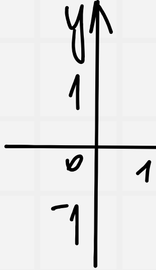
SUSY TARASCA @ THE BEACH

Alfredo Raya

IFM-UMSNH



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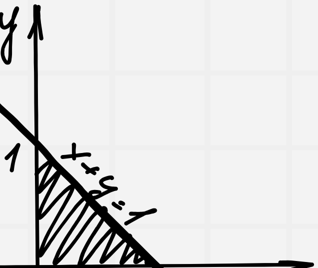


SUSY!

At the interface of High Energy and Condensed Matter Physics

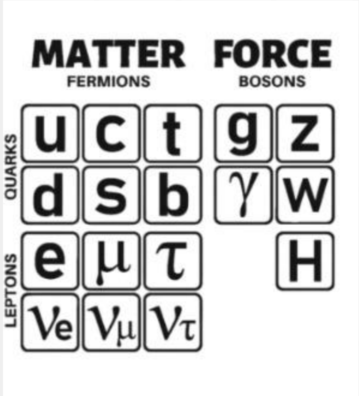
01

Fundamentals

SUSY and SUSY-QM

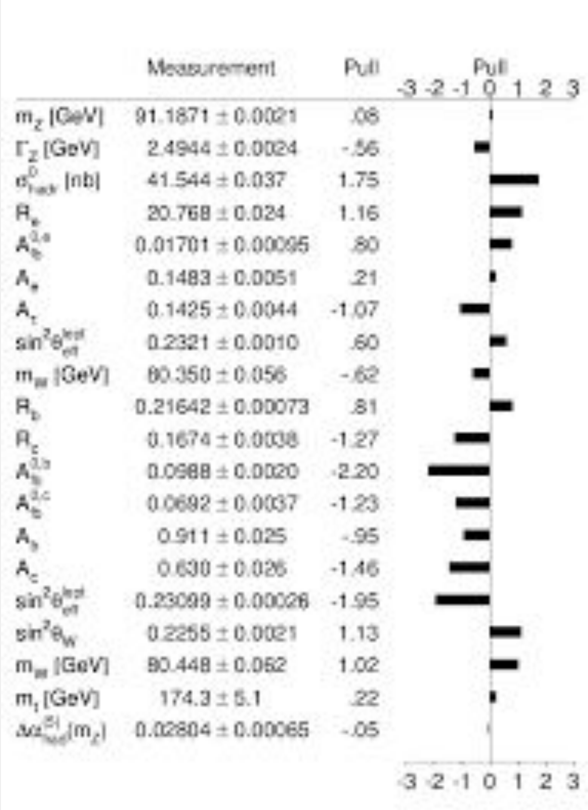
Standard Model of Particle Physics

Most successful theory. Many free parameters.



fdy

Great success with many free parameters

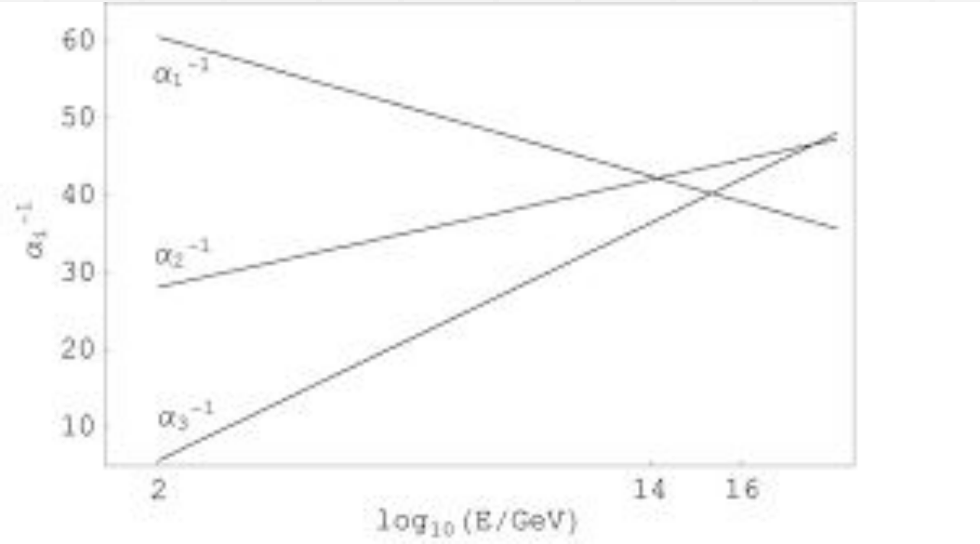


	Measurement	Pull
m_Z [GeV]	91.1871 ± 0.0021	.08
Γ_Z [GeV]	2.4944 ± 0.0024	-.56
σ^0_{hadr} [nb]	41.544 ± 0.037	1.75
R_e	20.768 ± 0.024	1.16
$A^{0,e}_{fb}$	0.01701 ± 0.00095	.80
A_e	0.1483 ± 0.0051	.21
A_τ	0.1425 ± 0.0044	-1.07
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.60
m_W [GeV]	80.350 ± 0.056	-.62
R_b	0.21642 ± 0.00073	.81
R_c	0.1674 ± 0.0038	-1.27
$A^{0,b}_{fb}$	0.0988 ± 0.0020	-2.20
$A^{0,c}_{fb}$	0.0692 ± 0.0037	-1.23
A_b	0.911 ± 0.025	-.95
A_c	0.630 ± 0.026	-1.46
$\sin^2\theta^{lept}_{eff}$	0.23099 ± 0.00026	-1.95
$\sin^2\theta_W$	0.2255 ± 0.0021	1.13
m_W [GeV]	80.448 ± 0.062	1.02
m_t [GeV]	174.3 ± 5.1	.22
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.05

$5 = 2$

fdy

Grand Unification



Unification of the fundamental interactions

- Almost the same strength
- Pairwise unification
- Can there be exact unification?

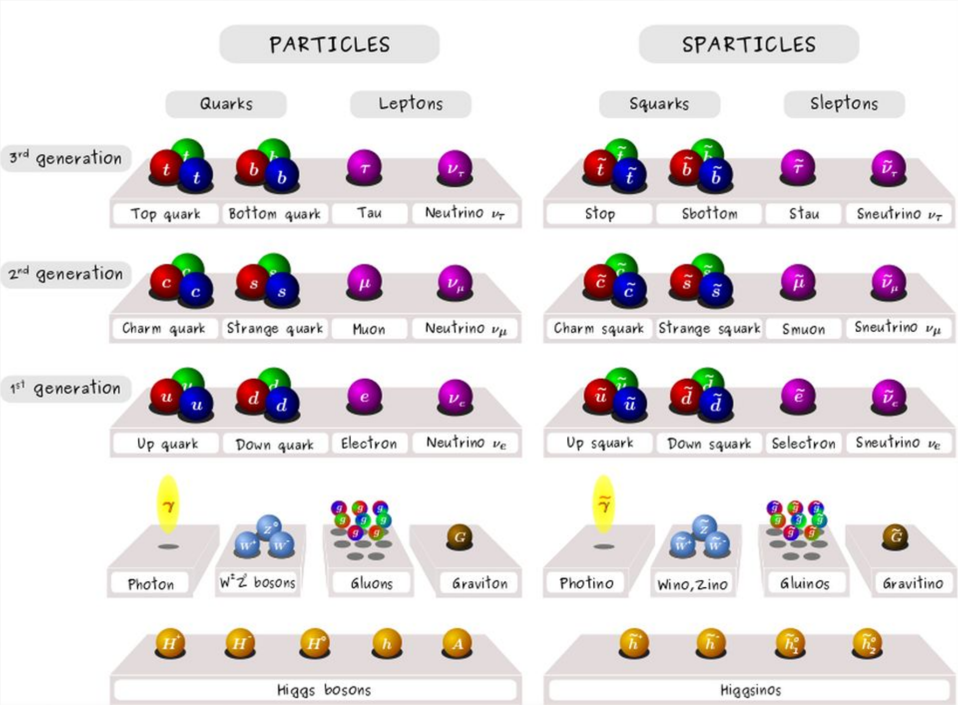
SUSY comes to rescue

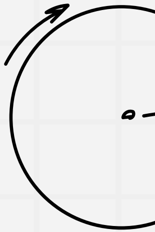
5 = 2

fdy

Minimal Supersymmetric Standard Model

$$Q^A_\alpha |B\rangle = |F\rangle$$
$$Q^A_\alpha |F\rangle = |B\rangle$$





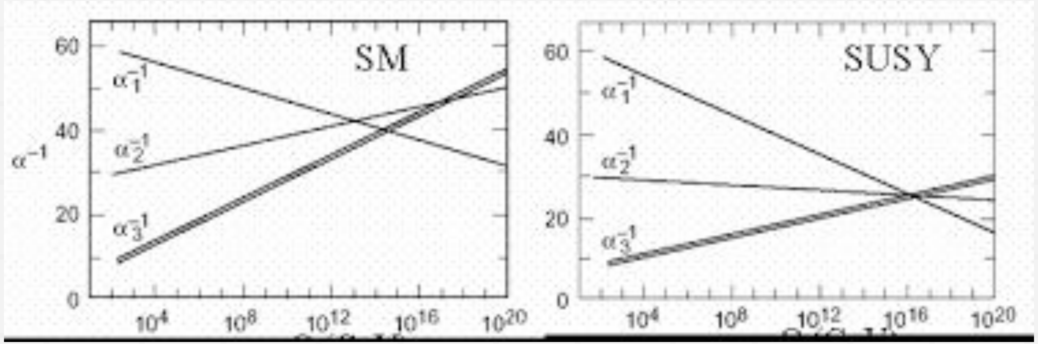
5 = 2

$\int f\,dy$

Minimal Supersymmetric Standard Model

$$
\begin{aligned}
Q^A_\alpha |B\rangle &= |F\rangle \\
Q^A_\alpha |F\rangle &= |B\rangle
\end{aligned}
$$

NoetherSym



$S = 2$

SUSY is broken

No sparticle has been observed

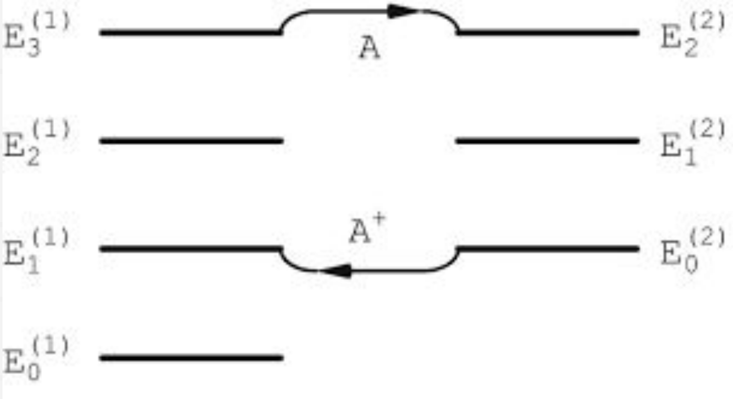
- How is SUSY broken?
- SUSY-QM
- Beyond the MSSM

From high-energy physics to other realizations

SUSY-QM

(0+1)-QFT

- 2 Schrödinger Hamiltonians
- Quasi-isospectrality



02

Dirac Theory

Planar Fermions

Relativistic Wave Equation

Trying to unify QM and SR, the Klein Gordon equation emerged as the first attempt

- Negative Energy States
- Probabilistic Interpretation of the wave function

Need a better idea!

$$(\partial_\mu \partial^\mu + m^2)\phi = 0$$

$$\rho \sim \phi^* \partial_t \phi - \phi \partial_t \phi^*$$

Relativistic Wave Equation

Dirac:

- Need a Hamiltonian linear in space derivatives
- Its squared is equivalent to the KG equation
- Has a neat probabilistic interpretation
- No negative energy states

Oops!

$$H_D = \vec{\alpha} \cdot \vec{p} + \beta m$$

α_i, β matrices

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}, \qquad |\psi|^2 \geq 0$$

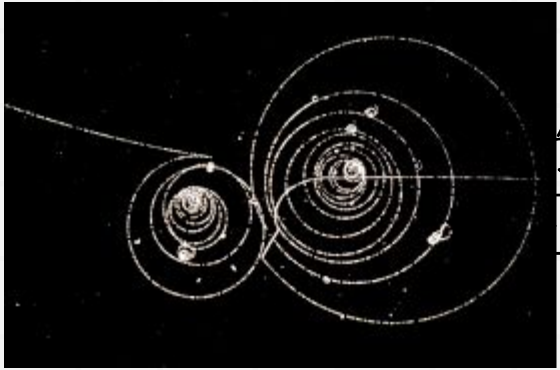
$$E = m_0, m_0, -m_0, -m_0$$

Relativistic Wave Equation

Dirac:

- QM+SR = antimatter
- More intelligent equation that its creator

$$\psi^{>}_{\uparrow\downarrow}, \qquad \psi^{<}_{\uparrow\downarrow},$$



Relativistic Wave Equation in 2D

$$H_D = \sigma_x p_x + \sigma_y p_y + \sigma_3 m$$

Dirac:

- 2x2 Dirac Matrices
- 2 Irreducible inequivalent representations
- 1 Spin orientation in each
- Mass breaks parity
- No chiral symmetry $\gamma_5 = \pm iI$

Relativistic Wave Equation in 2D

Dirac: $$H_D = \alpha_x p_x + \alpha_y p_y + \beta m + \alpha_x \alpha_y m_0$$

- 4x4 Dirac Matrices
- 1 reducible representation
- 2 Spin orientations, 2 positive to negative energy solution
- Dirac mass preserves parity
- 2 chiral transformations, $\gamma_3 \gamma_5$
- New mass term $m_0 \frac{1}{2} \gamma_3 \gamma_5$

Relativistic Wave Equation in 2D

Graphinos

- 4x4 Dirac Matrices
- Uniform magnetic field
- Dirac and Haldane masses

SHernández Ortiz, G Murguía, AR JPCM24(2012)015304

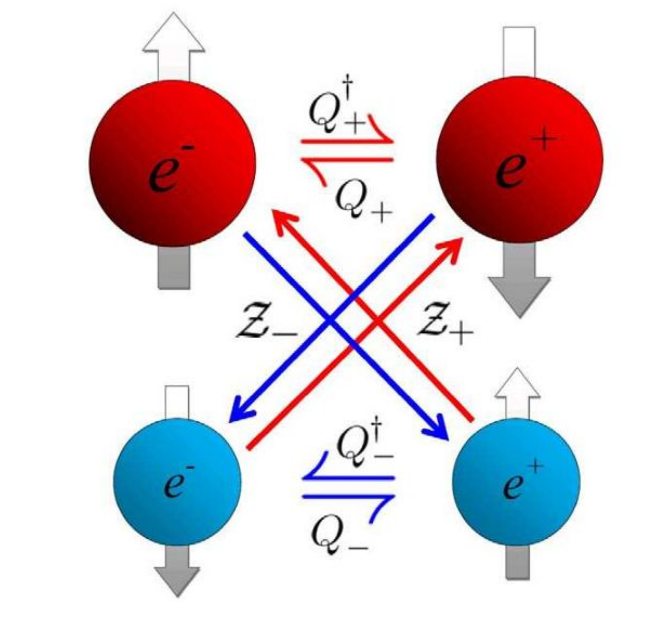


Figure 2. Action of the operators $\mathcal{Z}_\pm$ in the reducible representation of graphinos. These operators reveal an exact SUSY-QM structure in the massless limit and hardly break such a structure if a Haldane mass term is included.

03
Graphene

The Age of Quantum Materials

Graphene

Graphene

- 1 atom thick layer
- Genuine 2D crystal
- Remarkable properties
- Low energy, massless Dirac eq.

$$H_g = v_F \vec{\sigma} \cdot \vec{p}$$

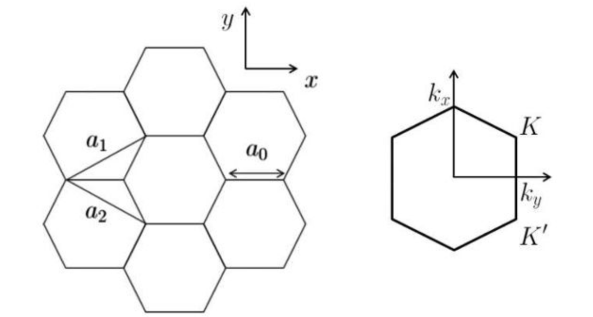
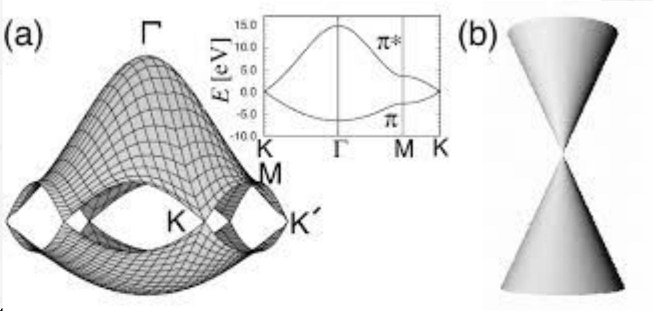


Figure 1. Crystallographic structure of graphene (left) and the Dirac points in the reciprocal k-space (right). The lattice parameters are $a_0 = 1.42$ Å, $\mathbf{a}_1 = a_0\sqrt{3}(1/2, \sqrt{3}/2)$ and $\mathbf{a}_2 = a_0\sqrt{3}(-1/2, \sqrt{3}/2)$.



Graphene

Propagator

Ritus Method

Propagator

Propagator

- Non-diagonal in momentum space
- Spectral representation
- Schwinger Method
- Ritus method
- Very few field configurations known

Ritus Method

Ritus Method

- Eigenfunctions of $(\gamma\Pi)^2$

$$(\gamma\cdot\Pi)^2\mathbb{E}_p = p^2\mathbb{E}_p$$

- Ritus functions

$$\begin{aligned}\int \mathrm{d}^d z\,\bar{\mathbb{E}}_{p'}(z)\mathbb{E}_p(z) &= \mathbb{I}\,\delta(p-p'),\\ \int \mathrm{d}^d p\,\mathbb{E}_p(z')\bar{\mathbb{E}}_p(z) &= \mathbb{I}\,\delta(z-z'),\end{aligned}$$

- Propagator

$$S(z,z') = \int \mathrm{d}^d p\,\mathbb{E}_p(z)\,\frac{1}{\gamma\cdot\bar{p}-m}\,\bar{\mathbb{E}}_{p'}(z')$$

Ritus Method

Ritus Method

- Perpendicular magnetic field (Landau-like gauge)

$$[-\partial_x^2 + (p_2 + e\mathcal{W}_0(x))^2 - e\sigma\mathcal{W}_0'(x)]F_{k,p_2,\sigma} = kF_{k,p_2,\sigma}$$

- Uniform Magnetic Field and exponentially decaying magnetic field
  G. Murguía, A. Sánchez, E. Reyes, AR, Am. J. Phys. 78, 700–707 (2010)

Ritus Method

Ritus Method $(\gamma \cdot \Pi)^2 \mathbb{E}_p(z) = p^2 \mathbb{E}_p(z)$

$$\mathbb{E}_p^{(\mathcal{A})}(z) = \begin{pmatrix} E_{p,+1}(z) & 0 \\ 0 & E_{p,-1}(z) \end{pmatrix}$$

$$\mathbb{E}_p^{(\mathcal{A})}(z) = E_{p,+1}(z)\Delta(+) + E_{p,-1}(z)\Delta(-)$$

$$\Delta(\pm) = \frac{\mathbb{I} \pm i\gamma^1\gamma^2}{2}$$

Ritus Method

Ritus Method $(\gamma \cdot \Pi)^2 \mathbb{E}_p(z) = p^2 \mathbb{E}_p(z)$

$$E_{p,\sigma}(z) = N_\sigma e^{-i(p_o t - p_2 y)} F^{\sigma}_{k,p_2}(x)$$

$$[\partial_x^2 - (-p_2 + eW(x))^2 + e\sigma W'(x) + k] F^{\sigma}_{k,p_2}(x) = 0,$$

$$V_{\pm}(x) = (-p_2 + eW(x))^2 \pm eW'(x)$$

Ritus Method

Ritus Method $(\gamma \cdot \Pi)^2 \mathbb{E}_p(z) = p^2 \mathbb{E}_p(z)$

$$B(x) = Be^{-\hat{\alpha}x} \quad \Rightarrow \quad W(x) = -\frac{B}{\hat{\alpha}}\left(e^{-\hat{\alpha}x} - 1\right)$$

$$k_n = \hat{p}_2^2 - (\hat{p}_2 - n\hat{\alpha})^2$$

$$\begin{aligned} E_{p,+1}(z) &= \frac{\hat{\alpha}}{2\pi}\left(\frac{2n!(s-n)}{\Gamma(2s-n+1)}\right)^{1/2} e^{-ip_0t+ip_2y} e^{-\varrho/2}\varrho^{(s-n)} \\ &\quad \times L_n^{2(s-n)}(\varrho), \\ E_{p,-1}(z) &= \frac{\hat{\alpha}}{2\pi}\left(\frac{2(n-1)!(s-n)}{\Gamma(2s-n)}\right)^{1/2} e^{-ip_0t+ip_2y} \\ &\quad \times e^{-\varrho/2}\varrho^{(s-n)} L_{n-1}^{2(s-n)}(\varrho). \end{aligned} \tag{31}$$

Ritus Method

Ritus Method $$(\gamma \cdot \Pi)^2 \mathbb{E}_p(z) = p^2 \mathbb{E}_p(z)$$

$$\langle \bar{\psi}\, \psi \rangle = \mathrm{Tr}\{iS(z, z)\},$$

$$\langle \bar{\psi}\, \psi \rangle_{\mathcal{A}} = i \sum\!\!\!\!\!\!\int d^3 p \frac{m}{\bar{p}^2 - m^2} [|E_{p,+1}(z)|^2 + |E_{p,-1}(z)|^2]$$

$$\begin{aligned}
\langle \bar{\psi}\, \psi \rangle_{\mathcal{A}} = \frac{m\hat{\alpha}^2}{2\pi} \Bigg\{ & \sum_{\varsigma=0}^{\infty} \frac{1}{|m|} \left(\frac{\varsigma}{\Gamma(2\varsigma + 1)} \right) e^{-\varrho} \varrho^{2\varsigma} [L_0^{2\varsigma}(\varrho)]^2 \\
& + \sum_{n=1}^{\infty} \sum_{\varsigma=n+1}^{\infty} \frac{e^{-\varrho} \varrho^{2(\varsigma-n)}}{\sqrt{\hat{\alpha}^2(2\varsigma n - n^2) + m^2}} \\
& \times \Bigg[\left(\frac{n!(\varsigma - n)}{\Gamma(2\varsigma - (n-1))} \right) [L_n^{2(\varsigma-n)}(\varrho)]^2 \\
& + \left(\frac{(n-1)!(\varsigma - n)}{\Gamma(2\varsigma - n)} \right) [L_{(n-1)}^{2(\varsigma-n)}(\varrho)]^2 \Bigg] \Bigg\},
\end{aligned}$$

$$\langle \bar{\psi}\, \psi \rangle_{\mathcal{A}}^{n=0,\sigma=1} = \frac{e}{4\pi} B e^{-\hat{\alpha} x}\, \mathrm{sgn}(m),$$

Ritus Method

Ritus Method

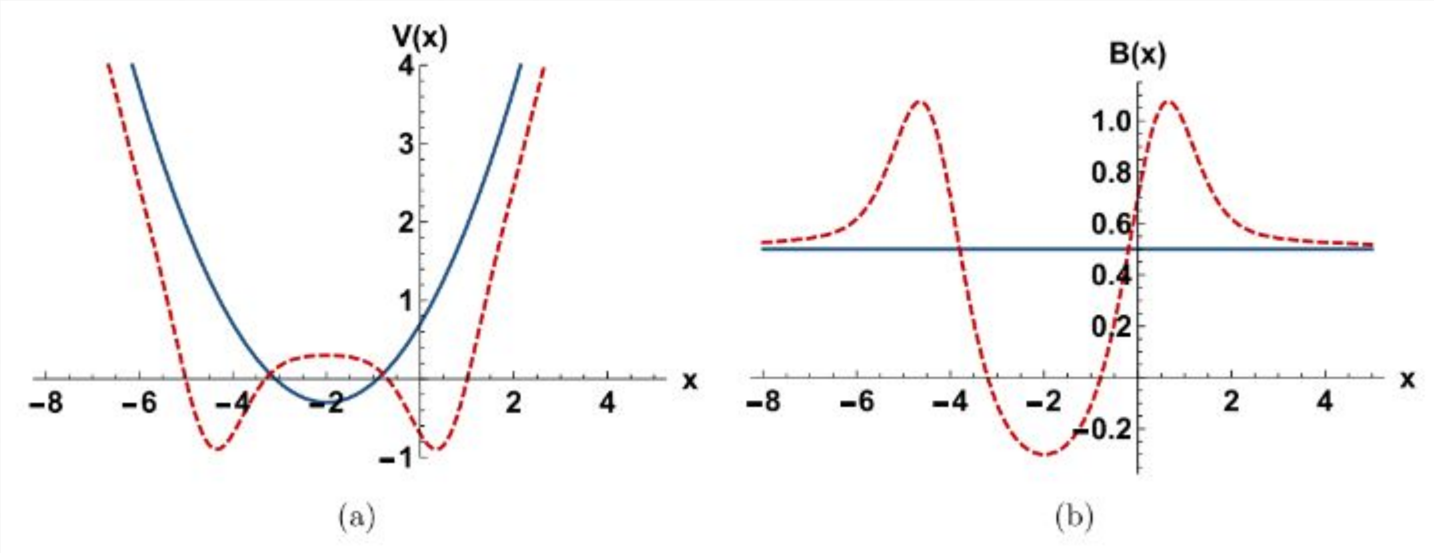
- Perpendicular magnetic field (Landau-like gauge)

$$[-\partial_x^2 + (p_2 + e\mathcal{W}_0(x))^2 - e\sigma\mathcal{W}_0'(x)]F_{k,p_2,\sigma} = kF_{k,p_2,\sigma}$$

- Second Order SUSY for these fields
  Y. Concha, E. Díaz-Bautista, AR, Phys. Scr. 97, 095203 (2022)

Ritus Method

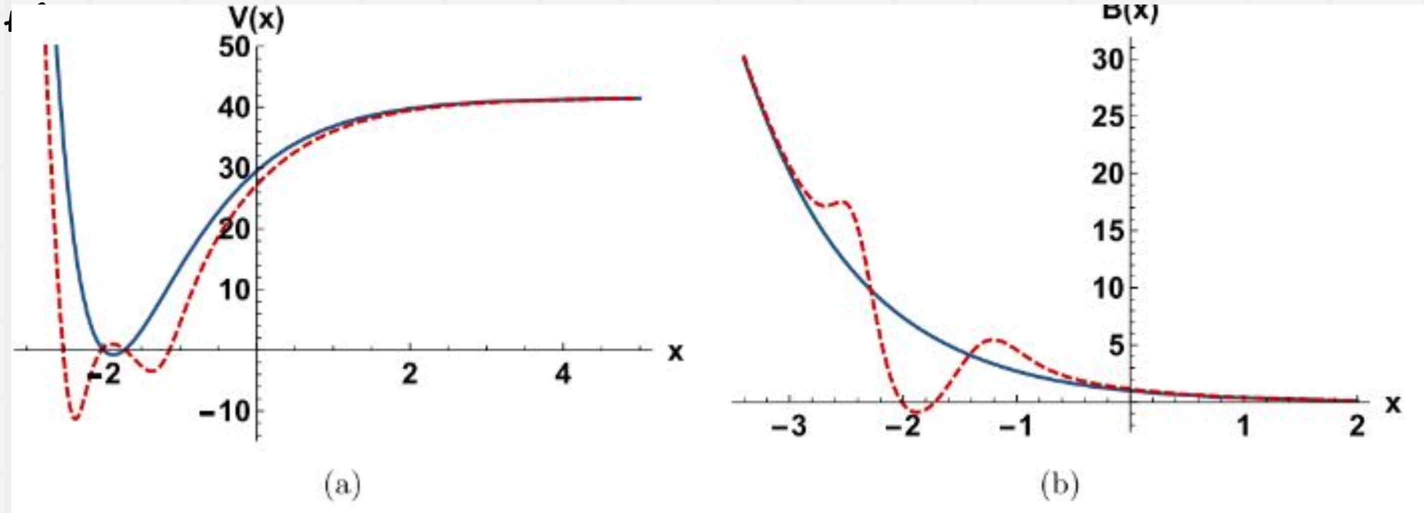
Ritus Method



Y. Concha, E. Díaz-Bautista, AR, Phys. Scr. 97, 095203 (2022)

Ritus Method

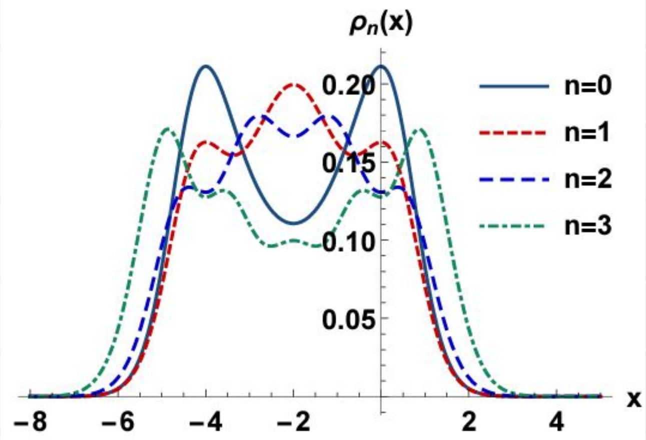
Ritus Method

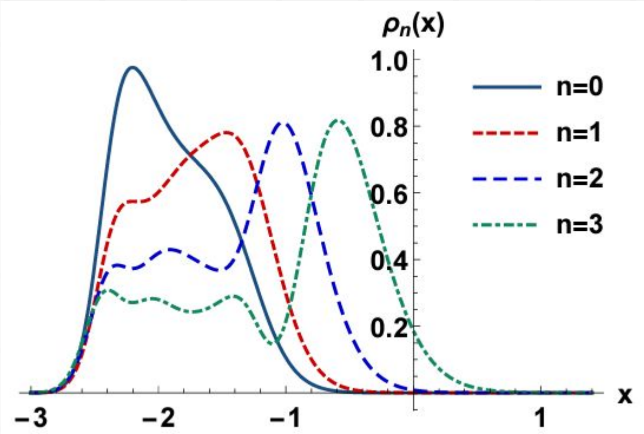


(a) (b)

Y. Concha, E. Díaz-Bautista, AR, Phys. Scr. 97, 095203 (2022)

Ritus Method





Y. Concha, E. Díaz-Bautista, AR, Phys. Scr. 97, 095203 (2022)

05

Final Remarks

Good to be in vacations while working!

n-th Latin American Workshop on Electromagnetics Effects in QCD

Thanks!