# Uncovering Quantum Field Theory and the Standard Model

#### From Fundamental Concepts to Dynamical Mechanisms

with Uwe-Jens Wiese, University of Bern, Switzerland

First textbook about particles and fields with an author working in Mexico (second in Latin America) [H. Nastase (Romanian in São Paulo): "Introduction to QFT", 2019]

Respect for the Standard Model: Most precise scientific theory!

Origin: Lecture notes by Uwe-Jens from Aachen, 1992

Start for me in Berlin: visit by Uwe-Jens in September 2003, walk near the Brandenburg Gate, agreement to convert it into a joint textbook project.



Uwe-Jens Wiese in his office in Bern

Sporadic progress during >10 years, in 2009 I moved to Mexico

Sabbatical semester in Bern, 2016/7 (bad luck: stolen laptop).

Book proposal sent to *Cambridge University Press*,5 chapters and one appendix. Approved by 6 referees.Contract: Maximum of 600 pages, deadline: July 1, 2028.

Another sabbatical semester in Bern, summer 2018

Progress, but distraction with new research projects, termination still behind the horizon.

More theory and concepts than phenomenology

Focus on non-perturbative perspective  $\neq$  previous literature



Discussion about fermion generations (Chapter 17).

Upon request, we included gauge fixing, but not SUSY.

#### Pandemics:



Frequent video calls

Uwe-Jens: Intense work, *e.g.* on canonical fermion fields.

Helpful comments on specific chapters by *Oliver Bär*, Debasish Banerjee, *Detlev Buchholz, Wilfried Buchmüller,* Klaus Fredenhagen, *Urs Gerber, Carlo Giunti,* Kieran Holland, Gurtej Kanwar, *Martin Lüscher,* Alessandro Mariani, *Colin Morningstar, Mike Peardon,* Michele Pepe, João Pinto Barros, *Lilian Prado,* Simona Procacci, Christopher Smith, *Rainer Sommer,* Youssef Tammam, Christiane Tretter, *Christof Wetterich, Edward Witten.*  Early version: conceptual introduction of 14 pages. Martin Lüscher: reader who does not know e.g. gauge theory gets lost very soon. We only assume previous knowledge of Quantum Mechanics and Special Relativity.

## New structure: **Ouverture, Intermezzo, Finale** 26 Chapters is 4 Parts: Link to "Le quattro stagioni" by Antonio Vivaldi [QFT, SM, Strong Interaction, Beyond SM]. $\sim$ 3 semesters

Finally submitted in December 2022, "takes on average 9 months", hope for it to be faster, since we used the Cambridge style file throughout.

**No way:** summer 2023: edited pdf with  $O(10^4)$  modifications (lots of commas, some errors, a few corrections: "Klein-Gordon" vs. "Clebsch-Gordan"), convergence after some cycles of revision.

We thank *Sunantha Ramamoorthy*, competent and cooperative, but she could not avoid a strange last-minute change of fonts (exponents, indices etc. displaced). Reduction from 772 to 732 pages.

Cover page: chalk drawing of  $\beta$ - and  $\pi^0$ -decay, by *Nadiia Vlasii* 



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#### Ranking of the most frequent names in the authors index:

10 entries: Callen, Lüscher, 't Hooft, Wilczek

9 entries: Weinberg

7 entries: Einstein, Feynman, Wilson (Referee: "Wilson could have written such a book, but ...")

6 entries: Gross, Leutwyler, Witten

5 entries: Hasenfratz, Pauli, Schwinger

4 entries: Bogoliubov, Coleman, Dashen, Dirac, Gasser, Gell-Mann, Glashow, Niedermayer, Politzer, Polyakov, Symanzik, Wess, Yang, Zimmermann, Zinn-Justin, Zumino

## Meißner-Rusetsky problem:

4.9 Chiral Perturbation Theory at One Loop Low-Energy Effective Field Theory of QCD not be needed. Note only that the integration over the  $\delta$ -function Here, the first, second and third terms correspond to the s-, t- and u-channel contrithe number of particles in the intermediate state. Here, the insectively. It can be straightforwardly checked that the t- and u-channel contri-butions, respectively. It can be straightforwardly checked that the t- and u-channel contributions emerge from the disconnected parts. the number of particles in the power counting. As we know, the tree-level what remains is to substitute the  $O(p^2)$  expression for the scattering amplitude, given Next, let us turn to the optimizer. Now, let us demonstrate that analytich are of order  $p^2$  in chiral counting. Now, let us demonstrate that analytich are of order  $p^{+}$  in chiral countries amplitude at order  $p^{+}$  (the meson-meson training allow one to evaluate the amplitude at order  $p^{+}$  (the meson-mesonby Eq. (4.69), tarity allow one to evaluate the powers of p only, so that any  $O(p^3)$  term is abuilt amplitudes contain even powers of p only, so that any  $O(p^3)$  term is abuilt of the powers of p only.  $A(s,t,u) = \frac{s}{F_{\pi}^2}$ amplitudes contain even potent is able in again and the imaginary part at this order. It is straight this end, let us first calculate the imaginary part at this order, it is straight the straight and the imaginary part at the imaginary part at the imaginary part at the order  $p^2$ . (4.238) this end, let us instructional  $2 \rightarrow n$  are of order  $p^2$ . Indeed, since each p to see that all amplitudes  $2 \rightarrow n$  are of order  $p^2$ . into the right-hand side of Eq. (4.237) and calculate the left-hand side at  $O(p^4)$ . In to see that all amplitude pained by  $1/F_{\pi}$ , see, for example, Eqs. (4.88) and the Lagrangian is accompanied by  $1/F_{\pi}$ , see, for example, Eqs. (4.88) and into the digital interval of the calculation, note that the amplitude  $T^{(l,k)}(p_1,p_2;p_3,p_4)$  can be the Lagrangian is accompliance of  $(f/T|n)_c$  and  $(n|T^+|i)_c$  can be expanded in a power the connected amplitudes  $(f/T|n)_c$  that descend only on along a power decomposed into parts with total isospin I = 0, 1, 2: the connected amplitude officients that depend only on pion momenta and  $T^{ijkl}(p_1, p_2; p_3, p_4) = P_0^{ijkl}T^0(s, t, u) + P_1^{ijkl}T^1(s, t, u) + P_2^{ijkl}T^2(s, t, u)$ in this parameter, whit contain a heavy scale anymore.<sup>14</sup> It can be checked that is they do not contain a heavy scale anymore.<sup>14</sup> It can be checked that is that is, they do not expansion is proportional to  $1/F_{\pi}^{n}$ . Further, let us consider the us (4:239) term in this expansion is pre-relation in Eq. (4.235). The mass dimension of the *n*-particle phase space and exwhere the projectors onto the states with definite isospin subject to the constraint relation in Eq. (1999), from this, we may conclude that these to  $P_l^{ij,ri}P_l^{rs,kl} = \delta_{ll'}P_l^{ij,kl}$  are given by function are 2n and  $p^{2n}$  and  $p^{-4}$  in chiral counting, because there is no heavy scale present there of  $p^{2n}$  and  $p^{-4}$  in chiral counting.  $p^{or}$  and  $p^{-}$  in clinic controls, since the left-hand side of Eq. (4.234) is dimensionless, it is immediately seen  $P_0^{ij,kl} = \frac{1}{2} \,\delta^{ij} \delta^{kl} \,,$ the expansion of the *n*-particle amplitudes should start at  $M^2/F_{\pi}^n$ , where  $M \dim$ some small scale. Consequently,  $\langle f|T|n\rangle_{c} = O(p^{2})$  and  $\langle n|T^{\dagger}|i\rangle_{c} = O(p^{2})$ . Puts  $P_{1}^{ij,kl} = \frac{1}{2} \left( \delta^{ik} \delta^{jl} - \delta^{il} \delta^{jk} \right),$ everything together, one finally sees that, working at  $O(p^4)$  in the chiral expansion it suffices to truncate the unitarity condition, retaining only the terms with a  $P_2^{ij,kl} = \frac{1}{2} \left( \delta^{ik} \delta^{jl} + \delta^{il} \delta^{jk} \right) - \frac{1}{2} \delta^{ij} \delta^{kl} \,.$ (4.240) (after rewriting this condition in terms of the connected diagrams). The result ha following form: Therefore, the scattering amplitudes with a definite isospin at  $O(p^2)$  are  $T^{0}(s,t) = \frac{1}{F^{2}} (3s+t+u) = \frac{2s}{F^{2}_{\tau}},$  $\frac{1}{2i} \left( T^{ij,kl}(p_1, p_2; p_3, p_4) - (T^{kl,ij}(p_3, p_4; p_1, p_2))^* \right)$  $= \frac{1}{4} \int \sum_{\eta} \frac{d^3 \mathbf{q}_1}{(2\pi)^3 2 q_1^0} \frac{d^3 \mathbf{q}_2}{(2\pi)^3 2 q_2^0} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - q_1 - q_2)$  $T^{1}(s,t) = \frac{1}{F_{x}^{2}}(t-u)$ (4.241)  $\times T^{rs,tl}(q_1,q_2;p_3,p_4)(T^{rs,tj}(q_1,q_2;p_1,p_2))^*$  $T^{2}(s,t) = \frac{1}{F_{\pi}^{2}}(t+u) = -\frac{s}{F_{\pi}^{2}}$ These expressions should be substituted into Eq. (4.237), and the integrations and sum- $+\frac{1}{4}\int_{\infty}^{\infty}\frac{d^{3}\mathbf{q}_{1}}{(2\pi)^{3}2q_{1}^{0}}\frac{d^{3}\mathbf{q}_{2}}{(2\pi)^{3}2q_{2}^{0}}(2\pi)^{4}\delta^{(4)}(p_{1}-p_{3}-q_{1}-q_{2})$ mations over isospin indices should be carried out. To this end, it is convenient to expand the amplitudes in the partial waves with the use of Eq. (4.230) and the relations  $\times T^{rr,jl}(q_1,q_2;-p_2,p_4)(T^{rs,jk}(q_1,q_2;p_1,-p_3))^*$  $s = 4p^2$ ,  $t = -2p^2(1 - \cos \theta)$ ,  $u = -2p^2(1 + \cos \theta)$ . The result is (4.242)  $t_0^2(s) = -\frac{1}{32\pi F_{\pi}^2}$  $t_0^0(s) = \frac{s}{16\pi F_{\pi}^2}, \quad t_1^1(s) = \frac{s}{96\pi F_{\pi}^2},$  $(2\pi)^4 \delta^{(4)}(p_1 - p_4 - q_1 - q_2)$ and all other partial-wave amplitudes vanish at this order  $\times T^{rs,kj}(q_1,q_2;p_3,-p_2)(T^{rs,il}(q_1,q_2;p_1,-p_4))^*$ Using now the definition  $\frac{1}{2i} \left( T^{ij,kl}(p_1, p_2; p_3, p_4) - \left( T^{kl, ij}(p_3, p_4; p_1, p_2) \right)^* \right)$ As we shall see in the next sections, this argument is valid in dimensional regularization independent regularization sedemal. Using, e.g., cutoff regularization would destroy th source the young justs the role of the hard scale in the coefficients. In the following functional regularization (4.243)  $= \delta^{ij} \delta^{kl} \mathrm{Im} A(s,t,u) + \delta^{il} \delta^{jl} \mathrm{Im} A(t,u,s) + \delta^{il} \delta^{jk} \mathrm{Im} A(u,s,t) \,,$ 

U.-G. Meißner, A. Rusetsky, "Effective Field Theories", Cambridge University Press, 2022. Christian Schubert: Must be related to *Fermat's Last Theorem*.

## Confusion about the endorsers, at last 5 very positive statements



Recommendations by Edward Witten (Princeton), William Detmold (MIT), Wilfried Buchmüller (DESY), Poul Damgaard (Niels Bohr Inst.), Tereza Mendes (U. São Paulo)

# A gluon string could hold a mass of 20,000 kg, like a steel cable with a diameter of some cm, but $\sim 10^{13}$ times thinner.



14.4 A single Yang—Mills string is strong enough to support more than 100 people. A "Yang—Mills elevator" was installed by one of the authors as a Gedankenexperiment at the Erice workshop "From Quarks and Gluons to Hadrons and Nuclei" in 2011, in order to "elevate" our intuitive understanding for the strength of the strong force (Wiese, 2012).

**Quick Question 14.6.1 Overloaded Yang–Mills elevator** What happens when about 1000 (adult) people enter the Yang–Mills elevator?

#### 14.7 Roughening Transition

The string tension has been computed to high orders of the strong coupling expansion. The resulting contributions are associated with plaquette surfaces that are deformations of the

2024: announced by Cambridge University Press for January, then February ... inductive until June.

Summer 2024: editorial paralyzed by a cyber attack. Sad news: Sunantha died, substituted by Shanthy Jaganathan.

Promised for November, then December, finally published on January 2, 2025.

Happy ending, once in a lifetime event! Last chapter about Grand Unified Theories (GUT): Ende GUT, alles gut! Up to now (in < 5 months) 638 times sold Internet:  $\langle$  selling of acad. books  $\rangle \approx 500 \dots 1000$ 

## Part I: QUANTUM FIELD THEORY

#### **Ouverture: Concepts of Quantum Field Theory**

Point Particles versus Fields at the Classical Level Particles versus Waves in Quantum Theory Classical and Quantum Gauge Fields Ultraviolet Divergences, Regularization, and Renormalization Euclidean Quantum Field Theory versus Classical Statistical Mechanics

#### **1 Basics of Quantum Field Theory**

- 1.1 From Point Particle Mechanics to Classical Field Theory
- 1.2 Quantum Mechanical Path Integral
- 1.3 Path Integral in Euclidean Time
- 1.4 Spin Models in Classical Statistical Mechanics
- 1.5 Quantum Mechanics versus Classical Statistical Mechanics

- 1.6 Transfer Matrix
- **1.7 Lattice Field Theory**

# **2 Scalar Field Theory and Canonical Quantization**

- 2.1 Scalar Fields
- 2.2 Noether Current
- 2.3 From the Lagrangian to the Hamilton Density
- 2.4 Commutation Relations for the Scalar Field Operators
- 2.5 Hamilton Operator in Scalar Field Theory
- 2.6 Vacuum State and Vacuum Energy
- 2.7 Cosmological Constant Problem
- 2.8 Particle States and their Energies and Statistics
- 2.9 Momentum Operator

# **3 From Particles to Wavicles and Back**

3.1 Model for lons Forming a Crystal

3.2 Phonon Creation and Annihilation Operators
3.3 Quantum States of a Vibrating Crystal
3.4 Phonons as Wavicles
3.5 Explicit Breaking of Continuous Translation Symmetry
3.6 Debye Field Theory of the Vibrating Solid
3.7 From Wavicles Back to Particles
3.8 What is Space?

## 4 Perturbative Scalar Field Functional Integral in Dimensional Regularization

- 4.1 From Minkowski to Euclidean Space-Time
- 4.2 Euclidean Propagator and Contraction Rule
- 4.3 Perturbative Expansion of the Functional Integral
- 4.4 Dimensional Regularization
- 4.5 2-Point Function to 1 Loop
- 4.6 Mass Renormalization

4.7 Connected, Disconnected, and 1-Particle Irreducible Diagrams

- 4.8 Feynman Rules for the  $\lambda\phi^4$  Model
- 4.9 4-Point Function to 1 Loop
- 4.10 Dimensional Regularization of  $J(p^2)$
- 4.11 Renormalization of the Coupling
- 4.12 Renormalizability of Scalar Field Theories
- 4.13 Condition for Renormalizability

# **5** Renormalization Group

- 5.1 Locality and Hierarchies of Energy Scales
- 5.2 Renormalization Group Blocking and Fixed Points
- 5.3 Gaussian Fixed Points of Lattice Scalar Field Theory
- 5.4 Blocking from the Continuum to the Lattice
- 5.5 Perfect Lattice Actions on the Renormalized Trajectory
- 5.6 Wilson–Fisher Fixed Points
- 5.7 Renormalization of Scalar Field Theory in a Cut-off Regularization

5.8 Callan–Symanzik Equation 5.9  $\beta$ -Function and Anomalous Dimension to 1 Loop 5.10 Running Coupling 5.11 Infrared and Ultraviolet Fixed Points

### **6** Quantization of the Free Electromagnetic Field

6.1 Vector Potential and Gauge Symmetry

6.2 From the Lagrangian to the Hamilton Density

6.3 Hamilton Operator for the Photon Field

6.4 Gauss Law

- 6.5 Vacuum and Photon States
- 6.6 Momentum Operator of the Electromagnetic Field
- 6.7 Angular Momentum Operator and Helicity of Photons
- 6.8 Planck's Formula and the Cosmic Background Radiation
- 6.9 Gauge Fixing and Photon Propagator

## 7 Charged States in Scalar Quantum Electrodynamics

- 7.1 Complex Scalar Field with Global U(1) Symmetry
- 7.2 Scalar Quantum Electrodynamics
- 7.3 Charged Particles as Infraparticles
- 7.4 Superselection Sectors
- 7.5 Charged Particles in a Periodic Volume
- 7.6 C-periodic Boundary Conditions

#### 8 Canonical Quantization of Free Weyl, Dirac, and Majorana Fermions

#### 8.1 Massless Weyl Fermions

- 8.2 Momentum, Angular Momentum, and Helicity of Weyl Fermions
- 8.3 Fermion Number, Parity, and Charge Conjugation
- 8.4 Cosmic Background Radiation of Neutrinos
- 8.5 Massive Dirac Fermions
- 8.6 Massive Majorana Fermions

- 8.7 Massive Weyl Fermions
- 8.8 **Redundant Particle Labels** and the Pauli Principle as
  - a "Gauss Law"
- 8.9 Can We Supersede Gauge Symmetry?
- **9 Fermionic Functional Integrals** 
  - 9.1 Grassmann Algebra, Pfaffian, and Fermion Determinant
  - 9.2 Dirac Equation
  - 9.3 Weyl and Majorana Equations
  - 9.4 Euclidean Fermionic Functional Integral
  - 9.5 Euclidean Lorentz Group
  - 9.6 Charge Conjugation, Parity, and Time Reversal for Weyl Fermions
  - 9.7 C, P, and T Transformations of Dirac Fermions
  - 9.8 CPT Invariance in Relativistic Quantum Field Theory
  - 9.9 Connections between **Spin and Statistics**
  - 9.10 Euclidean Time Transfer Matrix

### **10 Chiral Symmetry in the Continuum and on the Lattice**

- 10.1 Chiral Symmetry in the Continuum
- **10.2 Lattice Fermion Doubling Problem**
- 10.3 Nielsen-Ninomiya No-Go Theorem
- 10.4 Absence of Neutrinos on a Lattice
- 10.5 Wilson Fermions
- 10.6 Perfect Lattice Fermions and the **Ginsparg–Wilson Relation**
- 10.7 Overlap Fermions

## **11 Non-Abelian Gauge Fields**

- 11.1 Non-Abelian Gauge Fields at the Classical Level
- 11.2 Gauge Fixing and Faddeev–Popov Ghosts
- 11.3 Becchi–Rouet–Stora–Tyutin Symmetry
- 11.4 Nilpotency and BRST Cohomology
- 11.5 Aharonov–Bohm Effect as an Analogue of BRST Cohomology

11.6 Lattice Gauge Theory

11.7 Canonical Quantization of Compact U(1) Lattice Gauge Theory 11.8 Canonical Quantization of Non-Abelian Lattice Gauge Theory 11.9 Functional Integral for Compact U(1) Lattice Gauge Theory 11.10 Functional Integral for Non-Abelian Lattice Gauge Theory

## Part II: CONSTRUCTION OF THE STANDARD MODEL

## Intermezzo: Concepts of the Standard Model

The Standard Model: A Non-Abelian Chiral Gauge Theory Renormalizability of Non-Abelian Gauge Theories Triviality and Incorporation of Gravity Fundamental Standard Model Parameters Hierarchies of Scales and Approximate Global Symmetries **Local and Global Symmetries** Explicit versus Spontaneous Symmetry Breaking

## **Anomalies in Local and Global Symmetries** Power of Lattice Field Theory

## 12 Spontaneous Breakdown of Global Symmetries: From Condensed Matter to Higgs Bosons

- 12.1 Effective Scalar Fields for Cold Condensed Matter
- 12.2 Vacua in the  $\lambda |\Phi^4|$  Model
- 12.3 Higgs Doublet Model
- 12.4 Goldstone Theorem
- 12.5 Mermin–Wagner–Hohenberg–Coleman Theorem
- 12.6 Low-Energy Effective Field Theory
- 12.7 Hierarchy Problem
- 12.8 Solving the Hierarchy Problem with Supersymmetry?
- 12.9 Is Nature Natural?
- 12.10 Triviality of the Standard Model
- 12.11 Electroweak Symmetry Restoration at High Temperature

12.12 Extended Model with **Two Higgs Doublet** 

- 13 Local Symmetry and the Higgs Mechanism: From Superconductivity to Electroweak Gauge Bosons
  - 13.1 Higgs Mechanism in Scalar Electrodynamics
  - 13.2 Higgs Mechanism in the Electroweak Theory
  - 13.3 Identification of the Electric Charge
  - **13.4 Accidental Custodial Symmetry**
  - 13.5 Variants of the Standard Model with Modified Gauge Symmetry
  - 13.6 Scalar Electrodynamics on the Lattice
  - 13.7  $SU(2)_L$  Gauge-Higgs Model on the Lattice
  - **13.8 Small Electroweak Unification**
  - 13.9 Electroweak Symmetry Breaking in an SU(3) Unified Theory

## **14 Gluons: From Confinement to Deconfinement**

14.1 Gluons in the Continuum and on the Lattice

- 14.2 Quark Confinement and the Wegner–Wilson Loop
- 14.3 Character Expansion and Group Integration
- 14.4 Strong Coupling Limit of Lattice Yang-Mills Theory
- 14.5 Asymptotic Freedom and Natural Continuum Limit
- 14.6 How Strong is the Strong Force?
- 14.7 Roughening Transition
- 14.8 Systematic Low-Energy Effective String Theory
- 14.9 Lüscher Term as a Casimir Effect
- 14.10 Cosmological Constant Problem on the String World-Sheet
- 14.11 **Gluon Confinement** and the Fredenhagen–Marcu Operator
- 14.12 Glueball Spectrum
- 14.13 Polyakov Loop and Center Symmetry
- 14.14 Deconfinement at High Temperatures
- 14.15 Exceptional Confinement and Deconfinement in G(2) Yang-Mills Theory

#### **15 One Generation of Leptons and Quarks**

- 15.1 Electron and Left-Handed Neutrino
- 15.2 CP and T Invariance of Gauge Interactions
- 15.3 Fixing the Lepton Weak Hypercharges
- 15.4 Triangle Gauge Anomalies in the Lepton Sector
- 15.5 Witten's Global SU(2)<sub>L</sub> Gauge Anomaly in the Lepton Sector
- 15.6 Up and Down Quarks
- 15.7 Anomaly Cancellation between Leptons and Quarks
- 15.8 Electric Charges of Quarks and Baryons
- 15.9 Anomaly Matching
- 15.10 Right-Handed Neutrinos
- 15.11 Lepton and Baryon Number Anomalies
- 15.12 Gauge Anomaly-Free Technicolor Model

#### **16 Fermion Masses**

- 16.1 Electron and Down Quark Masses
- 16.2 Up Quark Mass
- 16.3 Neutrino Mass from a Dimension-5 Operator
- 16.4 Mass Hierarchies of Fermions
- 16.5 Neutrino Mass Term and Reconsideration of CP
- 16.6 Lepton and Baryon Number Violation by Higher-Dimensional Operators
- 16.7 Charge Quantization, Fermion Masses, and Consistency with Gravity

16.8 Dirac and Majorana Masses from Right-Handed Neutrino Fields

- 16.9 Seesaw Mass-by-Mixing Mechanism
- 16.10 Right-Handed Neutrinos and Electric Charge Quantization
- 16.11 Lepton–Baryon Mixing for  $N_{\rm c} = 1$

#### **17 Several Generations and Flavor Physics of Quarks and Leptons**

- 17.1 Electroweak versus Mass Eigenstates
- 17.2 Generation-Specific Lepton Numbers and Lepton Universality
- 17.3 Cabibbo-Kobayashi–Maskawa Quark Mixing Matrix
- 17.4 Flavor-Changing Neutral Currents and the GIM Mechanism
- 17.5 CP Violation with Neutral Kaons and B-Mesons
- 17.6 Pontecorvo-Maki-Nakagawa-Sakata Lepton Mixing Matrix
- 17.7 Neutrino Oscillations
- **17.8 Overview of Fundamental Standard Model Parameters**
- 17.9 Low-Energy Theory Perspective on the Standard Model Physics

#### Part III: STRONG INTERACTION

#### **18 Quantum Chromodynamics**

- 18.1 Deconstructing the Standard Model
- 18.2 Asymptotic Freedom

18.3 Structure of Chiral Symmetry
18.4 Dynamical Realization of Chiral Symmetry
18.5 Lattice QCD
18.6 Ginsparg-Wilson Relation and Lüscher's Lattice Chiral Symmetry
18.7 Under-Appreciated Fermionic Hierarchy Problem
18.8 Domain Wall Fermions and a Fifth Dimension of Space-Time

# **19 Topology of Gauge Fields**

- 19.1 Adler-Bell-Jackiw Anomaly
- 19.2 Topological Charge
- 19.3 Topology of a Gauge Field on a Compact Manifold
- 19.4 SU(2) Instanton
- 19.5 *θ*-Vacuum States
- 19.6 Analogy with Energy Bands in a Periodic Crystal
- 19.7 Some Questions Related to  $\boldsymbol{\theta}$
- 19.8 Atiyah-Singer Index Theorem

19.9 Zero-Mode of the SU(2) Instanton 19.10 **Index Theorem on the Lattice** 

## **20** U(1)<sub>A</sub>-Problem

- 20.1 Nature of the Problem
- 20.2 QCD in the Large- $N_{\rm c}$  Limit
- 20.3 Witten–Veneziano Formula for the  $\eta'$ -Meson Mass
- 20.4 Topological Susceptibility from Lattice Gauge Theory

#### 21 Spectrum of Light Baryons and Mesons

- 21.1 Isospin Symmetry
- 21.2 Nucleon and  $\Delta\text{-lsobar}$
- 21.3 Anti-Quarks and Mesons
- 21.4 Strange Hadrons
- 21.5 Gell-Mann–Okubo Baryon Mass Formula

21.6 Meson Mixing 21.7 Hadron Spectrum from Lattice QCD 21.8 Hadrons for  $N_{\rm c}=5$ 

#### 22 Partons and Hard Processes

22.1 Electron-Positron Annihilation into Hadrons 22.2 *R*-Ratio as Evidence for  $N_c = 3$ 22.3 Deep-Inelastic Electron-Nucleon Scattering 22.4 Deep-Inelastic Neutrino-Nucleon Scattering 22.5 Sum Rules

## **23 Chiral Perturbation Theory**

- 23.1 Effective Theory for Pions, Kaons, and the  $\eta$ -Meson
- 23.2 Masses of Pseudo-Nambu–Goldstone Bosons
- 23.3 Low-Energy Effective Theory for Nambu–Goldstone Bosons and Photons

- 23.4 Electromagnetic Corrections to the Nambu–Goldstone Boson Masses
- 23.5 Effective Theory for Nucleons and Pions
- 23.6 QCD Contributions to the W- and Z-Boson Masses

## 24 Topology of Nambu–Goldstone Boson Fields

- 24.1 Skyrmions
- 24.2 Anomaly Matching for  $N_{\rm f}=2$
- 24.3 G-Parity and its Explicit Breaking
- 24.4 Electromagnetic Decay of the Neutral Pion
- 24.5 Evidence for  $N_{\rm c}=3$  from  $\pi^0 \to \gamma\gamma$  ?
- 24.6 Wess-Zumino-Novikov-Witten Term
- 24.7 Intrinsic Parity and Its Anomalous Breaking
- 24.8 Electromagnetic Interactions of Pions, Kaons, and  $\eta\text{-}\mathrm{Mesons}$
- 24.9 Electromagnetic Interactions of Nambu–Goldstone Bosons for  $N_{\rm f} \geq 3$

24.10 Can One See the Number of Colors?

24.11 Techni-Baryons, Techni-Skyrmions, and Topological Dark Matter

# Part IV: SELECTED TOPICS BEYOND THE STANDARD MODEL

## 25 Strong CP-Problem

25.1 Rotating  $\theta$  into the Mass Matrix

25.2  $\theta$ -Angle in Chiral Perturbation Theory

25.3  $\theta\text{-Angle}$  at Large  $N_{\rm c}$ 

25.4 Peccei–Quinn Symmetry

25.5 U(1)<sub>PQ</sub> Symmetry Breaking and the Axion

25.6 Astrophysical and Cosmological Axion Effects

25.7 Elimination of the Weak  $SU(2)_L$  Vacuum-Angle

25.8 Is there an Electromagnetic CP-Problem?

#### **26 Grand Unified Theories**

- 26.1 Minimal SU(5) Model
- 26.2 Fermion Multiplets
- 26.3 Lepton-Quark Transitions and Proton Decay
- 26.4 Baryon Asymmetry in the Universe
- 26.5 Thermal Baryon Number Violation in the Standard Model
- 26.6 Topological Excitations as Cosmic Relics
- 26.7 't Hooft–Polyakov Monopole and Callan–Rubakov Effect
- 26.8 Dirac–Schwinger–Zwanziger Dyon Quantization Condition
- 26.9 Julia-Zee Dyon and Witten Effect
- 26.10 Fermion Masses and the Hierarchy Problem
- 26.11 Spin(10) Structure
- 26.12 Neutrino Masses in the Spin(10) GUT
- 26.13 Small Unification with SU(3), G(2), Spin(6), or Spin(7)
- 26.14 Grand or not so Grand Unification?

## FINALE

#### A Highlights in the Development of Particle Physics

A.1 **Development** of Experimental High-Energy Physics

A.2 Development of Quantum Field Theory and the Standard Model

#### **B Units, Hierarchies, and Fundamental Parameters**

B.1 Man-Made versus Natural UnitsB.2 Energy Scales and Particle Masses

C Structure of Minkowski Space-Time

C.1 Lorentz Transformations

C.2 Gradient as a 4-Vector and d'Alembert Operator

## **D** Relativistic Formulation of Classical Electrodynamics

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References(15 pages)Author Index(299 authors)

Subject Index (7 pages, structure with sub-subjects)

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