The propagator of a charged vector boson in the presence of an external magnetic field using the Ritus Eigenfunction method

M. E. Monreal[1] & A. Sánchez[1]

[1]Physics Department, School of Sciences, Universidad Nacional Autónoma de México

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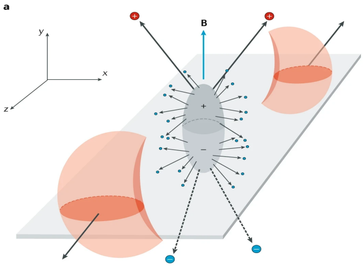
Intense Magnetic Fields

- Presence of intense magnetic fields in high energy physics.
  Magnetic field intensity in magnetars between $10^{-2}m_\pi^2$ on the surface[1] and up to $10^0 m_\pi^2$ in its core[2]. In heavy ion colisions $\sim 1 - 10m_\pi^2$.[3]



E. Kharzeev, & J. Liao, (2021) **3**, Nat. Rev. Phys., 55-63.

[1] R. Turolla, S. Zane, & A. L. Watts, (2015), Rep. Prog. Phys. **78**, 116901.
[2] G. Chanmugam, (1992), Annu. Rev. Astron. Astrophys. , **30**, 143-184.
[3] Y. Zhong, et. al, (2014), Adv. High Energy Phys., **193039**.

URCA mechanism in Neutron Stars (NS)

URCA processes are β decays responsible for the cooling of NS through the process of neutrino emission[4]

[4] L. B. Leinson, (2002) Nucl. Phys. A **707**, 543-560.
[5] A. Y. Potekhin, et. al., (2020), Mon. Not. R. Astron. Soc. **496**, 5052
[6] H. C. Das, et. al., (2024), Phys. Rev. D **109**, 123018
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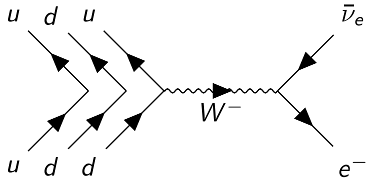

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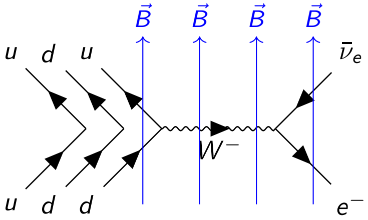

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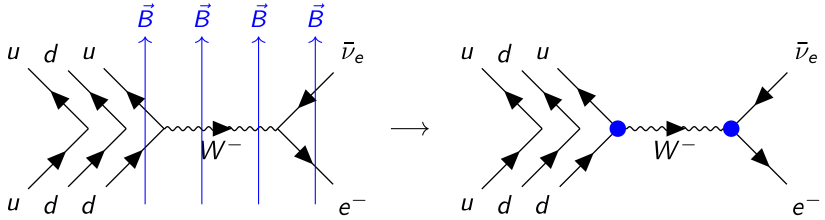

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Neutron Star Kicks

"Kicks" are anisotropic movements from the NS of around $1000 km/s$. Certain mechanisms have been proposed to explain the origin of these phenomenon. One of these mechanisms relates to the high intensity magnetic fields that affect the mean free path of the neutrinos that propel the star[8].

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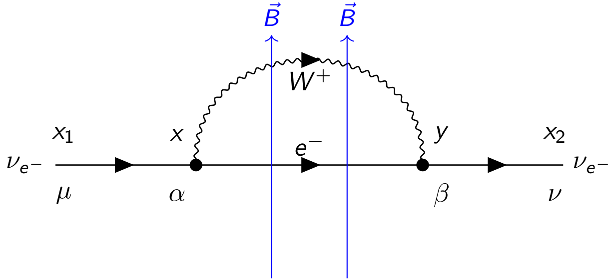

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W Boson Propagator

A conventional way of building the propagator is

- Standard Model Lagrangian.
- Equation of Motion and Gauge Condition.
- Wave-like ansatz.
- Choose a set of polarization vectors.
- Canonical quantization of classical field.
- Propagator definition.

W Boson Propagator

$$
\begin{aligned}
D_F^{\mu\nu}(x,y) &\equiv \langle 0|\, T\left(\hat{W}^{\mu-}(x)\hat{W}^{\nu+}(y)\right)|0\rangle \\
&= i\int \frac{d^4p}{(2\pi)^4}\frac{\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_W^2}\right)}{p^2-m_W^2+i\varepsilon}e^{-ip\cdot(x-y)} \\
&= i\left(-g^{\mu\nu}+\frac{\hat{p}_x^\mu\hat{p}_y^{\nu*}}{m_W^2}\right)\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{p^2-m_W^2+i\varepsilon}
\end{aligned}
$$

where

$$
\hat{p}_x^\mu = \left(i\partial_x^0, i\partial_x^1, i\partial_x^2, i\partial_x^3\right), \qquad \hat{p}_y^{\mu*} = \left(-i\partial_y^0, -i\partial_y^1, -i\partial_y^2, -i\partial_y^3\right).
$$

W Boson Propagator in a Magnetic Field

Mostly analogous methodology

- SM Lagrangian (including A^{μ}_{clas}).
- Equation of Motion (Proca Equation in a Magnetic Field) and Gauge Condition.
- (Mostly) Wave-like ansatz.
- Deriving sets of polarization vectors.
- Canonical quantization of the classical field.
- Propagator definition.

SM Lagrangian, EoM and Gauge Condition in a Magnetic Field

The Lagrangian that describes the $W^{\pm}$ bosons dynamics and interactions with the photon field A^{μ} is[10]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\left(\partial^{\mu}W^{\nu+} - \partial^{\nu}W^{\mu+}\right)\left(\partial_{\mu}W^{-}_{\nu} - \partial_{\nu}W^{-}_{\mu}\right) + m_W^2 W^{\mu+}W^{-}_{\mu}\\ &+ ie\left[F^{\mu\nu}W^{+}_{\mu}W^{-}_{\nu} - \left(\partial_{\mu}W^{+}_{\nu} - \partial_{\nu}W^{+}_{\mu}\right)A^{\mu}W^{\nu-} + \left(\partial_{\mu}W^{-}_{\nu} - \partial_{\nu}W^{-}_{\mu}\right)A^{\mu}W^{\nu+}\right]\\ &+ e^2\left(A^{\mu}W^{+}_{\mu}A^{\nu}W^{-}_{\nu} - A^{\mu}A_{\mu}W^{\nu+}W^{-}_{\nu}\right).\end{aligned}$$

The EoM will be

$$\left[\left(D_{\alpha}D^{\alpha} + m_W^2\right)g^{\mu\nu} + 2ieF^{\mu\nu}\right]W^{-}_{\nu} = 0.$$

where the gauge condition

$$D_{\mu}W^{\mu-} = 0.$$

has been used and $D^{\alpha} \equiv \partial^{\alpha} + ieA^{\alpha}$.

[10] SCHWARTZ, M., *Quantum Field Theory and the Standard Model*, United Kingdom, 2014, Ed. Cambridge University Press, Fifth Edition.

EoM Diagonalization

After defining the rotation matrices

$$N_{\alpha}{}^{\mu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \implies \left(N^{-1}\right)_{\alpha}{}^{\mu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

one gets

$$\left[\left(\hat{\Pi}^2 - m_W^2\right) g_{\alpha}{}^{\beta} - 2eB\tilde{F}_{\alpha}{}^{\beta}\right] \tilde{W}_{\beta}^{-} = 0,$$

where $\hat{\Pi}^{\alpha} \equiv iD^{\alpha}$, $\tilde{F}_{\alpha}{}^{\beta} \equiv iN_{\alpha}{}^{\sigma}\frac{F_{\sigma}{}^{\rho}}{B}\left(N^{-1}\right)_{\rho}{}^{\beta}$ and $\tilde{W}_{\alpha}^{-} \equiv N_{\alpha}{}^{\mu}W_{\mu}^{-}$.

Projection Matrix

Defining

$$\tilde{\Delta}^{\mu}{}_{\nu}(s_3) \equiv \begin{pmatrix} 1-s_3^2 & 0 & 0 & 0 \\ 0 & s_3\left(\frac{1+s_3}{2}\right) & 0 & 0 \\ 0 & 0 & -s_3\left(\frac{1-s_3}{2}\right) & 0 \\ 0 & 0 & 0 & 1-s_3^2 \end{pmatrix},$$

that can be identified as a spin projection matrix with eigenvalues $s_3 = \pm 1, 0$. Then, the EoM can be rewritten as

$$\sum_{s_3} \left[\left(\hat{\Pi}^2 - m_W^2 \right) + 2eBs_3 \right] \tilde{W}_{\alpha}^{s_3 -} = 0,$$

where $\tilde{W}_{\alpha}^{s_3 -} \equiv \tilde{\Delta}_{\alpha}{}^{\beta}(s_3)\tilde{W}_{\beta}^{-}$.

EoM Solution in a Magnetic Field

A choice of gauge is made $A^\mu = \left(0, 0, Bx^1, 0\right)$ so that the resulting magnetic field is homogeneous and goes in the z direction. The proposed ansatz is

$$\tilde{W}_\alpha^{s_3-}(x) = e^{-ir\left(Ex_0+p^2x_2+p^3x_3\right)} X_n^r(x^1) \tilde{\tilde{\varepsilon}}_\alpha^{s_3-},$$

Plugging the ansatz in the EoM one gets *Weber's differential equation*

$$\left[\frac{\partial^2}{\partial\rho_r^2} - \frac{1}{4}\rho_r^2 + \frac{1}{2} + n\right] X_n^r(\rho_r) = 0,$$

where

$$n = \frac{E^2 - (p^3)^2 - m_W^2 + 2eBs_3}{2|eB|} - \frac{1}{2} = \ell - 1 + \hat{s}_3,$$

$$\ell \equiv \frac{E^2 - (p^3)^2 - m_W^2}{2|eB|} - \frac{1}{2}, \qquad \text{and} \qquad \hat{s}_3 \equiv s_{eB}s_3,$$

and which solutions are parabolic cylinder functions: $X_n^r(\rho) = N_n D_n(\rho)$.

Gauge condition solution

Rewriting the gauge condition using the rotation matrices

$$\sum_{s_3} D^\mu W_\mu^{s_3 -} = \sum_{s_3} D^\alpha g_\alpha{}^\mu W_\mu^{s_3 -} = \sum_{s_3} D^\alpha \left(N^{-1}\right)_\alpha{}^\beta N_\beta{}^\mu W_\mu^{s_3 -} = \sum_{s_3} \tilde{D}^\mu \tilde{W}_\mu^{s_3 -} = 0,$$

where $\tilde{D}^\mu \equiv D^\nu \left(N^{-1}\right)_\nu{}^\mu$.
Then, plugging the ansatz here, one gets

$$\tilde{\mathbb{E}}^{r,\alpha}_{n-\hat{s}_3,\mu}(\rho_r)\tilde{\bar{p}}^{\mu *}\tilde{\bar{\varepsilon}}^{-}_{\alpha} = 0,$$

where

$$\tilde{\bar{p}}^{\alpha *} \equiv \left(E, i s_{eB}\sqrt{|eB|\left(\ell - 1 + \frac{1+s_{eB}}{2}\right)}, -i s_{eB}\sqrt{|eB|\left(\ell - 1 + \frac{1-s_{eB}}{2}\right)}, p^3\right),$$

$$\tilde{\mathbb{E}}^{r,\alpha}_{n-\hat{s}_3,\mu}(\rho_r) \equiv \sum_{s_3} X^r_{n-\hat{s}_3}(\rho_r)\tilde{\Delta}^\alpha{}_\mu(s_3) e^{-ir\left(Ex_0 + p^2 x_2 + p^3 x_3\right)}.$$

Polarization Vectors

To completely know every factor in the ansatz, it is necessary to calculate a set of polarization vectors from the previous equations and conditions $\tilde{\omega}^{\mu-}$. Additionally, the orthonormalization condition

$$\int dx^1 J_0\left(W_i^-, W_j^+\right) = 4\pi E\delta_{ij},$$

will help us define each entry of the polarization vectors. These vectors satisfy the polarization sum relation

$$\sum_{\lambda=1}^{3} \tilde{\tilde{\varepsilon}}_\lambda^{\alpha-}\left(\tilde{\tilde{\varepsilon}}_\lambda^{\beta-}\right)^* = -g^{\alpha\beta} + \frac{\tilde{\tilde{p}}^\alpha \tilde{\tilde{p}}^{\beta *}}{m_W^2}.$$

Canonical Quantization

Following conventional canonical quantization

$$\hat{\tilde{W}}^{\mu -}(x) = \sum\!\!\!\!\!\!\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E}}\sum_{\lambda=1}^{3}\left[\hat{a}^{\lambda}_{\tilde{\bar{p}}}\tilde{\mathbb{E}}^{+\mu}_{n,\ \alpha}(\rho_+)\tilde{\bar{\varepsilon}}^{\alpha -}_{\lambda}(\tilde{\bar{p}}) + \hat{b}^{\lambda\dagger}_{\tilde{\bar{p}}}\tilde{\mathbb{E}}^{-\mu}_{n,\ \alpha}(\rho_-)\left(\tilde{\bar{\varepsilon}}^{\alpha +}_{\lambda}(\tilde{\bar{p}})\right)^*\right],$$

where

$$\sum\!\!\!\!\!\!\int \frac{d^3p}{(2\pi)^3} \equiv \frac{1}{2\pi}\sum_{\ell=0}^{\infty}\int\frac{dp^2}{2\pi}\int\frac{dp^3}{2\pi}.$$

The commutation relations are defined as

$$\left[\hat{a}^{\lambda}_{\bar{p}}, \hat{a}^{\lambda'\dagger}_{\bar{p}'}\right] = (2\pi)^3\,\delta_{\ell\ell'}\delta_{\lambda\lambda'}\delta\left(p_2 - p_2'\right)\delta\left(p_3 - p_3'\right) = \left[\hat{b}^{\lambda}_{\bar{p}}, \hat{b}^{\lambda'\dagger}_{\bar{p}'}\right].$$

W Boson Propagator in a Magnetic Field

Using the propagator definition

$$\tilde{D}_B^{\mu\nu}(x,y) \equiv \langle 0 | \mathcal{T} \left\{ \hat{\tilde{W}}^{\mu -}(x) \left(\hat{\tilde{W}}^{\nu -}(y) \right)^{\dagger} \right\} |0\rangle \,,$$

one arrives to the result

$$\tilde{D}_B^{\mu\nu}(x,y) = i \sum\!\!\!\!\!\!\int \frac{d^4 p}{(2\pi)^4} \left(-g^{\alpha\beta} + \frac{\bar{\tilde{\mathcal{P}}}^{\alpha} \bar{\tilde{\mathcal{P}}}^{\beta *}}{m_W^2} \right) \left[\frac{\tilde{\mathbb{E}}^{+\mu}_{n,\ \alpha}(\rho_+) \left(\tilde{\mathbb{E}}^{+}_{n,\beta}{}^{\nu}(\rho'_+) \right)^*}{\bar{\tilde{\mathcal{P}}}^2 - m_W^2 + i\varepsilon} \right]$$

where

$$\bar{\tilde{\mathcal{P}}}^{\alpha *} \equiv \left(p^0, i s_{eB} \sqrt{|eB| \left(\ell - 1 + \frac{1 + s_{eB}}{2} \right)}, -i s_{eB} \sqrt{|eB| \left(\ell - 1 + \frac{1 - s_{eB}}{2} \right)}, p^3 \right) .$$

Comparison: With and without magnetic field

$$D^{\mu\nu}(x,y) = i\int \frac{d^4p}{(2\pi)^4}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_W^2}\right)\frac{e^{-ip\cdot(x-y)}}{p^2 - m_W^2 + i\varepsilon}$$

$$\downarrow$$

$$\tilde{D}_B^{\mu\nu}(x,y) = i\sum\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4}\left(-g^{\alpha\beta} + \frac{\tilde{\bar{\mathcal{P}}}^\alpha \tilde{\bar{\mathcal{P}}}^{\beta *}}{m_W^2}\right)\left[\frac{\tilde{\mathbb{E}}^{+\mu}_{n,\ \alpha}(\rho_+)\left(\tilde{\mathbb{E}}^{+}_{n,\beta}{}^{\nu}(\rho'_+)\right)^*}{\tilde{\bar{\mathcal{P}}}^2 - m_W^2 + i\varepsilon}\right]$$

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$$\left. + \frac{\tilde{\bar{\Pi}}_x^{\alpha}\tilde{\bar{\Pi}}_y^{\beta *}}{m_W^2}\frac{\tilde{\mathbb{E}}^{+\mu}_{n-\hat{s}_3,\ \alpha}(\rho_+)\left(\tilde{\mathbb{E}}^{+\ \ \nu}_{n-\hat{s}_3,\beta}(\rho'_+)\right)^*}{\tilde{\bar{\mathcal{P}}}^2 - m_W^2 + i\varepsilon}\right]$$

with $\tilde{\bar{\Pi}}_x^{\alpha} \equiv \left(i\partial_x^0, \frac{i}{\sqrt{2}}\left(\partial_x^1 - i\left(\partial_x^2 + ieBx^1\right)\right), \frac{i}{\sqrt{2}}\left(\partial_x^1 + i\left(\partial_x^2 + ieBx^1\right)\right), i\partial_x^3\right)$.

Schwinger Parametrization

As a final remark on the representations of the propagator, a novel methodology has been developed to write the $W^{\pm}$ boson propagator in the presence of a magnetic field using the Schwinger parameter

$$\frac{1}{x} = -i \int_0^{\infty} ds \; e^{isx},$$

and parting from the presented Ritus Eigenfunction representation of the propagator.

From Ritus Eigenfunctions to Schwinger Parameter

After summing over all Landau Levels and integrating over the p^2 parameter

$$\tilde{D}_B^{\mu\nu}(x,y) = \int \frac{dp^0}{2\pi} \int \frac{dp^3}{2\pi} \int_0^\infty \frac{ds}{2\pi} e^{-is\left[(p^3)^2-(p^0)^2+m_W^2\right]} \frac{-ieB}{2\sin(eBs)}$$
$$\times \left[\left(-g^{\alpha\beta}\right) \sum_{s_3} \tilde{\Delta}^{\mu}{}_{\alpha}(s_3)\tilde{\Delta}^{\nu}{}_{\beta}(s_3) e^{2ieBs_3 s} + \frac{\tilde{\tilde{\Pi}}_x^{\alpha}\tilde{\tilde{\Pi}}_y^{\beta *}}{m_W^2} \sum_{s_3,s_3'} \tilde{\Delta}^{\mu}{}_{\alpha}(s_3)\tilde{\Delta}^{\nu}{}_{\beta}(s_3') \right]$$
$$\times\, \Omega(x,y) e^{-i\left[p^0(x_0-y_0)+p^3(x_3-y_3)\right]} e^{\frac{ieB}{4}\cot(eBs)\left[(x_1-y_1)^2+(x_2-y_2)^2\right]}$$

where $\Omega(x,y) = e^{\frac{ieB}{2}(x_1+y_1)(x_2-y_2)}$ is the Schwinger Phase. This procedure has been done before by another author[11]. However, this is where they stop.

[11] A. I. Nikishov. Vector boson in the constant electromagnetic field. J. Exp. Theor. Phys., 93(2):197–210, August 2001.

From Ritus Eigenfunctions to Schwinger Parameter

After applying the differential operators on the exponential functions

$$\frac{\tilde{\bar{\Pi}}_x^{\mu}\tilde{\bar{\Pi}}_y^{\nu *}}{m_W^2}\Omega(x,y)e^{-i\left[p^0(x_0-y_0)+p^3(x_3-y_3)\right]}e^{\frac{ieB}{4}\cot(eBs)\left[(x_1-y_1)^2+(x_2-y_2)^2\right]}=$$

$$=\Omega(x,y)e^{-i\left[p^0(x_0-y_0)+p^3(x_3-y_3)\right]}e^{\frac{ieB}{4}\cot(eBs)\left[(x_1-y_1)^2+(x_2-y_2)^2\right]}\frac{\tilde{\mathbb{X}}^{\mu\nu}+\tilde{\mathbb{B}}^{\mu\nu}}{m_W^2},$$

where

$$\tilde{\mathbb{X}}^{\mu\nu}\equiv\begin{pmatrix} (p^0)^2 & -p^0X_+\frac{eB}{2}c_{eB} & -p^0X_-\frac{eB}{2}c_{eB}^* & p^0p^3 \\ -p^0X_-\frac{eB}{2}c_{eB} & X_+X_-\frac{(eB)^2}{4}c_{eB}^2 & X_-^2\frac{(eB)^2}{4}|c_{eB}|^2 & -p^3X_-\frac{eB}{2}c_{eB} \\ -p^0X_+\frac{eB}{2}c_{eB}^* & X_+^2\frac{(eB)^2}{4}|c_{eB}|^2 & X_+X_-\frac{(eB)^2}{4}(c_{eB}^*)^2 & -p^3X_+\frac{eB}{2}c_{eB}^* \\ p^0p^3 & -p^3X_+\frac{eB}{2}c_{eB} & -p^3X_-\frac{eB}{2}c_{eB}^* & (p^3)^2 \end{pmatrix}$$

$$\tilde{\mathbb{B}}^{\mu\nu}\equiv\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{ieB}{2}c_{eB} & 0 & 0 \\ 0 & 0 & -\frac{ieB}{2}c_{eB}^* & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and $X_\pm\equiv(X_1\pm iX_2)/\sqrt{2}$ with $X_i\equiv x_i-y_i$; and $c_{eB}\equiv i+\cot(eBs)$ is defined.

From Ritus Eigenfunctions to Schwinger Parameter

After identifying the Gaussian integrals

$$e^{i\frac{eB}{4}\cot(eBs)(X_i)^2} = \int_{-\infty}^{\infty} dp^i \; e^{-ip^i X_i} e^{-i(p^i)^2\frac{\tan(eBs)}{eB}} \sqrt{\frac{i\tan(eBs)}{\pi eB}},$$

$$X_i e^{i\frac{eB}{4}\cot(eBs)(X_i)^2} = \int_{-\infty}^{\infty} dp^i \; e^{-ip^i X_i} e^{-i(p^i)^2\frac{\tan(eBs)}{eB}} \frac{2i}{\sqrt{\pi}}\left(\frac{i\tan(eBs)}{eB}\right)^{\frac{3}{2}},$$

$$(X_i)^2 e^{i\frac{eB}{4}\cot(eBs)(X_i)^2} = \int_{-\infty}^{\infty} dp^i \; e^{-ip^i X_i} e^{-i(p^i)^2\frac{\tan(eBs)}{eB}} \frac{-2i}{eB\cot(eBs)} \times\left[(p^i)^2\frac{2}{\sqrt{\pi}}\left(\frac{i\tan(eBs)}{eB}\right)^{\frac{3}{2}} - \sqrt{\frac{i\tan(eBs)}{\pi eB}}\right],$$

From Ritus Eigenfunctions to Schwinger Parameter

The propagator can be rewritten as

$$\tilde{D}_B^{\mu\nu}(x,y) = \Omega(x,y)\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left[p_\parallel^2 + p_\perp^2 \frac{\tan(eBs)}{eBs} - m_W^2\right]}$$
$$\times \left[\left(-g^{\alpha\beta}\right) \sum_{s_3} \tilde{\Delta}^\mu{}_\alpha(s_3) \tilde{\Delta}^\nu{}_\beta(s_3) e^{2ieBss_3} + \frac{\tilde{\mathbb{P}}_t^{\mu\nu} + \tilde{\mathbb{T}}^{\mu\nu}}{m_W^2} \right]$$

where

$$\tilde{\mathbb{P}}_t^{\mu\nu} = \begin{pmatrix} (p^0)^2 & p^0 p_+ t_{eB} & p^0 p_- t_{eB}^* & p^0 p^3 \\ p^0 p_- t_{eB} & p_+ p_- t_{eB}^2 & (p_-)^2 |t_{eB}|^2 & p^3 p_- t_{eB} \\ p^0 p_+ t_{eB}^* & (p_+)^2 |t_{eB}|^2 & p_+ p_- (t_{eB}^*)^2 & p^3 p_+ t_{eB}^* \\ p^0 p^3 & p^3 p_+ t_{eB} & p^3 p_- t_{eB}^* & (p^3)^2 \end{pmatrix}$$

$$\tilde{\mathbb{T}}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{eB}{2} t_{eB} & 0 & 0 \\ 0 & 0 & \frac{eB}{2} t_{eB}^* & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where $p_\pm \equiv \left(p^1 \pm ip^2\right)/\sqrt{2}$ and $t_{eB} \equiv i\tan(eBs) + 1$.

From Ritus Eigenfunctions to Schwinger Parameter

Finally, rotating back using the N and N^{-1} matrices, one arrives at

$$D_B^{\mu\nu}(x,y) = \Omega(x,y) D_B^{\mu\nu}(x-y) \implies D_B^{\mu\nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} D_B^{\mu\nu}(p)$$

$$D_B^{\mu\nu}(p) = -\int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left[p_\parallel^2 + p_\perp^2 \frac{\tan(eBs)}{eBs} - m_W^2\right]} \Bigg[g_\parallel^{\mu\nu} + g_\perp^{\mu\nu}\cos(2eBs) + \varphi^{\mu\nu}\sin(2eBs)$$

$$- \frac{1}{m_W^2}\left[(p^\mu + (\varphi p)^\mu \tan(eBs))(p^\nu + (p\varphi)^\nu \tan(eBs)) - \frac{ieB}{2}(\varphi^{\mu\nu} - g_\perp^{\mu\nu}\tan(eBs)) \right] \Bigg]$$

where $(\varphi p)^\mu \equiv \varphi^{\mu\alpha} p_\alpha$ and $(p\varphi)^\mu \equiv p_\alpha \varphi^{\alpha\mu}$.

Summary and Results

- The W boson propagator was explicitly calculated in the presence of a magnetic field using the Ritus Eigenfunction methodology.
- This methodology and calculation is heavily inspired on a very well known way of arriving at the W boson propagator in a vacuum.
- A novel way of calculating the propagator in a magnetic field in the Schwinger parameter representation is presented.

Thank you for your attention!

W Boson Propagator in a Magnetic Field: Ritus Eigenfunction

Rotating back the propagator to a more useful representation

$$D_B^{\mu\nu}(x,y) = i \sum\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4} \frac{\mathbb{E}^{+\mu}_{n,\ \alpha}(\rho_+)\left[-g^{\alpha\beta} + \frac{\bar{\mathcal{P}}^{\alpha}\bar{\mathcal{P}}^{\beta *}}{m_W^2}\right]\left(\mathbb{E}_n^{+\dagger}(\rho'_+)\right)_{\beta}{}^{\nu}}{(p^0)^2 - E^2 + i\varepsilon}$$

where

$$\mathbb{E}^{+\mu}_{n,\ \alpha}(\rho_+) \equiv \sum_{s_3} X_n^+(\rho_+)\Delta^{\mu}{}_{\alpha}(s_3)e^{-i\left(p^0x_0+p^2x_2+p^3x_3\right)},$$

$$\left(\mathbb{E}_n^{+\dagger}(\rho'_+)\right)_{\beta}{}^{\nu} \equiv \sum_{s'_3} X_n^+(\rho'_+)\Delta_{\beta}{}^{\nu}(s'_3)e^{i\left(p^0y_0+p^2y_2+p^3y_3\right)},$$

and

$$\Delta^{\mu}{}_{\alpha}(s_3) \equiv \left(N^{-1}\right)^{\mu}{}_{\sigma}\tilde{\Delta}^{\sigma}{}_{\alpha}(s_3)N^{\alpha}{}_{\delta} = \begin{pmatrix} 1-s_3^2 & 0 & 0 & 0 \\ 0 & \frac{s_3^2}{2} & \frac{-is_3}{2} & 0 \\ 0 & \frac{is_3}{2} & \frac{s_3^2}{2} & 0 \\ 0 & 0 & 0 & 1-s_3^2 \end{pmatrix}$$