phys. Rev. D, 78:065047, 2008.

And finally, we introduce a new set of sources

$$S_{extra} = \int d^3x \, \Big[-\Xi^a_\mu D^{ab}_\mu(A^h)\eta^a + X^i \eta^a \bar{\omega}^{ai} + Y^i \eta^a \bar{\varphi}^{ai} + \bar{X}^{abi}\eta^a \omega^{bi} + \bar{Y}^{abi}\eta^a \varphi^{bi} \Big], \tag{23}$$

which are related to the A^h field[13]. All the new sources introduced are invariant under BRST, that is

$$s\rho_i = 0. \tag{24}$$

The full tree-level action to be considered is

$$\Sigma = S_{YMCS} + S^{LC}_{GF} - \int d^3x \, \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h)\omega^{bc}_\mu \right) + \int d^3x \left(\tau^a \partial_\mu A^{h,a}_\mu - \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b \right) + S_{\gamma^2} + S_{cond} + S_{ext} + S_{extra}, \tag{25}$$

[13] Capri *et al*. Phys. Rev. D, 96(5):054022, 2017.

This enlarged action is invariant under BRST

$$s\Sigma = 0, \tag{26}$$

and in the limit where the sources take their physical values

$$\Sigma|_{phys} = S^{RGZ}_{YMCS}. \tag{27}$$

The introduction of the sources gives another invariance (δ) to the action Σ, defined by the transformations

$$
\begin{aligned}
\delta\varphi^{ab} &= \omega_\mu^{ab}, & \delta\omega_\mu^{ab} &= 0, \\
\delta\bar{\omega}_\mu^{ab} &= \bar{\varphi}_\mu^{ab}, & \delta\bar{\varphi}_\mu^{ab} &= 0, \\
\delta N_\mu^{ai} &= M_\mu^{ai}, & \delta M_\mu^{ai} &= 0, \\
\delta V_\mu^{ai} &= U_\mu^{ai}, & \delta U_\mu^{ai} &= 0, \\
\delta Y^i &= X^i, & \delta Y^i &= 0, \\
\delta\bar{X}^{abi} &= \bar{Y}^{abi}, & \delta\bar{Y}^{abi} &= 0,
\end{aligned}
\tag{28}
$$

where $\delta\Phi = 0$ for all other fields and sources and $\delta^2 = 0$. It also possible to define the gauge parameter α and the auxiliary fields source ρ as doublets

$$
s\alpha = \chi, \quad s\chi = 0, \quad \delta\rho = \sigma, \quad \delta\sigma = 0,
\tag{29}
$$

We can then define an extended BRST operator Q, given by

$$Q = s + \delta, \quad Q^2 = 0, \tag{30}$$

so that the complete extended action Σ is left invariant

$$Q\Sigma = 0. \tag{31}$$

Fields	A	b	c	$\bar{c}$	ξ	$\bar{\varphi}$	φ	$\bar{\omega}$	ω	α	χ
d	1/2	3/2	-1/2	3/2	-1/2	1/2	1/2	1/2	1/2	0	0
$\Phi\Pi$-ghost	0	0	1	-1	0	0	0	-1	1	0	1
η-ghost	0	0	0	0	0	0	0	0	0	0	0
$U(f)$-number	0	0	0	0	0	-1	1	-1	1	0	0
Nature	B	B	F	F	B	B	B	F	F	B	F

Sources	Ω	L	K	$\mathcal{J}$	M	N	U	V	J	ρ	σ
d	5/2	7/2	7/2	5/2	3/2	3/2	3/2	3/2	2	2	2
$\Phi\Pi$-ghost	-1	-2	-1	0	0	-1	1	0	0	0	1
η-ghost	0	0	0	0	0	0	0	0	0	0	0
$U(f)$-number	0	0	0	0	-1	-1	1	1	0	0	0
Nature	F	B	F	B	B	F	F	B	B	B	F

Fields/Sources	τ	η	$\bar{\eta}$	Ξ	X	Y	$\bar{X}$	$\bar{Y}$
d	$3/2$	$1/2$	$1/2$	$3/2$	2	2	2	2
$\Phi\Pi$-ghost	0	0	0	0	1	0	-1	0
η-ghost	0	1	-1	-1	-1	-1	-1	-1
$U(f)$-number	0	1	-1	0	1	1	-1	-1
Nature	B	F	F	F	B	F	B	F

The extended action Σ presents a large set of symmetries and functional identities, like the Slavnov-Taylor

$$\mathcal{S}_Q(\Sigma) = \int d^3x \left[\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta K^a}\frac{\delta\Sigma}{\delta\xi^a} + b^a\frac{\delta\Sigma}{\delta\bar{c}^a} + \omega^{ai}\frac{\delta\Sigma}{\delta\varphi^{ai}} + \bar{\varphi}^{ai}\frac{\delta\Sigma}{\delta\bar{\omega}^{ai}} \right.$$
$$\left. + M^{ai}_\mu\frac{\delta\Sigma}{\delta N^{ai}_\mu} + U^{ai}_\mu\frac{\delta\Sigma}{\delta V^{ai}_\mu} + \sigma\frac{\delta\Sigma}{\delta\rho} + X^i\frac{\delta\Sigma}{\delta Y^i} - \bar{Y}^{abi}\frac{\delta\Sigma}{\delta\bar{X}^{abi}} \right] + \chi\frac{\partial\Sigma}{\partial\alpha} = 0, \tag{32}$$

the gauge-fixing condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^a_\mu - \alpha b^a - \frac{1}{2}\chi\bar{c}^a, \tag{33}$$

the ghost equation

$$\frac{\delta\Sigma}{\delta\bar{c}^a} - \partial_\mu\frac{\delta\Sigma}{\delta\Omega^a_\mu} = \frac{1}{2}\chi b^a, \tag{34}$$

The equations of motion of the lagrange multiplier τ^a

$$\frac{\delta\Sigma}{\delta\tau^a} - \partial_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} = 0, \tag{35}$$

a global $U(f)$ symmetry

$$\begin{aligned} U_{ij} = \int d^3x \Bigg[&\varphi^{ai}\frac{\delta}{\delta\varphi^{aj}} - \bar{\varphi}^{aj}\frac{\delta}{\delta\bar{\varphi}^{ai}} + \omega^{ai}\frac{\delta}{\delta\omega^{aj}} - \bar{\omega}^{aj}\frac{\delta}{\delta\bar{\omega}^{ai}} \\ &- M^{aj}_\mu\frac{\delta}{\delta M^{ai}_\mu} + V^{ai}_\mu\frac{\delta}{\delta V^{aj}_\mu} - N^{aj}_\mu\frac{\delta}{\delta N^{ai}_\mu} + U^{ai}_\mu\frac{\delta}{\delta U^{aj}_\mu} \\ &+ X^i\frac{\delta}{\delta X^j} + Y^i\frac{\delta}{\delta Y^j} - \bar{X}^{abj}\frac{\delta}{\delta\bar{X}^{abi}} - \bar{Y}^{abj}\frac{\delta}{\delta\bar{Y}^{abi}}\Bigg] = 0, \end{aligned} \tag{36}$$

a set of linearly broken identities

$$\frac{\delta\Sigma}{\delta\bar{\varphi}^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta M_\mu^{ai}} + g f^{abc} V_\mu^{bi} \frac{\delta\Sigma}{\delta\mathcal{J}_\mu^c} = -\rho\varphi^{ai} + Y^i\eta^a; \tag{37}$$

$$\frac{\delta\Sigma}{\delta\varphi^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta V_\mu^{ai}} - g f^{abc} \bar{\varphi}^{bi} \frac{\delta\Sigma}{\delta\tau^c} + g f^{abc} M_\mu^{bi} \frac{\delta\Sigma}{\delta\mathcal{J}_\mu^c} = -\rho\bar{\varphi}^{ai} - \sigma\bar{\omega}^{ai} + \bar{Y}^{bai}\eta^b; \tag{38}$$

$$\frac{\delta\Sigma}{\delta\bar{\omega}^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta N_\mu^{ai}} - g f^{abc} U_\mu^{bi} \frac{\delta\Sigma}{\delta\mathcal{J}_\mu^c} = \rho\omega^{ai} + \sigma\varphi^{ai} - X^i\eta^a; \tag{39}$$

$$\frac{\delta\Sigma}{\delta\omega^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta U_\mu^{ai}} - g f^{abc} \bar{\omega}^{bi} \frac{\delta\Sigma}{\delta\tau^c} + g f^{abc} N_\mu^{bi} \frac{\delta\Sigma}{\delta\mathcal{J}_\mu^c} = -\rho\bar{\omega}^{ai} - \bar{X}^{bai}\eta^b; \tag{40}$$

two distinct global ghost number identities for $\Phi\Pi$

$$\int d^3x \left[c^a \frac{\delta\Sigma}{\delta c^a} - \bar{c}^a \frac{\delta\Sigma}{\delta \bar{c}^a} + \omega^{ai} \frac{\delta\Sigma}{\delta \omega^{ai}} - \bar{\omega}^{ai} \frac{\delta\Sigma}{\delta \bar{\omega}^{ai}} - \Omega^a_\mu \frac{\delta\Sigma}{\delta \Omega^a_\mu} - 2L^a \frac{\delta\Sigma}{\delta L^a} - K^a \frac{\delta\Sigma}{\delta K^a} \right.$$
$$\left. + U^{ai}_\mu \frac{\delta\Sigma}{\delta U^{ai}_\mu} - N^{ai}_\mu \frac{\delta\Sigma}{\delta N^{ai}_\mu} + X^i \frac{\delta\Sigma}{\delta X^i} - \bar{X}^{abi} \frac{\delta\Sigma}{\delta \bar{X}^{abi}}\right] + \chi \frac{\partial\Sigma}{\partial\chi} = 0; \tag{41}$$

and η

$$\int d^3x \left[\eta^a \frac{\delta\Sigma}{\delta \eta^a} - \bar{\eta}^a \frac{\delta\Sigma}{\delta \bar{\eta}^a} - \Xi^a_\mu \frac{\delta\Sigma}{\delta \Xi^a_\mu} \right.$$
$$\left. - X^i \frac{\delta\Sigma}{\delta X^i} - Y^i \frac{\delta\Sigma}{\delta Y^i} - \bar{X}^{abi} \frac{\delta\Sigma}{\delta \bar{X}^{abi}} - \bar{Y}^{abi} \frac{\delta\Sigma}{\delta \bar{Y}^{abi}}\right] = 0, \tag{42}$$

a $\mathcal{R}_{ij}$ symmetry, corresponding to an exchange between the auxiliary fields and sources

$$\mathcal{R}_{ij} = \int d^3x \left[\varphi^{ai} \frac{\delta \Sigma}{\delta \omega^{aj}} - \bar{\omega}^{aj} \frac{\delta \Sigma}{\delta \bar{\varphi}^{ai}} + V_\mu^{ai} \frac{\delta \Sigma}{\delta U_\mu^{aj}} \right. \\ \left. - N_\mu^{aj} \frac{\delta \Sigma}{\delta M_\mu^{ai}} + \bar{X}^{abj} \frac{\delta \Sigma}{\delta \bar{Y}^{abi}} + Y^i \frac{\delta \Sigma}{\delta X^j} \right] = 0, \tag{43}$$

a η-ghost equation

$$\frac{\delta \Sigma}{\delta \bar{\eta}^a} - \partial_\mu \frac{\delta \Sigma}{\delta \Xi_\mu^a} = 0, \tag{44}$$

and a η anti-ghost equation

$$\int d^3x \left[\frac{\delta \Sigma}{\delta \eta^a} + g f^{abc} \bar{\eta}^b \frac{\delta \Sigma}{\delta \tau^c} - g f^{abc} \Xi_\mu^b \frac{\delta \Sigma}{\delta \mathcal{J}_\mu^c} \right] \\ = \int d^3x \left[X^i \bar{\omega}^{ai} - Y^i \bar{\varphi}^{ai} + \bar{X}^{abi} \omega^{bi} - \bar{Y}^{abi} \varphi^{bi} \right]; \tag{45}$$

a series of identities that mix the auxiliary fields $(\bar{\varphi}, \varphi, \bar{\omega}, \omega)$ with the ghosts $(\bar{\eta}, \eta)$

$$W^i_{(1)}\Sigma = \int d^3x \left(\bar{\omega}^{ai} \frac{\delta\Sigma}{\delta\bar{\eta}^a} + \eta^a \frac{\delta\Sigma}{\delta\omega^{ai}} + N^{ai}_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} + \rho \frac{\delta\Sigma}{\delta X^i} \right) = 0; \quad (46)$$

$$W^i_{(2)}\Sigma = \int d^3x \left(\bar{\varphi}^{ai} \frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a \frac{\delta\Sigma}{\delta\varphi^{ai}} + M^{ai}_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} - \rho \frac{\delta\Sigma}{\delta Y^i} + \sigma \frac{\delta\Sigma}{\delta X^i} \right) = 0; \quad (47)$$

$$W^i_{(3)}\Sigma = \int d^3x \left(\varphi^{ai} \frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a \frac{\delta\Sigma}{\delta\bar{\varphi}^{ai}} - g f^{abc} \frac{\delta\Sigma}{\delta\bar{Y}^{abi}} \frac{\delta\Sigma}{\delta\tau^c} - V^{ai}_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} + \rho \frac{\delta\Sigma}{\delta\bar{Y}^{aai}} \right) = 0; \quad (48)$$

$$\begin{aligned} W^i_{(4)}\Sigma = \int d^3x \Bigg(& \omega^{ai} \frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a \frac{\delta\Sigma}{\delta\bar{\omega}^{ai}} \\ & + g f^{abc} \frac{\delta\Sigma}{\delta\bar{X}^{abi}} \frac{\delta\Sigma}{\delta\tau^c} + U^{ai}_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} + \rho \frac{\delta\Sigma}{\delta\bar{X}^{aai}} + \sigma \frac{\delta\Sigma}{\delta\bar{Y}^{aai}} \Bigg) = 0. \end{aligned} \quad (49)$$

The counterterm action Σ_{CT} is written as a perturbation of the classical action $(\Sigma + \epsilon\Sigma_{CT})$ and must obey

$$\mathcal{W}_i \Sigma_{CT} = 0, \tag{50}$$

where $\mathcal{W}_i$ stands for all the functional operators of the identities of Σ. It can be written as

$$\Sigma_{CT} = \Delta + \mathcal{B}_Q \Delta^{(-1)}, \tag{51}$$

where Δ and $\mathcal{B}_Q \Delta^{(-1)}$ are local, integrated polynomials bounded by dimension 2 and with ghost number 0 and -1, respectively.

The most general Σ_{CT} which respects the symmetry constraints is the non-trivial term

$$\Delta = \int d^3x \left[a_0 \epsilon_{\mu\rho\nu} \left(\frac{1}{2} A^a_\mu \partial_\rho A^a_\nu + \frac{g}{3!} f^{abc} A^a_\mu A^b_\rho A^c_\nu \right) + a_1 J \right] . \tag{52}$$

The a_1 source term must be taken at the physical value, being just an additive constant; and the a_0 term is the Chern-Simons action, which does not contribute to the counterterm, since it is not locally invariant under BRST.

Therefore, the counterterm action is trivial and the YMCS theory in linear covariant gauges remain finite when Gribov copies and condensates are taken into account.

Concluding Remarks

- The Quantum Action Principle allows to prove the finite character of YMCS theory without explicit computations;
- The restriction of YMCS theory to the Gribov region, within the RGZ framework, does not spoil the finiteness of the theory;
- YMCS is a good laboratory to study how the interplay between different mass parameters affect the confining/deconfined character of the theory;

Thank you!

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