Neutral pion screening masses in a magnetized medium

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ICN, UNAM, México

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[1] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Overview

- Why is strong-field physics important?
- Interplay between strong magnetic fields and QCD.
- Linear sigma model with quarks (LSMq)
- Analysis of the neutral pion screening masses
- Results
- Summary and perspectives

Strong-(Electromagnetic)Field Physics

- High energy physics (heavy ion collisions)
- Astrophysics (neutron stars particularly magnetars)

Strong-Field Physics

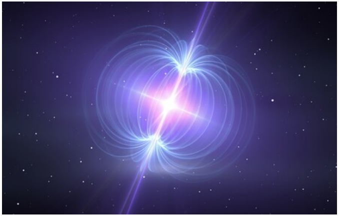


Figure: Magnetar ($10^{13} - 10^{15}$ G.)

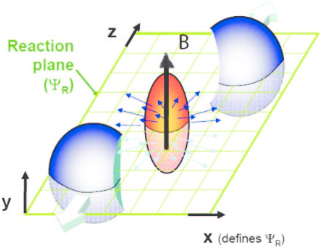


Figure: Heavy ion collisions ($10^{18} - 10^{19}$ G.)

Comparison of different magnetic fields

- Earth's magnetic field: $0.6\ G$
- Common comercial magnet: $100\ G$
- Strongest magnetic field produced in labs: $4.5 \times 10^5\ G$
- Magnetars: $(10^{13} - 10^{15}) G$
- Heavy ion collisions: $(10^{18} - 10^{19})\ G$

Interplay between strong magnetic fields and QCD

- Magnetic catalysis at zero temperature. enhancement of the condensate $< \bar{q}q >$ with B
- Inverse magnetic catalysis around T_C. decrease of the condensate $< \bar{q}q >$ as a function of B near T_C
- Chiral magnetic effect. Creation of a current J which is collinear to B as a consequence of the axial anomaly
- Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

Why screening masses of neutral pions?

Since the dynamics of chiral symmetry breaking is dominated by pions, the lightest of all quark-antiquark bound states, it then becomes important to explore how the pion properties like the pole mass and screening masses are affected by the presence of magnetic fields.

In this work we will study the effect of a constant magnetic field B in the screening masses of neutral pions.

Screening Mass

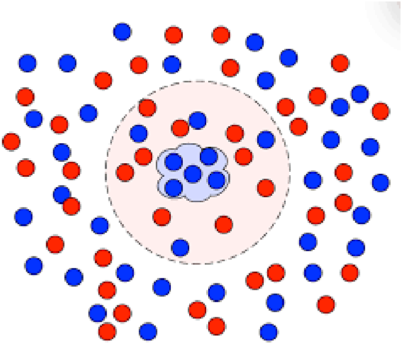


Figure: Debye mass (inverse of Debye length)

$$V \propto \frac{1}{r} \to \frac{1}{r} e^{-r/\lambda}$$

Screening Mass

In QFT we don't see the screening from the potential like in the previous example from classical electrodynamics, instead we pay attention to correlation functions:

$$G(x) \propto e^{-m_{scr}x}$$

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2=p_\perp^2=0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_\perp^2=0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_3^2=0} = 0$$

Comparison with LQCD and the NJL model

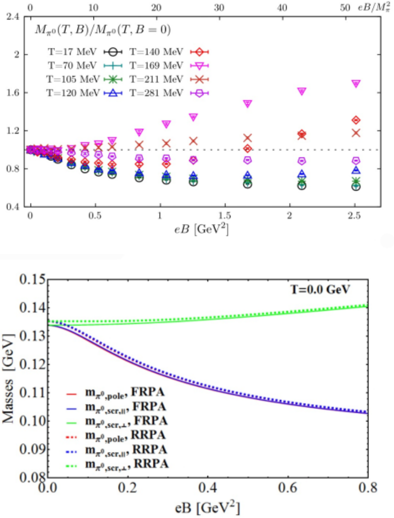


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

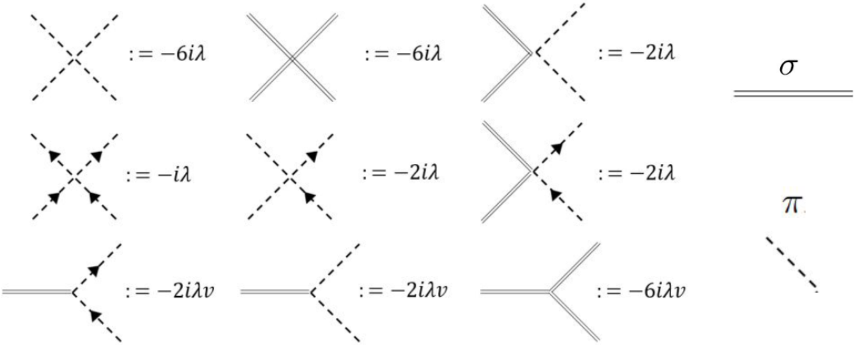
LSMq Lagrangian and features

$$\mathcal{L} =\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi \\ - ig\bar{\psi}\gamma^5\bar{\psi}\vec{\tau}\cdot\vec{\pi}\psi - g\bar{\psi}\psi\sigma\,.$$

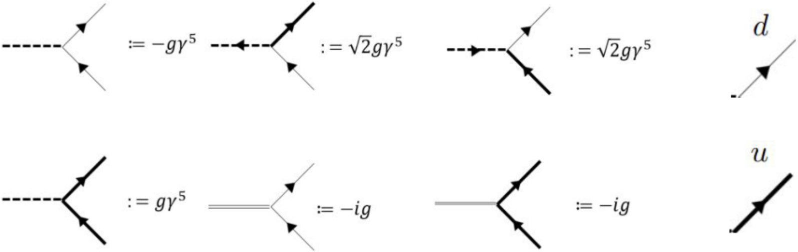
- it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- $m_f(v) = gv$.
- $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

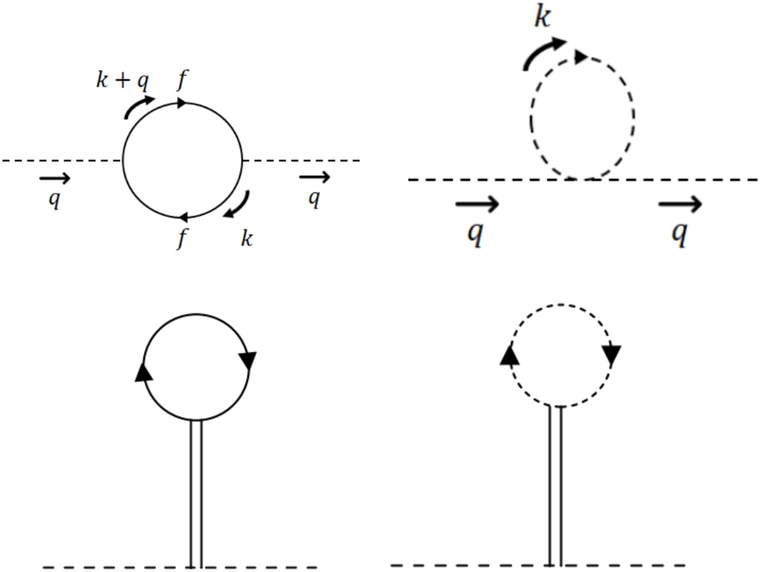
Feynman rules for boson-boson interactions



Feynman rules for boson-fermion interactions



Relevant Feynman diagrams



Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

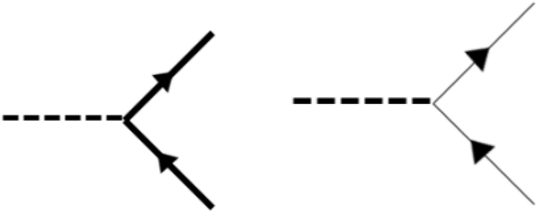


Figure: π_0-u quark vertex=gγ^5; π_0-d quark vertex=-gγ^5

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right)\right]$$

$$\times \left\{ (m_f + \not{p}_\parallel)\Big(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs) \Big) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$iS(p) \to i \frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

Neutral pion self-energy (fermion contribution)

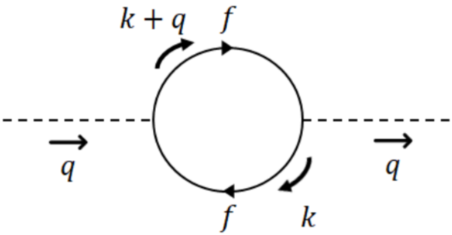


Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^5 iS(k)\gamma^5 iS(k+q)\right] + c.c..$$

Neutral pion self-energy (fermion contribution)

$$
\begin{aligned}
-i\pi_{f\bar{f}} = -4g^2 \int_0^\infty \int_0^\infty \frac{ds ds'}{\cos(qBs)\cos(qBs')} \\
\times \int \frac{d^4k}{(2\pi)^4} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)} e^{is'\left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon\right)} \\
\left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs)\cos qBs'} \right\}
\end{aligned}
$$

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Contour to perform the u integral

The u integral can be done analytically following the prinpal value prescription

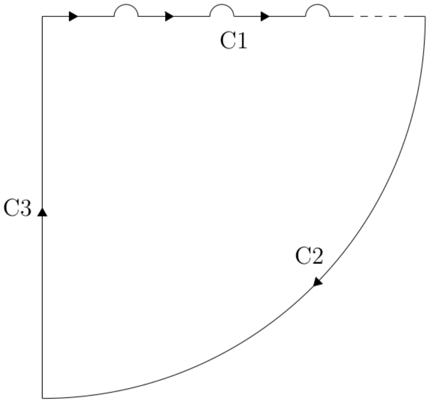


Figure: Contour to perform the u integral

Analysis

$$f(p_0, p_\perp, p_\parallel, B) = \pi_{f\bar{f}} - \lim_{B\to 0} \pi_{f\bar{f}}$$

We start with the simplest case ($p_\perp^2 = p_0^2 = 0$). This is the 'longitudinal' screening mass which is found by solving the equation:

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

Final expression

$$\Re f = lim_{\epsilon\to 0}\Bigg(- \frac{4g^2}{(4\pi)^2}\int_0^1 dv\Bigg\{\Big(- 2v(1-v)p_3^2\Big)$$

$$\times\left[\frac{\pi}{2}\frac{\sin(\frac{a\pi}{qB})}{\cosh(\frac{\epsilon\pi}{qB}) - \cos(\frac{a\pi}{qB})} - \tan^{-1}\left(\frac{\sin(\frac{a\pi}{qB})e^{\left(\frac{-\epsilon\pi}{qB}\right)}}{1 - e^{\left(\frac{-\epsilon\pi}{qB}\right)}\cos\left(\frac{a\pi}{qB}\right)}\right)\right]$$

$$+ qB\left[\frac{\epsilon\pi}{2qB} - \ln\sqrt{2\cosh\left(\frac{\epsilon\pi}{qB}\right) - 2\cos\left(\frac{a\pi}{qB}\right)}\right]$$

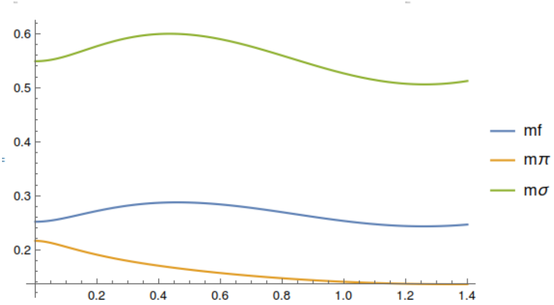
$$- \frac{qB}{\pi}\left[\Re\Big(Li_2\Big[e^{-(ia+\epsilon)\frac{\pi}{qB}}\Big]\Big)\right]\Bigg\}\Bigg)$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} = 2.75; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi^2} = 15$$

Magnetic field dependence of masses



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[2] S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

Fermionic contribution for different g values

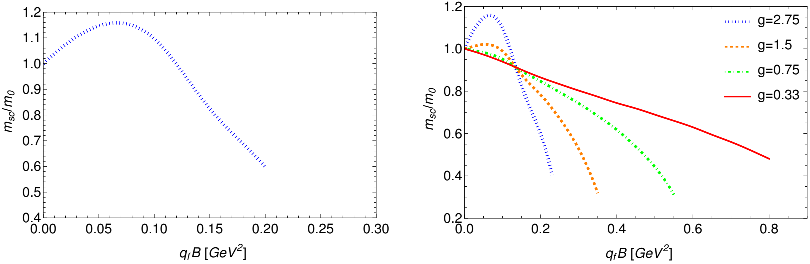

Figure: 'Longitudinal' SC mass as function of B for $g = 2.75$ (left). 'Longitudinal' SC mass as function of B for different values of g.

[3]

[3] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Fermionic contribution + Tadpoles

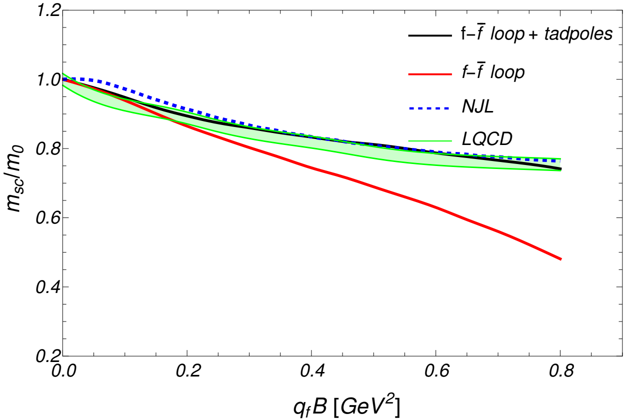

Figure: 'Longitudinal' Screening mass as a function of B, $g = 0.33, \lambda = 2.5$.

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[4] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2=p_\perp^2=0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_\perp^2=0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_3^2=0} = 0$$

Small magnetic field approximation

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)\right]$$
$$\times \left\{(m_f + \not{p}_\parallel)\Big(\cos(qBs) - \gamma^1\gamma^2 \sin(qBs)\Big) + \frac{\not{p}_\perp}{\cos(qBs)}\right\}$$

$$iS(p) \to iS^0(p) + iS^1(p) + iS^2(p)$$

,

$$iS^0(p) = i\frac{m_f + \not{p}}{p^2 - m^2}$$

$$iS^1(p) = |q_f B|\gamma^1\gamma^2 \frac{m_f + \not{p}_\parallel}{(p^2 - m^2)^2} sign(q_f B)$$

$$iS^2(p) = -2i|q_f B|^2 \frac{p_\perp^2(\not{p}_\parallel + m_f) - (m_f^2 - p_\parallel^2)\not{p}_\perp}{(p^2 - m_f^2)^4}$$

Self Energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^5(iS^0(q) + iS^1(q) + iS^2(q))\gamma^5$$
$$\times(iS^0(q-p) + iS^1(q-p) + iS^2(q-p))]$$
$$= -i\pi^{00}_{f\bar{f}} - i\pi^{11}_{f\bar{f}} - i\pi^{02}_{f\bar{f}} - i\pi^{20}_{f\bar{f}}\,.$$

There are no odd-powers in the magnetic field, there are just even powers

Feynman parameters

$$\frac{1}{A_1^{m1} A_2^{m2} \cdots A_n^{mn}} = \int_0^1 dx_1 dx_2 ... dx_n \delta(\sum x_i - 1) \frac{\prod x_i^{mi-1}}{(\sum x_i A_i)^{\sum mi}}$$
$$\times \frac{\Gamma(m_1 + ... + m_n))}{\Gamma(m_1) \cdots \Gamma(m_n)} .$$

Momentum integrals

$$\int \frac{d^4\ell}{(2\pi)^4}\,[\ldots] \to \int \frac{d^2\ell_\perp}{(2\pi)^2} \int \frac{d^2\ell_\parallel}{(2\pi)^2}\,[\ldots]\,.$$

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-d/2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-d/2}$$

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-d/2-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-d/2-1}$$

$$\int \frac{d^d\ell_E}{(2\pi)^d} \frac{1}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n-d/2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-d/2}$$

$$\int \frac{d^d\ell_E}{(2\pi)^d} \frac{\ell_E^2}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-d/2-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-d/2-1}$$

Final expressions

$$\left(\pi^{11}_{f\bar{f}}\right)_{\perp} = \frac{g^2(q_f B)^2}{4\pi^2}\int_0^1 dx x(1-x)\left[\frac{2m_f^2 + x(1-x)p_{\perp}^2}{(x(1-x))p_{\perp}^2 + m_f^2)^2}\right]$$

$$\left(\pi^{02}_{f\bar{f}} + \pi^{20}_{f\bar{f}}\right)_{\perp} = -\frac{g^2(q_f B)^2}{6\pi^2}\int_0^1 dx x(1-x)^3\left[\frac{3m_f^2 p_{\perp}^2 + x(1-x)p_{\perp}^4}{(x(1-x))p_{\perp}^2 + m_f^2)^3}\right]$$

$$\begin{aligned} f(p_0, p_{\perp}, p_{\parallel}, B) =& \pi_{f\bar{f}} - \lim_{B\to 0}\pi_{f\bar{f}} \\ =& \pi^{11}_{f\bar{f}} + \pi^{02}_{f\bar{f}} + \pi^{20}_{f\bar{f}} \end{aligned}$$

$$p_0^2 - p_{\perp}^2 - p_3^2 - m_{\pi}^2 - \Re f(p_0^2, p_{\perp}^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$

Transverse Mass

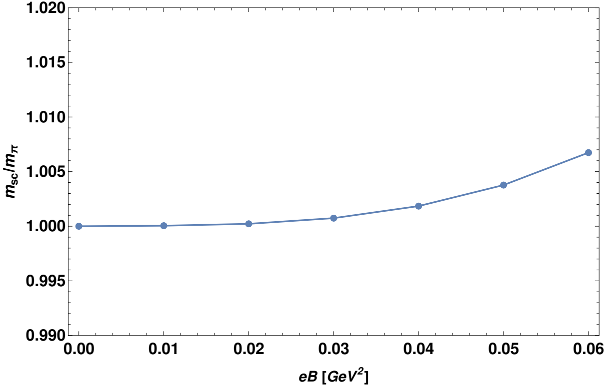

Figure: 'Transverse' Screening mass as a function of B, $g = 0.33, \lambda = 2.5$.

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[5]Masses taken from S. S. Avancini, M. Coppola, N. N. Scoccola and J. C. Sodré, Phys. Rev. D **104**, no.9, 094040 (2021), and S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252 .

Summary and perspectives

Summary:

- We have calculated the neutral pion self-energy in the LSMq.
- We have obtained the 'longitudinal' screening mass as a function of B for a general B, and the 'transverse' screening mass for small B.
- We have compared our results with LQCD and NJL, and we have found a nice agreement only when we have a magnetic field dependence on the couplings and masses.

Perspectives:

- We will complete the study of the 'transverse' screening mass as a function of B beyond small B.
- We will study the case where $T \neq 0$.

Thank You

Pole Mass

$$C(t) = \int d^3x < \mathcal{O}(\mathbf{x}, t)\mathcal{O}(\mathbf{0}, 0) >$$

$$C(t) = \sum_n | < 0|\mathcal{O}|n > |^2 e^{-E_n t}$$

$$C(t) \approx Z_0 e^{-m_{pole} t}$$

Pole and screening masses (definitions)

$$E^2 = u_\perp^2 \mathbf{q}_\perp^2 + u_\parallel^2 q_3^2 + m_{\pi^0,pole}^2$$

$$m_{\pi^0,scr.\perp} = \frac{m_{\pi^0,pole}}{u_\perp}$$

$$m_{\pi^0,scr.\parallel} = \frac{m_{\pi^0,pole}}{u_\parallel}$$

$$u_\perp \equiv u_\perp(B,T)\ ,\ \ u_\parallel \equiv u_\parallel(T)$$

Pole and screening masses (special cases)

- **(i)T=0, B=0**

$$u_{\perp} = u_{\parallel} = 1$$

$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} = m_{\pi^0,scr,\perp}$$

- **(ii)T≠0, B=0**

$$u_{\perp} = u_{\parallel} = u \neq 1$$

$$m_{\pi^0,pole} \neq m_{\pi^0,scr,\perp} = m_{\pi^0,scr,\parallel}$$

$u < 1$ in order to satisfy causality

Pole and screening masses (special cases)

- **(iii)B≠0, T=0**

$$u_\perp \neq u_\parallel \;\; but \;\; u_\parallel = 1$$

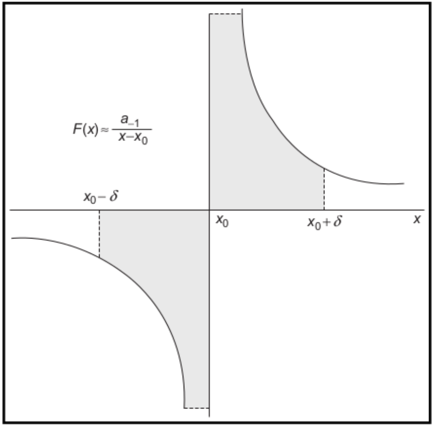
$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

- **(iv)B≠0, T≠0**

$$u_\perp < u_\parallel < 1$$

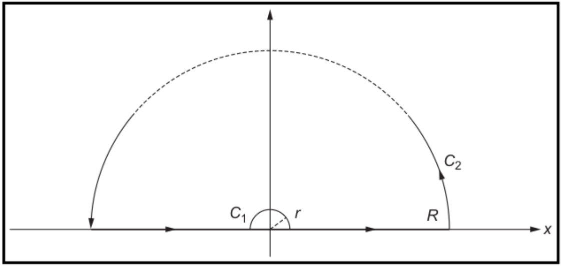
$$m_{\pi^0,pole} < m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

Principal value prescription for evaluating contours 1



$$f(z) = \frac{a_1}{(z-z_0)} + a_0 + \ldots$$

Principal value prescription for evaluating contours 2



$$f(z) = \frac{a_1}{(z - z_0)} + a_0 + \ldots \;\; ; \;\; z - z_0 = re^{i\theta} \; ; \; dz = ire^{i\theta} \;\; ; \;\; r = \delta$$

$$I_{over} = \int_{\pi}^{0} d\theta i \delta e^{i\theta} \left(\frac{a_{-1}}{\delta e^{i\theta}} + a_0 + \ldots \right) \to -i\pi a_{-1}$$

$$I_{under} = \int_{\pi}^{2\pi} d\theta i \delta e^{i\theta} \left(\frac{a_{-1}}{\delta e^{i\theta}} + a_0 + \ldots \right) \to i\pi a_{-1}$$

Principal value prescription for evaluating coutours 3

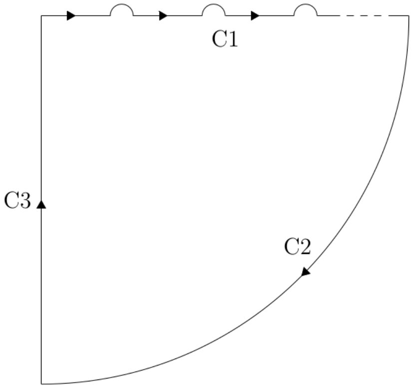
$$P.V. \int f(z)dz + I_{over} + \int_{C2} f(z)dz = 2\pi i \sum residues(other \;\; than \;\; z_0)$$

$$P.V. \int f(z)dz = -I_{over} - \int_{C2} f(z)dz + 2\pi i \sum residues(other \;\; than \;\; z_0)$$

$$P.V. \int f(z)dz = -I_{under} - \int_{C2} f(z)dz + 2\pi i \sum residues(other\, than\, z_0) + 2\pi i a_{-1}$$

$$2\pi i a_{-1} - I_{under} = -I_{over}$$

Typical contour of integration



$$I \equiv \int_0^\infty du e^{-u\epsilon} e^{-iau} \left(\cot(|q_f B|u) - \frac{1}{|q_f B|u} \right) .$$

$$I_C = \oint_C du e^{-u\epsilon} e^{-iau} \left(\cot(|q_f B|u) - \frac{1}{|q_f B|u} \right) ,$$

Neutral pion self-energy (fermion contribution)

$$\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp(-ix) \exp(-u\epsilon) \Bigg[\frac{m_f^2}{\tan(qBu)}$$

$$- \frac{qB}{\sin^2(qBu)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)}$$

$$- \frac{iqB}{\sin(qBu)} - \frac{i}{u \tan(qBu)} \Bigg]$$

where x is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

$B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B\to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2}\int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon} \times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

Fermionic contribution with g_{eff} as a function of B

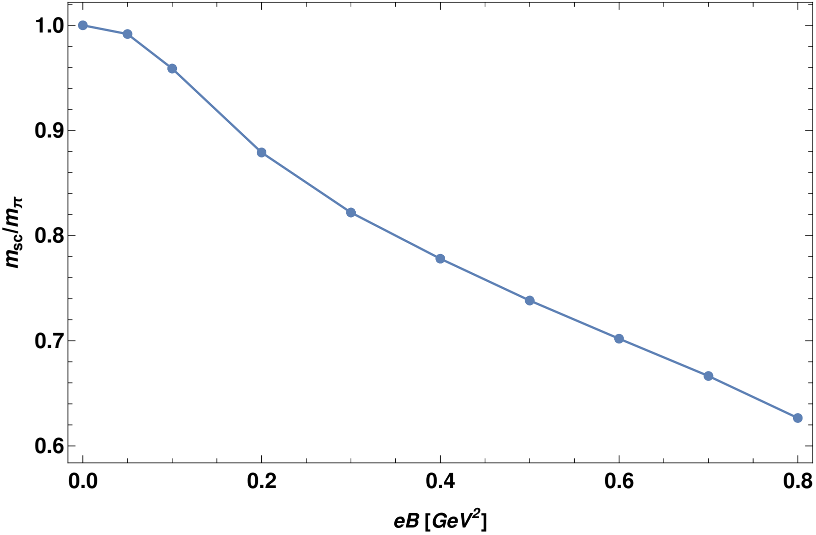


Figure: 'Longitudinal' Screening mass as a function of B, $g_{eff}(B) = 0.3 + 1.2\exp[-(7B)^2]$.