Temperature Fluctuations in a Relativistic gas: Pressure corrections and posible consequences in the deconfinement transition

M. Loewe

Universidad San Sebastián, Facultad de Ingeniería,2nd latinamerican workshop on electromagnetic effects in QCD. Santiago, November 2025

This talk is based on :

Jorge David Castaño-Yepes, M. Loewe, Enrique Muñoz, Juan Cristobal Rojas; ”Temperature fluctuations in a relativistic gas: Pressure corrections and posible consequences in the deconfinement transition”, Physical Review D 110, 056014 (2024).

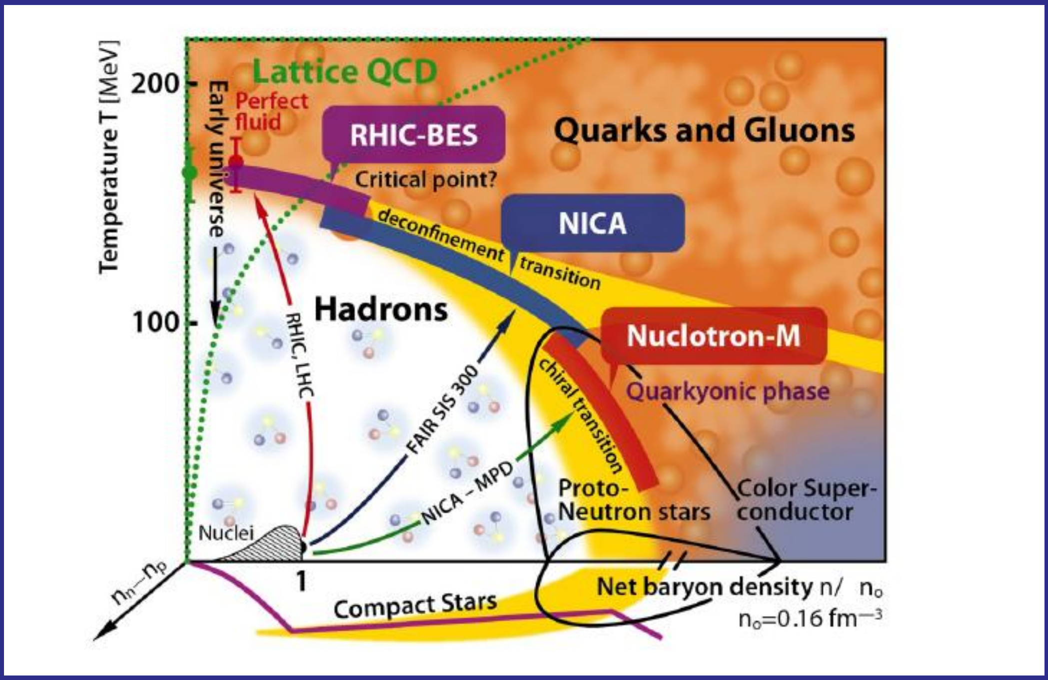
Previous articles where these ideas have been implemented in a different context

a) Jorge David Castaño-Yepes, M. Loewe, Enrique Muñoz, Juan Cristobal Rojas, Renato Zamora ; “QED fermions in a noisy magnetic field background”, Physical Review D 107 (2023), 096014;

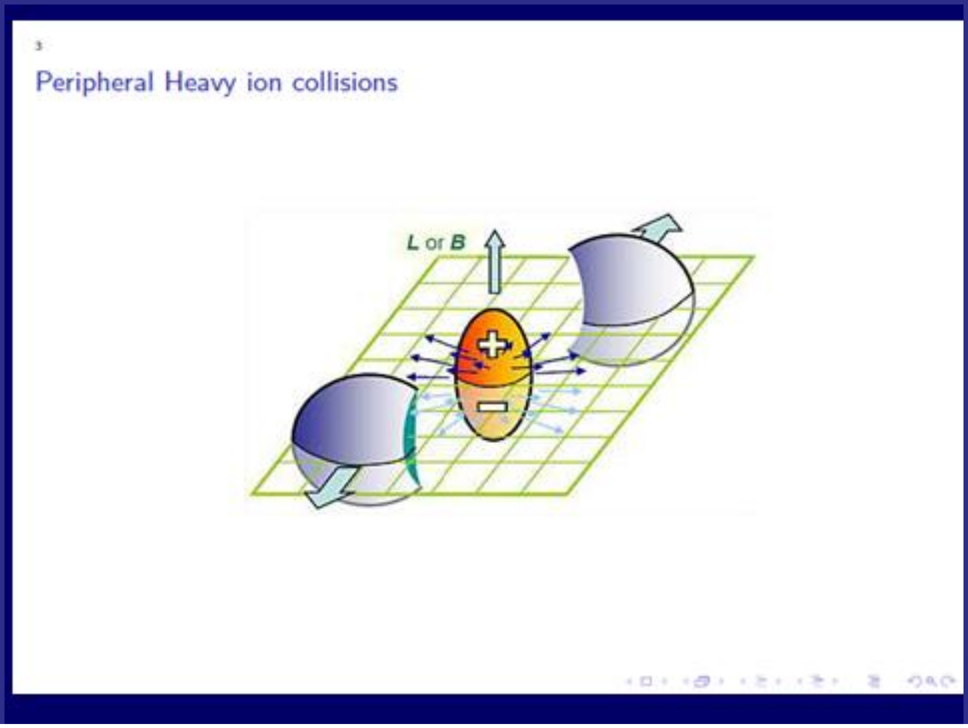
a) b) Jorge David Castaño-Yepes, M. Loewe, Enrique Muñoz, Juan Cristobal Rojas” QED fermions in a noisy magnetic field background: The effective action approach”, Physical Review D 108 (2023)116013 .

I acknowledge support form the Fondecyt projects1200483, 1190192, 1220035.

During the last years the community has tried to understand the QCD phase diagram under extreme conditions



Temperature and Magnetic Fields in peripheral heavy ion collisions. It is hard to believe that the field strength will be constant in the collision plane….Also it is not obvious that we are in thermodynamic equilibrium



Replica trick I

- The replica method was introduced through the spin glass model:
$$H = -\sum_{i,k} J_{ik}\sigma_i\sigma_k,$$
where the J_{ik} are uncorrelated Gaussian random variables with zero mean and variance $\overline{J_{ik}^2} = K_{ik}$.
- It is necessary to obtain the proper averaged thermodynamic potential
$$F = -kT\langle \ln Z\rangle_{\mathsf{ave}}\ .$$

- In order to average over macroscopic samples wherein a vast number of different configurations of the J_{ik} are operative, they introduced the so-called "replica trick":[1]

$$\ln Z = \lim_{n \to 0} \frac{Z^n - 1}{n}$$

- The average is computed before taking the limit $n \to 0$. It was introduced by Parisi as a method to average the free energy, defined via the logarithm of the partition function $\ln Z$, of a system over quenched (or frozen) disorder.

[1] Mézard M, Parisi G, Virasoro M. 1987. Spin glass theory and beyond: An **An Introduction to the Replica Method and Its Applications**. World Scientific, Singapore. 476pg.

It is used in

- Spin glasses,
- Polymer networks,
- Z_n field theory,
- intermittency of turbulence,
- Euclidean random matrices,
- granular matter
- AdS/CFT
- etc.

We assume a non-equilibrium scenario, where temperature is not defined uniformly through the whole system, but smaller regions may still be pictured as nearly-thermalized subsystems. Therefore, we model this situation by an ensemble of subsystems whose individual temperatures $T = T_0 + \delta T$ are subjected to stochastic fluctuations with zero mean $\overline{\delta T} = 0$, but finite variance $\overline{\delta T^2} = \Delta$. In terms of the inverse temperature

$$\beta = (T_0 + \delta T)^{-1} = T_0^{-1} - \frac{\delta T}{T_0^2} = \beta_0 + \delta\beta.$$

$$\beta = (T_0 + \delta T)^{-1} = T_0^{-1} - \frac{\delta T}{T_0^2} = \beta_0 + \delta\beta$$

Which also means

$$\delta\beta = -\frac{\delta T}{T_0^2},$$
$$\overline{\delta\beta} = -T_0^{-2}\overline{\delta T} = 0,$$
$$\overline{\delta\beta^2} = T_0^{-4}\overline{\delta T^2} = \beta_0^4\Delta = \Delta_\beta$$

These statistical features are captured by a Gaussian distribution with zero mean.

$$dP[\delta\beta] = \frac{d(\delta\beta)}{\sqrt{2\pi\Delta_\beta}} e^{-\frac{\delta\beta^2}{2\Delta_\beta}}.$$

$$\overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

We can apply this idea to any thermodynamical quantity. For example to the grand potential

$$\bar{\Omega}(\mu, \mathcal{V}, T) = -T\overline{\ln Z}(\mu, \mathcal{V}, T)$$

Notice that the relations of the previous slight can be extended to

$$\delta\beta = -\frac{\delta T}{T_0^2},$$
$$\overline{\delta\beta} = -T_0^{-2}\overline{\delta T} = 0,$$
$$\overline{\delta\beta^2} = T_0^{-4}\overline{\delta T^2} = \beta_0^4\Delta = \Delta$$

$$\overline{\delta\beta} = \overline{\delta\beta^{2j-1}} = 0,$$
$$\overline{\delta\beta^2} = \Delta_\beta,$$
$$\overline{\delta\beta^{2j}} = \Delta_\beta^j(2j-1)!!.$$

- We will discuss termal fluctuations for the relativistic Fermi gas, the photon gas and the "gluon" gas. Finally, and application in the frame of the bag model will be considered

In general, let us consider the grand canonical partition funtion

$$Z(\mu, \mathcal{V}, T) = \mathrm{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}]$$

The statistical average over temperature distribution is obtained via the replica method

$$\overline{\ln Z} = \lim_{n\to 0}\frac{1}{n}\left(\overline{\mathrm{Tr}\left[\exp\left\{-(\beta_0+\delta\beta)\sum_{r=1}^{n}\hat{K}^{(r)}\right\}\right]} - 1\right)$$

Where each replica 1 ≤ r ≤ n has an associated operator $\hat{K}^{(r)}$

Expanding the exponential in powers of the fluctuation and taking the statistical Average of each term

$$\overline{Z^n} = \overline{\mathrm{Tr}\left[\exp\left\{-(\beta_0+\delta\beta)\sum_{r=1}^{n}\hat{K}^{(r)}\right\}\right]} = \int dP[\delta\beta]\mathrm{Tr}\left[e^{-\beta_0\sum_{r=1}^{n}\hat{K}^{(r)}}\left(1+\sum_{j=1}^{\infty}\frac{(-1)^j(\delta\beta)^j}{j!}\left(\sum_{r=1}^{n}\hat{K}^{(r)}\right)^j\right)\right]$$

$$= \mathrm{Tr}\left[e^{-\beta_0\sum_{r=1}^{n}\hat{K}^{(r)}}\left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\left(\sum_{r=1}^{n}\hat{K}^{(r)}\right)^{2j}\right)\right]$$

$$= \left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\frac{\partial^{2j}}{\partial\beta_0^{2j}}\right)\mathrm{Tr}\left[e^{-\beta_0\sum_{r=1}^{n}\hat{K}^{(r)}}\right].$$

It is remarkable that this power expansión can be expressed as temperature derivatives of the partition function

$$Z_0^n = \mathrm{Tr}\left[e^{-\beta_0\sum_{r=1}^{n}\hat{K}^{(r)}}\right]$$

In fact we find

$$\overline{Z^n} = \left(1 + \sum_{j=1}^{\infty} \frac{(\Delta_\beta/2)^j}{j!} \frac{\partial^{2j}}{\partial \beta_0^{2j}}\right) Z_0^n$$
$$= \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] Z_0^n,$$

In this way we get for the statistical average of the grand potential

$$-\beta_0 \bar{\Omega} = \overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n} = \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] \lim_{n\to 0} \frac{Z_0^n - 1}{n}$$
$$= \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] \lim_{n\to 0} \frac{e^{n \ln Z_0} - 1}{n}$$
$$= \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] \ln Z_0.$$

The idea is to consider only the first order in the fluctuation Δ

$$\overline{\ln Z/Z_0} = \frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2} \ln Z_0 + O(\Delta_\beta^2)$$
$$= \beta_0(\mathcal{PV} - (\mathcal{PV})_{ig}),$$

Remember: the equation of state for the ideal gas is

$$\beta_0 (\mathcal{P}\mathcal{V})_{ig} = \ln Z_0$$

Therefore, up to order O(Δ^2), we have an excess of pressure due to the average effect of temperature fluctuations

$$\delta\mathcal{P} \equiv \mathcal{P} - \mathcal{P}_{\mathrm{ig}} = \frac{\Delta_\beta}{2\mathcal{V}\beta_0} \frac{\partial^2}{\partial\beta_0^2} \ln Z_0 + O(\Delta^2)$$

In the article we have shown that this excess of pressure is positive (too long to be presented here)

Relativistic Fermi gas with termal noise

$$\hat{H} - \mu\hat{N} = \int d^3x \hat{\psi}^\dagger(\mathbf{x})\gamma^0[\boldsymbol{\gamma}\cdot(-\mathrm{i}\nabla) + m - \gamma^0\mu]\hat{\psi}(\mathbf{x})$$
$$\equiv \hat{K}.$$

The partition function for the n-replicas of this ideal gas (represented by Grassmann fields $\Psi_r(x)$, r the replica index, $1 \leq r \leq n$,) is well known

$$\begin{aligned} Z_{F0}^n &= \prod_{r=1}^{n} \int \mathcal{D}[\psi_r^\dagger, \psi_r] \exp\left[-\int_0^{\beta_0} d\tau \sum_{r=1}^{n} \psi_r^\dagger(\mathbf{x},\tau)\gamma^0(\gamma^0(\partial_\tau - \mu) + \boldsymbol{\gamma}\cdot\mathbf{p} + m)\psi_r(\mathbf{x},\tau)\right] \\ &= \det\left[\partial_\tau - \mu + \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0\right]^n \\ &= \exp\left\{n\mathrm{Tr}\ln\left[\partial_\tau - \mu + \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0\right]\right\} \\ &= \exp\left(n\ln Z_{F0}\right), \end{aligned}$$

The partition function for the fermion gas

$$\begin{aligned}\ln Z_{F0} &= \mathrm{Tr}\ln\left[\partial_\tau - \mu + \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0\right]\\ &= \mathcal{V}\int\frac{d^3p}{(2\pi)^3}\sum_{k\in\mathbb{Z}}\mathrm{tr}\ln\left[\mathrm{i}\omega_k - \mu + \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0\right]\end{aligned}$$

Where we diagonalized the operator in Matsubara momentum space

$$\omega_k = (2k+1)\pi/\beta_0 \quad (k\in\mathbb{Z})$$

After some calculations (for details, please refer to the article) we find

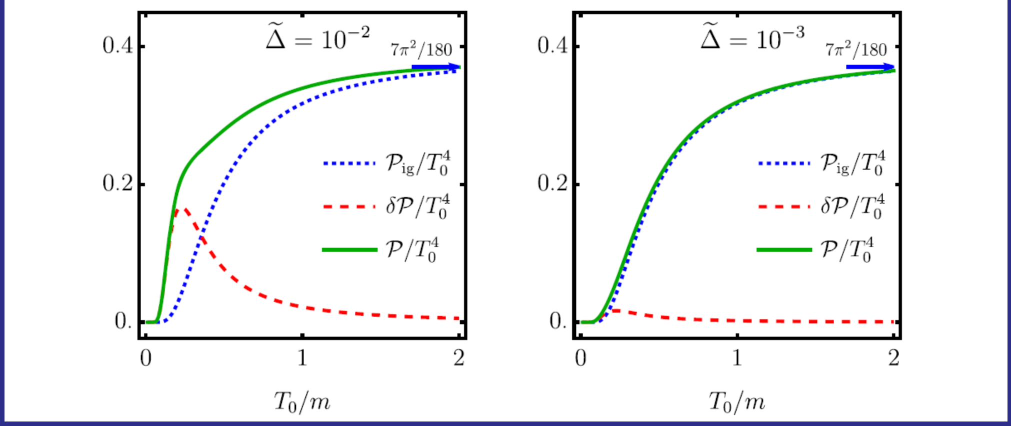
$$\begin{aligned}\ln Z_{F0} &= 2\mathcal{V}\int\frac{d^3p}{(2\pi)^3}\sum_{k\in\mathbb{Z}}\{\ln\left[\mathrm{i}\omega_k - \mu + E_{\mathbf{p}}\right]\\ &\quad + \ln\left[\mathrm{i}\omega_k - \mu - E_{\mathbf{p}}\right]\}\\ &= 2\mathcal{V}\int\frac{d^3p}{(2\pi)^3}\left\{\ln\left(1+e^{\beta_0(\mu-E_{\mathbf{p}})}\right)\right.\\ &\quad \left.+\ln\left(1+e^{\beta_0(\mu+E_{\mathbf{p}})}\right)\right\}.\end{aligned}$$

$$
\begin{aligned}
\overline{\ln Z_F} &= \lim_{n\to 0} \frac{\overline{Z_F^n} - 1}{n} = \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right] \lim_{n\to 0} \frac{Z_{F0}^n - 1}{n} \\
&= \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right] \lim_{n\to 0} \frac{e^{n \ln Z_{F0}} - 1}{n} \\
&= \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right] \ln Z_{F0}.
\end{aligned}
$$

Equation of state: we get an explicit expression for the excess pressure

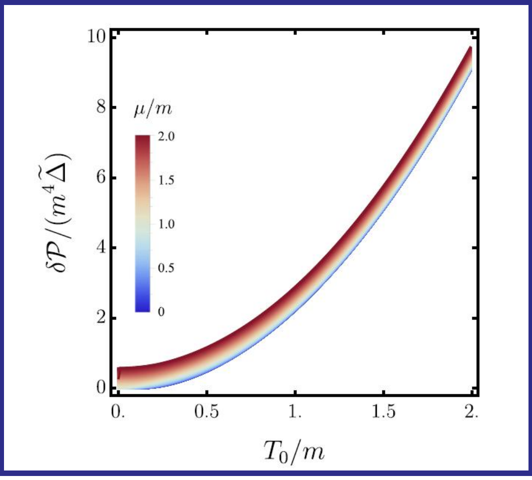
$$
\begin{aligned}
\delta\mathcal{P} = \frac{\Delta_\beta}{\beta_0} \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} (E_{\mathbf{p}} + s\mu)^2 n_{\mathrm{F}}\left(\frac{E_{\mathbf{p}} + s\mu}{T_0}\right) \\
\times \left[1 - n_{\mathrm{F}}\left(\frac{E_{\mathbf{p}} + s\mu}{T_0}\right)\right],
\end{aligned}
$$

For μ = 0



For different values of μ.

$\tilde{\Delta} \equiv \Delta/m^2$



A short parenthesis: In we have exlored an alternative way

PHYSICAL REVIEW D **98,** 114002 (2018)

Superstatistics and the effective QCD phase diagram

Alejandro Ayala,[1,2] Martin Hentschinski,[3] L. A. Hernández,[1,2] M. Loewe,[4,2,5] and R. Zamora[6,7]

$$B(\hat{H}) = \int_0^\infty f(\beta) e^{-\beta \hat{H}} d\beta$$

$$Z = \mathrm{Tr}[B(\hat{H})] = \int_0^\infty B(E) dE,$$

$$f(\beta) = \frac{1}{\Gamma(N/2)} \left(\frac{N}{2\beta_0}\right)^{N/2} \beta^{N/2-1} e^{-N\beta/2\beta_0}$$

We have repeated the exercise for a photon gas and a gluon gas with termal noise.

Some steps:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

We might use the Fadeev-Popov technique and, eventually you find

$$\begin{aligned} Z_{B0} &= \mathrm{Tr} e^{-\beta_0 \hat{H}} \\ &= \int \mathcal{D}[A_\mu] \delta[\mathcal{F}] \det\left(\frac{\partial \mathcal{F}}{\partial \alpha}\right) e^{\int_0^{\beta_0} d\tau \int d^3x \mathcal{L}} \end{aligned}$$

With a típica family of covariant gaugesl

$$\mathcal{F}[A_\mu] = \partial^\mu A_\mu - f(x,\tau) = 0$$

So that

$$\mathcal{F}[A_\mu - \partial^\mu \alpha] = \partial^\mu A_\mu - f(x,\tau) - \partial^2 \alpha$$

Which means

$$\det\left(\frac{\partial \mathcal{F}}{\partial \alpha}\right) = \det\left(-\partial^2\right)$$

After some steps

$$\begin{aligned}
\ln Z_{B0} &= -\mathrm{Tr}\ln[-\partial^2] = -\mathrm{Tr}\ln[-\partial_\tau^2 - \nabla^2] \\
&= -\mathcal{V}\int \frac{d^3p}{(2\pi)^3}\sum_{k\in\mathbb{Z}} \ln[\omega_k^2 + \mathbf{p}^2] \\
&= -\mathcal{V}\int \frac{d^3p}{(2\pi)^3}\sum_{k\in\mathbb{Z}} \{\ln[|\mathbf{p}| + \mathrm{i}\omega_k] + \ln[|\mathbf{p}| - \mathrm{i}\omega_k]\} \\
&= -\mathcal{V}\int \frac{d^3p}{(2\pi)^3} [\ln(1 - e^{-\beta_0|\mathbf{p}|}) + \ln(e^{\beta_0|\mathbf{p}|} - 1)] \\
&= -2\mathcal{V}\int \frac{d^3p}{(2\pi)^3}\left[\frac{\beta_0}{2}|\mathbf{p}| + \ln(1 - e^{-\beta_0|\mathbf{p}|})\right]. \qquad (34)
\end{aligned}$$

In this case

$$\omega_k = 2k\pi/\beta_0, \quad \text{for} \quad k\in\mathbb{Z}$$

In the previous expresión we can susbtracted the linear divergent term(vacuum energy). Proceeding in this way we find for the grand potential

$$
\begin{aligned}
\Omega_0^B &= -T_0 \ln Z_{B0} = \nu_B \frac{\mathcal{V}T_0^4}{2\pi^2}\int_0^\infty dx\, x^2 \ln\left(1 - e^{-x}\right) \\
&= -\nu_B \frac{\mathcal{V}T_0^4}{6\pi^2}\int_0^\infty dx \frac{x^3}{e^x - 1} \\
&= -\nu_B \mathcal{V}\frac{\pi^2 T_0^4}{90},
\end{aligned}
$$

$$
\mathcal{P}_{\text{ig}}^B = \nu_B \frac{\pi^2 T_0^4}{90}.
$$

As expected

$$
\begin{aligned}
\overline{\ln Z_B} &= \lim_{n\to 0}\frac{\overline{Z_B^n} - 1}{n} \\
&= \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right]\lim_{n\to 0}\frac{Z_{B0}^n - 1}{n} \\
&= \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right]\ln Z_{B0}.
\end{aligned}
$$

Finally we have again an excees of pressure due to termal fluctuations

$$\delta\mathcal{P}^B = \mathcal{P} - \mathcal{P}^B_{\mathrm{ig}} = \frac{\Delta_\beta}{2\beta_0\mathcal{V}}\frac{\partial^2}{\partial\beta_0^2}\ln Z_{B0}$$
$$= \nu_B\frac{\pi^2}{15}\Delta_\beta\beta_0^{-6} = \nu_B\frac{\pi^2}{15}\Delta T_0^2 > 0,$$

For the case of gluons, keep in mind that we are dealing with a gas of free gluons!!

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{a,\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

Following the usual procedure

$$Z_G = \int \mathcal{D}[A_a^\mu]\delta[\mathcal{F}^a]\det\left(\frac{\delta\mathcal{F}^a}{\delta\alpha^c}\right)e^{\int_0^{\beta_0} d\tau \int d^3x\mathcal{L}}.$$

In this case, however

$$\det\left(\frac{\delta\mathcal{F}^a}{\delta\alpha^c}\right) = \det\left(-\partial^2\delta_c^a + gf^{abc}\partial_\mu A_b^\mu\right)$$

$$Z_G = \int \mathcal{D}[A_a^\mu]\det\left(-\partial^2\delta_c^a + gf^{abc}\partial_\mu A_b^\mu\right) \times e^{\int_0^{\beta_0} d\tau \int d^3x(\mathcal{L}-\frac{1}{2\xi}(\partial_\mu A_a^\mu)^2)}.$$

We handle the determinant through the introduction of ghost fields

Finally

$$Z_G = \int \mathcal{D}[A_a^\mu] \int \mathcal{D}[\bar{\eta}_a, \eta_a] e^{\int_0^{\beta_0} d\tau \int d^3x \mathcal{L}_{\text{eff}}[A_a^\mu, \bar{\eta}_a, \eta_a]}$$

with

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\frac{1}{2} A_a^\mu (-\delta_b^a g_{\mu\nu} \partial^2) A_b^\nu + \bar{\eta}_a (-\delta_b^a \partial^2) \eta_b \\ & - \frac{g^2}{4} (f^{eab} A_{a\mu} A_{b\nu})(f^{ecd} A_c^\mu A_d^\nu) \\ & - g f^{abc} (\partial_\mu A_{a\nu}) A_b^\mu A_c^\nu - g \bar{\eta}_a f^{abc} \partial^\mu A_{b\mu} \eta_c .\end{aligned}$$

And now we take g= 0, since life is hard!

$$\begin{aligned}\overline{\ln Z_G} &= \lim_{n \to 0} \frac{\overline{Z_{G0}^n} - 1}{n} \\ &= \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] \lim_{n \to 0} \frac{e^{n(N^2-1)\ln Z_{B0}} - 1}{n} \\ &= (N^2 - 1) \exp\left[\frac{\Delta_\beta}{2} \frac{\partial^2}{\partial \beta_0^2}\right] \ln Z_{B0} .\end{aligned}$$

Bag Model. A quite basic approach to hadrons. Inside the bag we have QCD (free) degrees of freedom and outside we have hadrons (pions) that try to compress the bag.

For the gas of pions

$$\mathcal{P}_{\text{Had}} = 3\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\text{Had}}$$

For the plasma pase: v_F = 2x3x2=12 (quarks) and v_B = 2 x (3^2 – 1) = 16 (gluons)

$$\begin{aligned}\mathcal{P}_{\text{Plasma}} &= \left(\nu_B + \frac{7}{4}\nu_F\right)\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\text{Plasma}} - B \\ &= \frac{37\pi^2}{90} T_0^4 + \delta\mathcal{P}^{\text{Plasma}} - B.\end{aligned}$$

B ~200MeV, the bag constant

The critical temperature T_c

$$3\frac{\pi^2 T_c^4}{90} = \frac{37\pi^2}{90} T_c^4 + \delta\mathcal{P}^{\mathrm{Net}} - B,$$

Where we have defined hte net excess pressure as

$$\delta\mathcal{P}^{\mathrm{Net}} = \delta\mathcal{P}^{\mathrm{Plasma}} - \delta\mathcal{P}^{\mathrm{Had}} = \delta\mathcal{P}^{G} - \delta\mathcal{P}^{\mathrm{Had}} + \delta\mathcal{P}^{Q}$$
$$= 13\frac{\pi^2}{15}\Delta T_0^2 + \delta\mathcal{P}^{Q} > 0,$$

Solving for T_c

$$T_c = T_c^0\left(1 - \frac{\delta\mathcal{P}^{\mathrm{Net}}}{(T_c^0)^4}\right)^{1/4} \leq T_c^0,$$

$$T_c^0 = (45B/17\pi^2)^{1/4} \sim 144 \text{ MeV}$$

So, we have a kind of catalysis of the pase transition

Concerning our previous work on magnetic fluctuations, just a few words

The Model: QED in the presence of a classical and static magnetic field possesing random spatial fluctuations

$$A^\mu(x) \to A^\mu(x) + A^\mu_{\rm BG}(x) + \delta A^\mu_{\rm BG}(x)$$

We consider a white noise spatio-temporal fluctuation with respect to the mean value, i.e.

$$\begin{aligned} \langle \delta A^j_{\rm BG}(x) \delta A^k_{\rm BG}(x') \rangle_\Delta &= \Delta_B \delta_{j,k} \delta^{(4)}(x - x'), \\ \langle \delta A^\mu_{\rm BG}(x) \rangle_\Delta &= 0. \end{aligned}$$

As it is well known, these statistical properties are represented by a Gaussian functional distribution (which is natural because of the Central Limit theorem)

$$dP\left[\delta A^{\mu}_{\mathrm{BG}}\right] = \mathcal{N} e^{-\int d^4x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(x)\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}(x)\right]$$

In this way, we have the following decomposition

$$\mathcal{L} = \mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}},$$

$$\mathcal{L}_{\mathrm{FBG}} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\mathrm{BG}} - e A\!\!\!/ - m\right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$\mathcal{L}_{\mathrm{NBG}} = \bar{\psi}\left(-e\delta A\!\!\!/_{\mathrm{BG}}\right)\psi.$$

The statistical average is given in terms of the Gaussian functional measure.

Z^n is obtained by the incorporation of replica components for the fermion fields. $\psi(x) \to \psi^a(x)$ $1 \leq a \leq n.$

$$\langle Z^n[A]\rangle_\Delta = \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] \int \mathcal{D}[\delta A^\mu_{\mathrm{BG}}]\, e^{-\int d^4x \frac{\left[\delta A^\mu_{\mathrm{BG}}(x)\right]^2}{2\Delta_B}} \times e^{\mathrm{i}\int d^4x \sum_{a=1}^{n}\left(\mathcal{L}_{\mathrm{FBG}}[\bar{\psi}^a,\psi^a]+\mathcal{L}_{DBG}[\bar{\psi}^a,\psi^a]\right)}$$

$$= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{\mathrm{i}\bar{S}[\bar{\psi}^a,\psi^a;A]}$$

After perfoming the integral over the magnetic fluctuations

$$\mathrm{i}\,\bar{S}\left[\bar{\psi}^a,\psi^a;A\right] = \mathrm{i}\int d^4x\left(\sum_a \bar{\psi}^a\left(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m_f\right)\psi^a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$
$$- \frac{e^2\Delta_B}{2}\int d^4x\int d^4y\sum_{a,b}\sum_{j=1}^{3}\bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y).$$

It reminds us the NJL-action (or the Fermi theory)

We have ended up with an effective interaction between vector currents, associated to different replica, with a coupling constant proportional to the fluctuation amplitude Δ_B .

As a kind of conclusions:

The replica method seems to be an attractive way to explore situations beyond equilibrium in the dynamics of heavy ion collisions.

We have extended the scenarios to the presence of electric fields (appear in collisions between an heavy and light nuclei (Au-Cu, for example).

THANK YOU