Uncovering Quantum Field Theory and the Standard Model

From Fundamental Concepts to Dynamical Mechanisms

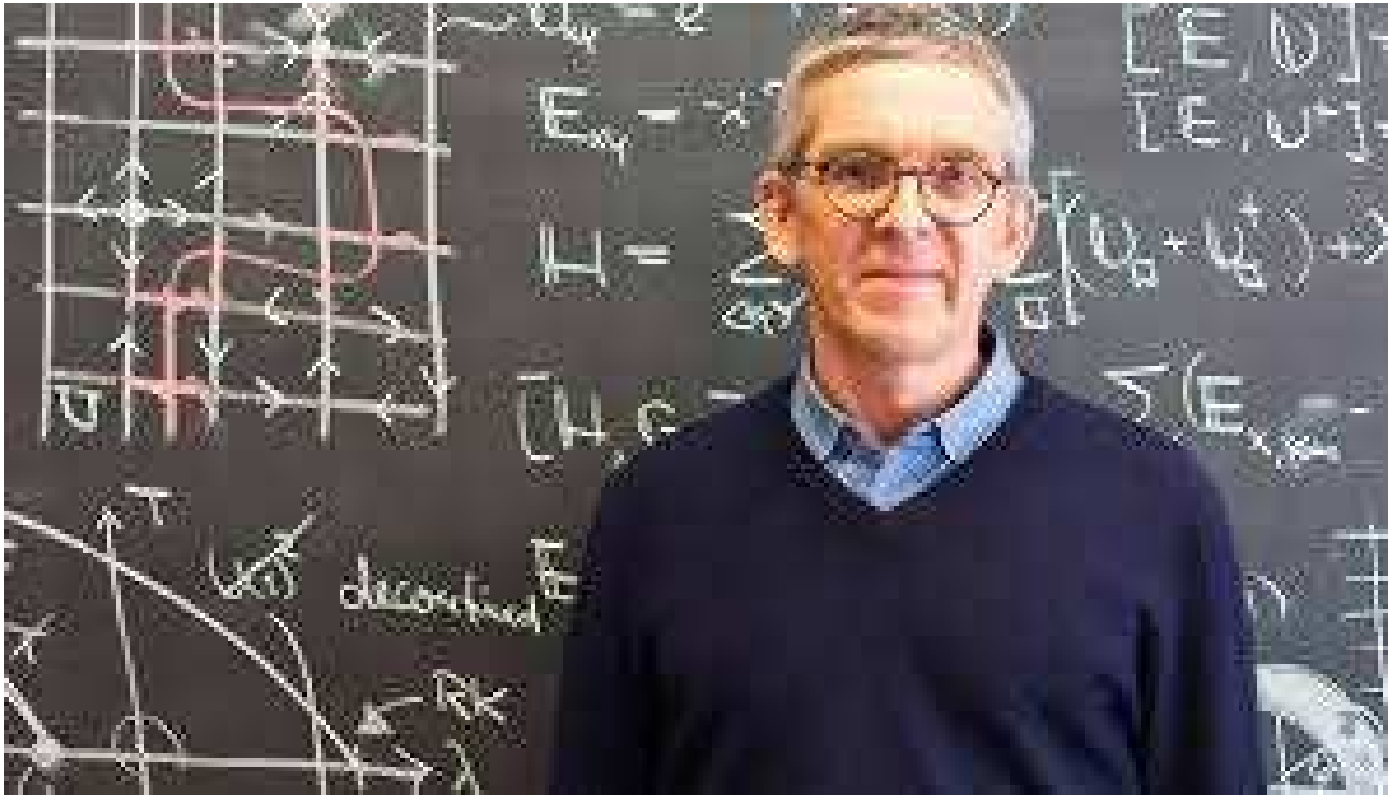
with Uwe-Jens Wiese, University of Bern, Switzerland

First textbook about particles and fields with an author working in Mexico (second in Latin America)
[H. Nastase (Romanian in São Paulo): “Introduction to QFT”, 2019]

Respect for the Standard Model: Most precise scientific theory!

Origin: Lecture notes by Uwe-Jens from Aachen, 1992

Start for me in Berlin: visit by Uwe-Jens in September 2003, walk near the Brandenburg Gate, agreement to convert it into a joint textbook project.



Uwe-Jens Wiese in his office in Bern

Sporadic progress during > 10 years, in 2009 I moved to Mexico

Sabbatical semester in Bern, 2016/7 (bad luck: stolen laptop).

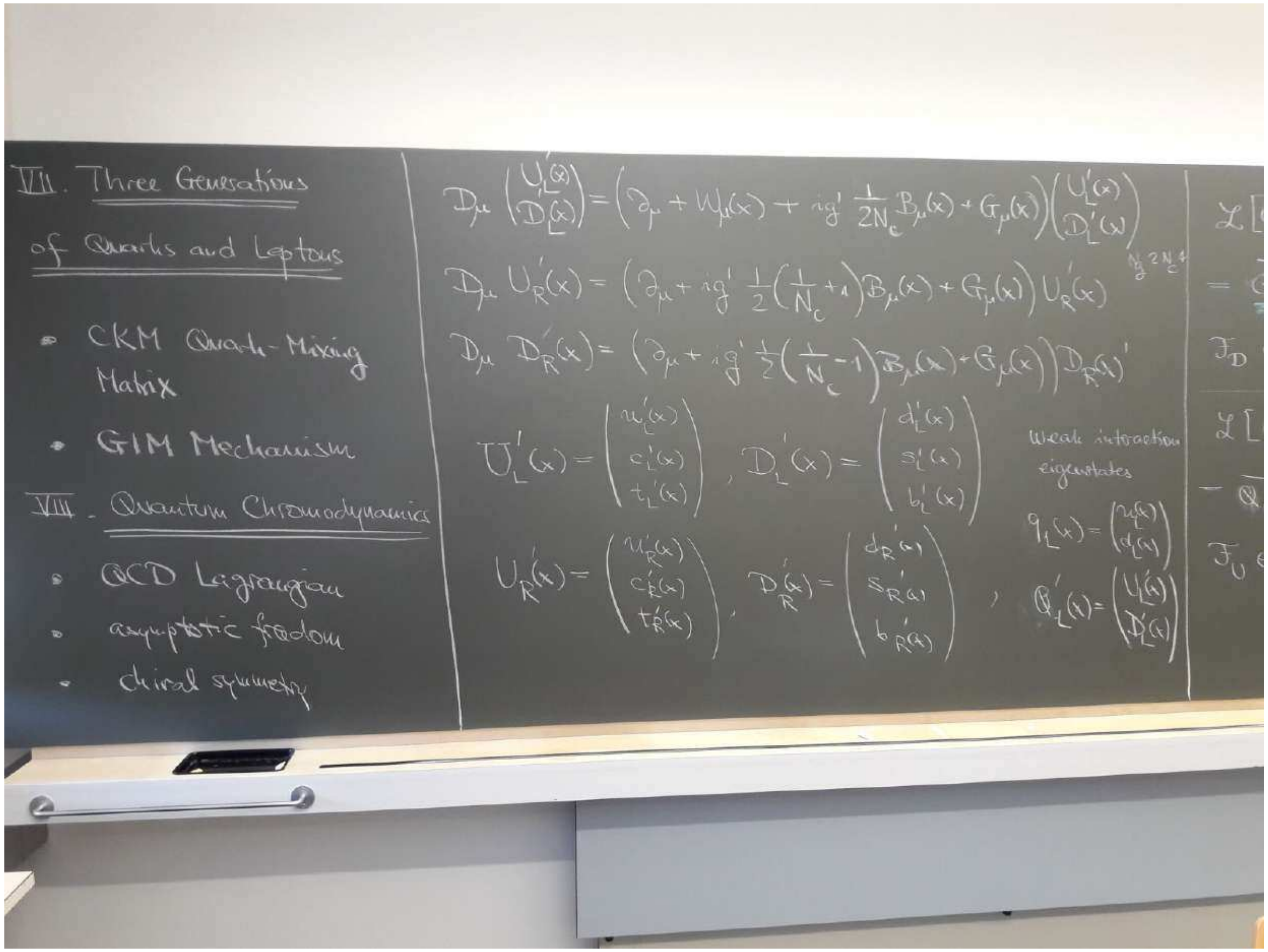
Book proposal sent to *Cambridge University Press*,
5 chapters and one appendix. Approved by 6 referees.
Contract: Maximum of 600 pages, deadline: July 1, 2028.

Another sabbatical semester in Bern, summer 2018

Progress, but distraction with new research projects,
termination still behind the horizon.

More theory and concepts than phenomenology

Focus on non-perturbative perspective $\neq$ previous literature



Discussion about fermion generations (Chapter 17).

Upon request, we included gauge fixing, but not SUSY.

Pandemics:

Frequent video calls

Uwe-Jens: Intense work, $e.g.$ on canonical fermion fields.

Helpful comments on specific chapters by *Oliver Bär*, Debasish Banerjee, *Detlev Buchholz, Wilfried Buchmüller,* Klaus Fredenhagen, *Urs Gerber, Carlo Giunti,* Kieran Holland, Gurtej Kanwar, *Martin Lüscher,* Alessandro Mariani, *Colin Morningstar, Mike Peardon,* Michele Pepe, João Pinto Barros, *Lilian Prado*, Simona Procacci, Christopher Smith, *Rainer Sommer,* Youssef Tammam, Christiane Tretter, *Christof Wetterich, Edward Witten.*

Early version: conceptual introduction of 14 pages. Martin Lüscher: reader who does not know *e.g.* gauge theory gets lost very soon. We only assume previous knowledge of Quantum Mechanics and Special Relativity.

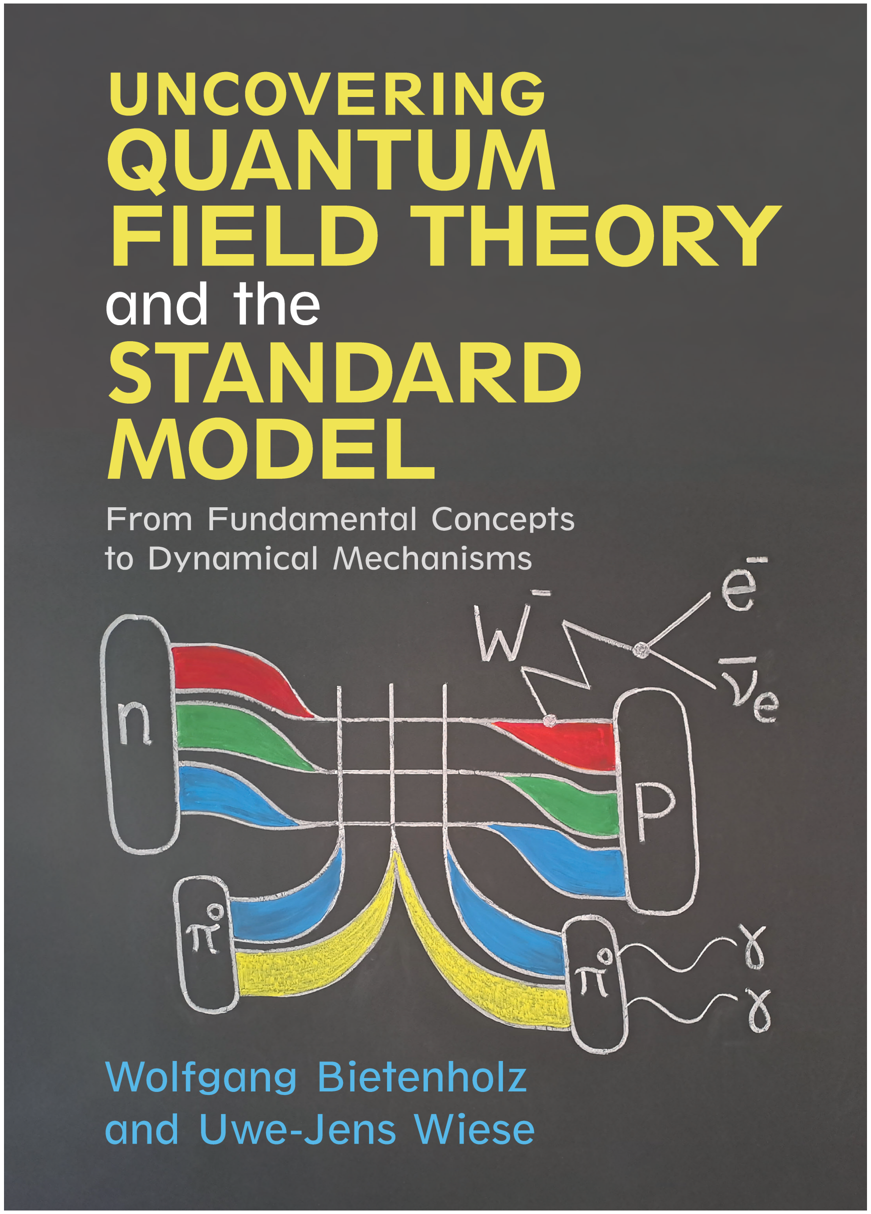
New structure: **Ouverture, Intermezzo, Finale**
26 Chapters is 4 Parts: Link to "Le quattro stagioni" by Antonio Vivaldi [QFT, SM, Strong Interaction, Beyond SM]. $\sim$ 3 semesters

Finally submitted in December 2022, "takes on average 9 months", hope for it to be faster, since we used the Cambridge style file throughout.

No way: summer 2023: edited pdf with O(10^4) modifications (lots of commas, some errors, a few corrections: "Klein-Gordon" vs. "Clebsch-Gord**a**n"), convergence after some cycles of revision.

We thank *Sunantha Ramamoorthy*, competent and cooperative, but she could not avoid a strange last-minute change of fonts (exponents, indices etc. displaced). Reduction from 772 to 732 pages.

Cover page: chalk drawing of β- and π^0-decay, by *Nadiia Vlasii*



Ranking of the most frequent names in the authors index:

10 entries: Callen, Lüscher, 't Hooft, Wilczek

9 entries: Weinberg

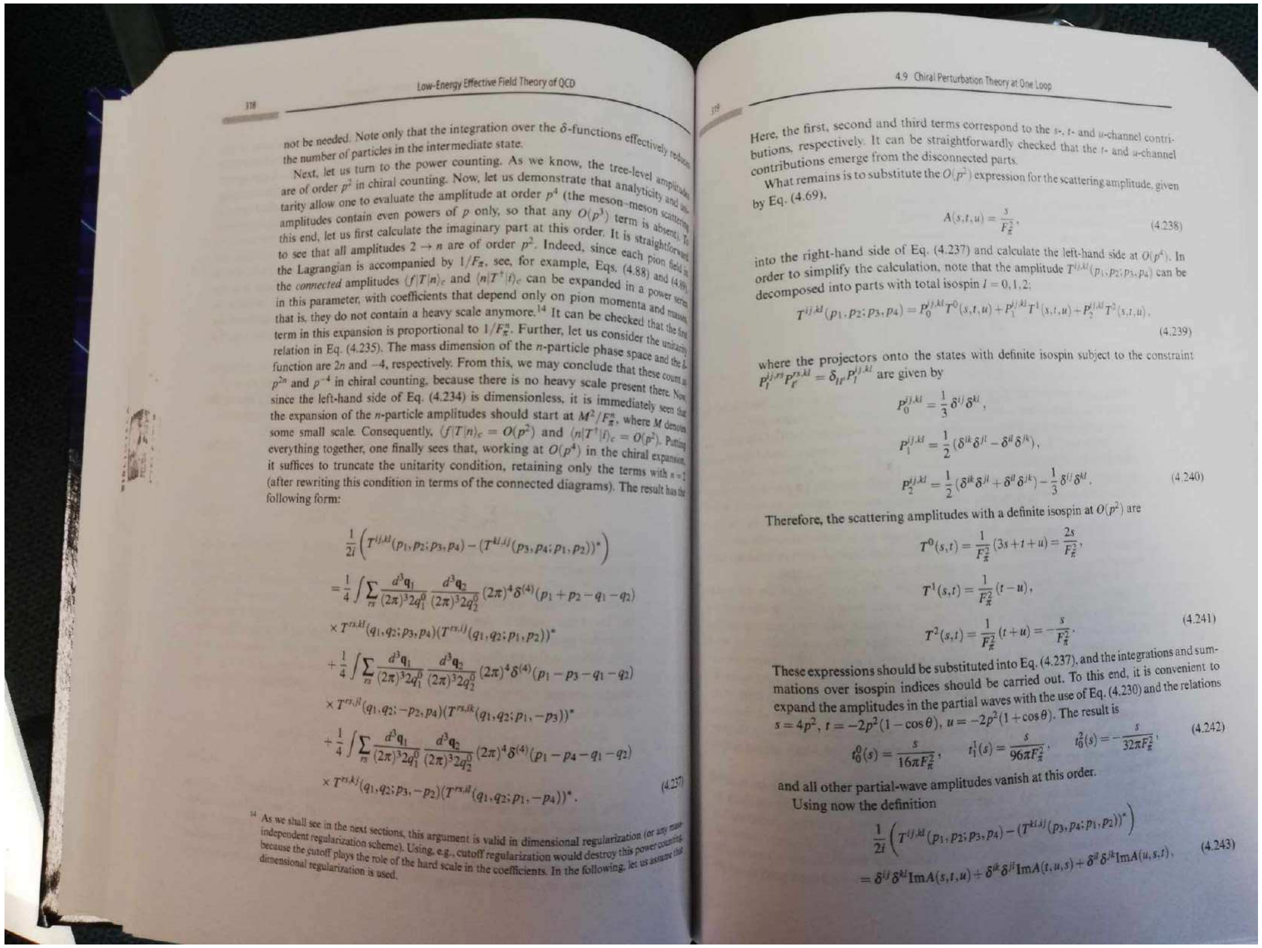
7 entries: Einstein, Feynman, Wilson
(Referee: "Wilson could have written such a book, but . . . ")

6 entries: Gross, Leutwyler, Witten

5 entries: Hasenfratz, Pauli, Schwinger

4 entries: Bogoliubov, Coleman, Dashen, Dirac, Gasser, Gell-Mann, Glashow, Niedermayer, Politzer, Polyakov, Symanzik, Wess, Yang, Zimmermann, Zinn-Justin, Zumino

Meißner-Rusetsky problem:



not be needed. Note only that the integration over the δ-functions effectively reduces the number of particles in the intermediate state.

Next, let us turn to the power counting. As we know, the tree-level amplitudes are of order p^2 in chiral counting. Now, let us demonstrate that analyticity and unitarity allow one to evaluate the amplitude at order p^4 (the meson–meson scattering amplitudes contain even powers of p only, so that any $O(p^3)$ term is absent). To this end, let us first calculate the imaginary part at this order. It is straightforward to see that all amplitudes $2 \to n$ are of order p^2. Indeed, since each pion field in the Lagrangian is accompanied by $1/F_\pi$, see, for example, Eqs. (4.88) and (4.89), the *connected* amplitudes $\langle f|T|n\rangle_c$ and $\langle n|T^\dagger|i\rangle_c$ can be expanded in a power series in this parameter, with coefficients that depend only on pion momenta and masses, that is, they do not contain a heavy scale anymore.[14] It can be checked that the first term in this expansion is proportional to $1/F_\pi^n$. Further, let us consider the unitarity relation in Eq. (4.235). The mass dimension of the n-particle phase space and the δ-function are $2n$ and -4, respectively. From this, we may conclude that these count as p^{2n} and p^{-4} in chiral counting, because there is no heavy scale present there. Now, since the left-hand side of Eq. (4.234) is dimensionless, it is immediately seen that the expansion of the n-particle amplitudes should start at M^2/F_π^n, where M denotes some small scale. Consequently, $\langle f|T|n\rangle_c = O(p^2)$ and $\langle n|T^\dagger|i\rangle_c = O(p^2)$. Putting everything together, one finally sees that, working at $O(p^4)$ in the chiral expansion, it suffices to truncate the unitarity condition, retaining only the terms with $n=2$ (after rewriting this condition in terms of the connected diagrams). The result has the following form:

$$\begin{aligned}
&\frac{1}{2i}\Big(T^{ij,kl}(p_1,p_2;p_3,p_4) - (T^{kl,ij}(p_3,p_4;p_1,p_2))^*\Big)\\
&= \frac{1}{4}\int\sum_{rs}\frac{d^3\mathbf{q}_1}{(2\pi)^3 2q_1^0}\frac{d^3\mathbf{q}_2}{(2\pi)^3 2q_2^0}(2\pi)^4\delta^{(4)}(p_1+p_2-q_1-q_2)\\
&\times T^{rs,kl}(q_1,q_2;p_3,p_4)(T^{rs,ij}(q_1,q_2;p_1,p_2))^*\\
&+\frac{1}{4}\int\sum_{rs}\frac{d^3\mathbf{q}_1}{(2\pi)^3 2q_1^0}\frac{d^3\mathbf{q}_2}{(2\pi)^3 2q_2^0}(2\pi)^4\delta^{(4)}(p_1-p_3-q_1-q_2)\\
&\times T^{rs,jl}(q_1,q_2;-p_2,p_4)(T^{rs,ik}(q_1,q_2;p_1,-p_3))^*\\
&+\frac{1}{4}\int\sum_{rs}\frac{d^3\mathbf{q}_1}{(2\pi)^3 2q_1^0}\frac{d^3\mathbf{q}_2}{(2\pi)^3 2q_2^0}(2\pi)^4\delta^{(4)}(p_1-p_4-q_1-q_2)\\
&\times T^{rs,kj}(q_1,q_2;p_3,-p_2)(T^{rs,il}(q_1,q_2;p_1,-p_4))^*.
\end{aligned} \tag{4.237}$$

[14] As we shall see in the next sections, this argument is valid in dimensional regularization (or any mass-independent regularization scheme). Using, e.g., cutoff regularization would destroy this power counting, because the cutoff plays the role of the hard scale in the coefficients. In the following, let us assume that dimensional regularization is used.

Here, the first, second and third terms correspond to the s-, t- and u-channel contributions, respectively. It can be straightforwardly checked that the t- and u-channel contributions emerge from the disconnected parts.

What remains is to substitute the $O(p^2)$ expression for the scattering amplitude, given by Eq. (4.69),

$$A(s,t,u) = \frac{s}{F_\pi^2}, \tag{4.238}$$

into the right-hand side of Eq. (4.237) and calculate the left-hand side at $O(p^4)$. In order to simplify the calculation, note that the amplitude $T^{ij,kl}(p_1,p_2;p_3,p_4)$ can be decomposed into parts with total isospin $I=0,1,2$:

$$T^{ij,kl}(p_1,p_2;p_3,p_4) = P_0^{ij,kl}T^0(s,t,u) + P_1^{ij,kl}T^1(s,t,u) + P_2^{ij,kl}T^2(s,t,u), \tag{4.239}$$

where the projectors onto the states with definite isospin subject to the constraint $P_I^{ij,rs}P_{I'}^{rs,kl} = \delta_{II'}P_I^{ij,kl}$ are given by

$$\begin{aligned}
P_0^{ij,kl} &= \frac{1}{3}\delta^{ij}\delta^{kl},\\
P_1^{ij,kl} &= \frac{1}{2}(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}),\\
P_2^{ij,kl} &= \frac{1}{2}(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}) - \frac{1}{3}\delta^{ij}\delta^{kl}.
\end{aligned} \tag{4.240}$$

Therefore, the scattering amplitudes with a definite isospin at $O(p^2)$ are

$$\begin{aligned}
T^0(s,t) &= \frac{1}{F_\pi^2}(3s+t+u) = \frac{2s}{F_\pi^2},\\
T^1(s,t) &= \frac{1}{F_\pi^2}(t-u),\\
T^2(s,t) &= \frac{1}{F_\pi^2}(t+u) = -\frac{s}{F_\pi^2}.
\end{aligned} \tag{4.241}$$

These expressions should be substituted into Eq. (4.237), and the integrations and summations over isospin indices should be carried out. To this end, it is convenient to expand the amplitudes in the partial waves with the use of Eq. (4.230) and the relations $s=4p^2$, $t=-2p^2(1-\cos\theta)$, $u=-2p^2(1+\cos\theta)$. The result is

$$t_0^0(s) = \frac{s}{16\pi F_\pi^2}, \qquad t_1^1(s) = \frac{s}{96\pi F_\pi^2}, \qquad t_0^2(s) = -\frac{s}{32\pi F_\pi^2}, \tag{4.242}$$

and all other partial-wave amplitudes vanish at this order.

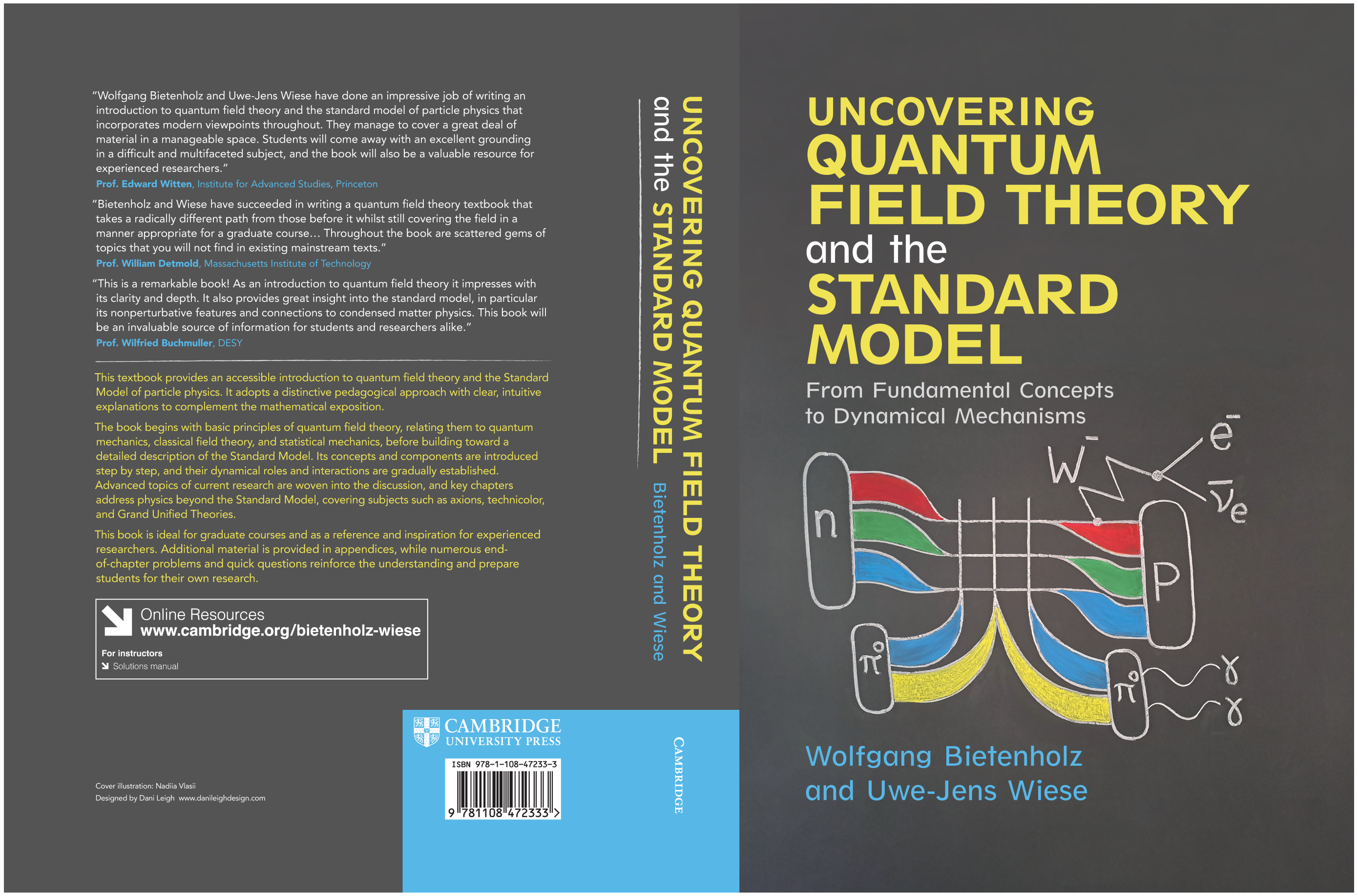
Using now the definition

$$\begin{aligned}
&\frac{1}{2i}\Big(T^{ij,kl}(p_1,p_2;p_3,p_4) - (T^{kl,ij}(p_3,p_4;p_1,p_2))^*\Big)\\
&= \delta^{ij}\delta^{kl}\mathrm{Im}A(s,t,u) + \delta^{ik}\delta^{jl}\mathrm{Im}A(t,u,s) + \delta^{il}\delta^{jk}\mathrm{Im}A(u,s,t),
\end{aligned} \tag{4.243}$$

U.-G. Meißner, A. Rusetsky, “Effective Field Theories”, Cambridge University Press, 2022.

Christian Schubert: Must be related to *Fermat’s Last Theorem*.

Confusion about the endorsers, at last 5 very positive statements



Recommendations by Edward Witten (Princeton), William Detmold (MIT), Wilfried Buchmüller (DESY), Poul Damgaard (Niels Bohr Inst.), Tereza Mendes (U. São Paulo)

A gluon string could hold a mass of 20,000 kg, like a steel cable with a diameter of some cm, but $\sim 10^{13}$ times thinner.

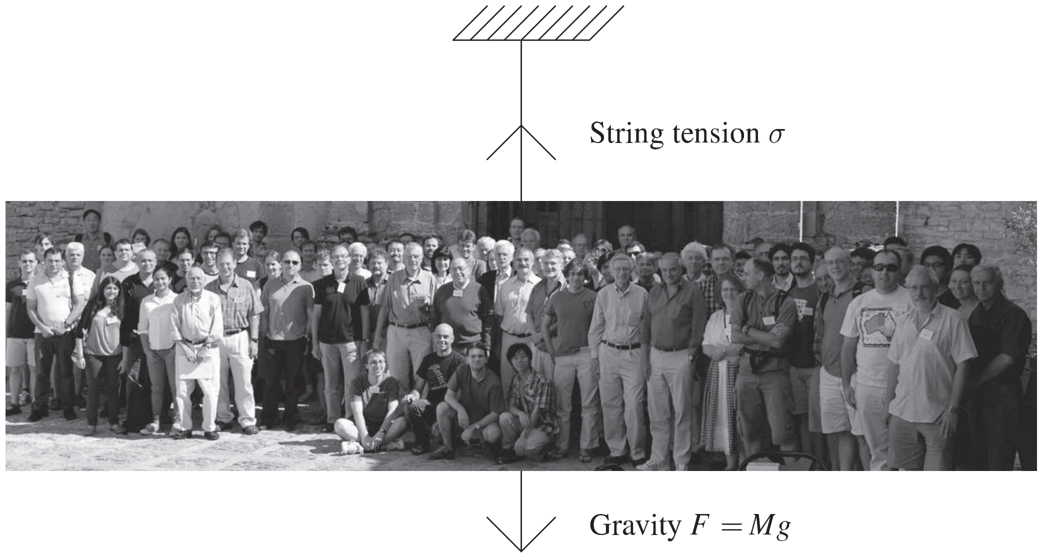


Fig. 14.4 A single Yang–Mills string is strong enough to support more than 100 people. A "Yang–Mills elevator" was installed by one of the authors as a Gedankenexperiment at the Erice workshop "From Quarks and Gluons to Hadrons and Nuclei" in 2011, in order to "elevate" our intuitive understanding for the strength of the strong force (Wiese, 2012).

> **Quick Question 14.6.1 Overloaded Yang–Mills elevator**
> What happens when about 1000 (adult) people enter the Yang–Mills elevator?

14.7 Roughening Transition

The string tension has been computed to high orders of the strong coupling expansion. The resulting contributions are associated with plaquette surfaces that are deformations of the

2024: announced by Cambridge University Press for January, then February ... inductive until June.

Summer 2024: editorial paralyzed by a cyber attack.
Sad news: Sunantha died, substituted by Shanthy Jaganathan.

Promised for November, then December, finally published on January 2, 2025.

Happy ending, once in a lifetime event!

Last chapter about Grand Unified Theories (GUT):

Ende GUT, alles gut!

Up to now (in < 5 months) 638 times sold

Internet: $\langle$ selling of acad. books $\rangle \approx 500 \ldots 1000$

Part I: QUANTUM FIELD THEORY

Ouverture: Concepts of Quantum Field Theory

Point Particles versus Fields at the Classical Level
Particles versus Waves in Quantum Theory
Classical and Quantum Gauge Fields
Ultraviolet Divergences, Regularization, and Renormalization
Euclidean Quantum Field Theory versus Classical Statistical Mechanics

1 Basics of Quantum Field Theory

1.1 From Point Particle Mechanics to Classical Field Theory
1.2 Quantum Mechanical Path Integral
1.3 Path Integral in Euclidean Time
1.4 Spin Models in Classical Statistical Mechanics
1.5 Quantum Mechanics versus Classical Statistical Mechanics

7 Charged States in Scalar Quantum Electrodynamics

7.1 Complex Scalar Field with Global U(1) Symmetry
7.2 Scalar Quantum Electrodynamics
7.3 Charged Particles as **Infraparticles**
7.4 Superselection Sectors
7.5 Charged Particles in a Periodic Volume
7.6 C-periodic Boundary Conditions

8 Canonical Quantization of Free Weyl, Dirac, and Majorana Fermions

8.1 Massless **Weyl Fermions**
8.2 Momentum, Angular Momentum, and Helicity of Weyl Fermions
8.3 Fermion Number, Parity, and Charge Conjugation
8.4 Cosmic Background Radiation of Neutrinos
8.5 Massive Dirac Fermions
8.6 Massive Majorana Fermions

12 Spontaneous Breakdown of Global Symmetries: From Condensed Matter to Higgs Bosons

14.2 Quark Confinement and the Wegner–Wilson Loop
14.3 Character Expansion and Group Integration
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14.5 Asymptotic Freedom and Natural Continuum Limit
14.6 **How Strong is the Strong Force?**
14.7 Roughening Transition
14.8 Systematic Low-Energy Effective String Theory
14.9 **Lüscher Term as a Casimir Effect**
14.10 Cosmological Constant Problem on the String World-Sheet
14.11 **Gluon Confinement** and the Fredenhagen–Marcu Operator
14.12 **Glueball Spectrum**
14.13 Polyakov Loop and Center Symmetry
14.14 Deconfinement at High Temperatures
14.15 Exceptional Confinement and Deconfinement in
G(2) Yang-Mills Theory

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16 Fermion Masses

17 Several Generations and Flavor Physics of Quarks and Leptons

17.1 Electroweak versus Mass Eigenstates
17.2 Generation-Specific Lepton Numbers and Lepton Universality
17.3 **Cabibbo-Kobayashi–Maskawa Quark Mixing Matrix**
17.4 Flavor-Changing Neutral Currents and the GIM Mechanism
17.5 CP Violation with Neutral Kaons and B-Mesons
17.6 **Pontecorvo-Maki-Nakagawa-Sakata Lepton Mixing Matrix**
17.7 Neutrino Oscillations
17.8 **Overview of Fundamental Standard Model Parameters**
17.9 Low-Energy Theory Perspective on the Standard Model Physics

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18.2 Asymptotic Freedom

18.3 Structure of Chiral Symmetry
18.4 Dynamical Realization of Chiral Symmetry
18.5 Lattice QCD
18.6 Ginsparg-Wilson Relation and **Lüscher's Lattice Chiral Symmetry**
18.7 Under-Appreciated **Fermionic Hierarchy Problem**
18.8 Domain Wall Fermions and a Fifth Dimension of Space-Time

19 Topology of Gauge Fields

19.1 Adler-Bell-Jackiw Anomaly
19.2 Topological Charge
19.3 Topology of a Gauge Field on a Compact Manifold
19.4 **SU(2) Instanton**
19.5 **θ-Vacuum States**
19.6 Analogy with **Energy Bands in a Periodic Crystal**
19.7 Some Questions Related to θ
19.8 **Atiyah-Singer Index Theorem**

Part IV: SELECTED TOPICS BEYOND THE STANDARD MODEL

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26 Grand Unified Theories

FINALE

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A.1 **Development** of Experimental High-Energy Physics
A.2 Development of Quantum Field Theory and the Standard Model

B Units, Hierarchies, and Fundamental Parameters

B.1 Man-Made versus Natural **Units**
B.2 Energy **Scales** and Particle Masses

C Structure of Minkowski Space-Time

C.1 Lorentz Transformations
C.2 Gradient as a 4-Vector and d'Alembert Operator

D Relativistic Formulation of Classical Electrodynamics

D.1 Current and Vector Potential
D.2 Field Strength Tensor
D.3 Maxwell Equations
D.4 Space-Time Scalars from Field Strength Tensors
D.5 Transformation of Electromagnetic Fields
D.6 Action and Euler–Lagrange Equation
D.7 Energy-Momentum Tensor

E From the Galilei to the Poincaré Algebra

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E.2 Poincaré and Lorentz Algebras

F Lie Groups and Lie Algebras

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F.2 Simple and Semi-Simple Lie Algebras
F.3 Representations of Lie Algebras
F.4 Lie Algebra so(3) and its Representations
F.5 Unitary Group SU(2) versus Orthogonal Group SO(3)
F.6 Unitary Group SU(n) and its Algebra su(n)
F.7 Group SU(3) and its Algebra su(3)
F.8 Permutation Group S_N
F.9 **su(n) Representations and Young Tableaux**
F.10 Tensor Product of su(n) Representations
F.11 Tensor Product of $\{3\}$ and $\{\bar{3}\}$ in su(3)
F.12 Orthogonal Group SO(n) and its Algebra so(n)
F.13 Symplectic Group Sp(n) and its Algebra sp(n)
F.14 Exceptional Group G(2) and its Algebra g(2)
F.15 Graphical Method for Tensor Product Reduction

G Homotopy Groups and Topology

G.1 Maps from S^d to S^n
G.2 Topological Charge in 2-d Abelian Gauge Theory
G.3 Homotopy Groups of Lie Group Manifolds

H Monte Carlo Method

H.1 Concept of a **Markov Chain**
H.2 **Detailed Balance**
H.3 **Ergodicity** and its Implications
H.4 Convergence to the Stationary Distribution
H.5 Metropolis Algorithm
H.6 Error Analysis
H.7 **Critical Slowing Down**
H.8 Supercritical Slowing Down and **Sign Problems**
H.9 Complexity Classes and the Severity of Sign Problems

“Wolfgang Bietenholz and Uwe-Jens Wiese have done an impressive job of writing an introduction to quantum field theory and the standard model of particle physics that incorporates modern viewpoints throughout. They manage to cover a great deal of material in a manageable space. Students will come away with an excellent grounding in a difficult and multifaceted subject, and the book will also be a valuable resource for experienced researchers.”

Prof. Edward Witten, Institute for Advanced Studies, Princeton

“Bietenholz and Wiese have succeeded in writing a quantum field theory textbook that takes a radically different path from those before it whilst still covering the field in a manner appropriate for a graduate course. The presentation is clear and is interestingly modulated by the authors’ backgrounds in lattice approaches to field theory, explaining difficult concepts in concise and interesting ways. Throughout the book are scattered gems of topics that you will not find in existing mainstream texts, where the authors eloquently delve into these selected topics in deeper detail.”

Prof. William Detmold, Massachusetts Institute of Technology

“This is a remarkable book! As an introduction to quantum field theory it impresses with its clarity and depth. It also provides great insight into the standard model, in particular its nonperturbative features and connections to condensed matter physics. This book will be an invaluable source of information for students and researchers alike.”

Prof. Wilfried Buchmüller, DESY and University of Hamburg

“This is a marvelous book that manages to synthesize the developments over more than fifty years in quantum field theory, statistical mechanics, the Standard Model of particle physics, critical phenomena, and the renormalization group. The authors have a deep understanding of the subject and manage to present the material with explanations and insight rather than heavy use of equations. It serves as an excellent complete textbook for graduate courses but also as a reference book for researchers in the field.”

Prof. Poul H. Damgaard, Niels Bohr Institute

“A modern and empowering text on quantum field theory. The book is very successful in engaging its readers right from the start, with the main relevant topics covered in a solid and motivating way, highlighting open questions, and providing interesting historical remarks and exercises. The authors’ approach is novel and stimulating and the book will be very valuable for students.”

Prof. Tereza Mendes, University of São Paulo