Data-driven magnetic dipole moment of the rho meson

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Seminario Altas Energías ICN - IF **12/03/25**

Outline

- *Motivation*
  - Radiative processes
  - Lessons from the W
  - Electromagnetic vertex VVγ
  - $e^+ e^- \to \pi^+ \pi^- 2\pi^0$
  - First analysis, the ρ MDM
- *New analysis, what is new?*
  - Modeling $e^+ e^- \to \pi^+ \pi^- 2\pi^0$
  - Channels and model tests
  - Magnetic dipole moment from Babar data
- *Conclusions*

Motivation

The magnetic dipole moment

It carries information on the nature of the particle itself. Related to the spin via the gyromagnetic ratio (g)

$$\vec{\mu} = g_\mu \frac{e\bar{h}}{2m_\mu c}\vec{S}$$

$$g_\mu = 2$$

Spin 1/2

Normal:
Dirac 30's

Measurements

Electron (g - 2)/2 = 0.001159

Proton g = 2.79285

Neutron g = - 1.91304

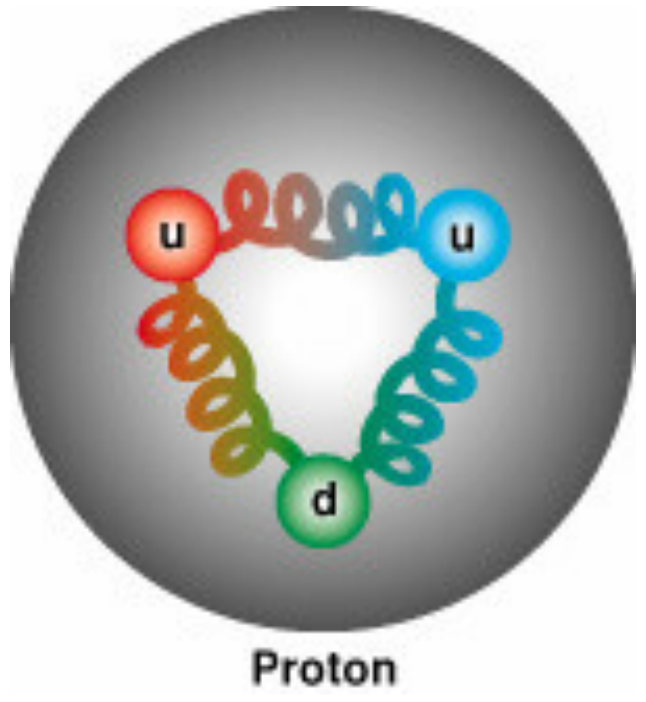
No fundamental states?

gneutron/gproton = - 0.6849

Motivation

Hadron structure and the quark model

Quark model: Spin 1/2 particles, the lighters : u (2/3), d (-1/3), s (-1/3)



Baryons (qqq)

Proton, made up of quarks uud $\qquad \mu_p = \frac{1}{3}(4\mu_u - \mu_d).$

Neutron, made up of quarks ddu $\qquad \mu_n = \frac{1}{3}(4\mu_d - \mu_u).$

gneutrón/gprotón = - 2/3= - 0.66

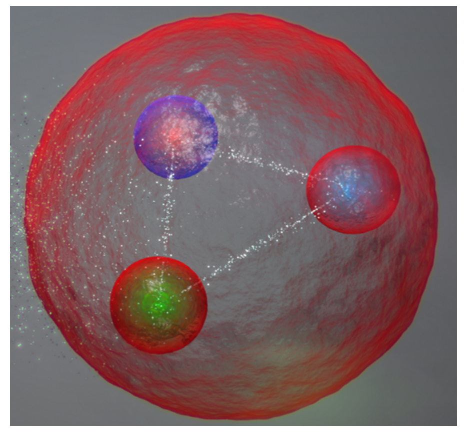
vs

gneutrón/gprotón = - 0.6849

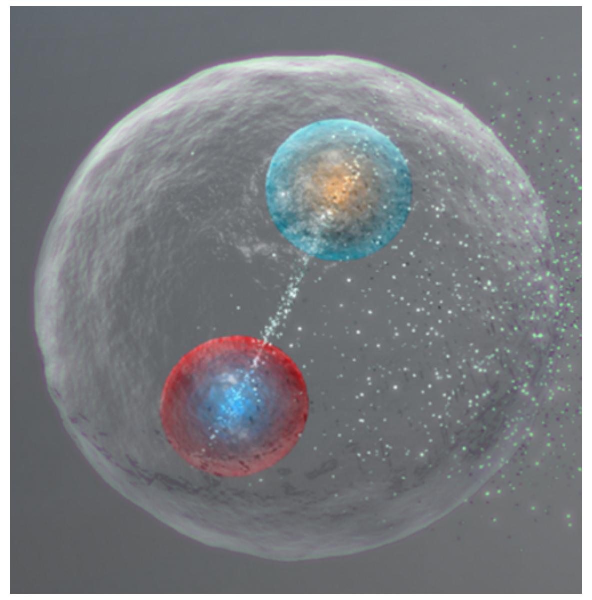
Explains the magnitude and relative sign. One of the first successes of the model.

Motivation

Mesons (vector) and their unestable feature

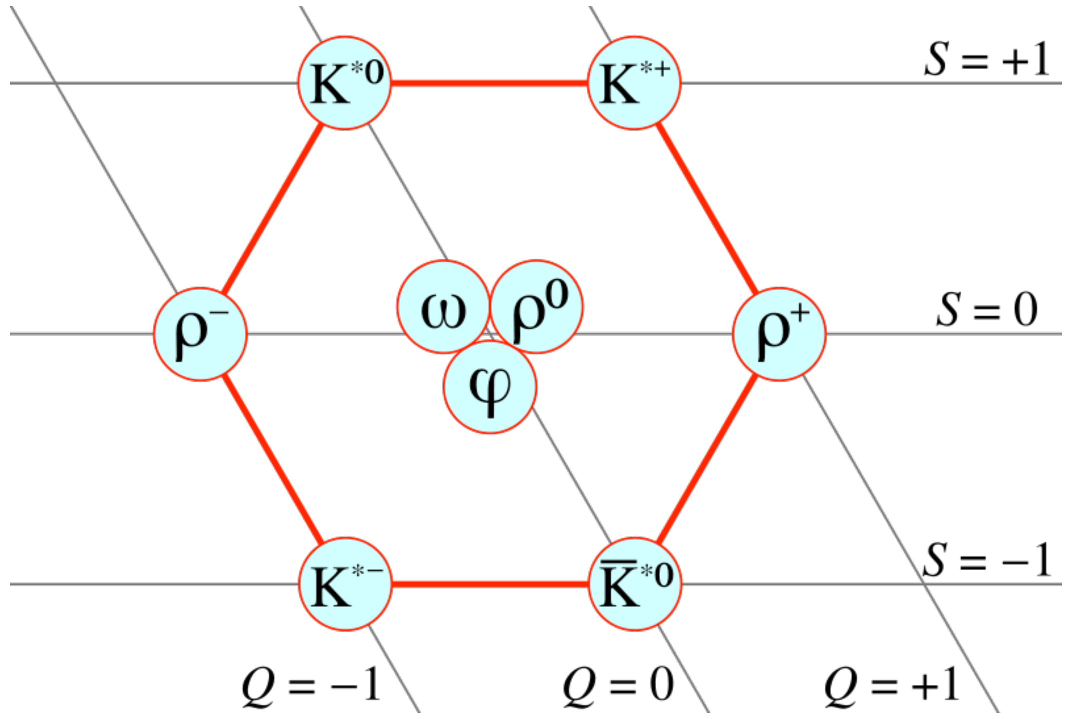


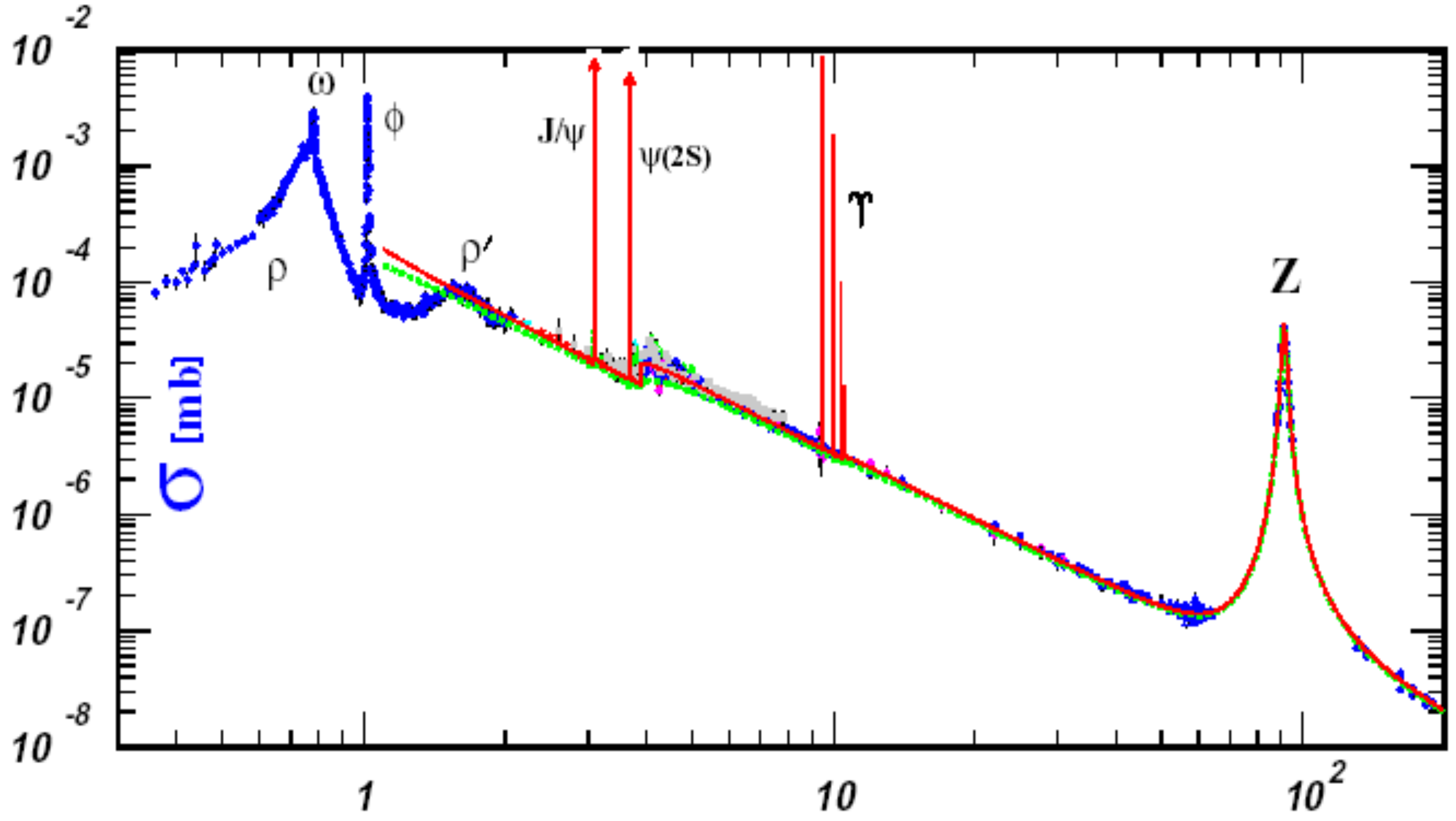
Baryons (qqq)



Mesons $(q\bar{q})$

Spin= 1





Example: ρ meson

$$J^{PC} = 1^{--}$$
Masa$= 775.49 \pm 0.34$ MeV
Ancho de decaimiento $(\Gamma) = 149.1 \pm 0.8$ MeV
Principal modo de decaimiento: $\rho \to \pi\pi$ $(\sim 100\,\%)$

$\tau = \hbar/\Gamma_\rho$

$\tau \sim 10^{-23}$ seg.

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G. Toledo

Motivation

The extremely short lifetime of vector mesons has prevented the measurement of their magnetic dipole moment (MDM)

Radiative process are an alternative to determine the mdm

V. I. Zakharov, L. A. Kondratyuk and L. A. Ponomarev, Sov. J. of Nucl. Phys. 8 456(1969)

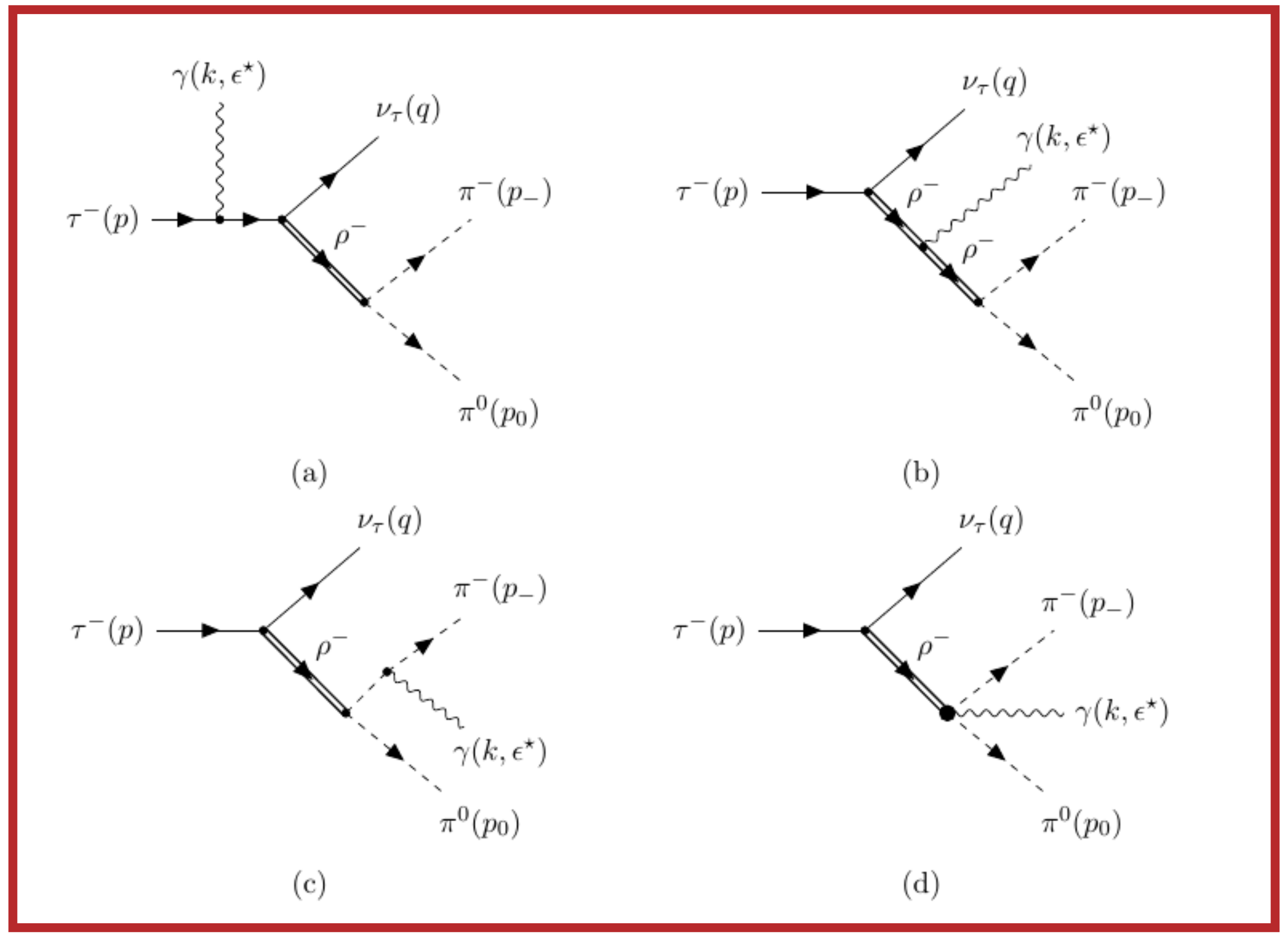
Radiation emitted off the particle carries information of the electromagnetic structure

$\rho \to \pi\pi\gamma$,

$\tau \to \nu\rho\gamma$

$\tau \to \nu\pi\pi\gamma$

G. Lopez Castro and G. Toledo Sanchez, Phys. Rev. D **56**, 4408(1997); Phys. Rev. D **60**, 053004(1999).
G. Lopez Castro and G. Toledo Sanchez, Phys. Rev. D **61**, 033007 (2000).



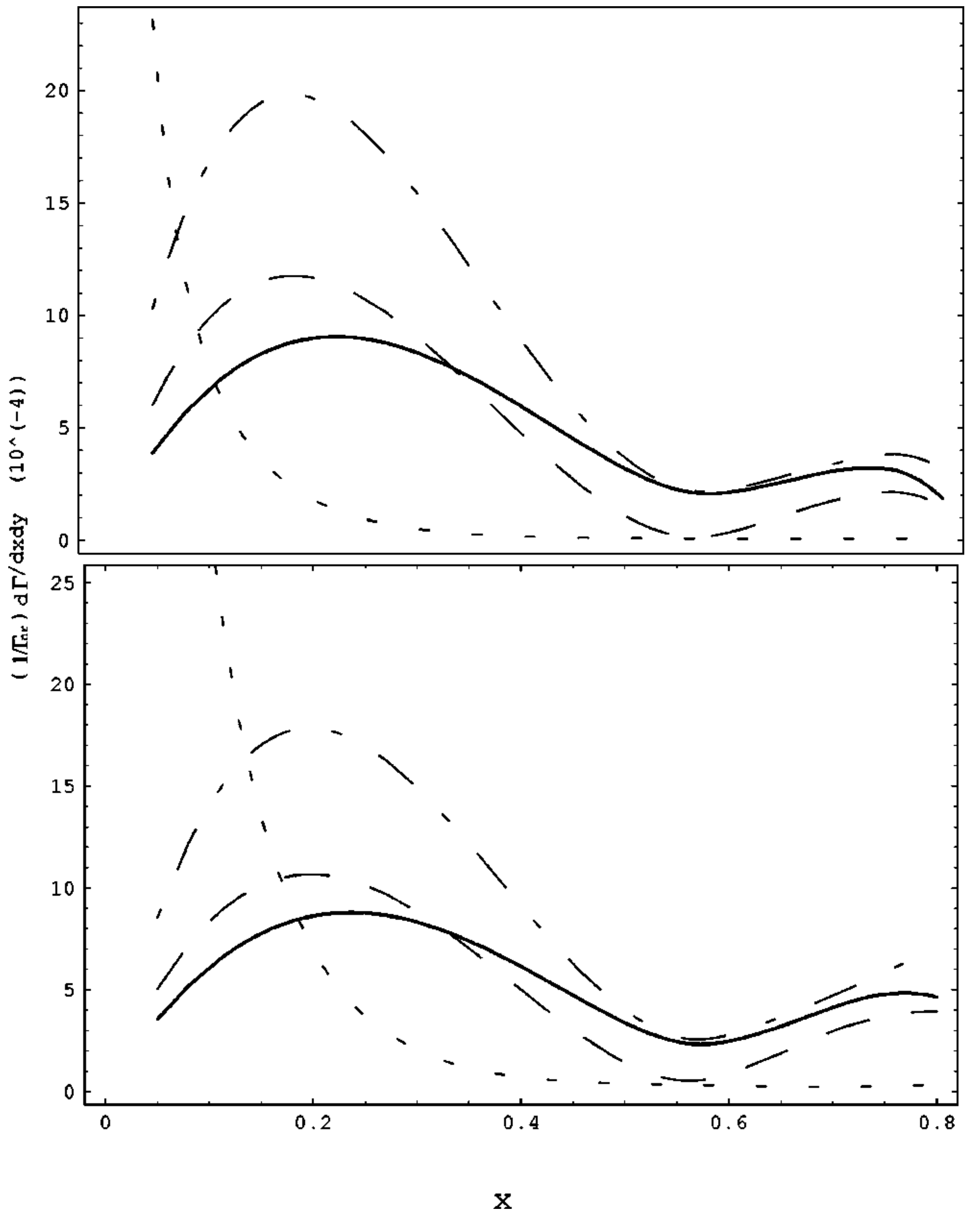


FIG. 1. Differential photon spectrum for $\tau \to \rho\nu\gamma$ decay. See

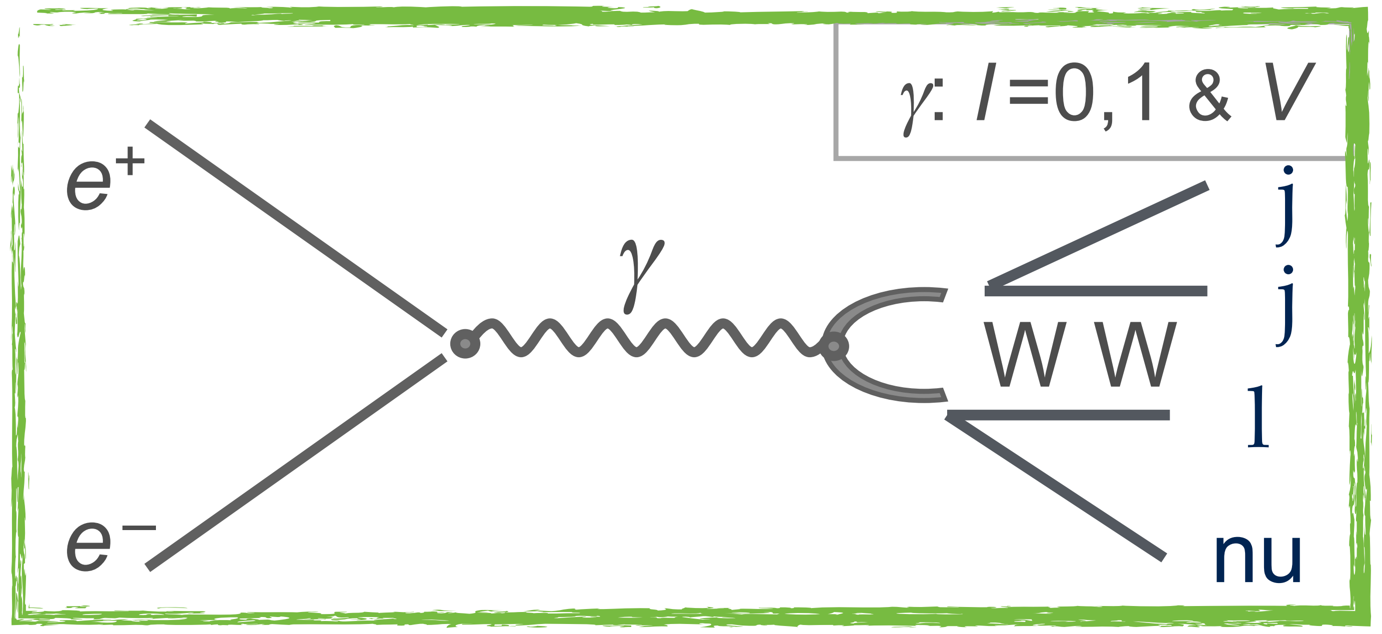
There is no experimental information on neither of these decays

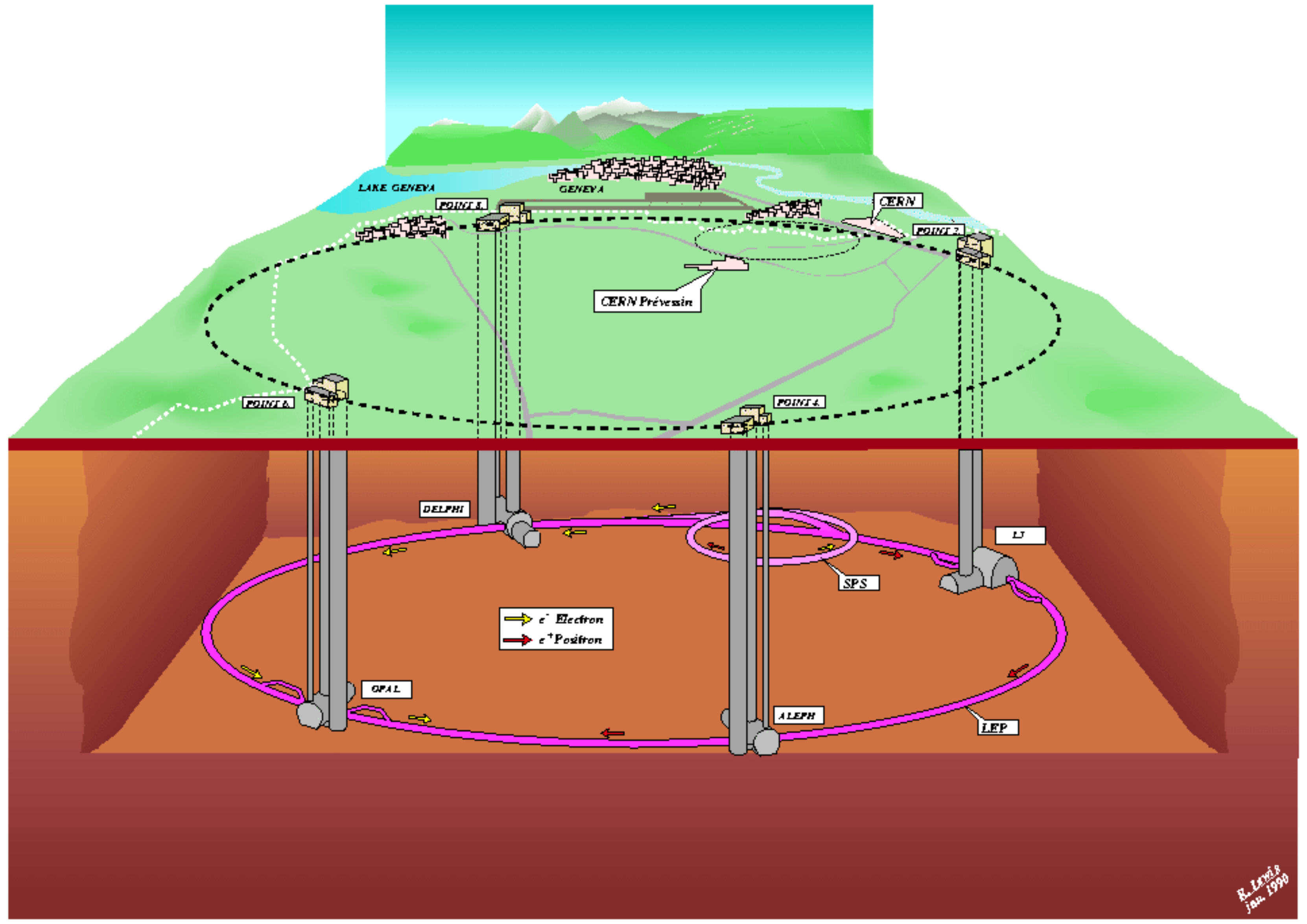
Lessons from the W

The extremely short lifetime prevents of applying standard precession techniques

W gauge boson MDM measured by DELPHI (LEP2)

Delphi Col. Eur. Phys. J. C 66 35(2010)

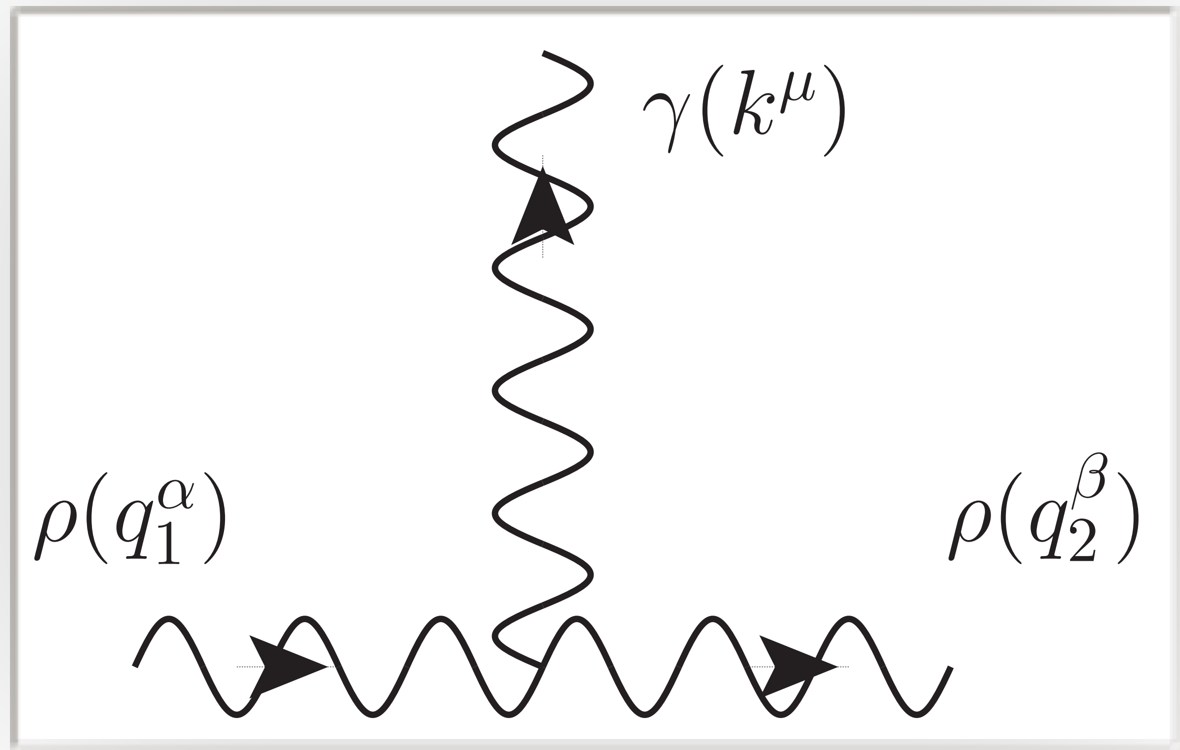




Tests the gauge structure of the standard model.
So far it is in agreement with the SM prediction

Electromagnetic vertex

The electromagnetic current is related to the vertex



$$J_\mu^{em}(x) = (\rho^{em}(x), \vec{J}^{em}(x))$$

$$\langle V(q_2)|J_\mu^{em}(0)|V(q_1)\rangle = \eta_{1\nu}\eta_{2\lambda}^{*}\Gamma_\mu^{\nu\lambda}(q_1,k) \equiv J_\mu(q_1,k)$$

$$\Gamma_{\nu\lambda\mu} = \alpha(k^2)g_{\nu\lambda}(q_1+q_2)_\mu + \beta(k^2)(g_{\mu\nu}k_\lambda - g_{\mu\lambda}k_\nu) - \gamma(k^2)(q_1+q_2)_\mu k_\nu k_\lambda.$$

Límite $k \to 0$

- electric charge
- Magnetic dipole moment
- electric quadrupole

$Q_V = \alpha(0)$ is the electric charge (in e units)

$\mu_V = \beta(0)$ is the magnetic dipole moment (in $e/2M_V$ units)

$X_{EV} = 1 - \beta(0) + 2\gamma(0)$ Electric quadrupole (in e/M_V^2 units).

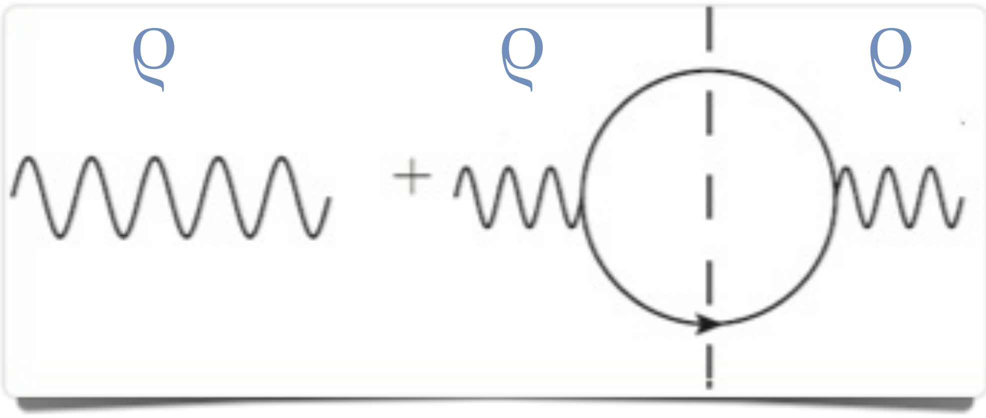
K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B282, 253 (1987).

J. F. Nieves and P. B. Pal, Phys. Rev. D 55 3118(1997).

Unstability

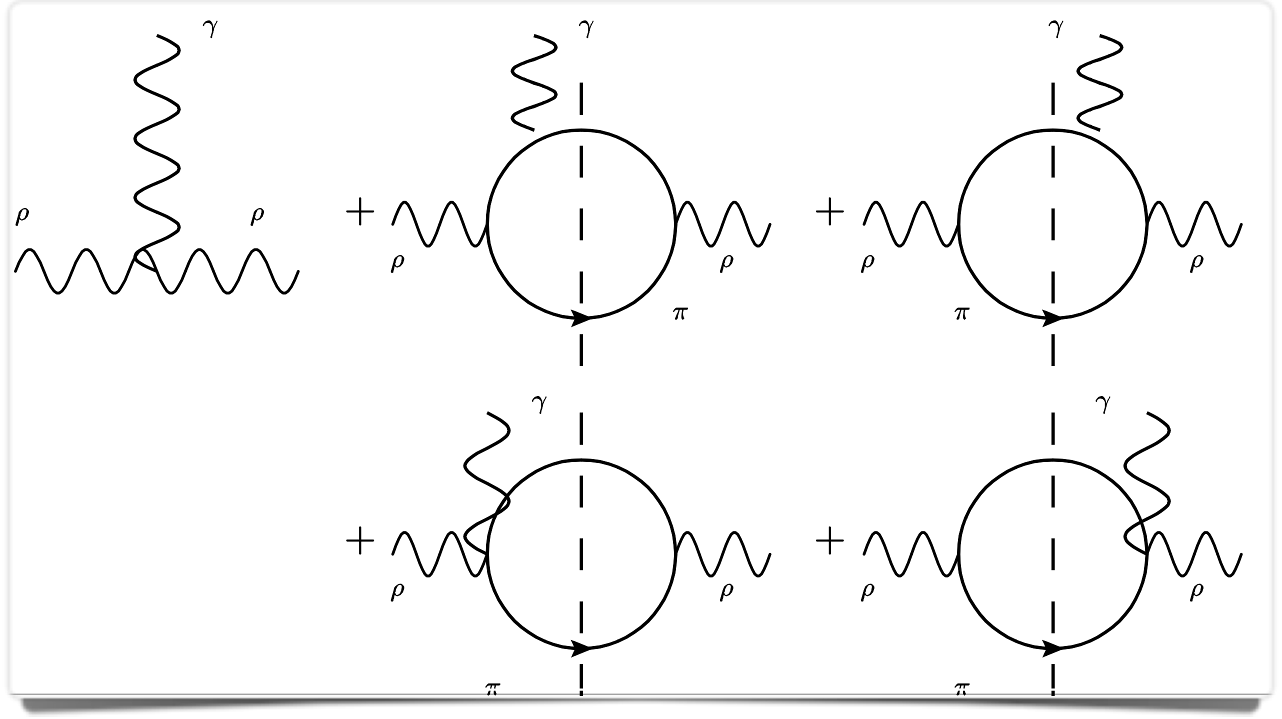
- In QFT the width arises naturally from the absortive part of the loops.
- The Ward identity is fulfilled order by order in PT.

Propagator.



G. López Castro y G. Toledo Sánchez, 2000 Phys. Rev. D 61 033007.

Vertex



Finite width effect on the multipoles is small

Multipole	W boson	ρ meson	K^* meson
$\lvert\mathcal{Q}\rvert$ $[e]$	1	1	1
$\lvert\vec{\mu}\rvert$ $[e/2M_V]$	2.0	$2-0.0091$	$2-0.0047$
$\lvert X_E\rvert$ $[e/M_V^2]$	$1-4.23\times10^{-7}$	$1-0.0387$	$1-0.097$

D. García Gudiño and G. Toledo Sánchez, Phys. Rev. D 81, 073006 (2010).

consistent with
complex mass scheme

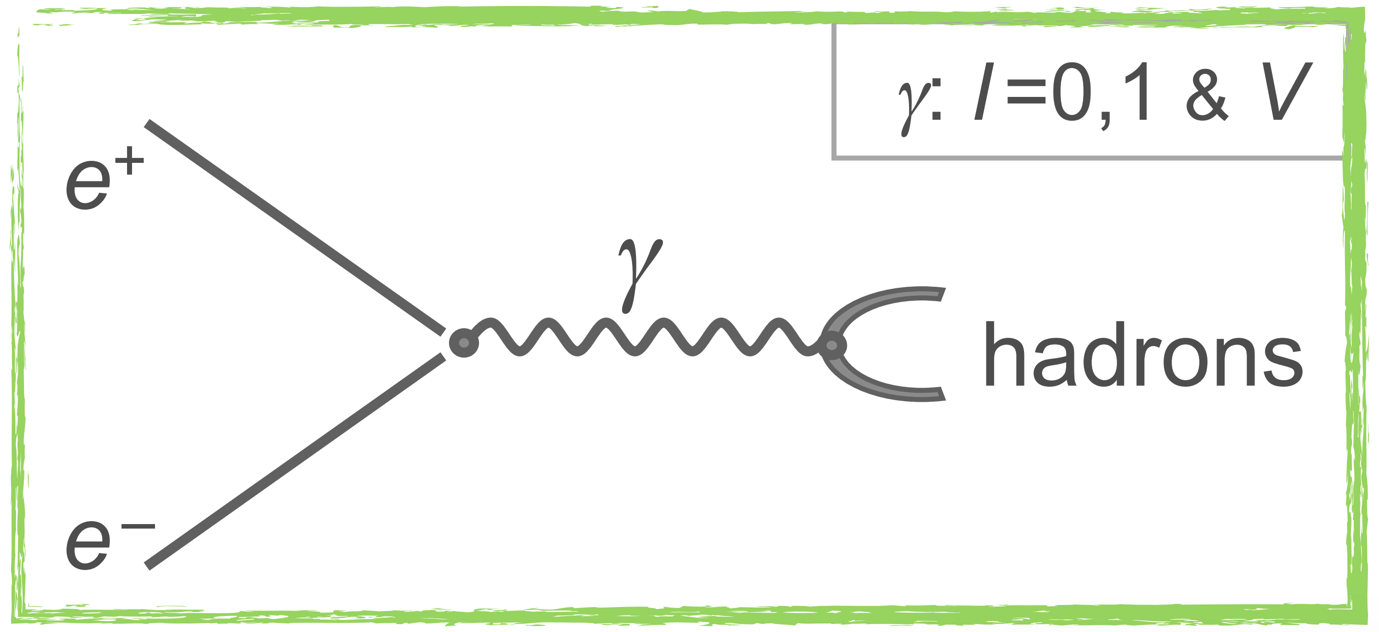
$$D^{\mu\nu}[q,V]=i\left(\frac{-g^{\mu\nu}+\frac{q^\mu q^\nu}{M_V-iM_V\Gamma}}{q^2-M_V^2+iM_V\Gamma}\right)$$

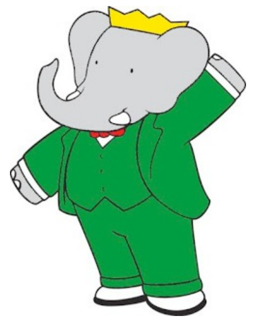
Momentum dependent width

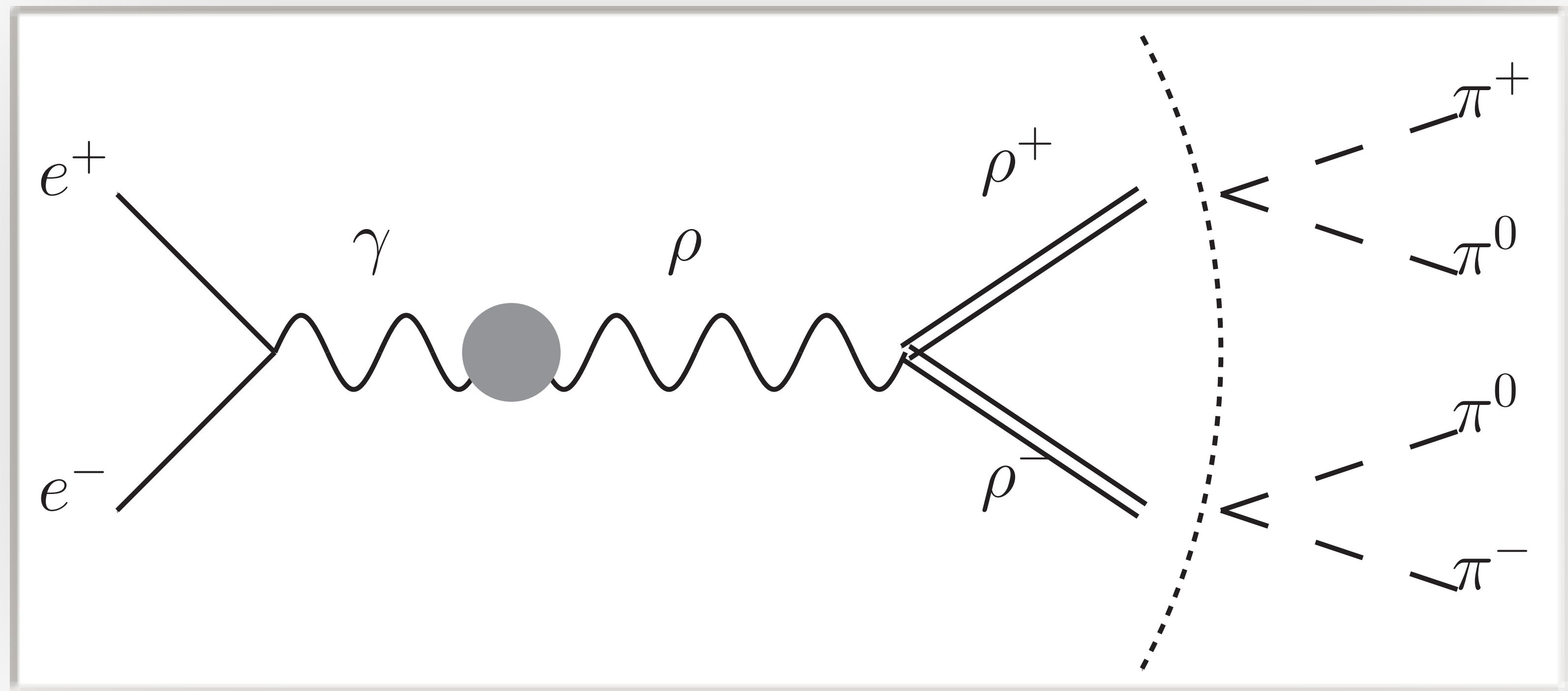
$$\Gamma_\rho(q^2)=\frac{\left(\sqrt{q^2}\right)^{-5}\left(\lambda\left[q^2,m_\pi^2,m_\pi^2\right]\right)^{3/2}}{m_\rho^{-5}\left(\lambda\left[m_\rho^2,m_\pi^2,m_\pi^2\right]\right)^{3/2}}\Gamma_\rho$$

Lessons from the W (continued)

Electroproduction of hadrons may be helpful for the rho ? BABAR (SLAC)





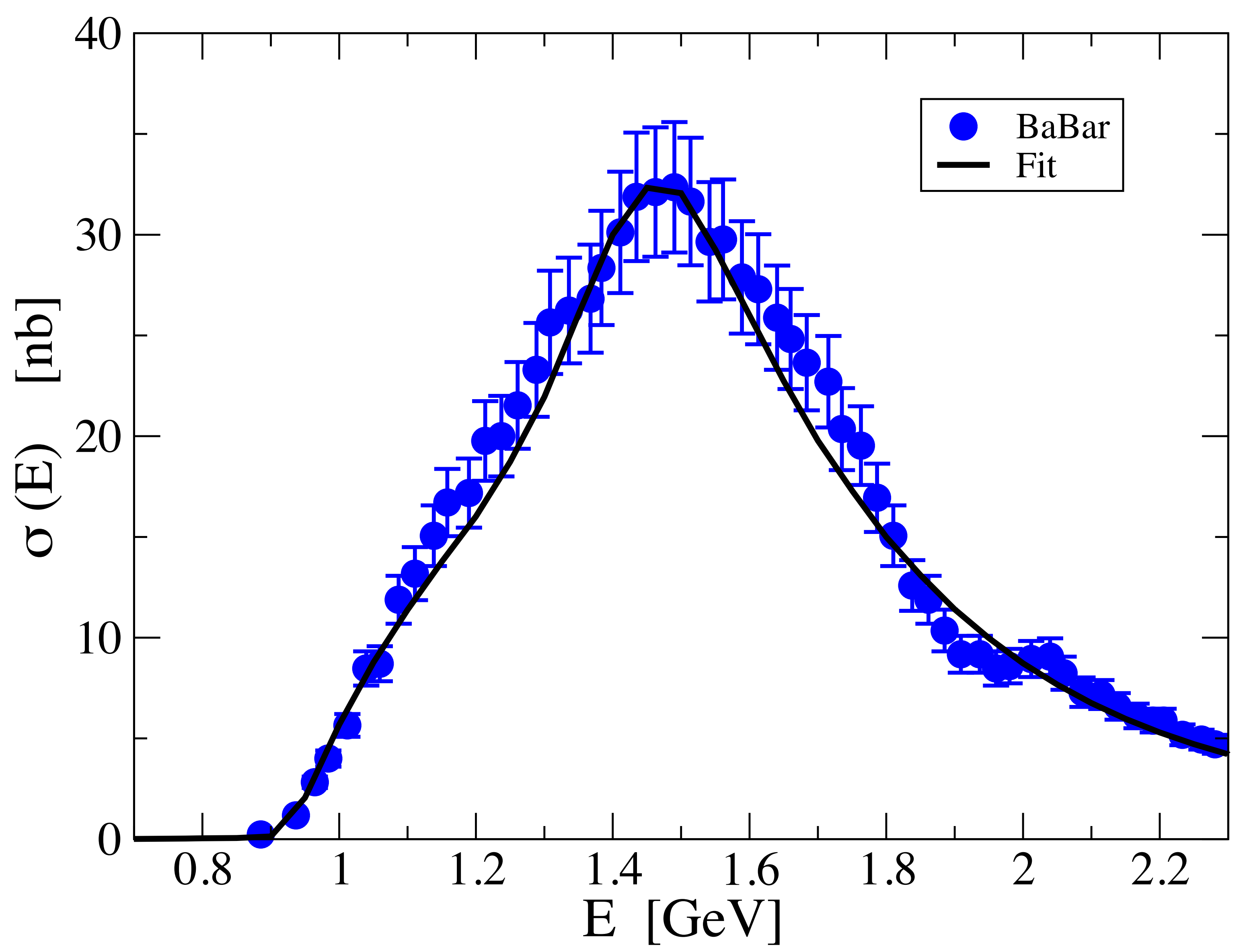


Carries the MDM info

First analysis

Preliminary data BABAR coll.

V. P. Druzhinin, arxiv: 0710.3455 (2007).



D. García Gudiño and G. Toledo Sánchez, Determination of the magnetic dipole moment of the rho meson using 4 pion electroproduction data, Int. J. Mod. Phys. Conf. Ser. **35**, 1460463 (2014), arXiv:1305.6345 [hep-ph].

D. G. Gudiño and G. T. Sánchez, Determination of the magnetic dipole moment of the rho meson using four-pion electroproduction data, International Journal of Modern Physics A **30**, 1550114 (2015).

Total cross section data from the preliminary analysis of BaBar, we have assigned a 10% systematic error bar (symbols). Provided all the parameters involved in our description are determined from other observables, we performed a fit considering the MDM as the only free parameter.

$$\mu_\rho = 2.1 \pm 0.5 \, [\frac{e}{2m_\rho}]$$

The quoted error bar:

Uncertainties coming from the couplings of the different channels

Model assumptions

Preliminary data

Scarce information on the ρ' meson

New analysis

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e-Print:2406.14676 [hep-ph]

Total cross section data from BaBar

J. P. Lees *et al.* (BaBar), Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section using initial-state radiation at BABAR, Phys. Rev. D **96**, 092009 (2017),

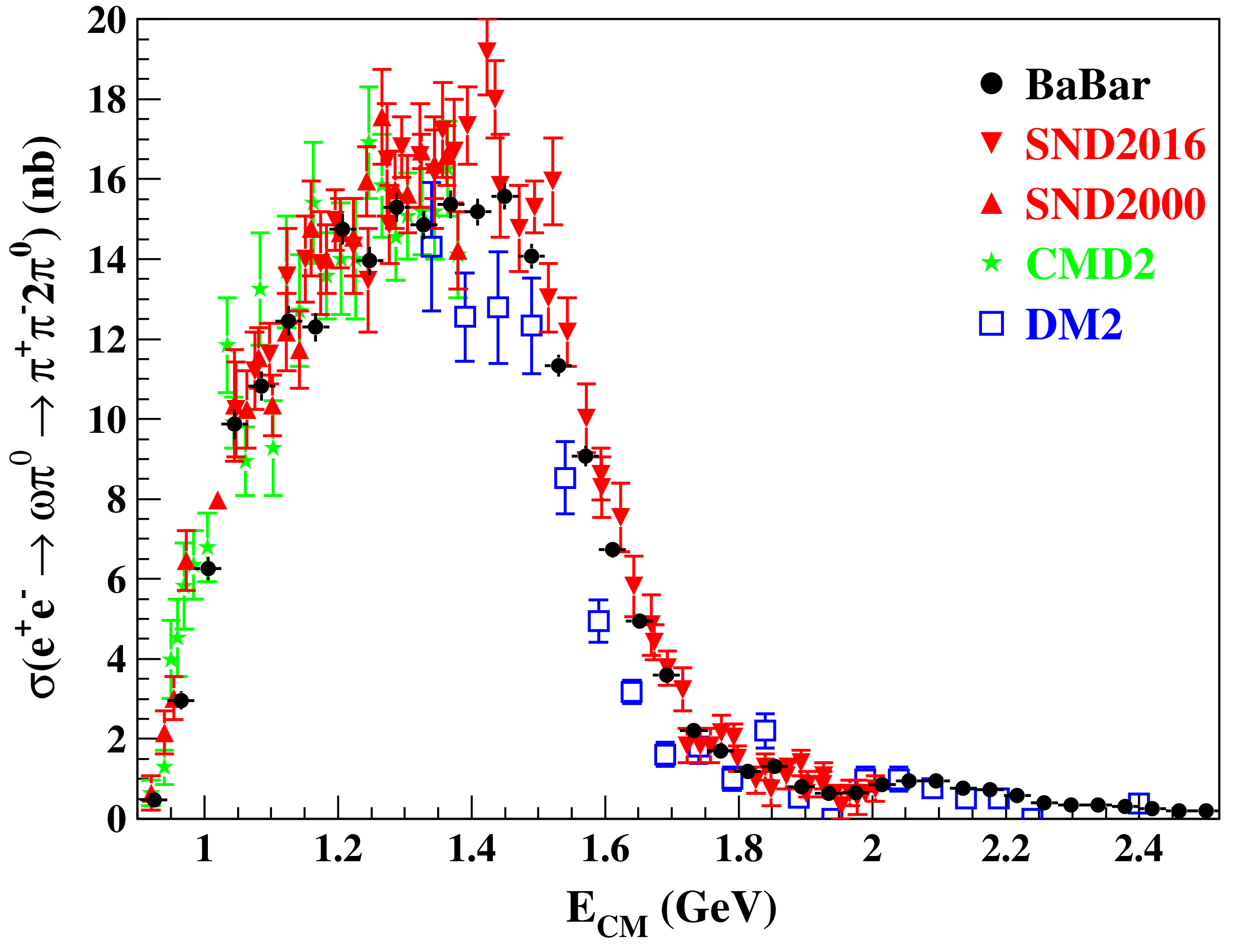


FIG. 13. The measured $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-2\pi^0$ cross sections from different experiments [41–44] as a function of E_{CM} with statistical uncertainties. Data measured in other decays than $\omega \to \pi^+\pi^-\pi^0$ are scaled by the appropriate branching ratio.

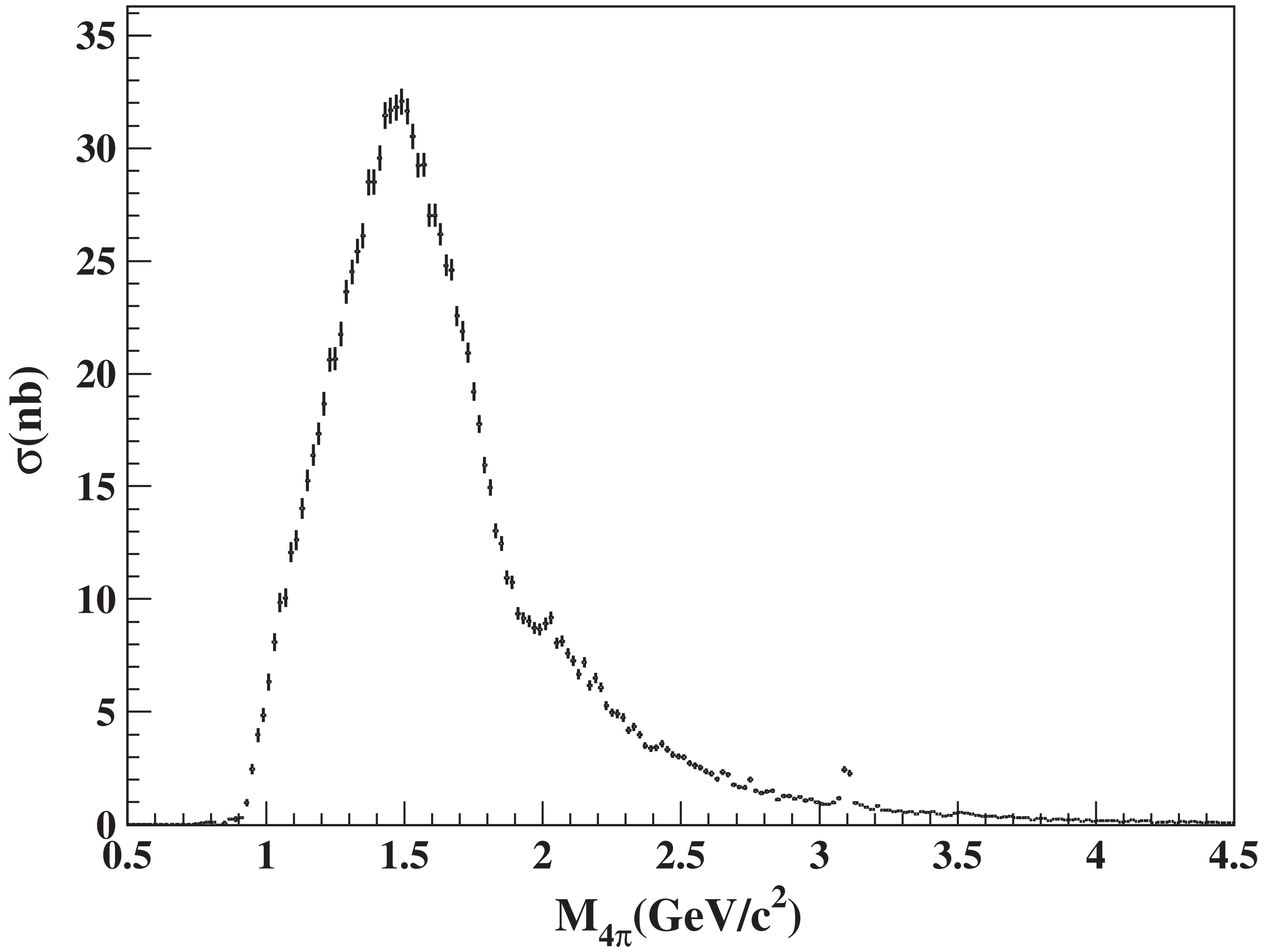


FIG. 9. The measured dressed $\pi^+\pi^-2\pi^0$ cross section (statistical uncertainties only).

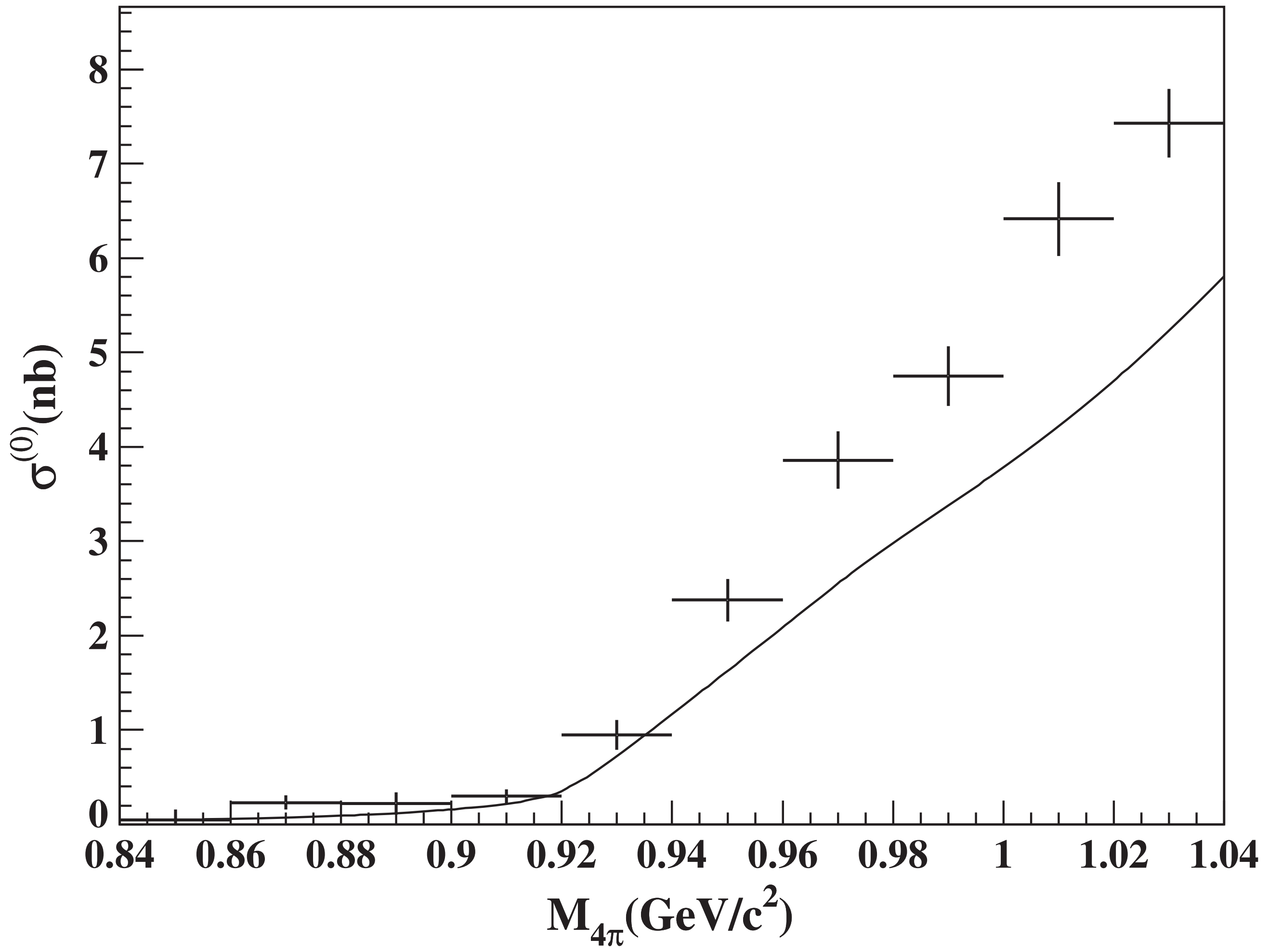


FIG. 11. The low-energy part of the vacuum polarization corrected measured undressed cross section (points with statistical uncertainties) compared to the theoretical prediction (line) from Ref. [36].

Improvements on previous analysis

Uncertainties coming from the couplings of the different channels ✓

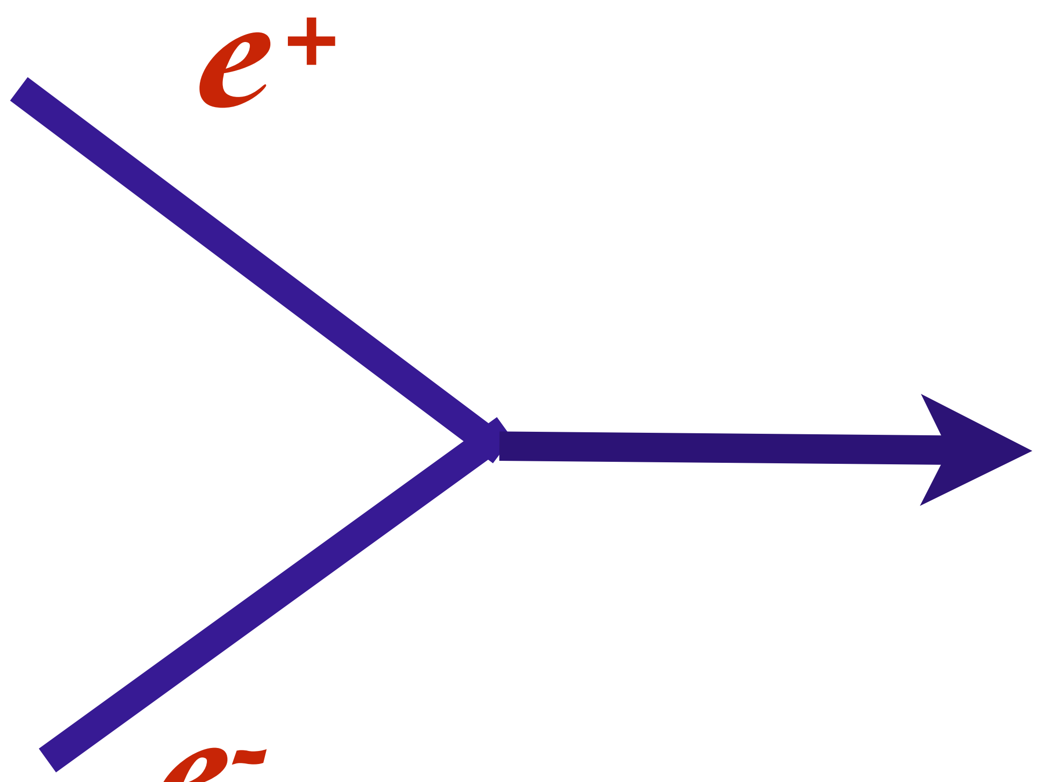
Model assumptions ✓

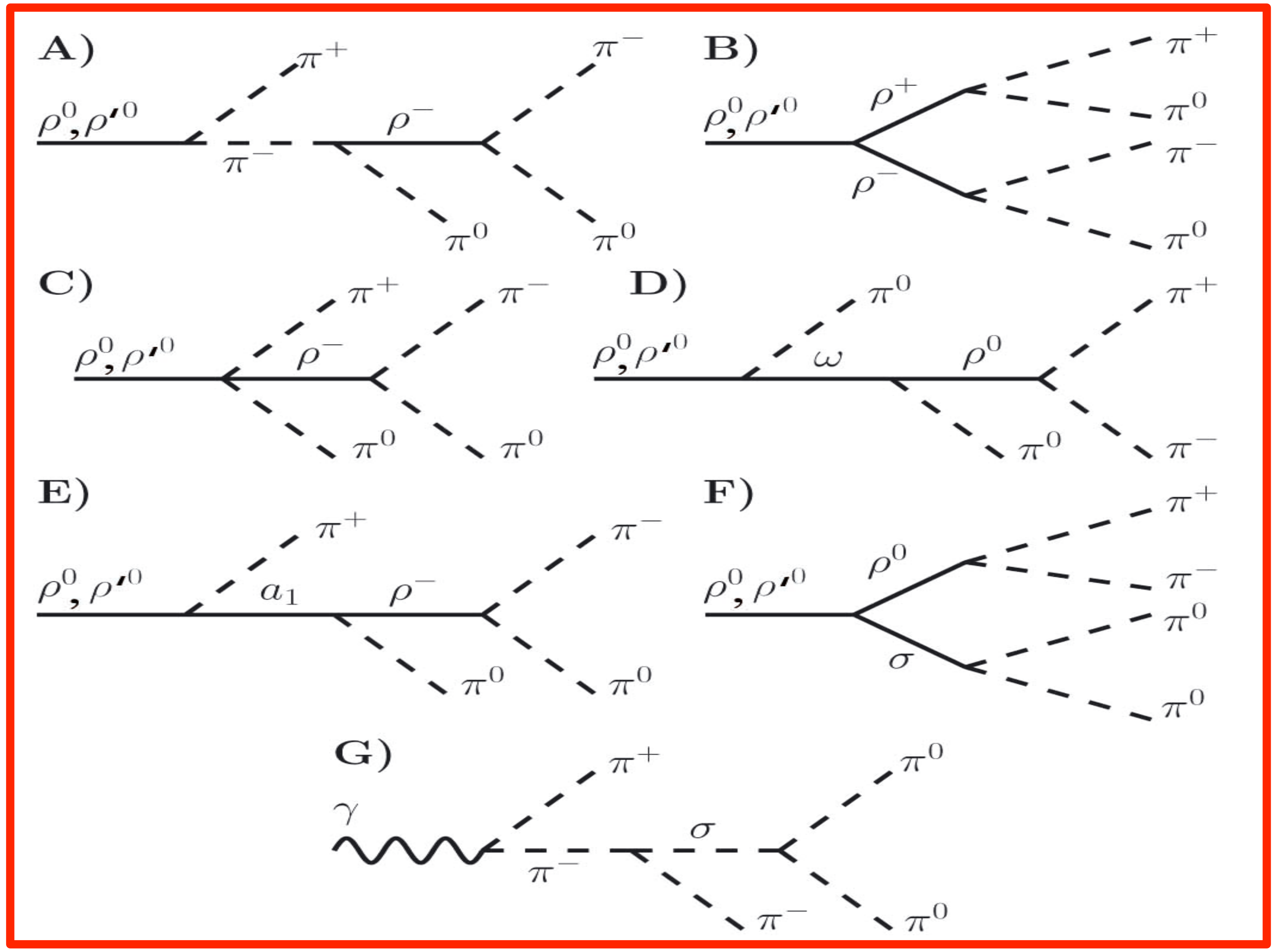
Definite data on this process ✓

Improved information on the ρ' meson ✓

Modeling $e^+ e^- \to \pi^+ \pi^- 2\pi^0$

We consider the Vector Meson Dominance apprach (VMD)





The channels that we have considered here, include the exchange of the π, ω, a1, σ, f(980), ρ and ρ' mesons.

$$\mathcal{L} = \sum_{V=\rho,\,\rho',\,\omega} \frac{e\, m_V^2}{g_V} V_\mu A^\mu + \sum_{V=\rho,\,\rho'} g_{V\pi\pi}\, \epsilon_{abc}\, V_\mu^a\, \pi^b\, \partial^\mu \pi^c$$
$$+ \sum_{V=\rho,\,\rho'} g_{\omega V\pi}\, \delta_{ab}\, \epsilon^{\mu\nu\lambda\sigma}\, \partial_\mu \omega_\nu\, \partial_\lambda V_\sigma^a\, \pi^b + g_{3\pi}\, \epsilon_{abc}\, \epsilon^{\mu\nu\lambda\sigma}\, \omega_\mu\, \partial_\nu \pi^a\, \partial_\lambda \pi^b\, \partial_\sigma \pi^c.$$

Modeling $e^+ e^- \to \pi^+ \pi^- 2\pi^0$

$$e^+(k_1)e^-(k_2) \to \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)\pi^0(p_4)$$

$$\mathcal{M} = \frac{-ie}{(k_1+k_2)^2} l^\mu h_\mu(p_1,p_2,p_3,p_4)$$

Leptonic current

$$l^\mu \equiv \bar{v}(k_2)\gamma^\mu u(k_1)$$

Four pion electromagnetic current current

$$h_\mu(p_1,p_2,p_3,p_4) = -h_\mu(p_3,p_2,p_1,p_4)$$ Charge conjugation

$$h_\mu(p_1,p_2,p_3,p_4) = h_\mu(p_1,p_4,p_3,p_2)$$ Bose-Symmetry

+ Gauge invariance

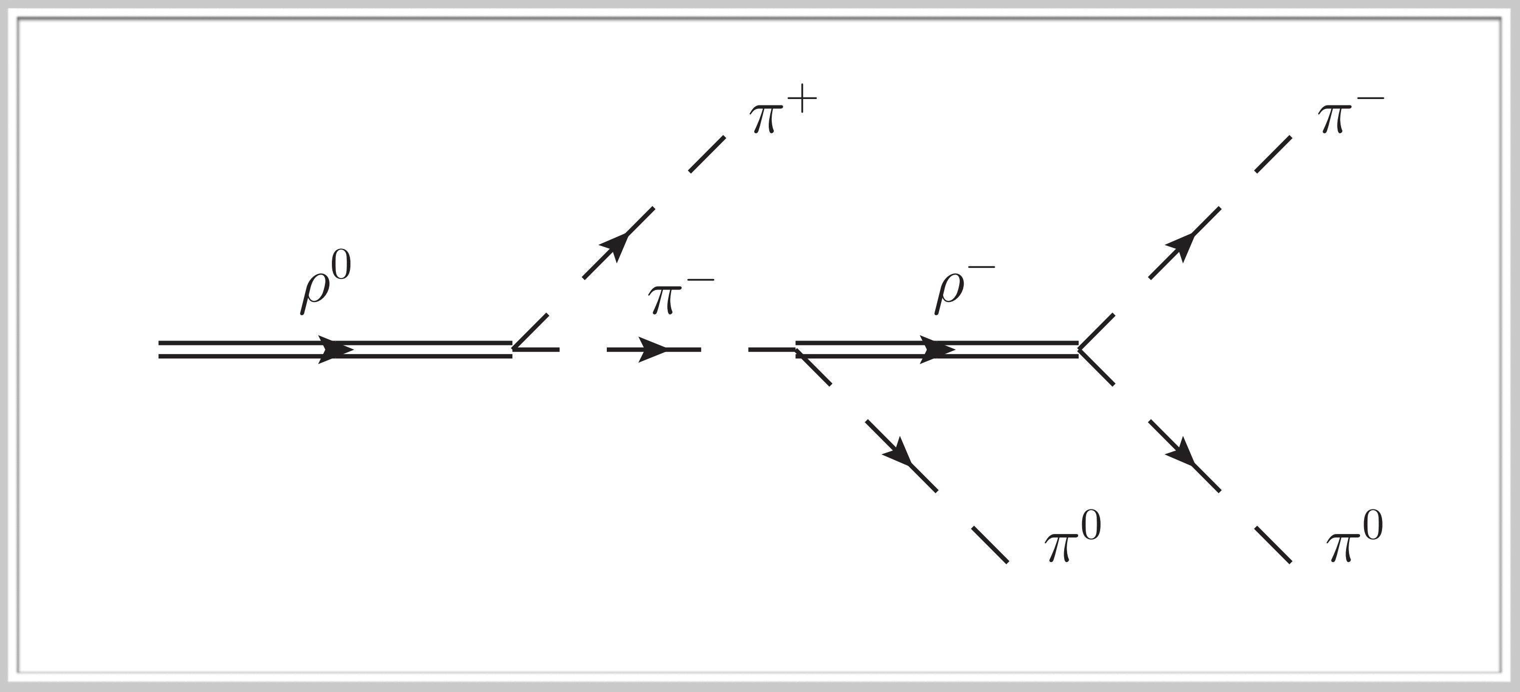
Previous studies S. I. Eidelman, Z. K. Silagadze and E. A. Kuraev; Phys. Lett. B 346 186(1995); G. Ecker and R. Unterdorfer, Eur. Phys. J. C 24 535(2002). H. Czyz, J. H. Kuhn and A. Wapienik, Phys. Rev. D 77 114005(2008); J. Juran and P. Lichard, Phys. Rev. D 78 017501(2011).

Written in terms of a reduced amplitude no longer restricted by the symmetries

$$h_\mu(p_1,p_2,p_3,p_4) = \mathcal{M}_{r\mu}(p_1,p_2,p_3,p_4) + \mathcal{M}_{r\mu}(p_1,p_4,p_3,p_2) - \mathcal{M}_{r\mu}(p_3,p_2,p_1,p_4) - \mathcal{M}_{r\mu}(p_3,p_4,p_1,p_2)$$

Here: We follow the same approach as in the previous analysis, but now we have explicit gauge invariant amplitudes with Charge conjugation and Bose- symmetry enforced

Channel A



$$\mathcal{M}_A^{\mu}(p_1,p_2,p_3,p_4) = -e\,\frac{g_{\rho\pi\pi}^3}{g_\rho}\,m_\rho^2\,D_\rho^{\alpha\mu}[q]\,(q-2p_1)_\alpha$$
$$S_\pi[q-p_1](q-p_1+p_2)_\gamma\,D_{\rho^-}^{\eta\gamma}[s_{43}]\,r_{43\eta},$$

With the following definitions sij ≡pi+pj rij ≡pi−pj

Vector propagator

$$D_V^{\alpha\mu}[p] = -iD_V[p]\left(g^{\alpha\mu} - \frac{p^\alpha p^\mu}{m_V^2 - i\,m_V\Gamma_V}\right)$$

$$D_V[p] \equiv 1/(p^2 - m_V^2 + i\,m_V\Gamma_V)$$

Energy dependent width

$$\Gamma_\rho(s) = \Gamma_\rho\left(\frac{m_\rho}{\sqrt{s}}\right)^5\left[\frac{\lambda(s,m_\pi^2,m_\pi^2)}{\lambda(m_\rho^2,m_\pi^2,m_\pi^2)}\right]^{3/2},$$

Pseudo-Scalar propagator

$$S_\pi[q] = i/(q^2 - m_\pi^2).$$

Similar amplitude for the rho' is added (180^0 phase)

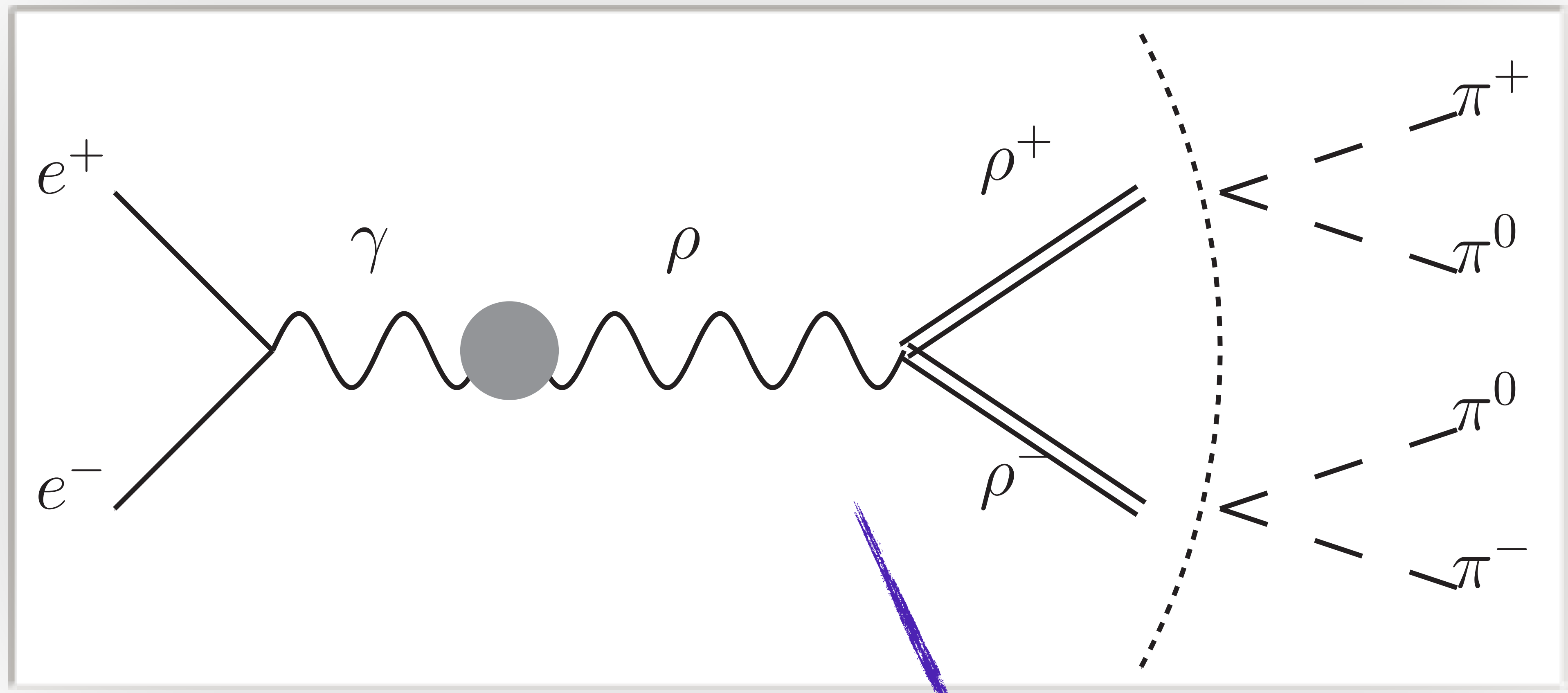
G. Ecker and R. Unterdorfer, Eur. Phys. J. C 24 535(2002).

H. Czyz, J. H. Kuhn and A. Wapienik, Phys. Rev. D 77 114005(2008)

In the previous analysis, given the scarce information on the rho', a VMD-like relation was used

$$\frac{m_{\rho'}^2}{g_{\rho'}}g_{\rho'\pi\pi} = \frac{m_\rho^2}{g_\rho}g_{\rho\pi\pi}$$

Channel B



Includes the ρργ vertex

$$\Gamma_{\alpha\lambda\delta} = \; g_{\lambda\delta}\, Q_{1\alpha} + \beta_0 \,(q_\delta\; g_{\alpha\lambda} - q_\lambda\; g_{\delta\alpha}) + s_{21\lambda}\; g_{\delta\alpha} - s_{43\delta}\; g_{\alpha\lambda},$$

The amplitude is:

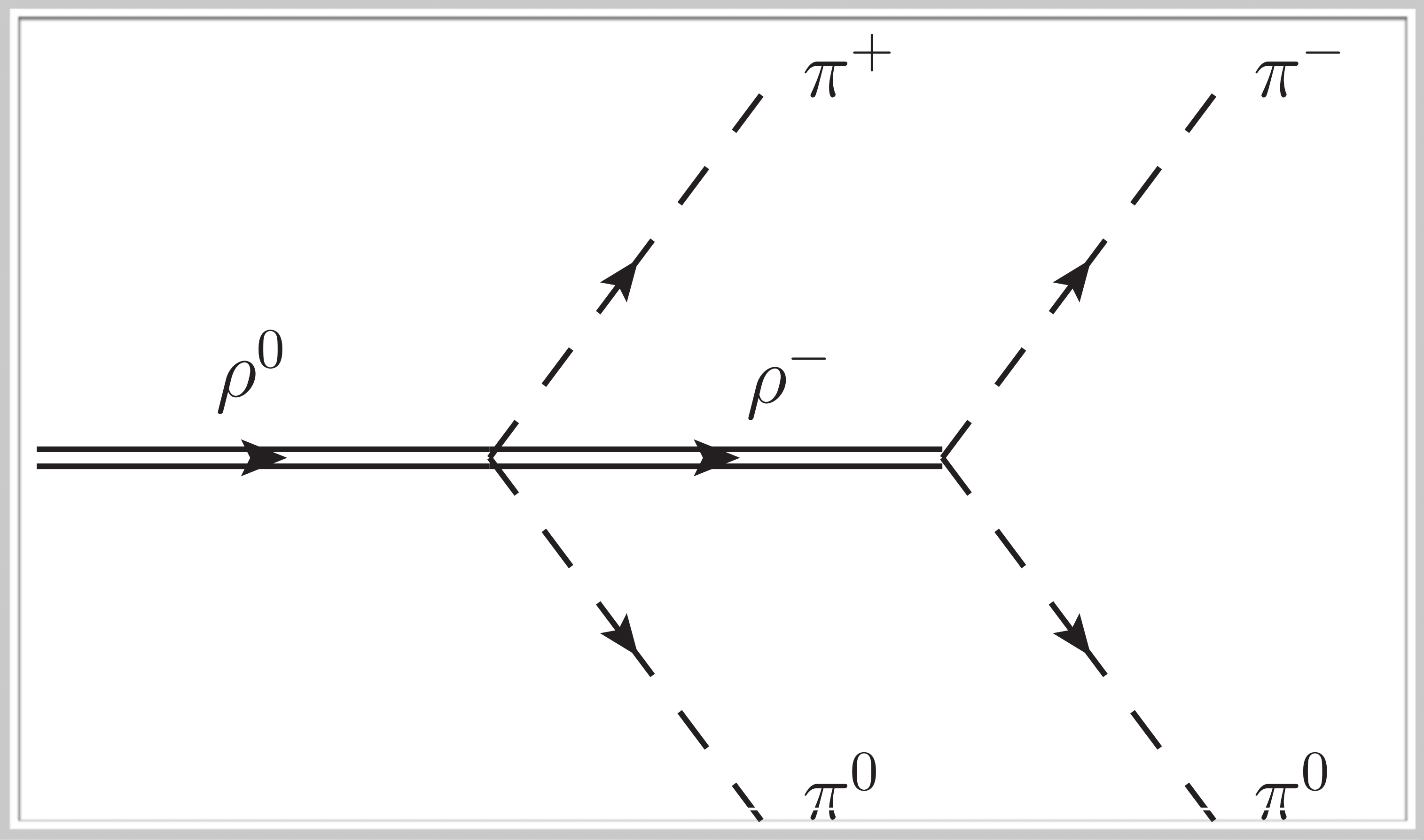
$$\mathcal{M}^\mu_B(p_1, p_2, p_3, p_4) = -e\,\frac{g^3_{\rho\pi\pi}}{g_\rho}\, m^2_\rho\, D^{\alpha\mu}_{\rho[q]}$$

$$r_{12\gamma}\, D^{\lambda\gamma}_{\rho^+}[s_{21}]\, \Gamma^1_{\alpha\lambda\delta}\, D^{\eta\delta}_{\rho^-}[s_{43}]\, r_{43\eta},$$

$$\Gamma^1_{\alpha\lambda\delta} = (1 + i\,\gamma)\,\Gamma_{\alpha\lambda\delta}$$

- Wherever the ρ meson appears, the ρ’ is also considered

Channel C



Gauge invariance of channels A, B y C fixes this contribution, applied for every form corresponding to the Bose and C symmetries.

$$q^\mu(\mathcal{M}_{rA\mu} + \mathcal{M}_{rB\mu} + \mathcal{M}_{rC\mu}) = 0.$$

Using a particular set of amplitudes, corresponding to the charge conjugation

$$\begin{aligned}\mathcal{M}^\mu_{ABC_{24}} &= \mathcal{M}^\mu_A(p_1,p_2,p_3,p_4) + \mathcal{M}^\mu_A(p_3,p_4,p_1,p_2)\\ &+\mathcal{M}^\mu_B(p_1,p_2,p_3,p_4)\\ &+\mathcal{M}^\mu_C(p_1,p_2,p_3,p_4) + \mathcal{M}^\mu_C(p_3,p_4,p_1,p_2).\end{aligned}$$

The explicit gauge invariant amplitude is:

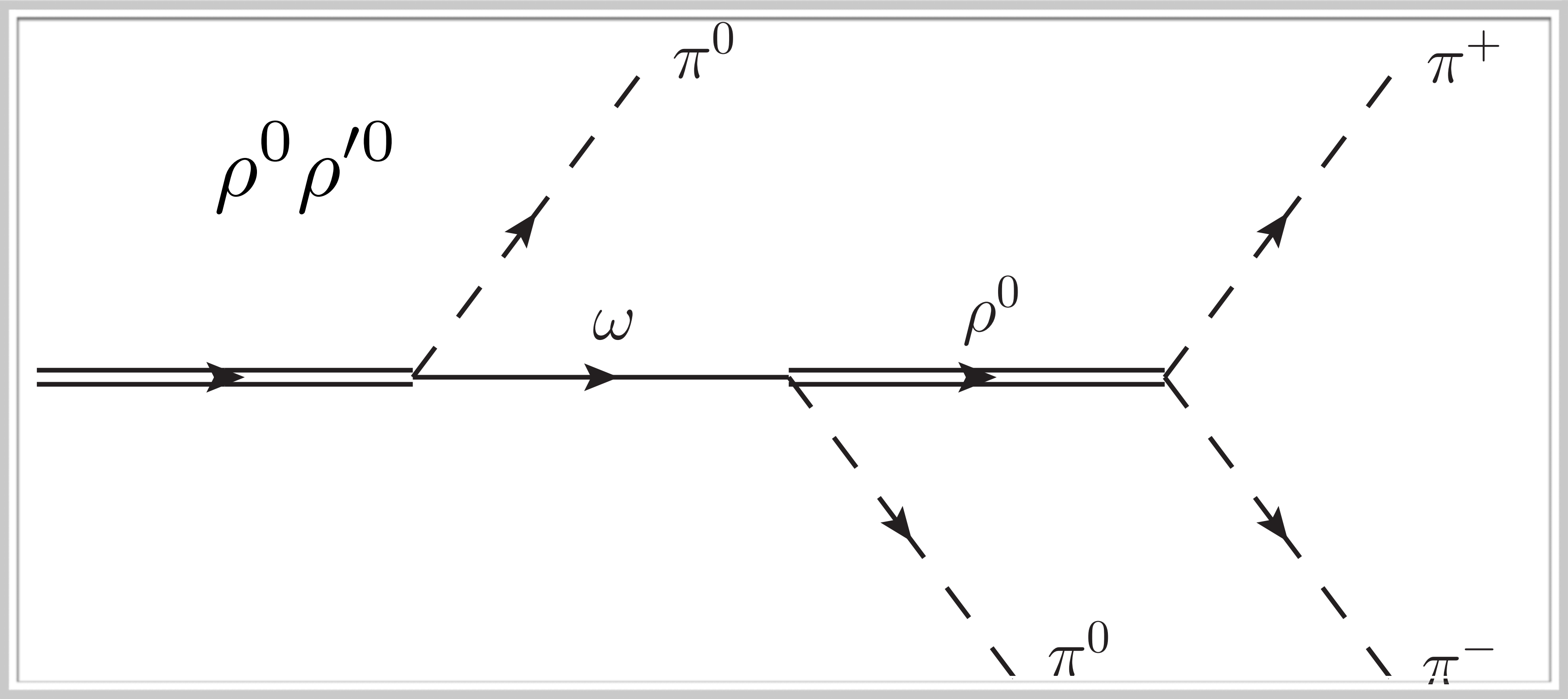
$$\begin{aligned}&\mathcal{M}^\mu_{ABC_{24}} = i\,e\,C\left\{L^\mu(x_1,x_3)\right.\\ &\left(D_{\rho^-}[s_{43}]\,r_{43}\cdot z_{12} - D_{\rho^+}[s_{21}]\,r_{12}\cdot z_{34}\right)\\ &+ r_{43}\cdot r_{12}\Big(D_{\rho^-}[s_{43}]\,L^\mu(Q_1,x_3) - D_{\rho^+}[s_{21}]\,L^\mu(Q_1,x_1)\Big)\\ &+ (1+i\,\gamma)\,D_{\rho^-}[s_{43}]\,D_{\rho^+}[s_{21}]\\ &\left.\beta_0\left(q\cdot r_{12}\,r^\mu_{43} - q\cdot r_{43}\,r^\mu_{12}\right)\right\},\end{aligned}$$

Gauge invariant tensor

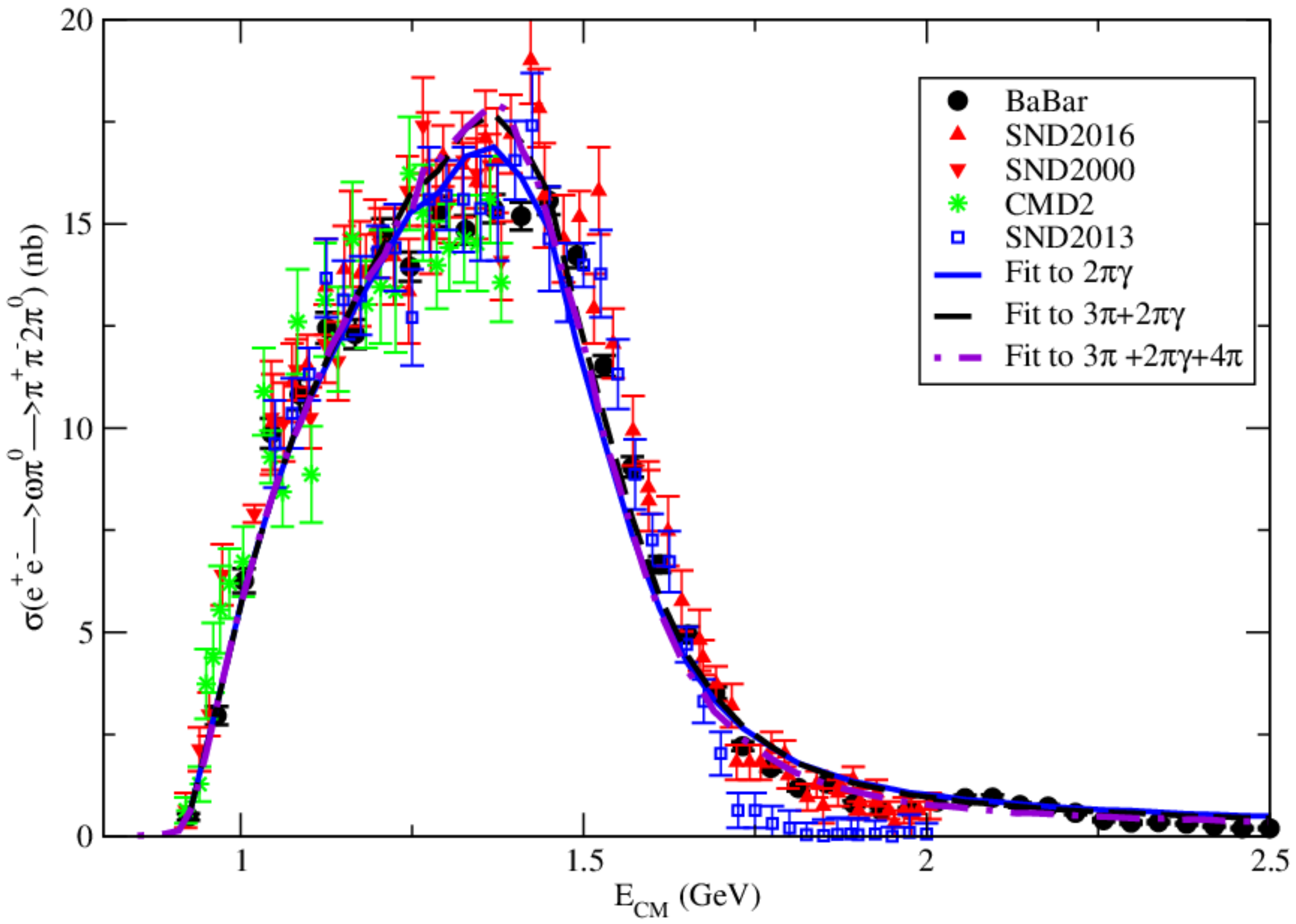
$$L^\mu(a,b) \equiv \frac{a^\mu}{a\cdot q} - \frac{b^\mu}{b\cdot q}.$$

Similar expression is obtained for the neutral pion exchange

Channel D



$$\mathcal{L}_\omega = g_{\omega\rho\pi}\delta_{ab}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu\partial_\lambda\rho^a_\sigma\pi^b$$



The explicit gauge invariant amplitude is:

$$\mathcal{M}^\mu_D(p_1,p_2,p_3,p_4) = -i\,e\left(C_d + e^{i\theta}\,C'_d\right)D_\omega[q-p_2]$$
$$\mathcal{A}[(q-p_2)^2]\;\epsilon_{\alpha\eta\beta\sigma}\,\epsilon^{\mu\gamma\chi\sigma}\,q_\gamma\,p_{2\chi}\,p_1^\alpha\,p_3^\eta\,p_4^\beta.$$

$$\mathcal{A}[(q-p_2)^2] = 6\,g_{3\pi}$$
$$+\,2\,g_{\rho\pi\pi}\,g_{\omega\rho\pi}\Big(D_{\rho^0}[s_{13}] + D_{\rho^+}[s_{41}] + D_{\rho^-}[s_{43}]\Big)$$
$$+\,2\,g_{\rho'\pi\pi}\,g_{\omega\rho'\pi}\Big(D_{\rho'}[s_{13}] + D_{\rho'}[s_{41}] + D_{\rho'}[s_{43}]\Big),$$

$$C_d = \frac{g_{\omega\rho\pi}}{g_\rho}\,m_\rho^2\,D_\rho[q], \qquad C'_d = \frac{g_{\omega\rho'\pi}}{g_{\rho'}}\,m_{\rho'}^2\,D_{\rho'}[q].$$

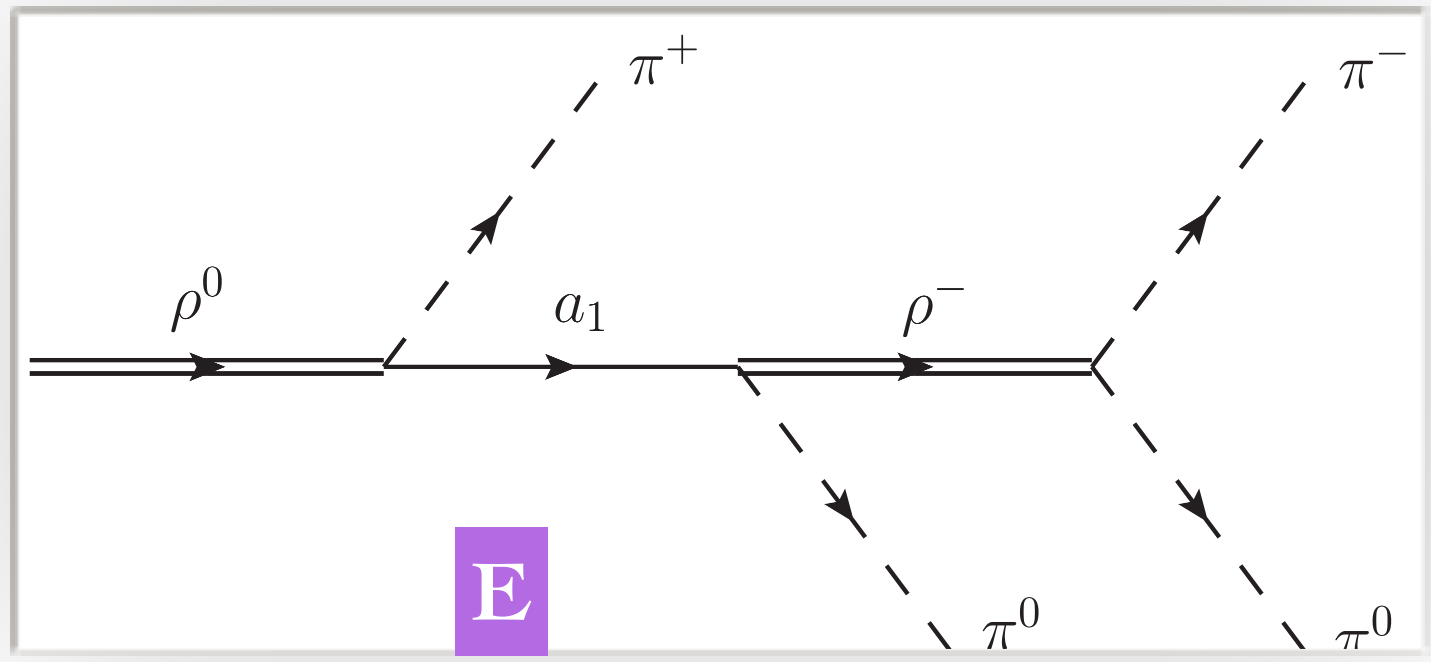
Fit to BaBar data for the ω channel plus a set of observables mentioned above. Improved precision wrt Avalos et al, Phys Rev D 107 056006 (2023) , where this channel was a prediction

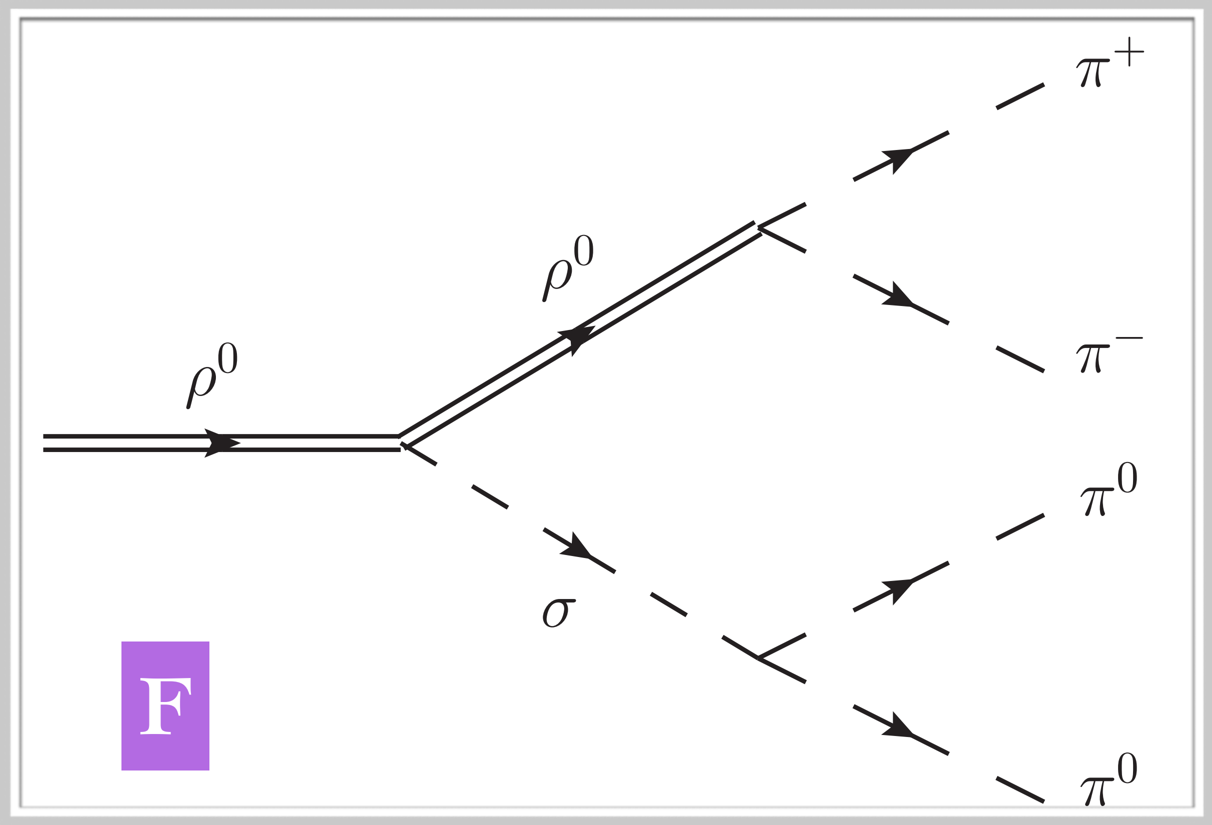
Channels E, F and G

$$\mathcal{L}_{a_1} = 2g_{a_1\rho\pi}(k\cdot q\rho_\mu a_1^\mu - \partial_\nu\rho^\mu\partial_\mu a_1^\nu)$$

N. Isgur, C. Morningstar, and C. Reader, Phys. Rev. D 39 1357(1989)

$$\mathcal{L}_S = g_{V_1V_2S}V_{1\mu}V_2^\nu S + g_{SP_1P_2}SP_1P_2$$





$\rho\to\sigma\gamma$

$$g_{\rho\rho\sigma} = -\left(\frac{em_\rho^2}{g_\rho q^2}\right)g_{\rho\sigma\gamma}.$$

$g_{\rho\sigma\gamma} = 0.63 \pm 0.15\ \mathrm{GeV}^{-1}$

$\sigma\to\pi\pi$

$g_{\sigma\pi\pi} = 3.69 \pm 1.6\ GeV$

$g_{a_1\rho\pi} = 3.25 \pm 0.3\ GeV$

The explicit gauge invariant amplitude are:

$$\mathcal{M}_E^\mu(p_1,p_2,p_3,p_4) = -i\,e\,C_a\,D_{\rho^-}[s_{43}]\,D_{a_1}[q-p_1]^{\beta}\,F^{\mu\alpha}(q-p_1,q)\,F_{\alpha\beta}(q-p_1,s_{43}),(2$$

$$C_a = (g_{a_1\rho\pi}^2\,g_{\rho\pi\pi}/g_\rho)m_\rho^2\,D_\rho[q]$$

$$\mathcal{M}_{F_\sigma}^\mu(p_1,p_2,p_3,p_4) = i\,e\,C_\sigma\,D_\sigma[s_{24}]\,D_{\rho^0}[s_{31}]\,F^{\mu\beta}(s_{31},q)\,r_{31\beta},$$

$$C_\sigma = (g_{\sigma\pi\pi}\,g_{\rho\rho\sigma}\,g_{\rho\pi\pi})/g_\rho)\,m_\rho^2\,D_\rho[q]$$

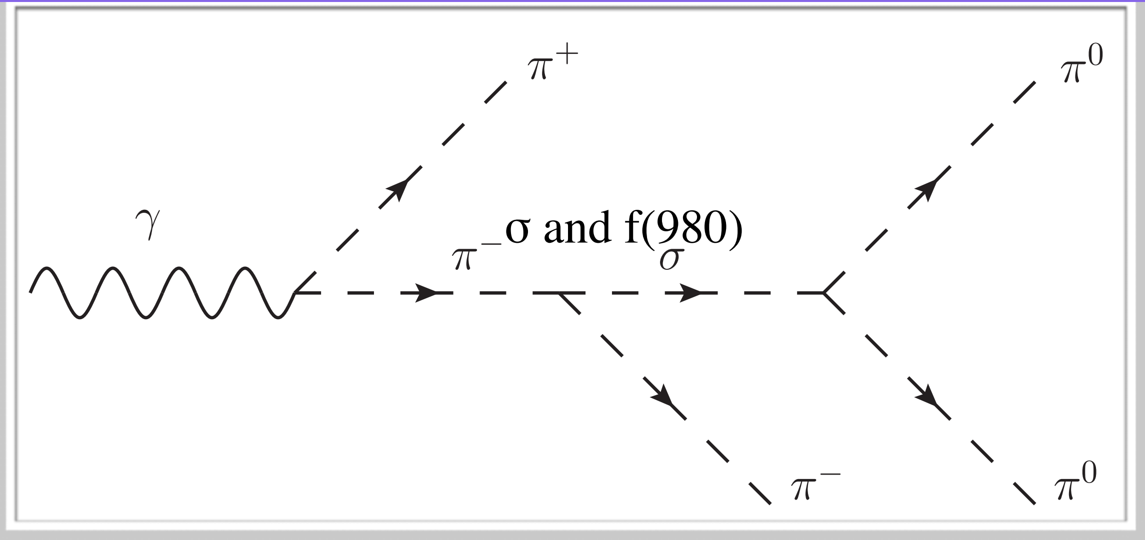
The gauge invariant tensor: $F_{\mu\alpha}(a,b) \equiv a\cdot b\,g_{\mu\alpha} - a_\mu b_\alpha.$

The corresponding coupling to ϱ′ is taken to be the same. As we will show later, this channel is very suppressed in the whole region of study and deviations from this assumption are expected to have a very small effect.

Channels E, F and G

Non-resonant channel

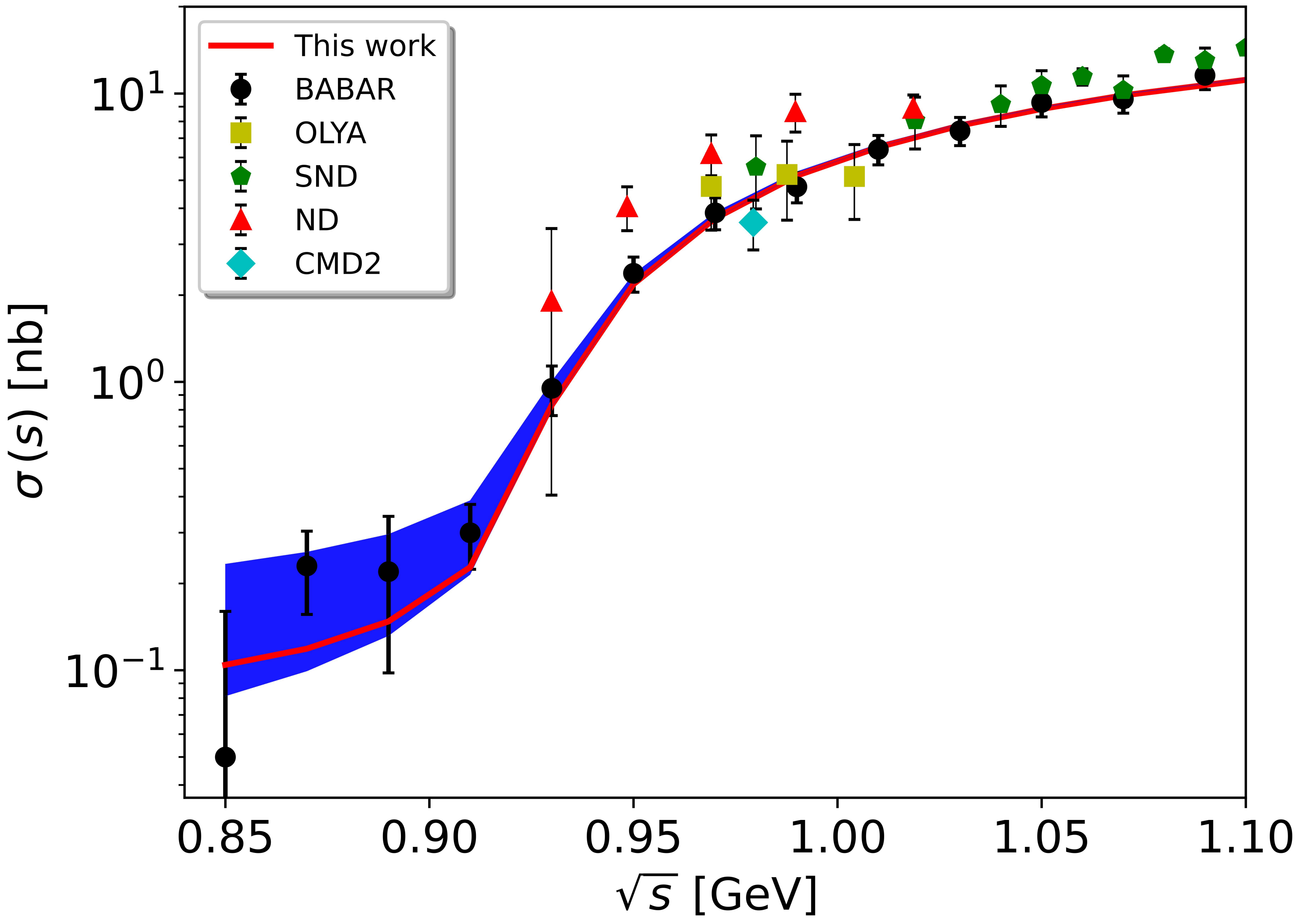
G



The explicit gauge invariant amplitude is:

$$\mathcal{M}^{\mu}_{G} = i\, e\, (g_{\sigma\pi\pi})^2\, D_{\sigma}[s_{42}]\, L^{\mu}(x_1, x_3).$$

The corresponding coupling to ϱ' is taken to be the same. As we will show later, this channel is very suppressed in the whole region of study and deviations from this assumption are expected to have a very small effect. For the f(980) we use the same coupling constants.



Total cross section $e^+e^- \to \pi^+\pi^-2\pi^0$ in the energy region from threshold to 1.1 GeV, compared to several experimental data.

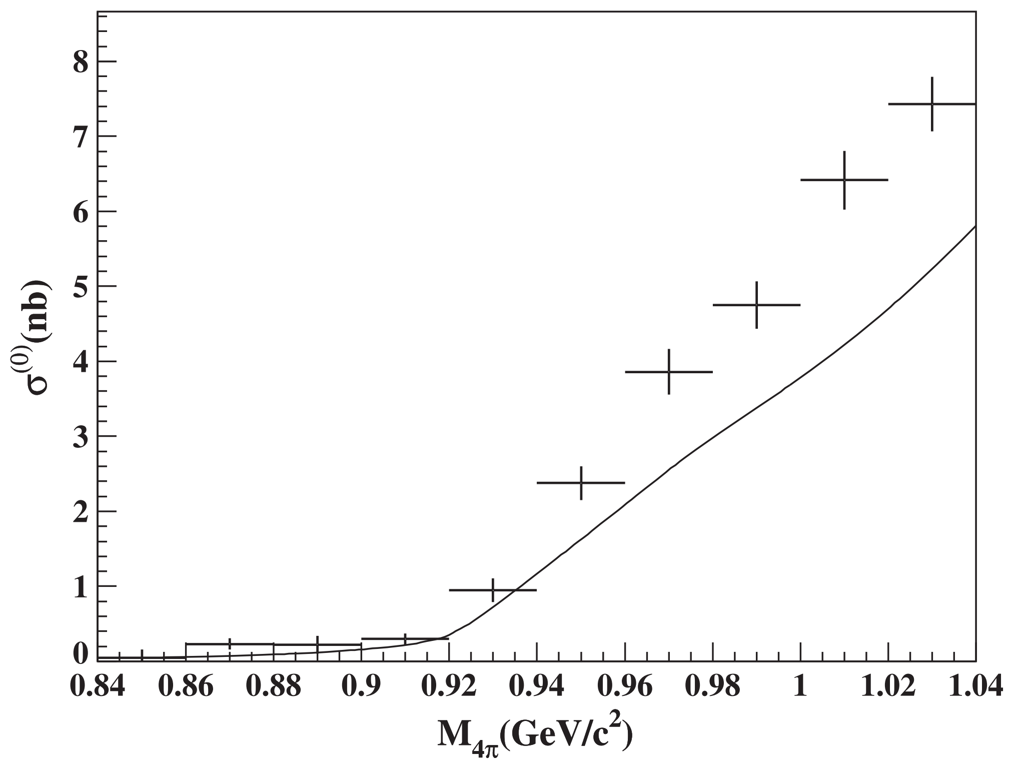


FIG. 11. The low-energy part of the vacuum polarization corrected measured undressed cross section (points with statistical uncertainties) compared to the theoretical prediction (line) from Ref. [36].

Babar Figure

Low energy region dominated by the ω and σ channels (D) and (G). Error (shaded area) dominated by the σ(600) parameters. In this region there is no effect due to variations of the parameters on channel (B)

Parameters analysis. decay modes and cross section data

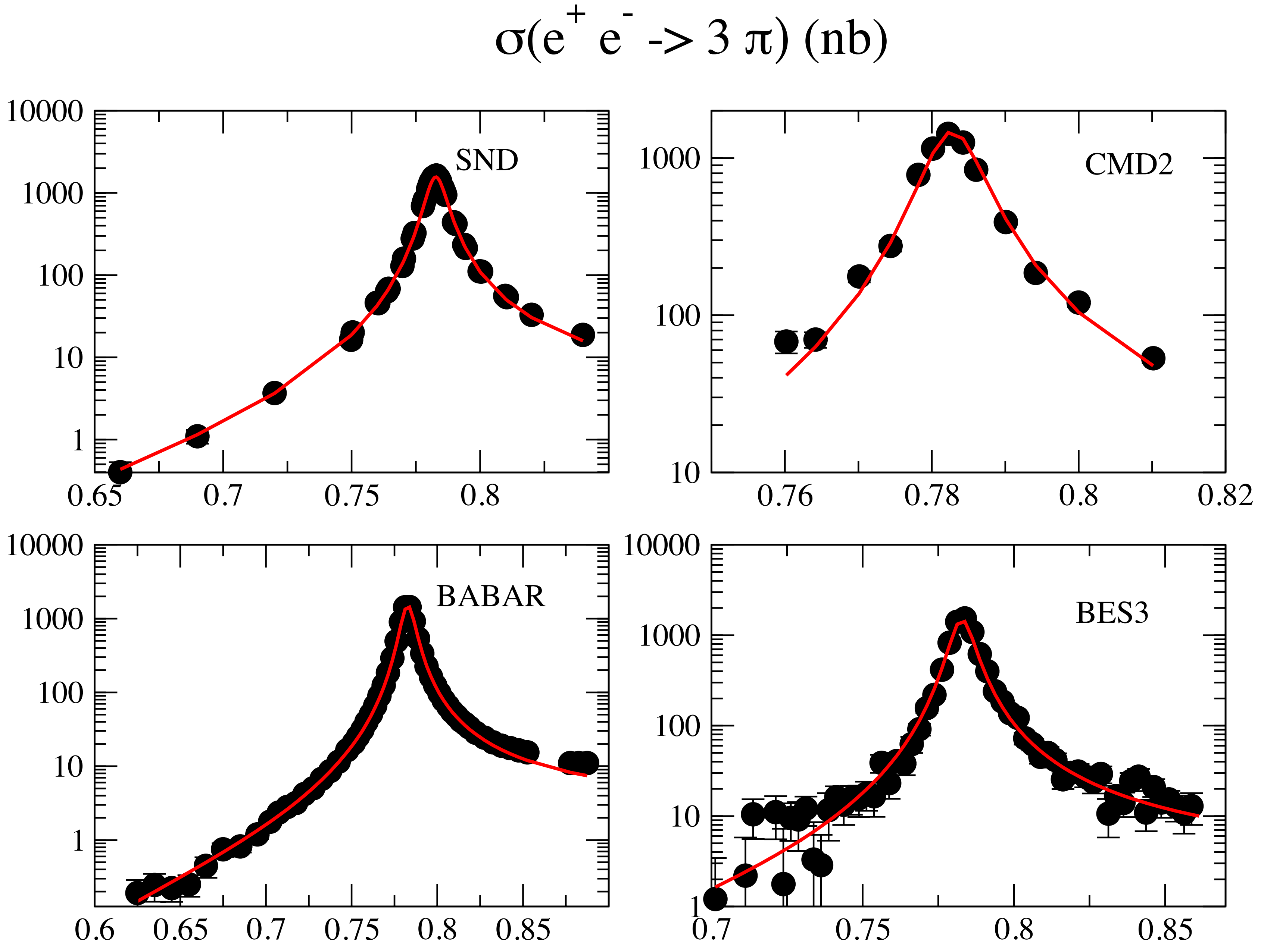
Avalos et al, Phys Rev D 107 056006 (2023)

We minimize the function

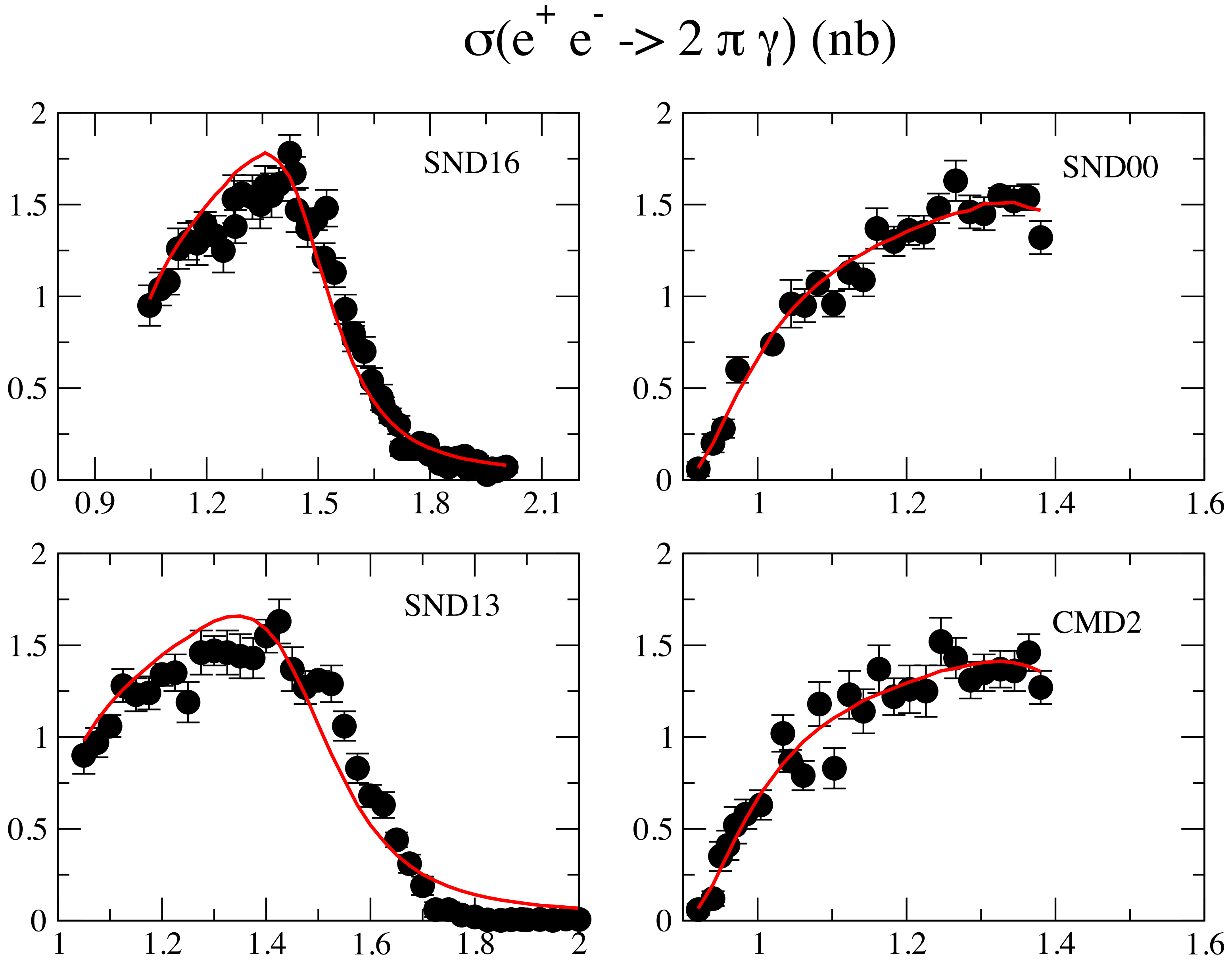
$$\chi^2(\theta) = \sum_{i=1}^{N} \frac{(y_i - \mu(x_i;\theta))^2}{E_i^2},$$

considering the couplings as free parameters, for the following data:

$\rho \to \pi\pi \quad \rho^0 \to e^+e^-, \ \mu^+\mu^- \quad \omega \to 3\pi \quad \rho \to \pi\gamma \quad \pi^0 \to \gamma\gamma. \ \omega \to \pi^0\gamma,$
$\omega \to e^+e^-, \ \mu^+\mu^- \quad e^+e^- \to \pi^0\pi^0\gamma \quad e^+e^- \to 3\pi$

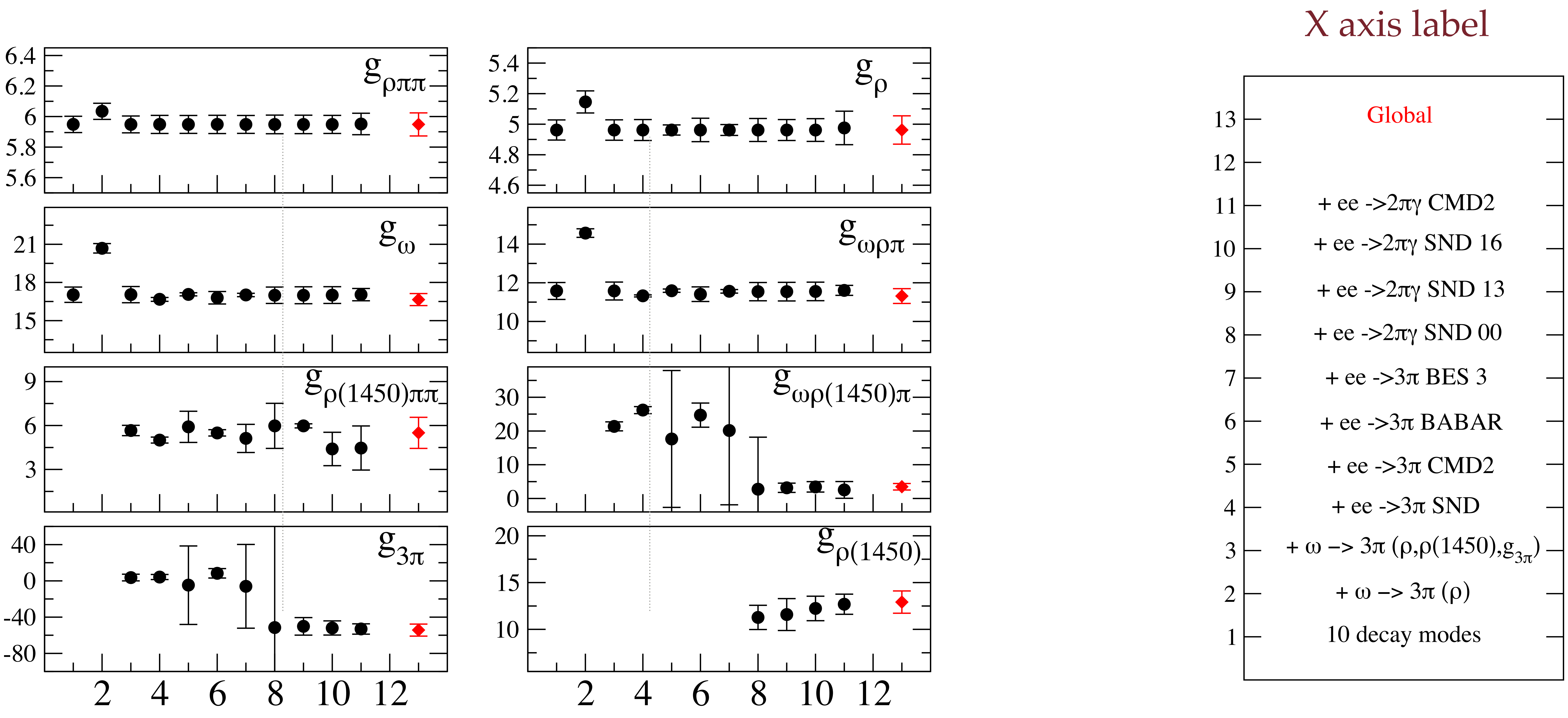


SND, BABAR, CMD2, BES 3



SND (00), (13), (16), CMD2

Couplings and data



Parameters behavior as more experimental data is added

Couplings

Parameter	Value
$g_{\rho\pi\pi}$	5.9485 ± 0.0776
g_{ρ}	4.9621 ± 0.0940
g_{ω}	16.624 ± 0.4727
$g_{\omega\rho\pi}$ (GeV^{-1})	11.294 ± 0.384
$g_{\rho'\pi\pi}$	5.7968 ± 0.4442
$g_{\omega\rho'\pi}$ (GeV^{-1})	3.613 ± 0.742
$g_{3\pi}$ (GeV^{-3})	-53.494 $\pm$ 7.1857
$g_{\rho'}$	12.845 ± 0.396
θ (in π units)	0.8967 ± 0.0416

$$\frac{m_{\rho'}^2 g_{\rho'}}{g_{\rho'\pi\pi}} = X \frac{m_\rho^2 g_\rho}{g_{\rho\pi\pi}}$$

X=1 -> $X = 1.3 \pm 0.4.$ ✓

Consistent within uncertainties

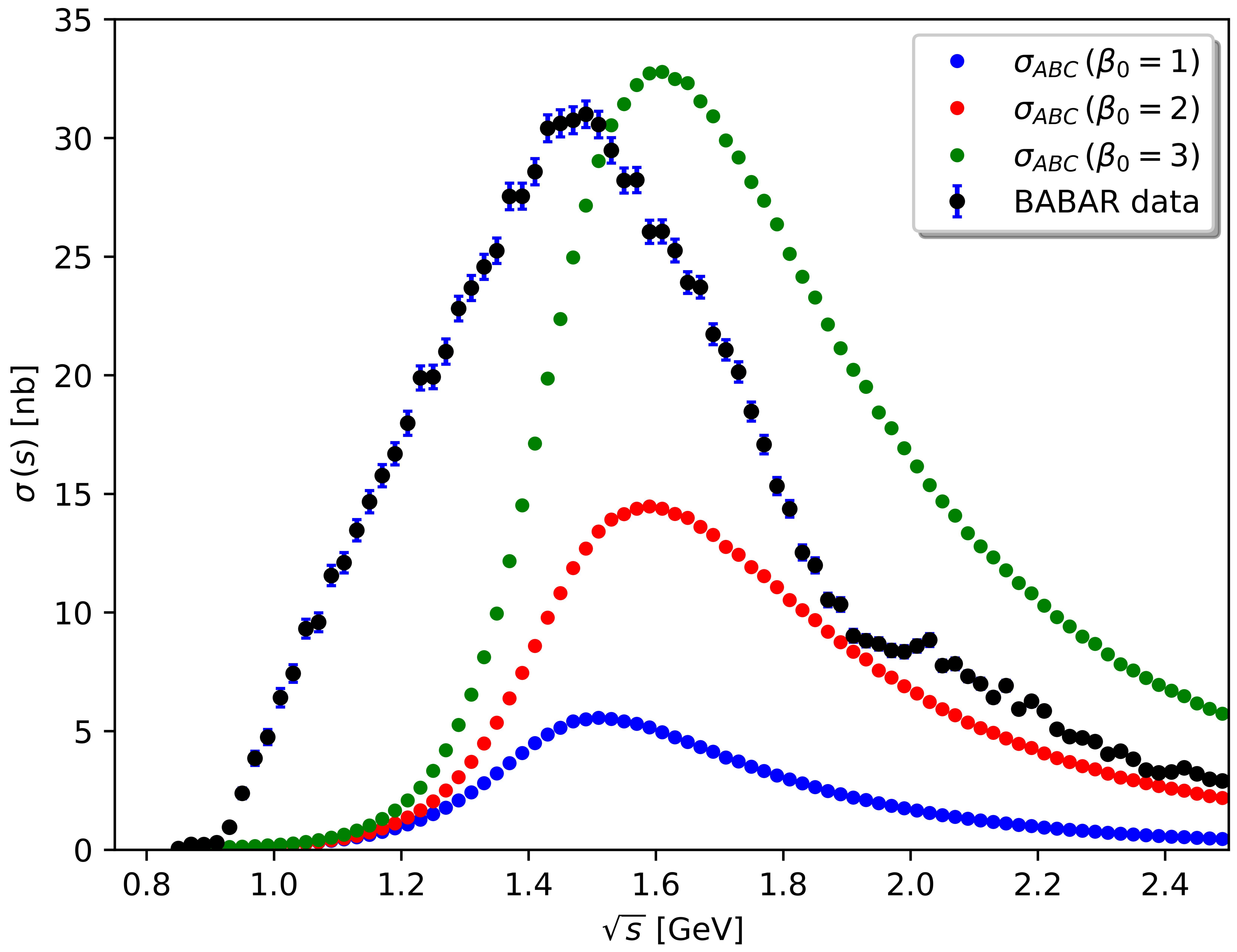
Magnetic dipole moment from Babar data

We compute the cross section of the process

$$\sigma(q^2) = \int_{s_{1-}}^{s_{1+}} ds_1 \int_{s_{2-}}^{s_{2+}} ds_2 \int_{u_{1-}}^{u_{1+}} du_1 \int_{u_{2-}}^{u_{2+}} du_2 \int_{t_{0-}}^{t_{0+}} dt_0 \int_{t_{1-}}^{t_{1+}} dt_1 \int_{t_{2-}}^{t_{2+}} dt_2 \frac{1}{4(2\pi)^8\sqrt{k_1.k_2}} |\mathcal{M}|^2 FEF.$$

The seven kinematical variables are chosen following Ref.[Kumar]. R. Kumar, Phys. Rev. 185, 1865-1875 (1969).

The integration is performed numerically using a Fortran code and the Vegas subroutine



A, B, and C channels contribution to the total cross section for $e^+e^- \to \pi^+\pi^- 2\pi^0$ and the BaBar experimental data. The strong dependence on the MDM is exhibited by choosing three values 1, 2 and 3.

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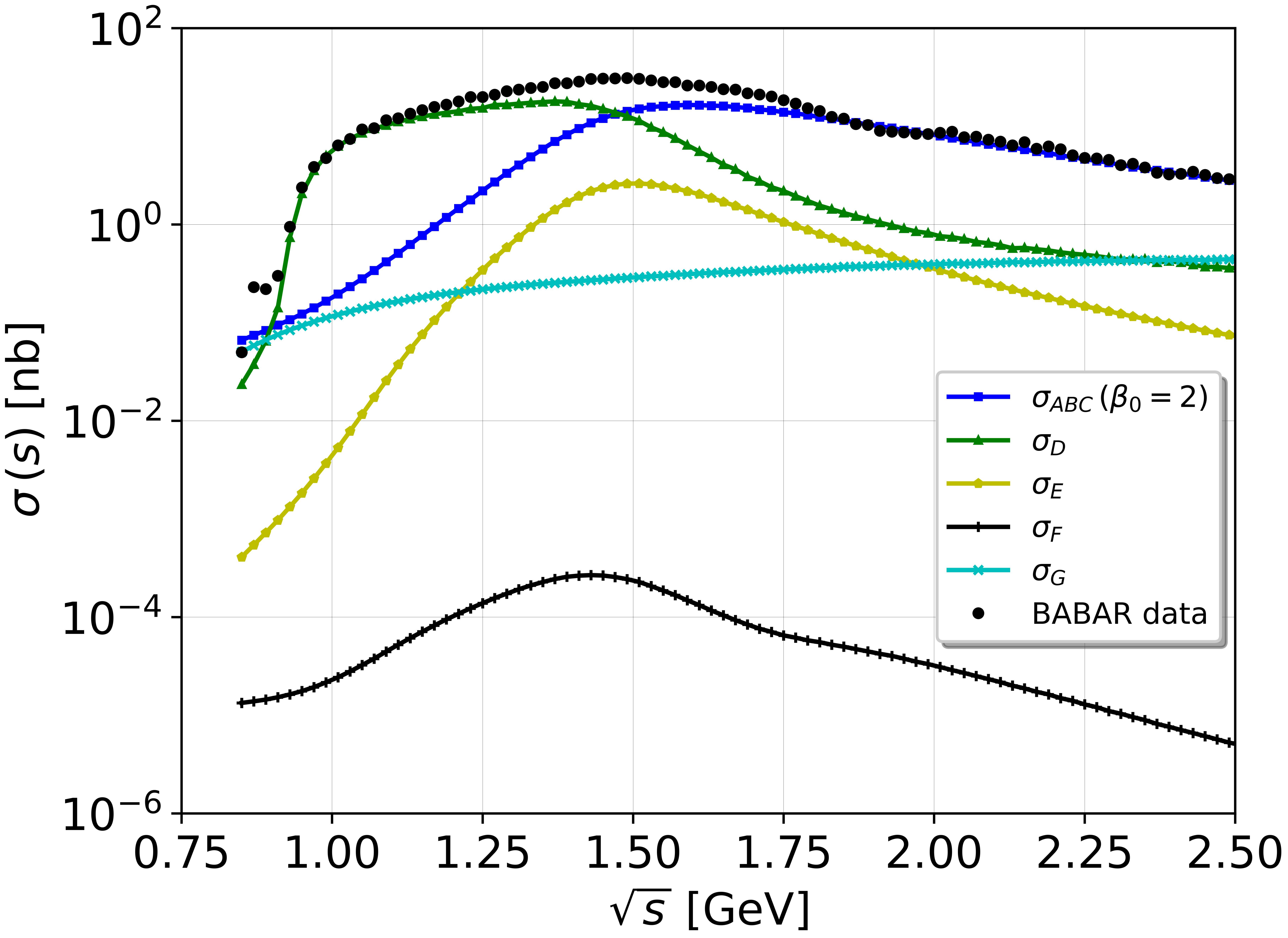
Individual channels contribution



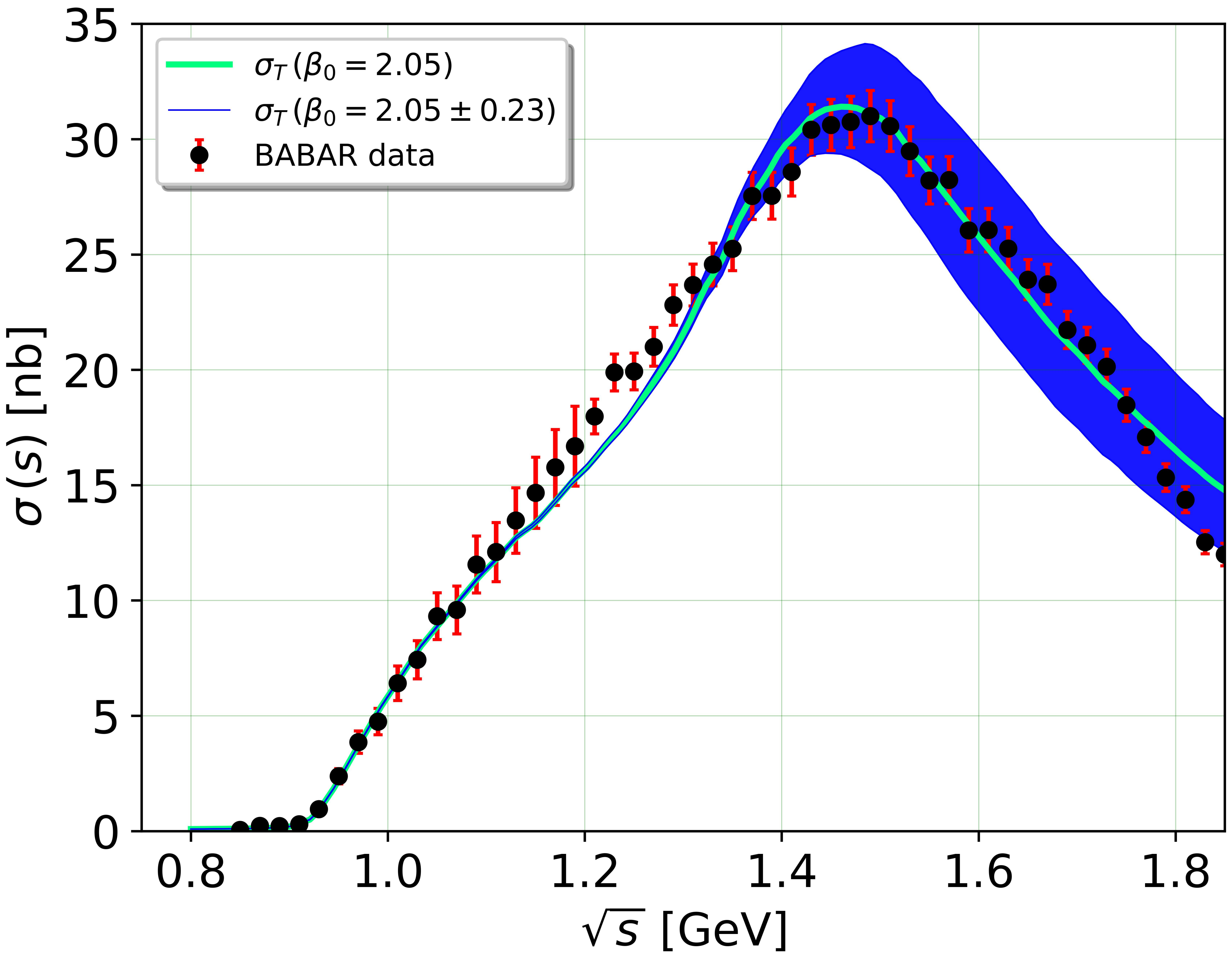
Individual channels contribution to the total cross section for $e^+e^- \to \pi^+\pi^-2\pi^0$ and the BaBar data.

Each channel includes the full reduced amplitudes for ϱ and ϱ' and their corresponding interferences, which are the dominant ones. The interferences among different channels are not shown but accounted in the analysis.

MDM, total cross section data

Provided all the parameters involved in our description are determined from other observables,

we fit the data considering β0 in the electromagnetic vertex as the only free parameter.



Fit to total cross section data from BaBar (symbols). The shaded area is the uncertainty including the one from electric charge form factor

From the fit

$$\beta_0 = 2.05 \pm 0.07 \qquad \chi^2/dof = 1.3$$

Electric charge form factor normalization for actual parameters

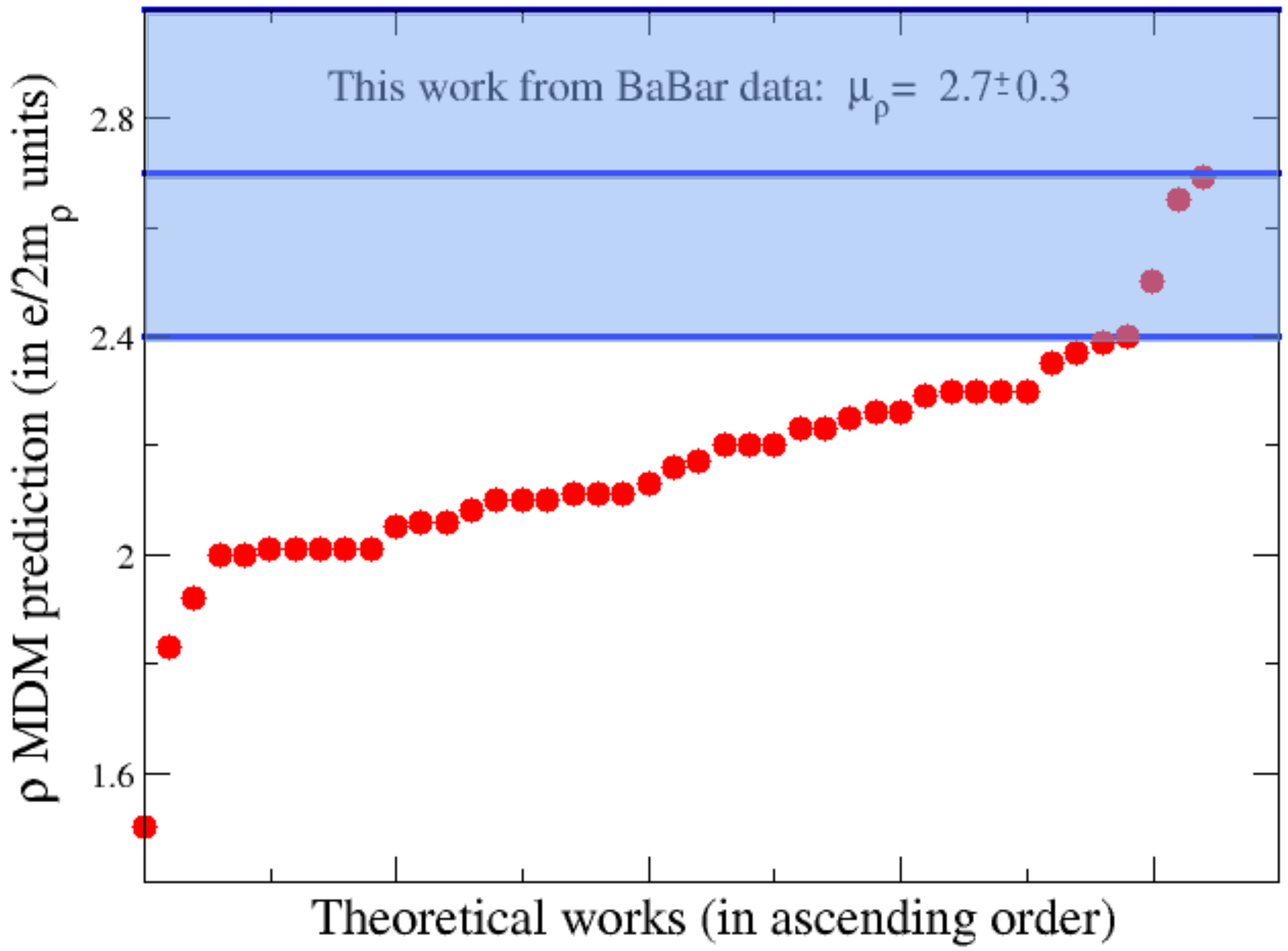
$$|F_\rho(0)| = \lim_{q^2\to 0}\left|\frac{g_{\rho\pi\pi}m_\rho^2}{g_\rho}D_\rho[q^2] - \frac{g_{\rho'\pi\pi}m_{\rho'}^2}{g_{\rho'}}D_{\rho'}[q^2]\right| = 1$$

0.75 ± 0.05

⇒ $\mu_\rho = 2.7 \pm 0.3$ in $(e/2m_\rho)$ units.

The quoted error bar takes into account the uncertainties coming from the electric charge form factor

Vs. Theoretical Predictions



SDE F.T Hawes et al PRC 59(1999)1743
Extended Bag model, Simonis V. 1803.01809
SDE MRL Zanbin Xing et al PRD 104 054038 (2021)
QCD Sum rules T.M. Aliev et al PLB 678 (2009)470
Lattice F.X. Lee et al PRD 78 094502(2008)

Conclusions

- ⋆ We obtained the magnetic dipole moment of the ρ meson using published data from the BaBar Collaboration for the $e^+e^- \to \pi^+\pi^-2\pi^0$ process, in the center of mass energy range from 0.9 to 1.8 GeV.

$$\mu_\rho = 2.7 \pm 0.3 \quad \text{in } (e/2m_\rho) \text{ units.}$$

- ⋆ We describe the $\gamma^* \to 4\pi$ vertex using a vector meson dominance model, including the intermediate resonant contributions relevant at these energies.

- ⋆ We improved on the previous extracted value, where preliminary data from the same collaboration was used, by considering published data, better grounded values of the parameters involved and explicit gauge invariant description of the process.

Thanks for your attention !